A Standard-Model Self-Consistent Multi-Body Bound-State Higgs Near the $t\bar{t}$ Threshold

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ABSTRACT

We consider a simple model in which a non-elementary electroweak neutral Higgs scalar (H) can arise primarily as a multi-body bound state. Only standard-model elementary particles have to be introduced explicitly. We work with a non-perturbative analytic $t\bar{t}$ scattering-amplitude unitarization scheme (basically a relativistic generalization of the usual Schrödinger equation) with a strong high-energy (multi-body) inelastic contribution added in, and with no divergences requiring arbitrary cutoffs or subtraction constants. The H itself is required to be simultaneously consistent in both mass and coupling with the exchange of the same H in the crossed channel when constructing the relativistic generalization of the input potential. The strong energy dependence of our amplitude near the $t\bar{t}$ threshold then leads to a bound-state H with standard-model coupling and mass calculated to be immediately below the $t\bar{t}$ threshold, a result which is expected to persist for a broad class of similar models and suggests that alternative solutions may be obtained with H immediately below the ZZ or W^+W^- thresholds. We also find that our high-energy $t\bar{t}$ scattering-amplitude multi-body inelastic contribution can be constructed explicitly from the exchange of a Regge trajectory passing through the H and generated dynamically in the same way (but for unphysical angular momentum). Finally, we find that our amplitude is in approximate agreement with the perturbative crossing-symmetric H-pole-only tree-graph amplitude at low energies, despite the highly non-perturbative origin of the H in our scheme.

The Higgs scalar (H) is usually taken to be an elementary particle in simple electroweak symmetry-breaking schemes, with arbitrary mass. In a more economical theory the H would not have to be put in by hand, but would arise dynamically, e.g. as a bound state with calculable mass. Building on an early preliminary exploration by Lee, Quigg and Thacker [1], the simplest bound-state models tend to give a massive TeV-scale Higgs; see e.g. ref. 2 and the references listed therein. It is difficult to find a less massive bound-state H without introducing somewhat arbitrary divergence-averting high-energy cutoffs or subtraction constants [3], or by making the theory more complicated, e.g. through the introduction of technicolor, supersymmetry, etc.

Most non-elementary-Higgs models consider the H as a bound state of a small number of particles. Here we explore the possibility that a major part of its binding might come from high-energy multi-body systems. We first consider a simple approximation for a unitarized top-antitop ($t\bar{t}$) quark scattering amplitude A where such multi-body effects are added in through a simple inelasticity contribution. The strong energy dependence of A near the $t\bar{t}$ threshold then leads to a self-consistent bound-state H solution, with a calculated H-mass $m_H \cong 2m_t$ and small standard-model $t\bar{t}H$ coupling; in this sense the $t\bar{t}$ channel itself plays an important dynamical role [4].

This result is similar to the one obtained in ref. 5 for coupled-channel $WW-ZZ$ scattering. Here, however, we find that we can construct a specific model for our high-energy multi-body inelasticity contribution by assuming Regge behavior [6] above an appropriate threshold, (calculated to be about 1 TeV) with parameters calculated self-consistently from a continuation of our equations to unphysical angular momenta [7].

We will neglect the effect of the spin of the t. This should not affect our main conclusions. Since the self-consistency of our dynamics relies primarily on very general features like the rapid energy dependence of our amplitude near the $t\bar{t}$ threshold, these conclusions should not be affected much by refinements like spin, although the actual numerical values of our H parameters may be modified somewhat. Moreover the t and $\bar{t}$ are nonrelativistic near the $t\bar{t}$ threshold and so a scalar H exchange would be dominated by a central spin-independent potential term there, with only negligible spin-orbit or other relativistic corrections. With the appropriate amplitude we can also treat high-energy Regge behavior as an effectively spinless problem in what follows.

The exchange of a single H in the crossed (t) channel gives a contribution to $A(s,t)$ of

$$W(t) = m_H^2\Gamma_H/(m_H^2 - t) \tag{1}$$

where $\sqrt{\Gamma_H} \propto Ht\bar{t}$ coupling and s, t, u are the usual Mandelstam variables. By carrying out a multiple iteration procedure, e.g. by starting with Eq.(1) and using S-matrix unitarity and analyticity properties in all channels, we are led to the diagrams of Fig. 1; Eq.(1) would itself be included within the first (simplest) term in the sum of Fig. 1(a), for example. Basically, Fig. 1(a) would be closely connected with unitarity at low s, whereas the multi-body intermediate states of Fig. 1(b) would be closely linked to high-s multi-body-production inelasticity in the direct (s)-channel. In general, of course, the lines in Fig. 1 would include more complicated systems beyond the t and H particles, and it is understood that we must add in corresponding diagrams with crossed lines where applicable. We must be careful to avoid double-counting, however, and subtract out any contributions from higher graphs that are already fully included in lower (simpler) graphs.

If θ is the center-of-mass scattering angle, the ℓth partial-wave projection

$$A_\ell(s) = \frac{1}{2}\int_{-1}^{1} d\cos\theta \; P_\ell(\cos\theta) A(s,t) \tag{2}$$

gives an amplitude with e.g. "right-hand" cuts in the complex-s plane running from branch points at $s \geq 4m_t^2$ to $s = \infty$, and "left-hand" cuts from $s < 4m_t^2$ to $s = -\infty$. These arise from Fig. 1 and can be seen quite readily for some of the simpler graphs. A projection of Eq.(1) with $\cos\theta = 1 + 2t/(s - 4m_t^2)$, for example, gives

$$W_\ell(s) = 2m_H^2 F \Gamma_H (s - 4m_t^2)^\ell \Gamma(\ell+1)\Gamma(3/2)/2^\ell \Gamma(\ell + 3/2)(2m_H^2 + s - 4m_t^2)^{\ell+1} \tag{3}$$

where

$$F = {}_2F_1(\frac{\ell+2}{2}, \frac{\ell+1}{2}; \ell + \frac{3}{2}; (s - 4m_t^2)^2/(2m_H^2 + s - 4m_t^2)^2)$$

is the usual hypergeometric function; Eq.(3) gives a branch point at $s = 4m_t^2 - m_H^2$. We expect $W_\ell(s)$ to give the dominant singularity structure for low-magnitude $s < 4m_t^2$. In practice we can approximate the contribution of W quite well for our purposes by setting $F \cong 1$; this reproduces the exact value and derivative of W at the physical $s = 4m_t^2$ threshold and is adequate for higher energies.

We can obtain a unitarized amplitude taking account of the above singularities by using Padé approximants [5] or the N/D method [8] (See the Appendix.) With an $F = 1$ approximation in Eq.(3), an equivalent simpler and more elementary alternative would be to apply the Cauchy integral formula to W_ℓ/A_ℓ, which has branch points and cuts at the same positions as $A_\ell(s)/W_\ell(s)$ itself. We then obtain the dispersion relation

$$W_\ell(s)/A_\ell(s) = 1 + (s - 4m_t^2 + 2m_H^2) \int_{-\infty}^{\infty} ds' \mathcal{I}m[W_\ell(s')/A_\ell(s')]/\pi(s' - 4m_t^2 + 2m_H^2)(s' - s), \tag{4}$$

where we have made a subtraction at $s = 4m_t^2 - 2m_H^2$, with the subtraction constant ($= 1$) adjusted to guarantee that $A_\ell \to W_\ell$ there, since A_ℓ is dominated by the pole in W_ℓ at that point in the $F = 1$ approximation. Strictly speaking, this approximation fails for $s \leq 4m_t^2 - m_H^2$ when evaluating W_ℓ itself. However, it can be argued, e.g. from the N/D method, that Eq.(4)

nevertheless continues to be a good approximation for $A_\ell(s)$ for $s \gtrsim 4m_t^2$, and also for any $s < 4m_t^2$ bound state arising from it, if we consistently set $F = 1$ everywhere (see the Appendix). In our case, this can be checked explicitly, e.g. by modifying the $F = 1$ approximation away from $s = 4m_t^2$ by taking

$$F = [m_{H'}^2(s + 2m_H^2 - 4m_t^2)/m_H^2(s + 2m_{H'}^2 - 4m_t^2)]^{\ell+1},$$

so that our equations continue to apply with $H \to H'$. We find that A_ℓ is not affected much by moderate changes in $m_{H'}$ for $s \gtrsim 4m_t^2$, even though W_ℓ may change drastically in certain regions of $s < 4m_t^2$.

With $F = 1$, W_ℓ and A_ℓ are both real for low-magnitude $s < 4m_t^2$ away from the W_ℓ pole at $s = 4m_t^2 - 2m_H^2$, and W_ℓ/A_ℓ is real at the pole itself, since $A_\ell \to W_\ell$ there. For low $s \geq 4m_t^2$, elastic S-matrix unitarity gives $\mathcal{I}mA_\ell^{-1}(s) = -\rho(s)$, where

$$\rho(s) = [(s - 4m_t^2)/s]^{1/2} \tag{5}$$

with our amplitude normalization; lower channels, such as W^+W^- and ZZ turn out to have a relatively small effect on our dynamics as long as m_H is away from their thresholds, and will therefore be neglected for the time being. For high $|s|$, on the other hand, $A_\ell(s)$ would be dominated by the inelastic Regge-type behavior arising from Fig. 1(b). We can therefore write

$$\mathcal{I}m[W_\ell(s)/A_\ell(s)] = -W_\ell(s)[\rho(s)\theta(s - 4m_t^2) + R(s)\theta(|s| - \Lambda)], \tag{6}$$

where $R(s)$ is related to the inelastic Regge contribution to $\mathcal{I}mA_\ell^{-1}(s)$ for very high $|s|$, and $\theta(y) = 1$ for any $y \geq 0$ and zero for $y < 0$ [7]. For lower $|s|$, $R(s)$ may also include the contribution of other intermediate-energy inelastic effects and singularities, but these could be approximately taken into account simply by using a Regge $R(s)$ with an appropriate modified Λ, which will in any case be fixed by self-consistency. In fact we will later find that such a Λ is

$>> 4m_t^2$, and that $R >> \rho$ for $|s| > \Lambda$, with an approximately ℓ-independent R. With $F = 1$, Eqs.(3)-(6) then give

$$1/A_\ell(s) = 1/W_\ell(s) - [I_\ell(s) + K](2m_H^2 + s - 4m_t^2)]^{\ell+2}/(s - 4m_t^2)^\ell. \qquad (7)$$

where the elastic contribution

$$I_\ell(s) = \int_{4m_t^2}^{\infty} ds'\rho(s')(s' - 4m_t^2)^\ell/\pi(2m_H^2 + s' - 4m_t^2)^{\ell+2}(s' - s). \qquad (8)$$

can be readily evaluated analytically for integer ℓ, and the remainder reduces to

$$K \cong \int_{\Lambda}^{\infty} ds'[R(s') + R(-s')]/\pi s'^3 \qquad (9)$$

for $\Lambda >> |s|$, when K is approximately independent of ℓ and s. We can readily check that Eq.(7), or Eqs.(4) and (6), correctly reproduce all the required properties of $A_\ell(s)$.

A particle i of mass m_i manifests itself as a pole contribution $\Gamma_i/(m_i^2 - s)$ to $A_\ell(s)$, with an extra imaginary part in the denominator if it is a resonance above the $t\bar{t}$ threshold. If it is a non-elementary bound state or resonance, it would arise from Eq.(4) or (7) if

$$ReA_\ell^{-1}(m_i^2) = 0, \qquad (10)$$

with pole residue Γ_i given by

$$\frac{1}{\Gamma_i} = -[\frac{\partial}{\partial s} ReA_\ell^{-1}(s)]_{s=m_i^2}. \qquad (11)$$

We find that, with $\Gamma_H << 1$, an $\ell = 0$, $i = H$ solution with m_H and Γ_H simultaneously consistent with the values in Eq.(1) or (3) is only possible with a K or Λ such that m_H is immediately below the $t\bar{t}$ threshold. A small Γ_H is in fact required by the standard model with our normalization, since it corresponds to $[m_H c^2/(246GeV)]^2/4\pi << 1$ for $m_H \cong 2m_t \cong 340GeV/c^2$ in that model. Specifically, with $\Gamma_H \cong 0.15$, for example, we must have $K \cong 1.5m_H^{-4}$ and $m_H \cong 2m_t\,(1 - 0.006)$.

The strong s-dependence of our amplitude near the $t\bar{t}$ threshold, or, equivalently, the large $I_o'(s) \cong 1/8m_H^5(4m_t^2 - s)^{1/2}$ contribution to Eq.(11) from Eq.(7) for $s \cong 4m_t^2$, plays a crucial dynamical role in making possible this self-consistent $i = H$. This general feature continues to apply for a broad class of models beyond the simple one considered here. We could, for example, multiply Eq.(3) by a constant taking on a broad range of values and yet maintaining $m_H \cong 2m_t$; on the other hand it would take a much more narrow range to obtain an H away from the $t\bar{t}$ threshold.

Returning to our original model, we find that the perturbative crossing-symmetric H-pole-only tree-graph amplitude

$$A(s,t) = m_H^2\Gamma_H/(m_H^2 - s) + W(t) \qquad (12)$$

gives an $\ell = 0$ amplitude at $s \approx 0$ within 20% of the one given by Eq.(7) if we again set $F = 1$ in Eq.(3). A more explicit perturbation-theory evaluation may also give additional constant terms of the same order in Eq.(12), but this does not affect our general conclusion. There is, of course, no reason why Eqs.(7) and (12) should agree exactly in a non-perturbative scheme such as the one we are using, and even a perturbative expansion might be expected to generate corrections comparable to the difference between Eqs.(7) and (12). The general success of electroweak theory at low energies, however, would lead us to expect the two amplitudes to have at least approximate agreement at $s \approx 0$.

An approximate way of adding in heavy-mass exchange contributions for $\ell = 0$ would be to add a constant c to $W_o(s)$ in Eq.(3). With increasing $c > 0$ we then find that we must have a decreasing K. We continue to find an $m_H \cong 2m_t$ self-consistent bound-state H solution with a small standard-model Γ_H, however, even in the elastic limit of $K \rightarrow 0$ or $R \rightarrow 0$ in Eq.(6). But the difference between Eq.(7) and the $\ell = 0$ projection of Eq.(12) then worsens; we find that $K = 0$ gives an $A_o(\cong \pi/2)$ much larger than the value

($\cong 3\Gamma_H << 1$) given by Eq. (12), contrary to what we might expect from the general phenomenological success of electroweak perturbation theory at low energies. But in fact it can be argued that the effect of heavy-mass exchange is already taken into account in an average "Regge-resonance duality" [8] sense by the more complex higher graphs of Fig. 1. We will therefore return to $c = 0$ in what follows.

Although $I_\ell(s)$ played an important role in the self-consistent calculation of H through the large size of $I'_o(s)$ for $s \cong 4m_t^2$ in Eq.(11), its effect is relatively unimportant for other purposes, even in Eq.(10) for $i = H$ and $s \cong 4m_t^2$. We can therefore safely set $I_\ell \cong 0$ in what follows. In particular, with our $m_H \cong 2m_t$ solution, we then find that Eqs.(7)-(11) also lead to an $\ell = 1$ state with $m_i \cong 2\sqrt{5}m_t \cong 750GeV/c^2$ and a width of $16m_t\Gamma_H/45 \cong 9.2GeV$, although the actual values are presumably more model dependent than our H-mass result. The added contribution to W_ℓ from the exchange of such a higher-ℓ state in the t channel is expected to be suppressed by the no-double counting requirement discussed above, however.

We have not as yet adopted any particular model for the inelasticity function $R(s)$ of Eqs.(6)-(9). In fact any inelastic effects leading to the approximately constant K required for our self-consistent H solution would be acceptable. We shall now see that $R(s)$ can be consistently calculated by the exchange of a Regge trajectory α passing through H, i.e. by the collective effect of H and all its orbital excitations. Our effective threshold Λ in Eqs.(6) and (9) turns out to be relatively large, so that we should be justified in neglecting the effect of lower-lying trajectories. On the other hand, the effects leading to diffraction, while dominant for extremely high s, are expected to arise from multi-α_H exchanges with "deferred thresholds" in our type of model, and should therefore not begin to modify single-α_H exchange significantly until we have $|s| >> \Lambda$, leading again to a negligible contribution to

K through Eq.(9); see e.g. ref. 10 and the references listed therein.

Eqs.(3)-(11) can be consistently continued to complex ℓ. A Regge pole contribution $b(s)/[\ell - \alpha(s)]$ to $A_\ell(s)$ in the ℓ plane then arises when

$$1/A_{\alpha(s)}(s) = 0 \tag{13}$$

with $A_\ell(s)$ given by Eq.(7) and (3), with $F \cong 1$. The corresponding residue $b(s)$ is then given by

$$\frac{1}{b(s)} = [\frac{\partial}{\partial\ell}A_\ell^{-1}(s)]_{\ell=\alpha(s)}. \tag{14}$$

Using $s - t$ crossing symmetry, α-dominant Regge exchange then gives

$$A(s,t) \cong B(t)[e^{-i\pi\alpha(t)} + 1](|s|/4m_t^2)^{\alpha(t)}/\sin\pi\alpha(t) \tag{15}$$

for large $|s|$ and small $|t|$, where

$$B(t) = \pi b(t)(t/4m_t^2 - 1)^{-\alpha(t)}\Gamma(\alpha + \frac{3}{2})/\Gamma(\frac{1}{2})\Gamma(\alpha + 1). \tag{16}$$

Eq.(15) arises basically from graphs of the Fig. 1(b) type.

To evaluate K from Eqs.(6),(9) and (15) we must first evaluate $\mathcal{I}m A_\ell^{-1}(s)$ from Eq.(15) using, e.g., the generalized complex-ℓ projection formula [7]

$$A_\ell(s) = \frac{1}{2}\int_{-1}^{1} d\cos\theta P_\ell(\cos\theta)A(s,t) - \frac{\sin\pi\ell}{\pi}\int_{-\infty}^{-1} d\cos\theta Q_\ell(-\cos\theta)A(s,t), \tag{17}$$

which is equivalent to the more familiar Froissart-Gribov formula and reduces to Eq.(2) for integer ℓ; it can be readily shown to give Eq.(3) from Eq.(1), for example. Since $A(s,t)$ is peaked at small $|t|$ for high $|s|$, the second integral of Eq.(17) gives a negligible contribution, however, and we can actually evaluate Eq.(17) analytically if we write $\alpha(t) \cong \alpha(0) + \alpha' t$ in the peaked factor $(|s|/4m_t^2)^{\alpha(t)}$ in Eq.(15) and replace all its other factors by their values at $t = 0$. For large $|s|/4m_t^2$ we then obtain the ℓ-independent result

$$\mathcal{I}m A_\ell^{-1}(s) \cong -4m_t^2\alpha' \sin^2\frac{\pi}{2}\alpha(0)(|s|/4m_t^2)^{1-\alpha(0)}\ell n(|s|/4m_t^2)/B(0). \tag{18}$$

Since we find that Eq.(18) gives $\mathcal{I}mA_\ell^{-1} >> \rho$ for $|s| > \Lambda$, Eqs.(6),(9) and (18) then give the ℓ-independent constant

$$K \cong \alpha'(\Lambda/4m_t^2)^{-1-\alpha(0)}\left\{\ell n(\Lambda/4m_t^2) + [1+\alpha(0)]^{-1}\right\}/2m_t^2 B(0)[1+\alpha(0)]. \quad (19)$$

If we now combine Eqs.(3),(7),(13),(14),(16) and (19) for $\ell < 0$ with our earlier self-consistent $i = H, \ell = 0$ results from Eqs.(3),(7),(10) and (11) we find that we must have $\Lambda \cong 44m_t^2 \cong 1TeV^2$. Since this Λ is $>> 4m_t^2$, as required for the large-$|s|$ approximations of Eqs.(15)-(19), we do in fact have a consistent model for the multiparticle inelastic function $R(s)$. Since the small-$|t|$ approximation used in going from Eq.(15) to Eq.(18) should actually overestimate $R(s)$ for very high $|s|$, we may actually be underestimating Λ, further strengthening our conclusion.

As discussed earlier, a future more detailed calculation should include the exchange of more massive higher$-\ell$ resonances in W, although we must at the same time subtract out the exchange of contributions to the same partial waves coming from the higher graphs of Fig. 1 to avoid double counting. The net contribution to W would have the general spectral form $\int dt' w(s,t')/(t'-t)$ with an explicit s dependence through $w(s,t')$. In the non-relativistic case this would correspond to an energy-dependent potential, which can be dealt with without introducing any divergences simply by allowing the overall strength of the potential to be different at different energies. For any given fixed energy $\sqrt{\bar{s}}$ we then have equations similar to the ones we had before, but with the replacements

$$W(t) \rightarrow W(t,\bar{s}) = m_H^2\Gamma_H/(m_H^2 - t) + \int dt' w(\bar{s},t')/(t'-t),$$

$$A(s,t) \rightarrow A(s,t,\bar{s}), W_\ell(s) \rightarrow W_\ell(s,\bar{s}) \quad \text{and} \quad A_\ell(s) \rightarrow A_\ell(s,\bar{s})$$

In effect, then, we have something like spinless exchange but with a coupling that varies parametrically with $\bar{s}$. The actual physical amplitude is then

$A(s,t,\bar{s})$ at $s = \bar{s}$. We can do the same with our relativistic equations. Eventually, of course, we must also include the full spin complications of the t and $\bar{t}$, or of any other external particles we may bring in.

We have so far not explicitly included channels like WW and ZZ which involve elementary particles beyond the t-quark. The effect of WW and ZZ is in fact negligible on our $m_H \cong 2m_t$ H-solution. However, the rapid energy dependence of an amplitude including their contribution near the WW and ZZ thresholds means that we can have, with a modified Λ, an alternative self-consistent H solution with m_H immediately below the WW, or the ZZ, threshold. In such a case it is the $t\bar{t}$ channel which now has a small effect on our H, so it would be more appropriate to start with the coupled-channel $WW - ZZ$ amplitude, with perhaps $m_W = m_Z$ in first approximation. A preliminary calculation with such an amplitude was in fact carried out in ref. 5; this did not include an explicit Regge model for high-energy inelastic multi-body contributions, however, and somewhat arbitrarily assumed that the effect of lower-lying Regge trajectories and the particles on them can be neglected. We shall explore both of these points in a later article.

APPENDIX

The equation (4) depended on the $F = 1$ approximation in Eq.(3). The N/D method provides a more general way of unitarizing $A_\ell(s)$, even in the absence of this approximation. To deal properly with the general$-\ell$ threshold behavior of A_ℓ we write

$$A_L(s)[(s_L + s)/(s - 4m_t^2)]^\ell \equiv E_\ell(s) = N(s)/D(s), \qquad (A1)$$

with $s_L \leq 4m_t^2 - m_H^2$,

$$D(s) = 1 - (s - s_o)\int_{-\infty}^{\infty} ds' T_\ell(s') N(s')/\pi(s' - s_o)(s' - s), \qquad (A2)$$

$$N(s) = \tilde{W}_\ell(s) D(s) + \int_{-\infty}^{\infty} ds' T_\ell(s') \tilde{W}_\ell(s') N(s')/\pi(s' - s), \qquad (A3)$$

$$\tilde{W}_\ell(s) = \frac{1}{\pi}\int_{-\Lambda}^{4m_t^2 - m_H^2} \frac{ds'}{s' - s} \mathcal{I}m[W_\ell(s')(\frac{s_L + s'}{s' - 4m_t^2})^\ell], \qquad (A4)$$

and

$$T_\ell(s) = \rho(s)[(s - 4m_t^2)/(s_L + s)]^\ell \theta(s - 4m_t^2) + \tilde{R}(s)\theta(|s| - \Lambda), \qquad (A5)$$

where $\tilde{R}$ is related to the inelastic Regge contribution to $\mathcal{I}mE_\ell^{-1}(s)$ for high $|s|$; Eqs.(A1)-(A3) then give an s_o-independent A_ℓ which satisfies the correct unitarity relation $\mathcal{I}mE_\ell^{-1}(s) = -T_\ell(s)$ for $s > 4m_t^2$ and $s < -\Lambda$, and has the correct left-hand cut discontinuity, etc., as given by $W_\ell(s)$, for $4m_t^2 - m_H^2 \geq s \geq -\Lambda$. In the usual elastic limit, $\tilde{R} \to 0$ and $\Lambda \to \infty$. Care must be exercised in dealing with the $s = -\Lambda$ boundary for finite Λ, however. With $F = 1$ and W_ℓ given by Eq.(3), Eqs.(A1)-(A5) reduce directly to Eq.(A7) with $s_L = 4m_t^2 - 2m_H^2$.

If we use Eq.(A2), Eq.(A3) reduces to a non-singular integral equation for $N(s)$

$$N(s) = \tilde{W}_\ell(s) + \int_{-\infty}^{\infty} ds' T_\ell(s') N(s')[(s' - s_o)\tilde{W}_\ell(s') - (s - s_o)\tilde{W}_\ell(s)]/\pi(s' - s_o)(s' - s). \qquad (A6)$$

which only needs to involve $N(s)$ and $\tilde{W}(s)$ in the $s \geq 4m_t^2$, $s < -\Lambda$ range. This is also the only range for which they are needed to evaluate $D(s)$ from Eq.(A2) for any s, or any $s < 0$ bound-state parameters, since $D = 0$ at a bound state, so that the $\tilde{W}_\ell(s)D(s)$ contribution to Eq.(A3) likewise vanishes there, leaving a contribution which again only requires $\tilde{W}(s)$ and $N(s)$ in the $s \geq 4m_t^2$, $s \leq -\Lambda$ range. This, in turn, means that any approximation for $\tilde{W}_\ell(s)$ only has to be valid within this range.

In a more complete treatment we must remember that the left-hand cut does not arise exclusively from Eq.(1). There are also contributions from the higher ladders of Fig. 1, a small number of which are hopefully sufficient for our purposes. An alternative approach would be to evaluate a finite number of Fig. 1 graphs within a given partial wave, and rearrange the resulting series in a Padé approximant [5].

REFERENCES

1. B.W. Lee, C. Quigg and H.B. Thacker, Phys. Rev. **D16**, 1519 (1977); See also R.N. Cahn and M. Suzuki, Phys. Lett. **B134**, 115 (1984).
2. D.A. Dicus and V.L. Teplitz, Phys. Rev. **D49**, 5735 (1994); see also G. Rupp, Phys. Lett. **B288**, 99 (1992).
3. D. Sivers and J.L. Uretsky, Phys. Rev. Lett. **68**, 1649 (1992).
4. Cf. e.g., J.P. Fatelo, J.-M. Gérard, T. Hambye, and J. Weyers, Phys. Rev. Lett. **74**, 492 (1995).
5. L.A.P. Balázs, Phys. Rev. **D48**, 1310 (1993).
6. See, e.g., S.C. Frautschi, Regge Poles and S-Matrix Theory, W.A. Benjamin, Inc., New York (1963).
7. L.A.P. Balázs, Phys. Rev. **132**, 867 (1963).

8. See, e.g., G.F. Chew, The Analytic S-Matrix, W.A. Benjamin, Inc., New York (1966).

9. See, e.g., P.D.B. Collins, Phys. Reports **1**, 103 (1971).

10. L.A.P. Balázs, Phys. Rev. **D15**, 309 (1977).

FIGURE CAPTION

Fig. 1. Generalized ladder diagram sums, e.g., for $t\bar{t} \rightarrow t\bar{t}$. The lines may include more complex systems beyond the $t, \bar{t}$ and H particles.

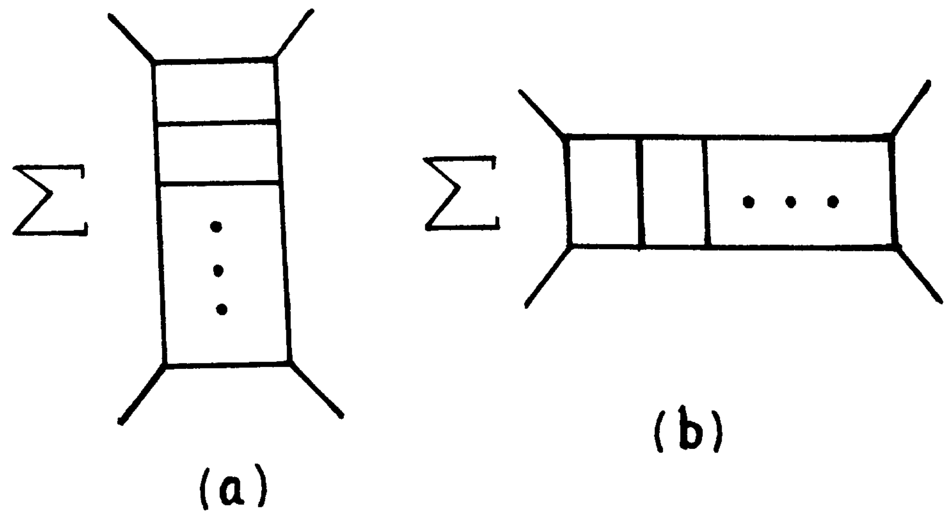

Fig. 1