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ABSTRACT

Within the one level R - matrix approach the relative intensities in the double fine structure of several radioactive decays of ^{252}Cf $\rightarrow$ ^{146}Ba + ^{106}Mo fission channel are calculated and compared with the experimental data. The internal wave functions are supposed to be given by the Wigner $\mathcal{D}$ - functions. The relative motion wave functions are calculated from a nucleus - nucleus double - folding model potential obtained with the M3Y interaction.

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1 Introduction

Cluster radioactivity [1], as a rare spontaneous decay mode of heavy nuclei has been intensively studied in recent years [2], [3], [4]. In this new type of radioactivity any emitted nuclear species with masses between A = 4 (α - particles) and $A \approx 60$ (fission fragments) are called "clusters". All nuclei with $Z > 40$ are unstable with respect to radioactive decay into two nuclear fragments (A_1, Z_1 and A_2, Z_2 with $A = A_1 + A_2$ and $Z = Z_1 + Z_2$), i.e. the energy release $Q = M(A,Z) - M_1(A_1, Z_1) - M_2(A_2, Z_2)$ is positive, however, only for certain combinations - (A_1, Z_1) plus (A_2, Z_2), the high value of the potential barrier (proportional to $Z_1 Z_2$) is almos t compensated by a high value of Q, and these decay modes may be detectable.

Moreover, Hourani and his co - workers [5] experimentally discovered the fine structure in the ^{14}C radioactivity, opening in this way a new area of research [6 - 8]. Spontaneous cluster emission being well established [1 - 13] it is tempting to search for induced cluster emission [14 - 20] and implicitely to study the "fine structure" in the induced or spontaneous fission.

During the last period of time many new experimental data concerning the spontaneous cold fragmentation (neutronless fission) of nuclei have been put into evidence, ranging from exotic decays with emissions of heavy clusters having masses from 12 to 34 atomic mass units and up to the cold fission of many actinide nuclei generating fragments with masses from around 70 and going to 166 atomic mass units. Consequently, tens of cases of heavy clusters emitted with nearly zero internal excitation energy are now experimentally known. They all confirmed the theoretical predictions based on the idea that cold rearrangements of large groups of nucleons from the ground state of the initial nucleus to the two ground states of the final fragments, can frequently be observed (see the review [2]).

Recently a first direct experiment of extremely cold (neutronless) fragmentation in the spontaneous fission of ^{252}Cf was done [18] using the multiple Ge - detector Compact Ball facility installed at Oak Ridge National Laboratory. Initially three definite neutronless channels (^{104}Mo + ^{148}Ba, ^{106}Mo + ^{146}Ba and ^{104}Zr + ^{148}Ce) were put into evidence and later a forth one (^{104}Mo + ^{148}Ba) has been observed. For these channels the ground state band gamma cascades were accurately counted using a triple gamma coincidence technique. In this way it was experimentally proved that the spontaneous decay with emission of light fragments, such as α -, ^{14}C, ^{20}O, ^{23}F, ^{24}Ne, ^{28}Mg and ^{32,34}Si (cluster radioactivity) and the neutronless spontaneous fission defined as a process where all the available energy goes into the total kinetic energy of the fragments (cold fission) have an analogous decay mechanism. Also, for the first time, a double fine structure, i.e. decay to the excited states of both fragments of the final channel were experimentally observed in analogy with the usual fine structure, i.e. decays only to the excited states of the dauther nuclei already known in alpha decay [21], [22], and ^{14}C - decay [5], [6], [8], [7].

In the last decade extensive experimental data have been obtained on low-lying states of negative parity. For the Ra-Th (Z$\sim$ 88, N$\sim$ 134) and Ba-Sm (Z$\sim$ 56, N$\sim$ 88) nuclei (the so called *"octupole"* or *"reflection asymmetric"* nuclei) low 3$^-$ states, parity doublets, alternating parity bands with enhanced dipole transitions have been found. The properties of these states differ from predictions made within the adiabatic model and nowadays they are explained as a consequence of octupole deformation. Based on a concept of symmetry breaking, the mean field calculations predict a reflection instability for nuclei around ^{222}Th and ^{146}Ba (see for review [25]). The main argument used in the analysis is the calculations of potential energy surface for a quadrupole-octupole deform ed mean field potential. For normally-deformed systems the condition for strong octupole coupling occurs for particle numbers associated with the maximum $\Delta N = 1$ interaction between the intruder subshell (l, j) and the normal parity subshell $(l-3, j-3)$. The calculated octupole minima are u sually very shallow with octupole barriers varying between 0.5 and 2.0 MeV, depending on the model. And, as a fact, the approach based on the shell correction method is striked against the problem of correct definition of a macroscopic part of the intrinsic dipole moment [26, 27] to order to describe the B(E1)-transitions. The discovery of superdeformed nuclei opens new

possibility to understand how the extreme deformations causes changes in the properties of nuclear structure, in particular, the interplay between single-particle and collective (octupole) aspects of nuclear dynamics [28, 29]. Recently attempts to analyze the energies as well as the B(E1) and B(E3) transitions probabilities from low lying collective negative parity states to the ground state using the realistic Gogny forces [30] have been made. Due to the realistic character of the force, numerical calculations are very time consuming and to cut them some assumption are usually made. It is not easy to extend both approaches for a description of collective excitations of quadrupole as wel l of octupole deformed nuclei.

One of the promising method for the description of the properties of the yrast and collective excitations near the yrast line is the cranked Hartree - Fock - Bogoliubov model together with the random phase approximation (CHFB + RPA approach) suggested in [31, 32] and developed in [33]. Starting from a generalized for octupole case CHFB + RPA approach [34] we could investigate the connection between the properties of the quadrupole + octupole deformed mean field and low - and high - lying excitations of one - phonon nature. The analysis of the energetic and the electromagnetic properties of the rotational states (yrast as well excited) could clarify the role of strong rotation for the octupole soft system. Another intriguing problem is the connection between classically chaotic system and corresponding quantum system [35, 36, 37]. Inclusion of an octupole term in addition to quadrupole term in the potential renders the classical single-particle motion nonintegrable. Depending on the strength of the octupole terms in case of superdeformed system the well pronounced new shell structure is found for nuclei as well as for metallic clusters [38, 39]. It is an exciting problem to study the occurrence of chaotic motion in the classical system and corresponding response of the quantum system under extreme conditions.

An alternative method to the above discussed method for describing the so called *"octupole"* or *"reflection asymmetric"* nuclei may be an approach based on the isomorphic shell model of atomic nuclei [40], which assumes a separate and different central potential for the nucleons of each shell, instead of a common central potential for all nucleons in a nucleus as is usually asasumed by the conventional shell model. An extension of the isomorphic shell model of atomic nuclei could in principle describe a phase transition from one center isomorphic shell model to many center (at least two center) one. Thus e.g. a two center isomorphic shell model could describe the *"octupole"* or *"reflection asymmetric"* nuclei.

There is very close relation between reflection asymmetric shapes and clustering in the system. From this point of view it is interesting to understand the mechanism of formation and decay of clusters from octupole-deformed system.

The theoretical models of heavy cluster decay are based, essentially, on Gamov's theory [41] which was the first success of quantum mechanics when applied to the α - decay phenomenon. The differences in approaches are related to the way of calculating the potential barrier defined by the (nuclear plus Coulomb) interaction potential acting between the emitted cluster and the residual nucleus. The decay energy always is taken to be equal to the experimental energy release of the decay [42].

All these theoretical treatments fit to a law for favored cluster transitions, analogous to the Geiger - Nuttal [44] law for favored α - decay, which emerges directly from the simplest JWKB [43] expression of the penetrability determined by the square well plus Coulomb interaction potential.

The unfavored transitions do not follow the Geiger - Nuttal law, because of the large variations of the reduced widths [21], [22], [45], [23], [46] which have a key role in the understanding of the decay process and require a precise knowledge of the structures of the initial and final quantum states. From such transitions we can learn much about the structure of atomic nuclei.

The theoretical study of α decay has provided a basic test for our understanding of several fundamental quantum phenomena, such as tunneling through the potential barrier, the clusterization process [47], [48], [21], [22], [23], [45], [64], [65], [66], and weak interaction models [49], [50], [52]. However, in spite of the effort invested, a detailed description of the α particle emission is not yet available.

By contrast to the case of the γ - or β - decay, where the changes in the nuclear structure are small and may be treated within perturbation theory, α - decay represents the simplest case of a series including phenomena like the heavy cluster decays [1], [3], [4] or fission [18], [19], when the transition has dramatic effects, generating in fact two new nuclei. While the fine structure of α - decay has been more or less understood [21], [22], few studies [6], [7], [8], [13], [10], [9], [11], [12], of the fine structure of heavy cluster decay are available. The clusterization mechanism proposed in Refs. [6], [7], [8] is analogous to that used in the alpha decay fine structure calculations [21], [22] i.e. the emitted cluster is formed from two fermion condesates, when working within the enlarged superfluid model [64], [65], [66] or more correlated groups of nucleons, when working within the OXBASH shell model code [7].

Unfortunately, no models have been proposed for treating the double fine structure in spontaneous (cold) fission.

It is the aim of this paper to calculate the relative intensities ($\mathcal{I}_{rel}$) for ^{106}Mo + ^{146}Ba channel and compare with the experimental intensities [70], [18] obtained as side feedings. The calculations are performed within the one level R - matrix approximation analogously to the calculations done in Refs.[6], [7], [8], [20] for the α, ^{14}C, ^{20}O and ^{34}Si decays. In the present calculations all the initial and final nuclei are supposed to be deformed nuclei with axial symmetry described by the Wigner $\mathcal{D}$ - functions. No other degrees of freedom will be included. The cluster residual nucleus scattering wave functions are generated by the Coulomb plus nuclear potential obtained within a double folding procedure from the realistic M3Y NN - potential [53], [80], [51], [52]. In this procedure one uses an effective interaction derived from the G - matrix elements based on the Reid soft - core NN potential [54] in the form assuming only OPEP force between the states with odd relative angular momentum [55].

2 Hindrance Factors

The experimental hindrance factor (HF) of any cluster decay is defined as a ratio between the Geiger - Nuttal [44] width ($\Gamma_{GN}(Q)$) divided by the width of the radioactive transition we are interested in [21]:

$$HF = \frac{\Gamma_{GN}(Q)}{\Gamma(Q)} \tag{1}$$

where Q stands for the energy release of the studied decay and [44]

$$lg\Gamma_{GN}(Q) = A + \frac{B}{\sqrt{Q}} \tag{2}$$

The theoretical hindrance factor is defined by eq. (1) in which the widths are replaced by their theoretical expressions. In the case of heavy deformed nuclei with axial symmetry the HF has the following expression [22] [45]:

$$HF^{(I_1K_1\pi_1\to I_{1c}K_{1c}\pi_{1c};I_{2c}K_{2c}\pi_{2c})} = \left[\sum_{l_c} F_{l_c} \mid b_{l_c K_c}^{(I_1K_1\pi_1\to I_{1c}K_{1c}\pi_{1c};I_{2c}K_{2c}\pi_{2c})}\mid^2\right]^{-1} \tag{3}$$

$$b_{l_c K_c}^{(I_1K_1\pi_1\to I_{1c}K_{1c}\pi_{1c};I_{2c}K_{2c}\pi_{2c})} = \frac{\alpha_{l_c K_c}^{(I_1K_1\pi_1\to I_{1c}K_{1c}\pi_{1c};I_{2c}K_{2c}\pi_{2c})}}{\alpha_{l=0\,K=0}^{(00+(g.s.)\to 00+(g.s.);00+(g.s.))}} \tag{4}$$

where

$$\alpha_{l_c K_c}^{(I_1K_1\pi_1\to I_{1c}K_{1c}\pi_{1c};I_{2c}K_{2c}\pi_{2c})} = \sum_{c'} \mathbf{K}_{cc'}\gamma_{l_{c'} K_{c'}}^{(I_1K_1\pi_1\to I_{1c'}K_{1c'}\pi_{1c'};I_{2c'}K_{2c'}\pi_{2c'})} \tag{5}$$

The factor γ_l^2 is the reduced width [46], [22], [21].

When dealing with the rotational bands of the $K^\pi = 0^+$ - states only, the HF corresponding to the decay from a double even nucleus to the channel consisting of two double even nuclei becomes

$$HF^{(00+\to I_1 0+;I_2 0+)} = \left[\sum_l F_l \left|\frac{\sum_{l'I_1'I_2'} \mathbf{K}_{lI_1I_2;l'I_1'I_2'}\gamma_{l'\,0}^{(00+\to I_1'0+;I_2'0+)}}{\sum_{l'I_1'I_2'} \mathbf{K}_{000;l'I_1'I_2'}\gamma_{l'\,0}^{(00+\to I_1'0+;I_2'0+)}}\right|^2\right]^{-1} = \left[\sum_l F_l \left|\frac{\mathbf{K}_{lI_1I_2;000}(1+F_{lI_1I_2})}{\mathbf{K}_{000;000}(1+F_{000})}\right|^2\right]^{-1} \tag{6}$$

where

$$F_{lI_1I_2} = \sum_{l'I_1'I_2'} \frac{\mathbf{K}_{lI_1I_2;l'I_1'I_2'}\,\gamma_{l'\,0}^{(00+\to I_1'0+;I_2'0+)}}{\mathbf{K}_{lI_1I_2;000}\,\gamma_{0\,0}^{(00+\to 00+;00+)}} \tag{7}$$

In the above equation (7), the summation runs over nonzero values of the mentioned quantum numbers. The ratio F_l stands for the ratio of the following penetrabilities ($P_l(Q)$):

$$F_l = \frac{P_l(Q)}{P_{l=0}(Q)} \tag{8}$$

Within the JWKB approximation P_l has the following expression:

$$P_l = R_c q_l(R_c)\exp\left(-\frac{2}{\hbar}\int_{R_c}^{r_0} q_l(r)dr\right) \tag{9}$$

in which "r_0" and "R_c" stand for the outer and inner turning points, respectively and

$$q_l(r) = \sqrt{2m_0 A_{red}(V_l^{coul+nucl} - Q)} \tag{10}$$

Here m_0 is the nucleon mass, A_{red} stands for the reduced mass number and $V_l^{coul+nucl}$ is the sum of the Coulomb and nuclear one body potential acting between the α - cluster and the daughter nucleus when studying the radial part of the Schroedinger equation.

Thus

$$F_l = \exp\frac{2}{\hbar}\int_{R_c}^{r_0}(q_{l=0}(r) - q_l(r))dr \tag{11}$$

Usually [2], [21], [22] the Coulomb part of this potential is replaced by point like Coulomb potential while the nuclear part by a Saxon - Woods one. Within these simple prescriptions in the case of α - decay the F_l - function has the following approximate expression [24]

$$F_l = \exp\left(-2.027l(l+1)Z^{-\frac{1}{2}}A^{-\frac{1}{6}}\right) \tag{12}$$

For deformed nuclei there is a matrix part of the penetrability [60], [61], [22], [23], [21] - ($\mathbf{K}_{cc'}$), responsible for the channel coupling, the so - called Fröman matrix. For double - even nuclei ground state bands the c - quantum numbers restricts to l, I_1, I_2 ($\mathbf{K}_{cc'} = \mathbf{K}_{l I_1 I_2, l' I_1' I_2'}$). Fröman matrix is generated by the anisotropic part of the double folded interaction potential between the emitted in the cluster radioactivity fragments.

3 Relative Intensities

The relative intensity ($\mathcal{I}_{rel}$) for a transition from the initial nucleus ground state $| I_i K_i \pi_i >$ to the channel state in which the first final fragment is in the state $| I_1 K_1 \pi_1 >$, while the second final fragment in the state $| I_2 K_2 \pi_2 >$ is defined as a ratio between the partial width of a given transition and the sum of partial widths over the channel quantum numbers defining all possible channels of a given fragmentation

$$\mathcal{I}_{rel}^{(I_i K_i \pi_i \to I_1 K_1 \pi_1; I_2 K_2 \pi_2)} = \frac{\Gamma^{(I_i K_i \pi_i \to I_1 K_1 \pi_1; I_2 K_2 \pi_2)}(Q_{c_0})}{\sum_c \Gamma^{(I_i K_i \pi_i \to I_{1_c} K_{1_c} \pi_{1_c}; I_{2_c} K_{2_c} \pi_{2_c})}(Q_c)} \tag{13}$$

where $c_0 = (I_1 K_1 \pi_1; I_2 K_2 \pi_2)$. Within the R - matrix approach [46] the partial width is a sum of all possible products between the penetrability ($P_l(Q)$),the Fröman matrix ($\mathbf{K}_{cc'}$) and the amplitude of the reduced width ($\gamma_l(I_i K_i \pi_i \to I_{1_c} K_{1_c} \pi_{1_c}; I_{2_c} K_{2_c} \pi_{2_c})$). Thus

$$\begin{aligned} \mathcal{I}_{rel}^{(I_i K_i \pi_i \to I_1 K_1 \pi_1; I_2 K_2 \pi_2)} &= \frac{\sum_l F_l b_l^2(I_i K_i \pi_i \to I_1 K_1 \pi_1; I_2 K_2 \pi_2) R(Q_{c_0}, Q_0)}{\sum_c \sum_{l_c} F_{l_c} b_{l_c}^2(I_i K_i \pi_i \to I_{1_c} K_{1_c} \pi_{1_c}; I_{2_c} K_{2_c} \pi_{2_c}) R(Q_c, Q_0)} \\ &= \frac{\left(HF^{(I_i K_i \pi_i \to I_1 K_1 \pi_1; I_2 K_2 \pi_2)}\right)^{-1} R(Q_{c_0}, Q_0)}{\sum_c \left(HF^{(I_i K_i \pi_i \to I_{1_c} K_{1_c} \pi_{1_c}; I_{2_c} K_{2_c} \pi_{2_c})}\right)^{-1} R(Q_c, Q_0)} \end{aligned} \tag{14}$$

where $b_{l_c}(I_i K_i \pi_i \to I_{1_c} K_{1_c} \pi_{1_c}; I_{2_c} K_{2_c} \pi_{2_c})$ and F_l ratios are defined in the eq. (4) and (8), respectively. $R(Q_c, Q_0)$ is the ratio $P_l(Q_c)/P_l(Q_0)$, where $P_l(Q_0)$ is the penetrability calculated for the transition between the ground state of the initial nucleus and the channel state where the fragment nuclei are in their ground states ($Q_0 = M(A,Z) - M_1(A_1, Z_1) - M_2(A_2, Z_2)$ and $Q_c = Q_0 - E^*_{I_{1_c} K_{1_c} \pi_{1_c}} - E^*_{I_{2_c} K_{2_c} \pi_{2_c}}$).

In the case the transition between the ground states is the most intese one the sum in the denominator can be approximated to one term only, namely to the favored width

$$\mathcal{I}_{rel} \approx \frac{\Gamma^{(00+ \to I_1 0+; I_2 0+)}(Q)}{\Gamma^{(00+ \to 00+, 00+)}(Q_0)} = \frac{R(Q, Q_0)}{HF^{(00+ \to I_1 0+; I_2 0+)}} \tag{15}$$

where

$$R(Q, Q_0) = \frac{P_{l=0}(Q)}{P_{l=0}(Q_0)} \tag{16}$$

4 The Fröman Matrix

The R - matrix decay width in the case of the one level approximation is defined by (see Ref. [46] Ch. 9, eq. 1.18a):

$$\Gamma_\lambda = 2(\alpha_\lambda^*, \mathbf{P}\alpha_\lambda) \tag{17}$$

where

$$\alpha_\lambda = (\mathbf{1} - \mathbf{R}^0\mathbf{L}^0)^{-1}\gamma_\lambda \tag{18}$$

Here $\mathbf{R}^0$ is the one - level part of the R - matrix (i.e. $\mathbf{R}^0 = (\gamma_\lambda \otimes \gamma_\lambda/(E_\lambda - E))$, $\mathbf{L}^0 = \mathbf{L} - \mathbf{B}$, $\mathbf{L} = \mathbf{S} + i\mathbf{P}$ and γ_λ is the vector defined by the whole set of the amplitudes of the reduced widths corresponding to the level λ. If the channel radius defines a sphere that includes every anisotropic structure of the interaction field acting between the channel fragments the matrices $\mathbf{L}$, $\mathbf{S}$ and $\mathbf{P}$ are diagonal matrices (see Ref. [46] Ch. 3, eq 4.4). If the channel radius defines a sphere that includes the initial nucleus volume, but outside this sphere there exist some anisotropic interaction potential acting between the channel fragments the matrices **L**, **S** and **P** become nondiagonal i.e. they should be replaced by $\mathbf{K}^{\dagger}\mathbf{L}\mathbf{K}$, $\mathbf{K}^{\dagger}\mathbf{S}\mathbf{K}$ and $\mathbf{K}^{\dagger}\mathbf{P}\mathbf{K}$ respectively. The matrix **K**, introduced first by Fröman [60] and Nosov [61], is responsible for the above anisotropy.

The matrix $(\mathbf{1} - \mathbf{R}^0\mathbf{L}^0)^{-1}$ may contribute essentially in the light nuclei region [20], while the matrix **K** gives its important contribution in the heavy deformed nuclei region.

4.1 Interaction Potential in the Case of Axially Symmetric Deformed Initial and Final Nuclei

We shall start writing down the double folded potential for the case when all the initial and the two final nuclei are axially symmetric deformed nuclei (see Refs. [80], [57], [58], [59]).

$$V_{M3Y}(\mathbf{R},\theta_i^{(1)},\theta_i^{(2)}) = \int d\mathbf{r_1} d\mathbf{r_2} \rho(\mathbf{r_1})\rho(\mathbf{r_2}) v(\mathbf{r_{12}}) =$$
$$\sum_{\lambda_1,\lambda_2,\lambda_3}\sum_{\mu_1,\mu_2,\mu_3} C^{\lambda_1\lambda_2\lambda_3}_{\mu_1\mu_2\mu_3} C^{\lambda_1\lambda_2\lambda_3}_{0\;0\;0} C^{\lambda_3\lambda_3\;0}_{\mu_3-\mu_3\;0} \mathcal{D}^{\lambda_1}_{\mu_1 0}(\theta_i^{(1)})\mathcal{D}^{\lambda_2}_{\mu_1 0}(\theta_i^{(2)}) Y_{\lambda_3-\mu_3}(R) V_{\lambda_1,\lambda_2,\lambda_3}(R) =$$
$$\tilde{V}_{000}(R) + V_1(\mathbf{R},\theta_i^{(1)},\theta_i^{(2)}) \tag{19}$$

Here the nuclear part of the effective M3Y - N - N interaction is of the form [55]

$$v(r) = v_{00}(r) + \mathcal{J}_{00}\delta(r) + v_{01}(r)\tau_1\cdot\tau_2 \tag{20}$$

with

$$v_{00}(r) = \left[7999\frac{\exp(-4r)}{4r} - 2134\frac{\exp(-2.5r)}{2.5r}\right] MeV \tag{21}$$

and

$$v_{01}(r) = \left[-4885.5\frac{\exp(-4r)}{4r} + 1175.5\frac{\exp(-2.5r)}{2.5r}\right] MeV \tag{22}$$

The δ - force term approximate the single nucleon exchange through the zero range pseudopotential ($\mathcal{J}_{00}$ = -262 MeV fm^2). The spin - spin and spin - isospin terms are neglected here. The nuclear densities are given by the following formula

$$\rho(\mathbf{r}) = \rho_0\left[1 + \exp\frac{1}{a}\left(r - \frac{R_0}{c}(1 + \beta_2\mathcal{Y}_{20}(cos\theta))\right)\right]^{-1} \tag{23}$$

with the constant ρ_0 fixed by normalizing the proton and neutron densities to the Z proton and N neutron numbers respectively, the diffusivity a = 0.63 fm and $R_0 = r_0A^{1/3}$ with r_0 = 1.19 fm. Here β_2 is the quadrupole deformation and c is the usual constant which ensures the volume conservation condition $\int_V d^3r = \frac{4\pi}{3}R_0^3$, from which it results

$$c(\beta_2) = \left[1 + \frac{3}{4\pi}\beta_2^2 + \frac{1}{14\pi}\beta_2^3\right]^{1/3} \tag{24}$$

The Coulomb component of the nucleus - nucleus potential is calculated by folding the Coulomb NN interaction with the deformed proton densities.

The $\tilde{V}_{000}(R) = (4\pi)^{-\frac{1}{2}}V_{000}(R)$ while

$$V_{\lambda_1,\lambda_2,\lambda_3}(R) = \frac{4}{\pi}(-1)^{\frac{\lambda_1+\lambda_2+\lambda_3}{2}}\hat{\lambda}_1\hat{\lambda}_2\int_0^\infty dx_1\rho_{\lambda_1}\int_0^\infty dx_2\rho_{\lambda_2}F^v_{\lambda_1,\lambda_2,\lambda_3}(x_1,x_2,x_3) \tag{25}$$

$$F^v_{\lambda_1,\lambda_2,\lambda_3}(x_1,x_2,x_3) = \int_0^\infty dqq^2v(q)\prod_{i=1}^{3} j_{\lambda_i}(qx_i) \tag{26}$$

Here $v(q)$ is the Fourier transform of the effective NN potential $v(r)$, ρ_{λ_i} are the multipoles of the nuclear densities, $j_{\lambda_i}(qx_i)$ are the spherical Bessel functions, $\mathcal{D}^{\lambda}_{\mu\mu'}(\theta_i)$ are the Wigner rotation matrices describing the orientation of the involved nuclei while $Y_{\lambda\mu}(\hat{R})$ is the λ^{th} - order spherical harmonics.

4.2 The Approximate Expression for the Fröman Matrix

Now we are able to write down, within the above approximations, the channel wave function for the case of the decay of a spin zero axially symmetric deformed nucleus into two axially symmetric deformed nuclei:

$$| \Psi >= \sum_{c'} \frac{1}{R} | \Phi_{c'} >< \Phi_{c'} | u_s^{(1)}(\mathbf{R}) | \Phi_c >= \tag{27}$$

$$\sum_{c'} \frac{1}{R} | \Phi_{c'} >< \Phi_{c'} | \exp(\frac{i}{\hbar} S_c^{(1)}(\mathbf{R})) | \Phi_c == \sum_{c'} \mathbf{K}_{cc'} \ \frac{1}{R} | \Phi_{c'} >$$

where the upper index (1) stands for the noncentral parts of the redial solution ($u_s^{(1)}$) and action ($S_c^{(1)}$), respectively. The $\mathbf{K}_{cc'}$ quantities are the elements of the Fröman - Nosov matrix, which within the three above approximations have the following expression

$$\mathbf{K}_{cc'}^{G} = \tag{28}$$

$$\sum_{MM'} C^{I_1 \ I_2 \ l}_{M \ -M \ 0} C^{I_1' \ I_2' \ l'}_{M' \ -M' \ 0} \int d\Omega_1 d\Omega_2 Y^*_{I_1' M'}(\Omega_1) Y^*_{I_2' -M'}(\Omega_2) e^{-G(B,\Omega_1,\Omega_2)} Y_{I_1 M}(\Omega_1) Y_{2-M}(\Omega_2)$$

The operator $G(B, \Omega_1, \Omega_2)$ contains the multipole terms from the nucleus - nucleus potential:

$$G(B, \Omega_1, \Omega_2) = \tag{29}$$

$$-\frac{1}{\hbar} \int_{R_1(\theta_i^{(1)},\theta_i^{(2)})}^{R_0(\theta_i^{(1)},\theta_i^{(2)})} dR \left(\sqrt{2m_0 A_{red} \left[V_{M3Y}(\mathbf{R}, \theta_i^{(1)}, \theta_i^{(2)}) - Q_0 \right]} \right)$$

$$+\frac{1}{\hbar} \left(\int_{R_1}^{R_0} dR \sqrt{2m_0 A_{red} \left[\tilde{V}_{000}(R) - Q_0 \right]} \right)$$

5 Relative Intensities in a Hybrid model for ^{252}Cf

To calculate the relative intensities for the case we are interested in is a very difficult task, however, within some rough approximations we can give estimations of these quantities.

We shall use in the following the formula (6) for the HF's entering the relative intensities expression:

$$\mathcal{I}_{rel}^{(I_1 0+)} = \sum_{I_2 K_2 \pi_2} \mathcal{I}_{rel}^{(I_i K_i \pi_i \to I_1 K_1 \pi_1; I_2 K_2 \pi_2)} = \tag{30}$$

$$= \frac{\sum_{I_2} \left(HF^{(00+ \to I_1)+; I_2)+)} \right)^{-1} R(Q_{I_1 I_2}, Q_{00})}{\sum_{I_1 I_2} \left(HF^{(00+ \to I_1)+; I_2)+)} \right)^{-1} R(Q_{I_1 I_2}, Q_{00})}$$

$$HF^{(00+ \to I_1 0+; I_2 0+)} = \left[\sum_l F_l \left| \frac{\sum_{l' I_1' I_2'} \mathbf{K}_{l I_1 I_2; l' I_1' I_2'} \gamma_{l' \ 0}^{(00+ \to I_1' 0-, I_2' 0+)}}{\sum_{l' I_1' I_2'} \mathbf{K}_{000; l' I_1' I_2'} \gamma_{l' \ 0}^{(00+ \to I_1' 0+; I_2' 0+)}} \right|^2 \right]^{-1} = \tag{31}$$

$$\left[\sum_l F_l \left| \frac{\mathbf{K}_{l I_1 I_2; 000}(1 + F_{l I_1 I_2})}{\mathbf{K}_{000;000}(1 + F_{000})} \right|^2 \right]^{-1}$$

where

$$F_{l I_1 I_2} = \sum_{l' \neq 0, I_1' \neq 0, I_2' \neq 0} \frac{\mathbf{K}_{l I_1 I_2; l' I_1' I_2'} \ \gamma_{l' \ 0}^{(00+ \to I_1' 0+ \ I_2' 0+)}}{\mathbf{K}_{l I_1 I_2; 000} \ \ \gamma_{0 \ 0}^{(00+ \to 00+ \ 00+)}} \tag{32}$$

and we shall neglect the $F_{l I_1 I_2}$ - quantities. This approximation could be a good approximation if taking into account the analogous quantities occuring in the alpha decay or some heavy cluster decay fine structure HF's, where for l≥2 the reduced wave amplitudes are less than 1%.

The results are reported in the Table 1. The experimental relative intensities are deduced from the *side feeding* values incorporating gamma - rays from other rotational bands built upon vibrational - or quasiparticle - states (including the statistical ones) or directly the cluster channels [70], [71]. The experimental relative intensities for α - decay fine structure of doubly even nuclei decrease with the increase of the relative motion angular momentum (l). The experimental relative intensities for ^{252}Cf $\to$ ^{146}Ba plus ^{106}Mo cluster decay presented in this paper seem to show a *"gaussian"* behaviour with a maximum for I $\approx$ 4. This fact roughly can be explained by the number of contributing terms to the relative intensities. For instance, for the spins of Ba nucleus: 0,2,4,6,8,.. the number of more or less equally contributing terms is: 4,10,13,9,2,..

The reduced Fröman matrix elements ($\tilde{\mathbf{K}}_{l I_1 I_2; l' I_1' I_2'} = \frac{\mathbf{K}_{l I_1 I_2; 000}}{\mathbf{K}_{000;000}}$) (see Table 2) are oscillating around the value $\approx$ 1.0.

The penetrability ratios $R_{l I_1 I_2} = F_l \cdot R(Q_{I_1 I_2}, Q_{00})$ are given in Table 3 for several dominant contributions. They restrict the number of contributing terms in the relative intensities expressions, as suggested above.

Of course, this is a crude approximation, however, due to the fact that for our kind of fragmentations, the accurate calculation of the reduced widths is not possible, a hybrid model should be constructed.

Within our model, the relative intensities are determined by the final state interaction only. This interaction is assumed to be described by the M3Y double folding approach, which generates both the one body penetrabilities and the Fröman matrix part of these quantities. In such a model we do not have informations concerning the rearangement mechanism. However, the first approach to the alpha decay fine structure of the double even nuclei has been analogous to ours [60].

In our calculations we used the accurate evaluation of the Q - values from the recent experimental mass tables [42], and for some of the fragmentations the masses were taken from the extended tables of Möller and Nix [67] generated within a macroscopic - microscopic model.

The deformation parameters which we used were also taken from the tables of Moller and Nix [67]. The calculated values of the penetrabilities and Fröman matrix elements are very sensitive to the β_2 - and r_0 - values entering the expressions of the nuclear densities, however, the quantities entering the relative intensities are always ratios of the penetrabilities or ratios of the Fröman matrix elements. Such ratios do not drastically depend on the above β_2 - and r_0 - values. In the calculations we did not include the octupole parameters β_3. Of course, the higher multipoles like the octupole can play an important role bringing about additional incertitudes. We used the previeous experience when we calculated the isotopic yields for the cold fission of the ^{252}Cf nucleus [68].

Our theoretical results for the relative intensities (see Table 1) do not exactly fit these experimental ones, but there are ways for improving the situation for both the experimental and the theoretical values. The experimental numbers could be adjusted by excluding the part coming from the gamma cascades, while the theoretical ones by incorporating in the theory: a) the clusterization process in the analogous way as it has been done in the alpha - [6], [7], [8], [22], [23], [21] or heavy cluster - decay [6], [7], [8] fine structure; b) the octupole component of the double folding potential; c) the spin - spin and spin - isospin components of the M3Y nucleon - nucleon interaction, d) coherent rotational state assumption for the channel states [77], [78], [79], [80], [81], [82], [83] which could improve the agreement with the experiment.

6 Conclusions

In this paper within a final state interaction hybrid model we calculated the relative intensities describing the double fine structure in the neutronless fission: $^{252}\mathrm{Cf} \rightarrow {}^{106}\mathrm{Mo} + {}^{146}\mathrm{Ba}$. The interaction in the final state is given by the double folding M3Y potential between the ^{106}Mo and ^{146}Ba nuclei.

Experimentally the half - life of ^{252}Cf is 2.54 years and the branchingratios for its alpha decay and spontaneous fission are 97% and 3%, respectively [69]. taking an energy of zero - point vibration [2] of about 1 MeV, we obtain a collision frequency ν with the fission barrier of $2.5 \cdot 10^{20}$ s^{-1} and a penetrability through this barrier of about $1.1 \cdot 10^{-30}$. Thus the partial half - life for the channel we are interested in is $5.04 \cdot 10^{16}$ sec. [68]. This ^{106}Mo + ^{146}Ba channel belongs to the most probable fragmentation channels having the cluster masses $98 \leq A_{light} \leq 110$ and $142 \leq A_{heavy} \leq 154$ and cluster charges $38 \leq Z_{light} \leq 44$ and $54 \leq Z_{heavy} \leq 60$, respectively (see also [68]). In these regions of nuclei both the light and heavy fragments have relatively large quadrupole and sometimes octupole deformations and therefore the potential barriers between the two final nuclei are significantly lowered. This fact leads to the increased penetrabilities and yields, respectively.

Our theoretical results do not fit the experimental ones, but there are ways for improving the situation for both the experimental and the theoretical values. The experimental numbers could be adjusted by excluding the part coming from the statistical gamma cascades, while the theoretical ones by incorporating in the theory: a) the clusterization process, b) the octupole component of the double folding potential; c) the spin - spin and spin

- isospin components of the M3Y nucleon - nucleon interaction, d) coherent rotational state assumption for the channel states which could improve the agreement with the experiment.

References

[1] H.J. Rose and G.A. Jones, Nature, **307** (1984) 245

[2] A. Sandulescu and W. Greiner, Rep. Progr. Phys. **55** (1992) 1423

[3] P. B. Price, Ann. Rev. Nucl. Part. Sci. **39** (1989) 19

[4] Yu.C. Zamiatin, V.L. Micheev, S.P. Tretyakova, V.I. Furman, S.G. Kadmensky and Yu. M. Chuvilsky , Fiz. Elem. Chastits At. Yadra 21 (1991) 537

[5] E. Hourani, L. Rosier, G.B. Ronsin, A. Elayi, A.C. Mueller, G. Rappenecker, G. Rotbard, G. Renou, A. Liebe and L. Stab, Phys. Rev., **C 44** (1991) 1424; L. Brillard, A.G. Elayi, E. Hourani, M. Hussonnois, J.F. Le Du, L.H. Rosier and L. Stab, C.R. Acad. Sci. Paris, **309** (1989) 1105

[6] O. Dumitrescu and C. Cioacă, *"Fine Structure in Cluster Decays of the Translead Nuclei"* Phys. Rev. **C 51** (1995) No.6; Preprint ICTP Trieste IC/93/122 (1994),

[7] O. Dumitrescu, *"Fine Structure of Cluster Decays"* Phys. Rev. **C 49** (1994) 1466; Preprint ICTP Trieste IC/93/164 (1993),

[8] O. Dumitrescu, *"Fine Structure in Cluster Decays"* Proc. International NATO Advanced Study Institute, Predeal (Sept. 1993), Romania, *"Frontier Topics in Nuclear Physics"*, Eds. W. Scheid and A. Sandulescu, **NATO - ASI Series B: Physics 334** (1994) 73

[9] M. Hussonnois, J.F. Le Du, L. Brillard and G. Ardisson, Phys. Rev., **C 44** (1991) 2884;

[10] G. Ardisson, M. Hussonnois, J.F. Le Du and L. Brillard, Proc. Int. School Seminar Heavy Ion Physics, Dubna 1989, p.336 (1990);

[11] G. Ardisson and M. Hussonnois, C.R. Acad. Sci. Paris, **310** (1990) 367;

[12] M. Hussonnois, J.F. Le Du, L. Brillard and G. Ardisson, J. Phys. G. **16** (1990) 177

[13] R.S. Sheline and I. Ragnarsson, Phys. Rev., **C 43** (1991) 1476

[14] W. Mollenkopf, J. Kaufmann, F. Goennenwein, P. Geltenbort, and A. Oed, J. Phys. G: Nucl. Part. Phys. **18** (1992) L203

[15] F. Goennenwein and B. Boersig, Nucl. Phys. **A 530** (1991) 27

[16] F. Goennenwein, Nucl. Instr. Meth. Phys. Reas. **A 316** (1992) 405

[17] J. Kaufmann, W. Mollenkopf, F. Goennenwein, P. Geltenbort, and A. Oed, Z. Phys: Hadrons and Nuclei, **A 341** (1992) 319

[18] J.H. Hamilton, *"Neutron Multiplicities in Spontaneous Fission and Nuclear Structure Studies"* Lecture at the International NATO Advanced Study Institute, Predeal (Sept 1993), Romania, *"Frontier Topics in Nuclear Physics"*, Eds. W. Scheid and A. Sandulescu, **NATO - ASI Series B: Physics 334** (1994) 101; see also R. Aryaeinejad *et al*, Phys. Rev. C **48** (1993) 566 and G.M. Ter - Akopyan *et al.* Phys. Rev. Lett. **73** (1994) 1477

[19] F. Goennenwein, Lecture at the International NATO Advanced Study Institute, Predeal (Sept. 1993), Romania, *"Frontier Topics in Nuclear Physics"*, Eds. W. Scheid and A. Sandulescu, **NATO - ASI Series B: Physics 334** (1994) 113

[20] M. Grigorescu, B.A. Brown and O. Dumitrescu, Phys. Rev., **C 47** (1993) 2666

[21] O. Dumitrescu, Fiz. Elem. Chastits At. Yadra 10 (1979) 377 (Sov. J. Part. Nucl. **10** (1979) 147)

[22] H.J. Mang, Ann. Rev. Nucl. Sci., **14** (1964) 1, Z. Phys., **148** (1957) 572, Phys.Rev., **119** (1960) 1069, Proc. Second International Conf. Clust. Phen. Nucl. College Park, Maryland (1975) H.J. Mang and J.O. Rasmussen,

[23] H.J. Mang, M.K. Poggenburg and J.O. Rasmussen, Phys. Rev.,**181** (1969) 1697; Kgl. Danske. Videnskab. Selskab. Mat. - Fys. Skrifter **2** (1962) No. 3

[24] J.O. Rasmussen, Phys. Rev.,**113** (1959) 1593; I. Perlman, J.O. Rasmussen, In *"Handbuch der Physik"* **42** (1957) 109

[25] W.Nazarewicz, Invited talk at Conference on Nuclear Shapes and Nuclear Structure at Low excitation Energies, Cargese, France, June3-7,1991. Preprint JIHIR 91- 03, University of Tennessee.

[26] Yu.V.Denisov, Yad.Fiz. **55**, 2647 (1992).

[27] J.Skalski, Phys.Rev.C. **49**, 2011 (1994).

[28] T.Nakatsukasa, S.Mizutori and K.Matsuyanagi, Prog.Theor.Phys. **87**, 607 (1992).

[29] R. Nazmitdinov and S.Aberg, Phys.Lett. **B289**, 238 (1992).

[30] J.L.Egido and L.M.Robledo, Nucl.Phys. **A545**, 589 (1992); V.Martin and L.M.Robledo, Phys.Rev.C. **49**, 188 (1994).

[31] E.R.Marshalek, Nucl.Phys. **A275**, 416 (1977).

[32] D.Janssen and I.N.Mikhailov, Nucl.Phys. **A318**, 390 (1979).

[33] J. Kvasil and R. G. Nazmitdinov, Sov.J.Part.Nucl. **17**

[34] J. Kvasil and R. G. Nazmitdinov, Nucl.Phys. **A439**

[35] M.C. Gutzwiller *Chaos in Classical and Quantum Mechanics* (Springer,1990).

[36] R.Arvieu et. al, Phys.Rev. **A35**, 2389 (1987).

[37] W.D.Heiss and R.G.Nazmitdinov, Phys.Rev.Lett. **73**, 1235 (1994).

[38] W.D.Heiss, R.G.Nazmitdinov and S.Radu, Phys.Rev.Lett. **72**, 2351 (1994); Phys.Rev.B (1995) in press.

[39] K.Arita, Phys.Lett. **B440**, 607 (1994).

[40] G.S. Anagnostatos, Can. J. Phys., **70** (1992) 361; Intern. J. Phys., **24** (1985) 579; Can. J. Phys., **51** (1973) 998; Atomkernenergie, **51** (1973) 207; Lett. Nouvo Cim., **22** (1978) 507; Lett. Nouvo Cim., **28** (1980) 573; Lett. Nouvo Cim., **29** (1980) 188; G.S. Anagnostatos and C.N. Panos, Phys. Rev., **C 26** (1982) 260; G.S. Anagnostatos *et al.* Atomkernenergie, **35** (1980) 60;

[41] G. Gamov, Zeit. Phys. **51** (1928) 24

[42] A.H.Wapstra and G.Audi, Nucl. Phys. **A 432** (1985) 1; A.H. Wapstra, G. Audi and R. Hoeckstra, At. Data and Nucl. Data Tables, **39** (1988) 281

[43] A. Messiah, *"Quantum Mechanics"*, North - Holland, Amsterdam, 1970; L.I. Schiff, *"Quantum Mechanics"*, McGrew - Hill Book Co., Inc. N.Y.,1955 H. Jeffreys, Proc. London Math. Soc. (2) **23** (1923) 428; G. Wentzel, Zeits. F. Phys. **38** (1926) 518; H.A. Kramers, Zeits. F. Phys. **39** (1926) 828; L. Brilluoin, Comptes Rendus, **183** (1926) 24; see also R.E. Langer, Phys. Rev., **51** (1937) 669

[44] H. Geiger and J. M. Nuttal, Phylos. Mag. **22** (1911) 613; **23** (1911) 613;

[45] O. Dumitrescu and A. Sandulescu, Nucl. Phys., **A 100** (1967) 456; Phys. Lett., **B 19** (1965) 404; Phys. Lett., **B 24** (1967) 212; Annales Acad. Sci. Fennicae. Series A. VI.Physica **265** (1968); M.I. Cristu, O. Dumitrescu, N.Y. Pyatov and A. Sandulescu, Nucl. Phys., **A 130** (1969) 31; O. Dumitrescu and D.G. Popescu, Revue Roumaine de Physique **18** (1973) 1065 and **21** (1976) 323; O. Dumitrescu, M. Petrascu, D.G. Popescu and A. Sandulescu, Revue Roumaine de Physique **18** (1973) 1231;

[46] A.M.Lane and R.G.Thomas, Rev. Mod. Phys., **30** (1958) 257

[47] Y. Akaishi, K. Kato, H. Noto, S. Okabe, Developements of Nuclear Cluster Dynamics, World Scientific Publishing Co. Pte. Ltd. 1989

[48] V.G. Neudatchin, Yu. F. Smirnov and N.F. Golovanova, Adv. Nucl. Phys., Eds.J.W. Negele andE. Vogt, **11** () chapter 1, page 1.

[49] M. Gari, Phys. Reports, **6 C** (1973) 317

[50] P.G. Bizzeti, *Weak Interactions in Nuclei*, Rivista del Nuovo Cimento **6** Nr.12 (1983) 1

[51] A. Bulgac, S. Holan, F. Carstoiu and O. Dumitrescu, Nuovo Cimento, **70 A** (1982) 142

[52] F. Carstoiu, O. Dumitrescu, G. Stratan and M.Braic, Nucl. Phys., **A 441** (1985) 221

[53] G. Bertsch, J. Borisowicz, H. McManus and W.G. Love, Nucl. Phys. **A 284** (1977) 399

[54] J.P. Elliot, A.D. Jackson, H. A. Mavromatis, E.A. Sanderson and B. Singh Nucl. Phys. **A 121** (1968) 241

[55] G.R. Satchler and W.G. Love, Phys. Reports **55** (1979) 123

[56] A. Bulgac, F. Carstoiu and O. Dumitrescu, Rev. Roum. Phys., **27** (1982) 331

[57] F. Carstoiu, O. Dumitrescu and L. Fonda, Nuovo Cimento, **A 70** (1982) 38

[58] I. Brandus, F. Carstoiu and O. Dumitrescu, Nuovo Cimento, **A 76** (1983) 15

[59] F. Carstoiu and R.J. Lombard, Annals Phys. (N.Y.), **217** (1992) 279

[60] N. Fröman and P.O. Fröman, *"JWKB Approximation"* (North - Holland Publ. Co., Amsterdam, 1965); P.O. Fröman, Kgl. Danske. Videnskab. Selskab. Mat. - Fys. Skrifter 1 (1957) No. 3

[61] V.G. Nosov, Sov. Phys. JETP **10** (1960) 631; Zh. Exp. Theor. Fiz. **37** (1959) 886; Dokl. Akad. Nauk SSSR **112** (1957) 414.

[62] P. Ring, J.O. Rasmussen and G. Massman, Fiz. Elem. Chastits At. Yadra 7 (1976) 916 (Sov.J. Part. Nucl., **7** (1976))

[63] F. Barranco, G.F. Bertsch, R.A. Broglia, E. Vigezzi, Nucl. Phys., **A 512**; F. Barranco, R.A. Broglia and G.F. Bertsch, Phys. Rev. Lett., **60** (1988) 507; G.F. Bertsch, F. Barranco and R.A. Broglia Windsurfing the Fermi Sea, Vol. 1, Eds. T.T.S. Kuo and J. Speth, Elesevier Science Publ. B. V., (1987); G.F. Bertsch, *"Collective Motion in Fermi Droplets"* in "Frontiers and Borderlines in Many Particle Physics" Eds. R.A. Broglia and J.R. Schrieffer, Amsterdam - North Holland (1989).

[64] O. Dumitrescu, Fiz. Elem. Chastits At. Yadra 23 (1992) 430 (Sov. J. Part. Nucl., **23** (1992) 187)

[65] O. Dumitrescu, Preprint ICTP Trieste **IC/91/72** (1991)); Nuovo Cimento **A 104** (1991) 1057 and Preprint ICTP - Trieste **IC/91/72** (1991); O. Dumitrescu and M Horoi, Nuovo Cimento, **A 103** (1990) 653; M. Apostol, I. Bulboaca, F. Carstoiu, O Dumitrescu and M. Horoi, Nucl. Phys., **A 170** (1987) 64; M. Apostol, I. Bulboaca, F Carstoiu, O. Dumitrescu and M. Horoi, Europhys. Letters, **4** (1987) 197

[66] O. Dumitrescu, Nuovo Cimento **A 104** (1991) 1057

[67] P. Möller and J.R. Nix, At. Data and Nucl. Data Tables, **39** (1988) 225

[68] A. Sandulescu, F. Carstoiu, A. Florescu, E. Stefanescu, W. Greiner, J.H. Hamilton and A.V. Ramayya, *"Isotopic Yields for the Cold Fission of* ^{252}Cf*"*, Priprint, Institute of Atomic Physics, Bucharest, Romania, FT - 93 (1995).

[69] A. Ritz, At. Data and Nucl. Data Tables, **47** (1991) 205

[70] Yu. Tz. Oganesyan, private communication.

[71] D. Bucurescu, private communication.

[72] P. W. Atkins and J. C. Dobson, Proc. Roy. Soc. London Ser. **A 321** (1971) 321

[73] M. Bouten, P. van Leuven and M. Rosina, Phys. Lett., **B 73** (1978) 27

[74] P. Carruters, Rev. Mod. Phys. **40** (1968) 411

[75] R. Jackiw, J. Math. Phys. (N.Y.), **9** (1968) 339

[76] P. Carruters and K.S. Dy, Phys. Rev., **147** (1960) 214

[77] L. Fonda, N. Mankoc - Borstnik and M. Rosina, Physics Reports, **158** (1988) 150

[78] O. Dumitrescu, N. Manckok - Borstnick and L. Fonda, Nuovo Cimento **A 58** (1980) 105

[79] F. Carstoiu, O. Dumitrescu and L. Fonda, Nuovo Cimento **A 70** (1982) 38

[80] I. Brandus, F. Carstoiu and O. Dumitrescu, Nuovo Cimento **A 76** (1983) 15

[81] N. Manckok - Borstnick, M. Rosina and L. Fonda, Nuovo Cimento **A 53** (1979) 440

[82] K. Alder and A. Winther, *"Electromagnetic Excitations"* (Amsterdam, 1975)

[83] R.A. Broglia and A. Winther, *"Heavy Ion Reactions, Lecture Notes"*; Vol. 1: *"Elastic and Inelastic Reactions"* (New York, N.Y. 1981)

Table Captions:

Table 1: The experimental relative intensities calculated as side feedings for the neutronless spontaneous fission $^{252}Cf \rightarrow {}^{146}Ba + {}^{106}Mo$ (see Ref. [18]) and the theoretical ones calculated in the present work.

The input parameters are: the decay energy $Q_0 = 217.33$ MeV; The quadrupole deformations: $\beta_{Ba} = 0.178$; $\beta_{Mo} = 0.326$.

Table 2: The reduced Fröman matrix elements ($\bar{\mathbf{K}}_{lI_1I_2;l'I_1'I_2'} = \frac{\mathbf{K}_{lI_1I_2;000}}{\mathbf{K}_{000;000}}$) calculated for several dominant cases. E(n) means 10^n.

Table 3: The penetrability ratios ($R_{lI_1I_2} = \frac{P_l(Q_{I_1I_2})}{P_0(Q_{00})}$) calculated for several dominant cases.

Table 1

I_1	$\mathcal{I}^{exp}_{rel-Ba}$ (%)	$\mathcal{I}^{theo}_{rel-Ba}$ (%)
0	0	24
2	10±20	67
4	66±38	8
6		0.25
8		0.12
10		<0.1
12		<0.1
14		<0.1

Table 2

l	I_1	I_2	$\bar{\mathbf{K}}_{lI_1I_2;000}$	l	I_1	I_2	$\bar{\mathbf{K}}_{lI_1I_2;000}$	l	I_1	I_2	$\bar{\mathbf{K}}_{lI_1I_2;000}$	l	I_1	I_2	$\bar{\mathbf{K}}_{lI_1I_2;000}$
0	0	0	1.000	2	2	0	2.996	4	4	0	1.478	6	6	0	0.693
2	0	2	3.535	0	2	2	2.044	2	4	2	1.516	4	6	2	0.495
4	0	4	2.408	2	2	2	-2.921	4	4	2	-1.378	6	6	2	-0.400
6	0	6	1.025	4	2	2	5.256	6	4	2	2.412	8	6	2	0.678
8	0	8	0.015	2	2	4	2.095	0	4	4	0.458	2	6	4	0.186
10	0	10	1.391	4	2	4	-1.904	2	4	4	-0.595	4	6	4	-0.148
12	0	12	1.479	6	2	4	3.333	4	4	4	0.668	6	6	4	0.159
14	0	14	0.039	4	2	6	0.950	6	4	4	-0.834	8	6	4	-0.197
16	0	16	0.819	6	2	6	-0.769	8	4	4	1.572	10	6	4	0.372
18	0	18	4.621	8	2	6	1.301	2	4	6	0.254	0	6	6	0.342

Table 3

l	I_1	I_2	$R_{lI_1I_2}$	l	I_1	I_2	$R_{lI_1I_2}$	l	I_1	I_2	$R_{lI_1I_2}$	l	I_1	I_2	$R_{lI_1I_2}$
0	0	0	1.0E(+0)	2	2	0	0.7E(+0)	4	4	0	0.4E(+0)	6	6	0	1.3E(-1)
2	0	2	0.7E(+0)	0	2	2	0.5E(+0)	2	4	2	2.7E(-1)	4	6	2	1.0E(-1)
4	0	4	0.4E(+0)	2	2	2	0.5E(+0)	4	4	2	2.5E(-1)	6	6	2	0.9E(-1)
6	0	6	1.1E(-1)	4	2	2	0.5E(+0)	6	4	2	2.3E(-1)	8	6	2	0.8E(-1)
8	0	8	2.2E(-2)	2	2	4	2.5E(-1)	0	4	4	1.3E(-1)	2	6	4	0.5E(-1)
10	0	10	0.4E(-2)	4	2	4	2.5E(-1)	2	4	4	1.3E(-1)	4	6	4	0.5E(-1)
12	0	12	0.5E(-4)	6	2	4	2.3E(-1)	4	4	4	1.2E(-1)	6	6	4	0.5E(-1)
14	0	14	1.1E(-6)	4	2	6	0.9E(-1)	6	4	4	1.1E(-1)	8	6	4	0.4E(-1)
16	0	16	2.0E(-8)	6	2	6	0.8E(-1)	8	4	4	1.0E(-1)	10	6	4	0.4E(-1)
18	0	18	0.4E(-16)	8	2	6	0.7E(-1)	2	4	6	0.5E(-1)	0	6	6	1.5E(-2)
l	I_1	I_2	$R_{lI_1I_2}$	l	I_1	I_2	$R_{lI_1I_2}$	l	I_1	I_2	$R_{lI_1I_2}$	l	I_1	I_2	$R_{lI_1I_2}$
8	8	0	0.4E(-1)	10	10	0	0.9E(-2)	12	12	0	2.2E(-3)	14	14	0	1.7E(-5)
6	8	2	0.4E(-1)	8	10	2	0.7E(-2)	10	12	2	2.1E(-3)	12	14	2	1.4E(- 5)
8	8	2	2.4E(-2)	10	10	2	6.1E(-3)	12	12	2	2.0E(-3)	14	14	2	1.2E(-5)
10	8	2	2.0E(-2)	12	10	2	5.4E(-3)	14	12	2	1.5E(-3)	16	14	2	0.9E(-6)
4	8	4	1.2E(-2)	6	10	4	3.8E(-3)	8	12	4	1.1E(-3)	10	14	4	0.8E(- 6)
6	8	4	1.2E(-2)	8	10	4	3.4E(-3)	10	12	4	1.0E(-3)	12	14	4	0.6E(- 6)
8	8	4	1.1E(-2)	10	10	4	3.3E(-3)	12	12	4	0.9E(-3)	14	14	4	0.6E(-6)
10	8	4	1.0E(-2)	12	10	4	1.0E(-3)	14	12	4	0.7E(-3)	16	14	4	0.5E(-6)
12	8	4	0.8E(-2)	14	10	4	0.8E(-3)	16	12	4	0.5E(-3)	18	14	4	0.2E(-6)
2	8	6	0.4E(-2)	4	10	6	0.4E(-3)	6	12	6	0.4E(-3)	8	14	6	0.4E(- 6)