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ABSTRACT

We derive observational consequences of the hypothesis that cosmic rays (CR's) of energy $> 10^{19}$eV originate in the same cosmological objects producing gamma-ray bursts (GRB's). Inter-galactic magnetic fields $\gtrsim 10^{-12}$G are required in this model to allow CR's to be observed continuously in time by producing energy dependent delays in the CR arrival times. This results in individual CR sources having very narrow observed spectra, since at any given time only those CR's having a fixed time delay are observed. Thus, the brightest CR sources should be different at different energies. The average number of sources contributing to the total CR flux decreases with energy much more rapidly than in a model of steady CR sources, dropping to one at $E_{\rm crit} \simeq 2 \times 10^{20}$ eV with very weak sensitivity to the inter-galactic magnetic field strength. Below $E_{\rm crit}$, a very large number of sources is expected, consistent with observations. Above $E_{\rm crit}$, a source may be observed with a flux considerably higher than the time-averaged CR flux from all sources, if a nearby GRB occurred recently. If such a source is present, its narrow spectrum may produce a "gap" in the total spectrum. These signatures should be detectable by the planned "Auger" CR experiment.

Subject headings: cosmic rays — gamma rays: bursts — magnetic fields

1. Introduction

The sources of gamma ray bursts (GRB's) and of cosmic rays (CR's) with energy $E > 10^{19}$eV are unknown. In particular, most of the sources of cosmic rays that have been proposed have difficulties in accelerating CR's up to the highest observed energies (Hillas 1984, Cronin 1992). Recent gamma ray and cosmic ray observations give increasing evidence that both phenomena are of cosmological origin [see Meegan *et al.* 1992, Paczyński 1992, Piran 1992 for GRB's; Bird *et al.* 1994, Yoshida *et al.* 1995, Waxman 1995b for CR's]. Although the source of GRB's is unknown, their observational characteristics impose strong constraints on the physical conditions in the γ-ray emitting region (Piran 1994, Mészáros 1995), which imply that protons may be accelerated in the γ-ray emitting region to energies $10^{20} - 10^{21}$eV (Waxman 1995a, Vietri 1995). In addition, the average rate (over volume and time) at which energy is emitted as γ-rays by GRB's and in

CR's above 10^{19}eV in the cosmological scenario is, remarkably, comparable (Waxman 1995a,b). These two facts suggest that GRB's and high-energy CR's may have a common origin.

An essential ingredient of a bursting model for cosmic rays is the time delay due to inter-galactic magnetic fields. The energy of the most energetic CR detected by the Fly's Eye experiment is in excess of 2×10^{20}eV (Bird *et al.* 1993), and that of the most energetic AGASA event is above 10^{20}eV (Hayashida *et al.* 1994). On a cosmological scale, the distance traveled by such energetic particles is small: $<$ 100Mpc for the AGASA event, $<$ 50Mpc for the Fly's Eye event (Aharonian & Cronin 1994). Thus, the detection of these events over a $\sim$ 5yr period can be reconciled with the rate of nearby GRB's ($\sim$ 1 per 50yr in the field of view of the CR experiments out to 100Mpc in a standard cosmological scenario; e.g., Cohen & Piran 1994) only if there is a large dispersion in the arrival time of protons produced in a single burst. The required dispersion, $\geq$ 50yr for 10^{20}eV proton, may be produced by inter-galactic magnetic fields (Waxman 1995a, Waxman & Coppi 1995). The deflection angle for a proton propagating a distance D in a magnetic field B with coherence length λ is $\theta_s \simeq 0.05^\circ (D/\lambda)^{1/2} (\lambda/10\mathrm{Mpc}) (B/10^{-11}\,\mathrm{G}) (E/10^{20}\mathrm{eV})^{-1}$, and the induced time delay is $\tau(E) \simeq 10^3\mathrm{yr}(D/100\mathrm{Mpc})^2 (\lambda/10\mathrm{Mpc}) (B/10^{-11}\,\mathrm{G})^2 (E/10^{20}\mathrm{eV})^{-2}$. Since the time delay is energy dependent, the large spread in proton energy, induced by random energy loss, results in a time broadening of the CR pulse over a time $\sim \tau(E)$. Thus, the CR's from a single burst can be received on Earth over a long time interval. Nevertheless, since the angular deflection is small, the individual sources are still detectable by measuring the arrival directions.

In this *Letter*, we examine the characteristics of CR sources that should be expected in a bursting source model, as a function of the time delay, dependent on the magnetic field. We find that there are characteristic signatures for such a model, which would allow to distinguish it from a scenario where the CR sources are steady, i.e., where the sources emit a constant flux on a time scale longer than the time delay of the lowest energy CR's that are relevant. In §2.1 we give a qualitative description of the bursting model properties, using an approximate analytic approach. In §2.2 we present Monte-Carlo simulations that demonstrate the properties discussed in §2.1. The various tests of the bursting model and the implications for future high energy CR experiments are discussed in §3.

2. Characteristics of Cosmic Ray Bursts Sources

We consider a Cosmic Ray Burst (CRB) taking place at a distance D from us, where a total number of protons $n_p(E)\mathrm{d}E$ of energy E is emitted at a single instant in time. The CR's arrive with a time delay t, relative to gamma-rays, with a probability density $p[t/\tau(D,E)]\mathrm{d}[t/\tau(D,E)]$, where $\tau(D,E) \propto D^2/E^2$ is the characteristic time delay. The time delay t at a fixed energy and distance varies randomly due to two effects. First, the magnetic field along a trajectory should have random variations; for example, if the magnetic fields originate in galaxies and are later ejected to the inter-galactic medium, the field strength along a trajectory should vary depending on the impact parameter to individual galaxies. In the absence of energy losses, the bursting source would produce a number of cosmic ray "images", and the cosmic rays in each image would be of a single energy which would decrease with time as $t^{-1/2}$. However, the random nature of the energy loss of a cosmic ray eliminates these images, and simply introduces a dispersion in the arrival times and arrival directions at a fixed energy. In general, the dispersion in arrival times t will be of order $\tau(D,E)$ (Waxman & Coppi 1995).

2.1. Analytic Model

We now perform a simple analytic calculation of the number of CR sources that should be seen at each energy and flux in a CRB model. For this purpose, we approximate the effect of energy losses as being negligible when CR's come from a distance $D < D_c(E)$, and eliminating all cosmic rays coming from $D > D_c(E)$ (for $E < 10^{20}$eV, this approximation is quite good and $D_c(E)$ corresponds to the distance where the initial proton energy necessary to have an observed energy E, after losses to electron-positron production, exceeds the threshold for pion production). We also assume that the sources are observed only during a time $\tau(D,E)$ with a constant flux

$$F(E,D) = \frac{n_p(E)}{4\pi D^2 \tau(D,E)} = \frac{n_p(E) D_c(E)^2}{4\pi D^4 \tau_c(E)} \quad , \tag{1}$$

where $\tau_c(E) = \tau(D_c(E),E)$. If the rate per unit volume of CRB's is ν, all emitting the same $n_p(E)$, then the average number of bursts at distance D observed at any time is $n(D,E)\,\mathrm{d}D = 4\pi\nu\tau_c(E)[D^4/D_c(E)^2]\,\mathrm{d}D$, giving a number of bursts at a given observed flux

$$n(F,E)\,\mathrm{d}F = \pi\nu D_c(E)^3 \tau_c(E) \left[\frac{F_c(E)}{F}\right]^{5/4} \frac{\mathrm{d}F}{F} \quad , \tag{2}$$

where $F_c(E) = n_p(E)/[4\pi D_c(E)^2 \tau_c(E)]$. The flux $F_c(E)$ is the minimum flux observed for the sources. In our simplified model, the number of sources drops to zero abruptly at $F_c(E)$ owing to the assumed distance cutoff $D_c(E)$ and the "top-hat" time profile. In reality, there should be a smooth turnover at $F_c(E)$ of the number of CRB sources from the $-5/4$ power-law slope at the bright end. This result for bursting sources is in contrast to the usual $-3/2$ Euclidean slope, which applies for steady sources of cosmic rays.

The total average number of sources above flux F is

$$N(F,E) = \frac{4\pi\nu}{5} D_c(E)^3 \tau_c(E) \left[\frac{F_c(E)}{F}\right]^{5/4} \quad , \tag{3}$$

and the average background flux resulting from all the sources is

$$B(E) = 4\pi\nu D_c(E)^3 \tau_c(E) F_c(E) = \nu n_p(E) D_c(E) \quad . \tag{4}$$

The background flux is dominated by sources with flux near $F_c(E)$, although the contribution from brighter sources decreases only as $F^{-1/4}$.

As the cosmic ray energy is increased, the average number of bursts observed above the turnover flux $F_c(E)$ decreases, and there is a critical energy $E_{\rm crit}$ where this average number of sources equals unity:

$$\frac{4\pi\nu}{5} D_c(E_{\rm crit})^3 \tau_c(E_{\rm crit}) = 1 \quad . \tag{5}$$

We can write the average number of sources in terms of $E_{\rm crit}$ as

$$N_c(E) \equiv N[F_c(E), E] = \left(\frac{E_{\rm crit}}{E}\right)^2 \left[\frac{D_c(E)}{D_c(E_{\rm crit})}\right]^5 \quad . \tag{6}$$

The number of sources N_c drops rapidly with energy, due to the strong dependence on the decreasing cutoff distance $D_c(E)$. The drop is especially rapid near 10^{20}eV, where $D_c(E)$ decreases quickly (see Fig. 2 below). Therefore, for $E < E_{\rm crit}$, the number of sources contributing to the flux is very large, and the total number of CR's received at any given time is near the average background $B(E)$. The brightest source has a typical flux $F_1(E) \sim F_c(E) N_c(E)^{4/5} = [B(E)/5](E/E_{\rm crit})^{2/5}[D_c(E)/D_c(E_{\rm crit})]^{-1}$, although there is a probability to observe a source with $F > F_1(E)$, $P \sim [F/F_1(E)]^{-5/4}$. At $E > E_{\rm crit}$, the total energy received in CR's will generally be much lower than the average $B(E)$, because there will be no burst within a distance $D_c(E)$ having taken place sufficiently recently. The few CR's may be the lucky survivors from sources further than $D_c(E)$, or they may have anomalously long time-delays as a result of crossing a region of high magnetic field (probably near a galaxy). There is, however, a probability $P \simeq N_c(E)$ of seeing one CR source with $E > E_{\rm crit}$ having a flux $\sim B(E)/N_c(E)$, or an even brighter one with probability decreasing as $F^{-5/4}$.

If the CR sources are steady, then the number of sources decreases with energy only as $D_c(E)^3$, i.e., much more slowly than predicted by eq. (6). This implies that for a given critical energy, the number of bright sources at $E < E_{\rm crit}$ predicted by a model of steady sources is much larger than that predicted for bursting sources.

Bursting CR sources should have narrowly peaked energy spectra, and therefore the brightest sources should be different at different energies. For example, if a bright source is observed at $E > E_{\rm crit}$, the burst must have taken place recently in order that the high energy cosmic rays are just arriving on Earth, so lower energy cosmic rays will not have arrived yet. Typically, there will be other brighter sources at $E < E_{\rm crit}$, corresponding to bursts that took place a longer time ago, and probably closer to us (since there is a longer time interval available). This is in marked contrast to a model of steady state sources, where the brightest source at high energies should also be the brightest one at low energies, its fractional contribution to the background decreasing to low energy only as $D_c(E)^{-1}$.

2.2. Numerical Results

We now present the results of a Monte-Carlo simulation of the total number of cosmic rays received from CRB's at some fixed time. For each realization we randomly draw the positions (distances from Earth) and times at which cosmological CRB's occured, assuming that the CRB's are homogeneously distributed standard candles with an average rate $\nu = 2.3 \times 10^{-8} h^3 \mathrm{Mpc}^{-3} \mathrm{yr}^{-1}$ (with $h = 0.75$) similar to that fitted to the observed flux distribution of GRB's assuming a no-evolution standard candle model (Cohen & Piran 1994). We assume an intrinsic cosmic ray generation spectrum $n_p(E) \propto E^{-2} \mathrm{d}E$, which produces a flux above 2×10^{19}eV consistent with the Fly's Eye and AGASA data (Waxman 1995b). We calculate the change of the spectrum due to interaction with the CMB photons in a method similar to that described in Waxman (1995b), except that for distances < 130Mpc we do not use the continuous energy loss approximation but rather an exact calculation of the energy loss, which includes fluctuations.

For the probability distribution of the time-delay for a cosmic ray of fixed energy from a source at a given distance, we use the form $p(\tau) = p_0(\tau/\tau_0)^{-\alpha-1}$ for $\tau > \tau_0$, $p(\tau)\mathrm{d}\tau = (4p_0/3)(\tau/\tau_0 - 1/4)$ for $\tau_0 > \tau > \tau_0/4$, and $p(\tau) = 0$ for $\tau < \tau_0/4$, where τ_0 is the characteristic time-delay and is proportional to D^2/E^2, and $p_0 = [\tau_0(3/8 + 1/\alpha)]^{-1}$. For $\tau < \tau_0$ the form of the probability function approximately matches results obtained by Waxman & Coppi (1995). Cosmic rays with large τ are the ones that have crossed regions of high magnetic field, and we assume there is a power-law distribution of magnetic field strengths, giving a power-law distribution of time delays. If the typical field in the inter-galactic medium is B and has coherence length λ, the typical deflection angle is $\theta \propto B\,(D\lambda)^{1/2}$. When intercepting a region with magnetic field $B' \gg B$ and coherence length λ', the deflection angle is $\theta' \propto B'\lambda'$, yielding a time delay $\tau' \propto \tau(B'\lambda'/B\lambda)^2(\lambda/D)$. If n is the number density of such regions, the interception probability is $n\pi\lambda'^2 D$, so the index α is $\alpha = -\log(\pi n \lambda'^2 D)/\log[(B'^2\lambda'^2)/(B^2\lambda D)]$. Here, we shall use as an example $\tau_1 \equiv \tau(D = 80\mathrm{Mpc}, E = 10^{20}\mathrm{eV}) = 10^3$ years, corresponding to $\lambda \approx 10$Mpc and $B \approx 10^{-11}$G. We also take $B'/B = 10^5$, $\lambda'/\lambda = 10^{-3}$, and $n = 10^{-2}\mathrm{Mpc}^{-3}$, giving typical parameters for spiral galaxies (this leads to $\alpha \simeq 1$, with a weak dependence on distance).

We have examined a total of 50 realizations. About 70% of these are similar to the realization presented in Fig. 1a: there is no source sufficiently nearby having occurred sufficiently recently, so the flux at high energies is below the average. In the other 30%, there are typically one or two bright sources dominating at $E > 10^{20}$eV, as in the realization presented in Fig. 1b. A source similar to the brightest one in Fig. 1b appears only 4% of the time (in this example, the source is at $z = 0.0056$ and occurred 51 years ago); the second brightest source at $E \simeq 10^{20}$ eV in Fig. 1b is more common. The analytic expression (3) for $N(F, E)$ provides a good approximation to the numerical results for the number of sources with flux F and spectral peak at E, except that sources at high energy are also present at fluxes below $F_c(E)$, coming from CRB's at distances higher than $D_c(E)$ for which some high energy cosmic rays still survive. The spectral shape of the individual sources is determined by the time-delay probability distribution we have assumed, and is slightly modified by the interaction with the microwave background (this is the reason why the shape of the spectra of different sources varies).

Fig. 2 shows $N_c(E)$, calculated from the average background and equations (4) and (6). Since our numerical model does not assume a sharp cutoff in the flux distribution, as we did above in the analytical model, $N_c(E)$ is here an indication of the number of sources above the turnover flux, which dominate the contribution to the average background. All the characteristics of the sources depend on the CRB rate ν and on the characteristic time delay $\tau_1 \equiv \tau(D = 80\mathrm{Mpc}, E = 10^{20}\mathrm{eV})$ only through their product $\nu\tau_1$, or, equivalently, through the critical energy $E_{\mathrm{crit}}(\nu\tau_1)$. For the parameters we have chosen, $\nu\tau_1 = 10^{-5}\mathrm{Mpc}^{-3}$, Fig. 2 shows that $E_{\mathrm{crit}} \simeq 1.4 \times 10^{20}$ eV. The dependence of E_{crit} on $\nu\tau_1$ is easily determined from Fig. 2, since $N_c \propto \nu\tau_1$ (see eqs. 5-6) and therefore the curve in Fig. 2 shifts vertically as $\nu\tau_1$. If $> 10^{19}$eV CR's are indeed produced by GRB's, then ν is determined by the GRB flux distribution. The time delay, however, depends on the unknown properties of the inter-galactic magnetic field, $\tau_1 \propto B^2\lambda$. As mentioned in §1, current data requires $\tau_1 \gtrsim 50$yr, or, equivalently, $E_{\mathrm{crit}} \gtrsim 10^{20}$ eV, which corresponds to $B\lambda^{1/2} \gtrsim 10^{-11}\mathrm{G\ Mpc}^{1/2}$. The current upper limit for the inter-galactic magnetic field, $B\lambda^{1/2} \leq 10^{-9}\mathrm{G\ Mpc}^{1/2}$ (Kronberg 1994, Vallee 1990), allows a much larger delay, $\tau_1 \leq 10^6$yr. However, the rapid decrease of $N_c(E)$ with energy near 10^{20}eV, implies that E_{crit} is not very sensitive to $\nu\tau_1$. Thus, for the range allowed for the GRB model, $5 \times 10^{-7}\mathrm{Mpc}^{-3} \leq \nu\tau_1 \leq 10^{-2}\mathrm{Mpc}^{-3}$, the critical energy is limited to the range $10^{20}\mathrm{eV} \leq E_{\mathrm{crit}} \leq 3 \times 10^{20}\mathrm{eV}$.

3. Discussion

We have analyzed a model where $> 10^{19}$eV CR's are produced by cosmological sources bursting at a rate comparable to GRB's. We have found that, in this model, the average number of CR sources contributing to the flux decreases with energy much more rapidly than in the case where the CR sources are steady. We have shown that a critical energy exists, $10^{20}\mathrm{eV} \leq E_{\mathrm{crit}} < 3 \times 10^{20}\mathrm{eV}$, above which a few sources produce most of the CR's, and that the observed spectra of these sources is very narrow: the bright sources at high energy should be totally absent in cosmic rays of substantially lower energy, since particles take longer to arrive the lower their energy. In contrast, a model of steady sources predicts that the brightest sources at high energies should also be the brightest ones at low energies.

Above E_{crit}, there is a significant probability to observe one source with a flux considerably higher than average. If such a source is present, its narrow spectrum may produce a "gap" in the overall spectrum, as in Fig. 1b. Recently, Sigl *et al.* (1995) argued that the observation of such an energy gap would imply that the sources of $> 10^{20}$eV CR's are different from the sources at lower energy, hinting that the highest energy CR's are produced by the decay of a new type of massive particles. We see here that this is not the case when bursting sources are allowed, owing to the time variability. If such an energy gap is present, our model predicts that most of the cosmic rays above the gap should normally come from one source.

If our model is correct, then the Fly's Eye event above 2×10^{20}eV suggests that we live at one of the times when a bright source is present at high energies. However, the absence of such a source can not be ruled out, since, for example, the probability to have detected the Fly's Eye event in the realization of Fig. 1a, where no bright source exists, is $\sim 3\%$. The highest energy AGASA event might more easily be produced by a common, faint source (like in Fig. 1a). Furthermore, notice that, given that Fly's Eye has detected only one cosmic ray with $E > 10^{20}$ eV, we already know that the AGASA cosmic ray had a low probability of being detected; within the measurement error, its energy might be not much above 10^{20} eV.

Given the present scarcity of ultra-high energy CR's, no solid conclusions can be drawn. However, with the projected Auger experiment (Cronin 1992), the number of detected CR's would increase by a factor ~ 50. If $E_{\rm crit}$ is $\sim 2 \times 10^{20}$eV, as predicted by our model, then a few bright sources above 10^{20}eV should be identified. In addition, for $E_{\rm crit} = 2 \times 10^{20}$eV our model predicts ~ 10 sources producing more then 5 events at 5×10^{19}eV, compared to ~ 100 such sources predicted in a steady source model with a similar E_{crit}.

The observed characteristics of high energy CR sources depend on the bursting rate ν and on the typical time delay τ_1 only through their product $\nu\tau_1$. However, ν and τ_1 may be measured separately, if the time delay is either very short, $\tau_1 \leq 50$yr, or very long, $\tau_1 \sim 10^6$yr. In the former case, time variability of high energy sources may be detected, while in the latter, which implies large magnetic fields, dispersion in CR arrival directions could be measured. The magnetic field of our galaxy can also have interesting observable effects: the images of CRB sources should appear elongated perpendicular to the direction of the magnetic field, with a predictable correlation of the cosmic ray position and energy. For example, a cosmic ray with $E = 3 \times 10^{19}$eV could be deflected by $\sim 10^\circ$ when arriving along the plane of the galaxy.

The positions of cosmic rays could also be correlated with those of nearby galaxies to see if the events producing them occur in normal stellar populations (Waxman, Fisher & Piran 1995). The identification with GRB's could then lead to further constraints on the nature of the objects producing these explosions.

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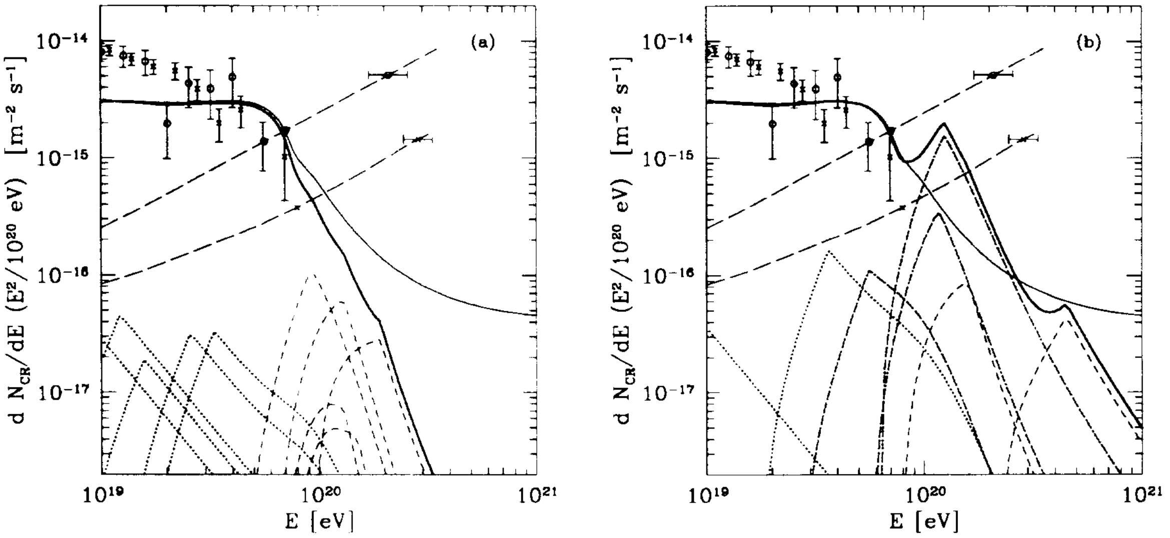

Fig. 1.— Results of two Monte-Carlo realizations of the bursting sources model with $\nu\tau_1 = 10^{-5}\mathrm{Mpc}^{-3}$: Thick solid line- overall spectrum in the realization, shown as the number of cosmic rays per unit $\log E$, times the energy. Thin solid line- average spectrum, obtained when the emissivity is spatially uniform and not due to discrete sources; notice that this curve is also proportional to $D_c(E)$, from eq. (4). Dotted lines- spectra of the five sources having the largest CR flux. Short dashed lines- spectra of the five sources that reach the highest fraction of the average flux. Filled circles- Fly's Eye data. Open circles- AGASA data (1σ errors are shown for the flux in bins with more than 1 detected events, and for the energy of the highest energy CR's). Long dashed lines- the intensity where each experiment should have detected on average one CR in each bin, where the bins are equally spaced in $\log E$ (the upper line corresponds to AGASA). The normalization of the average flux is chosen to fit the observations at $E > 2 \times 10^{19}$ eV [at lower energies, a contribution from iron cosmic rays from other sources is likely to be present (Bird *et al.* 1994, Waxman 1995b)].

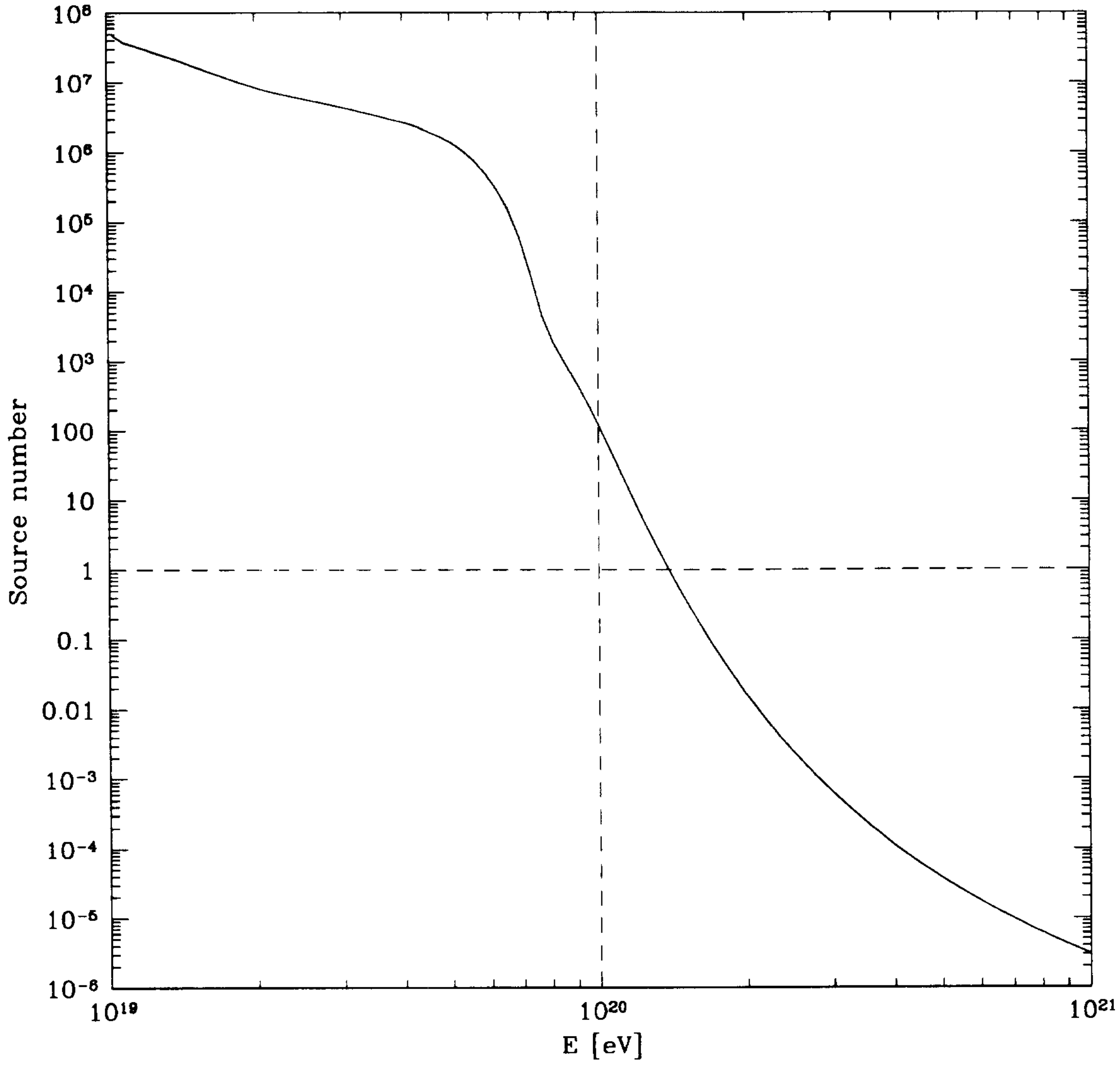

Fig. 2.— The average number of CR sources as function of energy, for bursting sources model with $\nu\tau_1 = 10^{-5}\mathrm{Mpc}^{-3}$ (the dashed lines are added only for visual aid).