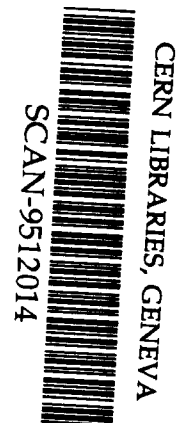


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D.V. Amelin, E.B. Berdnikov, S.I. Bitjukov, G.V. Borisov,  
V.A. Dorofeev, R.I. Dzhelyadin, Yu.P. Gouz, I.A. Kachaev,  
A.N. Karyukhin, Yu.A. Khokhlov, G.A. Klyuchnikov, V.F. Konstantinov,  
S.V. Kopikov, M.E. Kostrikov, V.V. Kostyukhin, V.D. Matveyev,  
A.P. Ostankov, D.I. Ryabchikov, A.A. Solodkov, O.V. Solovyanov,  
E.A. Starchenko, E.V. Vlasov, A.M. Zaitsev,  
*IHEP, Protvino*  
G. Sekhnaidze, E. Tskhadadze,  
*IPh, Tbilisi*

**PARTIAL-WAVE ANALYSIS OF THE REACTION**

$\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$  AT  $p_{\pi^-} = 36$  GeV/c.  
**STUDY OF  $a_2(1320)$  AND  $\omega_3(1670)$  MESONS**

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**Abstract**

Amelin D.V. et al. Partial-Wave Analysis of the Reaction  $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$  at  $p_{\pi^-} = 36$  GeV/c. Study of  $a_2(1320)$  and  $\omega_3(1670)$  mesons: IHEP Preprint 95-71. – Protvino, 1995. – p. 9, figs. 5, tables 4, refs.: 14.

Results of the partial-wave analysis of the reaction  $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$ , studied at  $p_{\pi^-} = 36$  GeV/c on  $2 \cdot 10^5$  event statistics, are presented. Masses and widths of  $a_2(1320)$  and  $\omega_3(1670)$  mesons are measured, as well as differential cross-sections  $d\sigma/dt$  in the  $a_2$  and  $\omega_3$  regions. It is shown that the relative phase between  $2^+$  and  $3^-$  is consistent with the two-resonance description. The estimated limits for the  $\pi_0(1300)$ ,  $\pi_2(1670)$ ,  $\omega_1(1420)$  and  $\omega_1(1600)$  cross sections are also presented.

**Аннотация**

Амелин Д.В. и др. Парциально-волновой анализ реакции  $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$  при  $p_{\pi^-} = 36$  GeV/c. Исследование мезонов  $a_2(1320)$  и  $\omega_3(1670)$ : Препринт ИФВЭ 95-71. – Протвино, 1995. – 9 с., 5 рис., 4 табл., библиогр.: 14.

Представлены результаты парциально-волнового анализа реакции  $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$  при  $p_{\pi^-} = 36$  ГэВ/с, проведенного на статистике  $2 \cdot 10^5$  событий. Измерены массы и ширины  $a_2(1320)$  и  $\omega_3(1670)$ , а также дифференциальные сечения  $d\sigma/dt$  в области масс  $a_2$  и  $\omega_3$ . Показано, что разность фаз волн  $2^+$  и  $3^-$  отвечает вкладу двух резонансов. Представлены также ограничения на сечения рождения  $\pi_0(1300)$ ,  $\pi_2(1670)$ ,  $\omega_1(1420)$  и  $\omega_1(1600)$  мезонов.

## Introduction

This paper presents the results of the partial-wave analysis for the charge-exchange reaction  $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$  on the Beryllium target at  $p_{\pi^-} = 36$  GeV/c. The experimental sample, consisting of  $\sim 2 \cdot 10^5$  events, was collected with Protvino VES detector [1] and processed by a modified version of the Illinois PWA program [2].

The  $\pi^+ \pi^- \pi^0$  mass spectrum, shown in Fig.1a, has a number of prominent structures: well-known  $\eta(548)$ ,  $\omega(783)$ ,  $a_2(1320)$  and a bump at mass  $\approx 1670$ , referred nowadays as  $\omega_3$  in PDG tables [9].

This paper is devoted to the consideration of the most intensive waves in the mass range  $0.9 < M_{3\pi} < 2.3$ . Other PWA results will be presented elsewhere.

We pay particular attention to the  $\omega_3$  signal, which was previously studied in a number of papers [3–6]. First it was identified as  $I = 0$  resonance [3], then quantum numbers  $J^P = 3^-$  [4, 5] and evidence for its phase motion [5] were reported. Our analysis is based on a significantly higher statistics.

## 1. Event Selection

The  $\pi^+ \pi^- \pi^0$  events were selected from the initial  $\pi^+ \pi^- \gamma \gamma$  sample consisting of  $8 \cdot 10^5$  events in accordance with the following criteria:

- invariant mass of the  $\gamma \gamma$  pair is in  $\pi^0$  mass range:  $115 < m_{\gamma\gamma} < 155$  MeV/c<sup>2</sup>,
- the  $\pi^0$  momentum is greater than 2.5 GeV/c,
- total energy of  $\pi^+$ ,  $\pi^-$  and  $\pi^0$  is in the range  $33 < E_{\text{tot}} < 38$  GeV,
- cuts for the Gottfried-Jackson angles are  $\cos \theta_{\pi^+} > -0.95$ ,  $\cos \theta_{\pi^-} < 0.9$ ,  $-0.95 < \cos \theta_{\pi^0} < 0.95$ .

Some installation-dependent conditions were also imposed. In particular, only events with tracks, identified as pions with the threshold Čerenkov counter and not identified as electrons by the energy release in the  $\gamma$ -detector, were selected. Events with target guard activity, which have probably isobar in the final state, were rejected.

As a result,  $2 \cdot 10^5$  events, matching all the criteria have been selected for the partial-wave analysis.

## 2. Partial-Wave Analysis

We are dealing with the three-meson PWA technique, based on the isobar model. In accordance with [2], each wave is denoted as  $J^P L M \eta(I, d)$ , where  $J$  is the total angular momentum of 3-meson system,  $P$  is the space parity,  $L$  is the orbital momentum of meson-dimeson system,  $M = |J_z|$  is the projection on the  $t$ -channel Gottfried-Jackson system,  $\eta = \pm 1$  denotes is the symmetry with respect to the reaction plane, which is exactly the naturality of the exchange particle in the lowest  $1/s$  order [10]. In parentheses  $I$  denotes the total isospin, and  $d$  stands for the meson-isobar decay mode.

Our analysis incorporates isobars with spin  $j \leq 2$ . For  $\rho(770)$ -meson the parametrization of [7] is used. The  $f_2(1270)$  resonance is parametrized as the relativistic Breit-Wigner function with the standard Blatt-Weisskopf factor [11] for  $j=2$  and  $R=5.2$  c/GeV.

For the  $(\pi^+\pi^-)_S$  wave amplitude we gave compared a number of parametrizations and conclude that broad  $f_0(1300)$  is really significant for the event description, while narrow  $f_0(975)$  is not. The so-called  $M$  solution of Au, Morgan and Pennington [8] with “subtracted”  $f_0(975)$  shows the best likelihood, and it was chosen for physical runs<sup>1</sup>

The general PWA procedure was applied to the mass region  $0.9 < M_{3\pi} < 2.3$  GeV/ $c^2$ , which was splitted in to 70 bins of 20 MeV/ $c^2$  width. The waves, included in the fit, are listed in Table 1. For each PWA run the pseudo-wave FLAT was also added in order to control the fit quality.

Table 1. Partial waves, included into analysis.

$I^G(J^{PC})$	List of amplitudes		
$1^-(0^{-+})$	$0^- S0+(1, f_0\pi)$ ,	$0^- P0+(1, \rho\pi)$	
$0^-(1^{--})$	$1^- P1+(0, \rho\pi)$		
$0^-(1^{+-})$	$1^+ S0+(0, \rho\pi)$ ,	$1^+ S1+(0, \rho\pi)$ ,	$1^+ S1-(0, \rho\pi)$
$1^-(1^{++})$	$1^+ S0+(1, \rho\pi)$ ,	$1^+ S1+(1, \rho\pi)$ ,	$1^+ S1-(1, \rho\pi)$
$1^-(2^{-+})$	$2^- S0+(1, f_2\pi)$		
$1^-(2^{++})$	$2^+ D1+(1, \rho\pi)$ ,	$2^+ D0-(1, \rho\pi)$ ,	$2^+ D1-(1, \rho\pi)$
$0^-(3^{--})$	$3^- F1+(0, \rho\pi)$ ,	$3^- F0-(0, \rho\pi)$	$3^- F1-(0, \rho\pi)$

The whole  $\pi^+\pi^-\pi^0$  statistics was splitted into 4 subsets by the value of the momentum transfer squared: a)  $0 < t' < 0.05$ ; b)  $0.05 < t' < 0.2$ ; c)  $0.2 < t' < 0.6$ ; d)  $0.6 < t' < 1.2$ , where  $t' = -(t - t_{\max})$ .

<sup>1</sup>This “subtraction” has been performed with the fit of the  $M$ -solution amplitude [8] by the sum of wide and narrow Breit-Wigner functions. The resulting  $(\pi^+\pi^-)_S$  amplitude is the difference  $F_M - C \cdot BW(m_0, \Gamma_0)$ . Fit parameters are  $m_0 = 983$  MeV/ $c^2$ ,  $\Gamma_0 = 37$  MeV.

The dominant waves are collected in Figs.1 and 2. We do not observe any resonant structures in  $1^-(0^{++})$  and  $1^-(2^{--})$  waves (Figs.1c, 1d). In waves with  $J^P = 1^+$  broad structures are observed, which correspond to the  $a_1(1260)$  and  $h_1(1170)$  mesons (Fig. 2). Their analysis requires careful study of the non-resonant background and will be presented elsewhere.

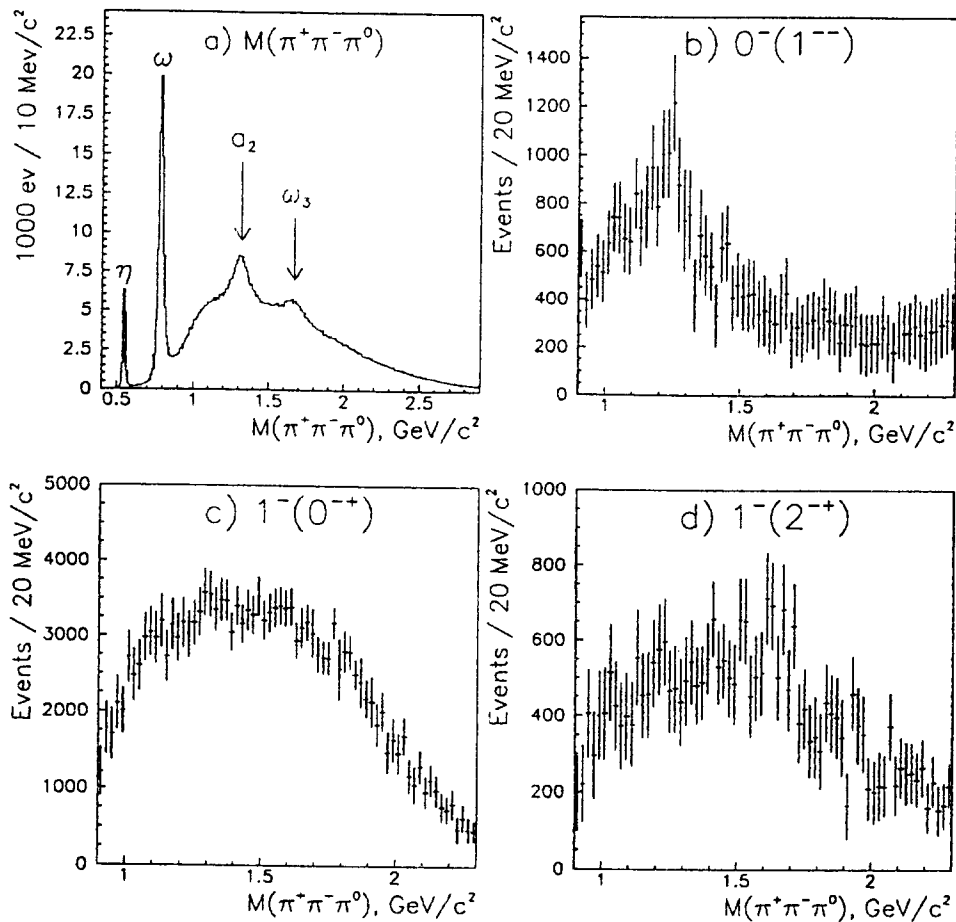


Fig. 1. a) Invariant mass of  $\pi^+\pi^-\pi^0$  system; b,c,d) summary of  $1^-$ ,  $0^-$  and  $2^-$  waves.

The  $2^+$  and  $3^-$  partial waves were fitted separately in each  $t'$  interval by the noncoherent sum of polynomial background and relativistic Breit-Wigner shape, folded with the VES resolution function. The last one was parametrized as the sum of two Gauss functions. The shape of resolution function was obtained with the help of a simulation procedure. This method was tested explicitly for the  $\omega(783)$  meson, then it was applied for  $a_2$  and  $\omega_3$  masses.

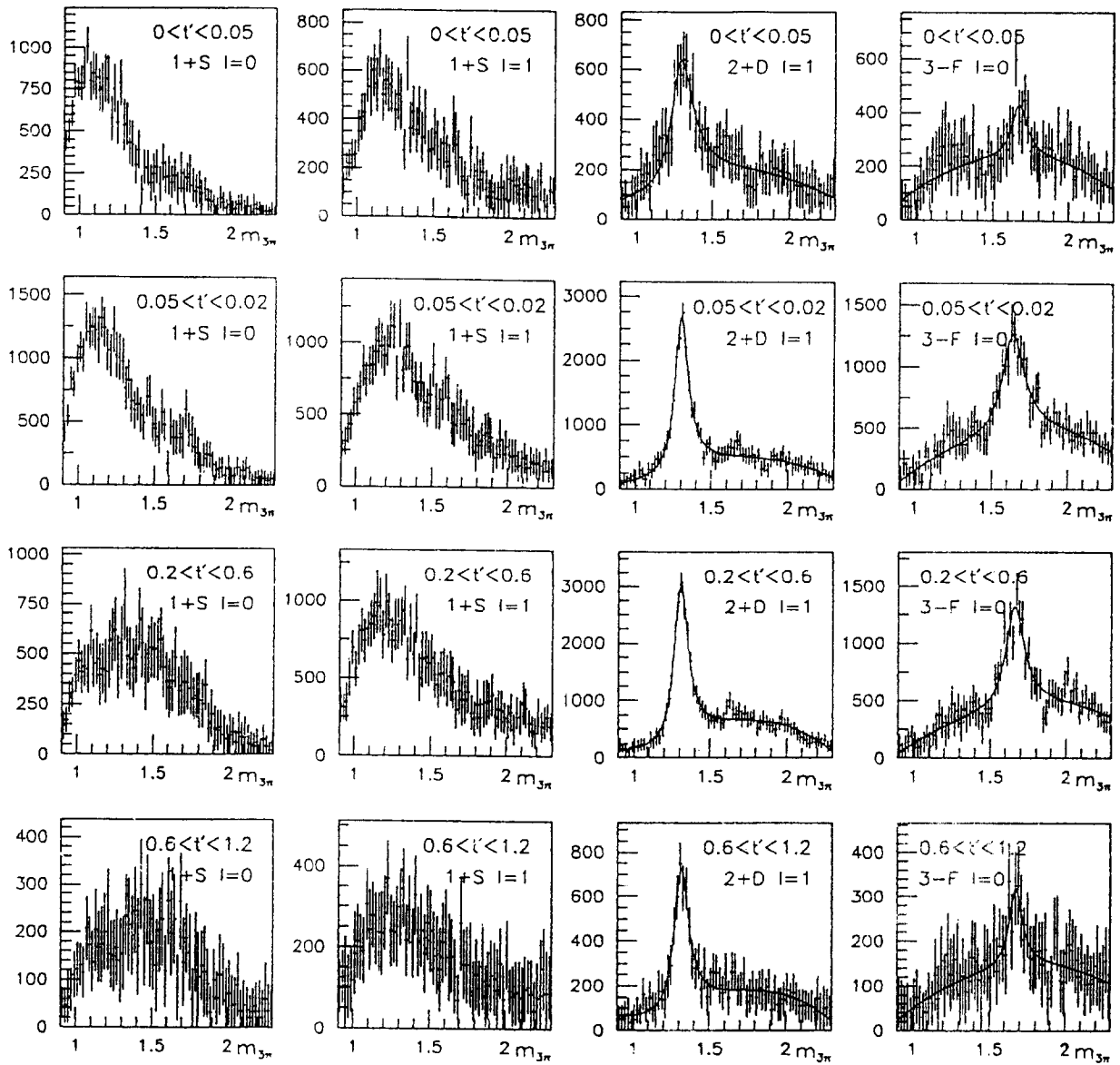


Fig. 2. Summary intensities of the dominant waves in the  $\rho\pi$ -system. Each row corresponds to the particular  $t'$  region.

The dynamical width is constructed as the sum

$$\Gamma_{\text{tot}}(m) = \sum_i \Gamma_i(m), \quad \Gamma_i(m) = b_i \Gamma_0 \left( \frac{p_i}{p_{i0}} \right)^{2l+1} \frac{m_0}{m} \cdot \frac{D_l(p_{i0}R)}{D_l(p_i R)}, \quad (1)$$

where index “ $i$ ” denotes a particular decay channel,  $b_i$  is its branching ratio, taken from PDG [9],  $m$  is the mass of initial state,  $p_i = p_i(m)$  is the relative momentum of decay products,  $p_{i0} = p_i(m_0)$ , function  $D_l(x)$  is the Blatt-Weisskopf factor for spin  $l$  [11],  $R =$

5.2 c/GeV is the hadron size parameter. Channels  $\rho\pi$ ,  $\eta\pi$ ,  $K\bar{K}$  and  $\omega\pi\pi$  were taken into account for the  $a_2$  fit, and only the  $\rho\pi$  channel was accounted for  $\omega_3$  fit.

Results of the fit are

$$\begin{aligned} M(a_2^0) &= 1311.3 \pm 1.6(\text{stat}) \pm 3.0(\text{syst}) \text{ MeV}/c^2, \\ \Gamma_0(a_2^0) &= 103 \pm 6(\text{stat}) \pm 3.3(\text{syst}) \text{ MeV}, \end{aligned}$$

with  $N(a_2) = (72.4 \pm 4) \cdot 10^3$  events, and

$$\begin{aligned} M(\omega_3) &= 1665.3 \pm 5.2(\text{stat}) \pm 4.5(\text{syst}) \text{ MeV}/c^2, \\ \Gamma_0(\omega_3) &= 149 \pm 19(\text{stat}) \pm 7(\text{syst}) \text{ MeV}, \end{aligned}$$

with  $N(\omega_3) = (23.4 \pm 2.6) \cdot 10^3$  events. The corresponding cross sections are

$$\begin{aligned} \sigma(a_2) \times \text{Br}(a_2 \rightarrow \pi^+\pi^-\pi^0) &= 5.4 \pm 0.6 \mu\text{b}, \\ \sigma(\omega_3) \times \text{Br}(\omega_3 \rightarrow \pi^+\pi^-\pi^0) &= 1.75 \pm 0.26 \mu\text{b}. \end{aligned}$$

The main source of our systematic errors is the absolute calibration of  $\gamma$ -detector. In our calibration procedure we rescale  $E_\gamma$ , so that the  $\omega(783)$  mass was fitted to its PDG value [9].

The resonant character of  $3^-$  signal is confirmed by the behaviour of non-diagonal  $\rho$ -matrix elements. In Fig. 3a the relative phase  $3^-F1+ / 2^+D1+$  is shown. It has a characteristic drop due to  $a_2$  and increase due to  $\omega_3$ . The smooth curve corresponds to the phase difference of fitted Breit-Wigner functions.

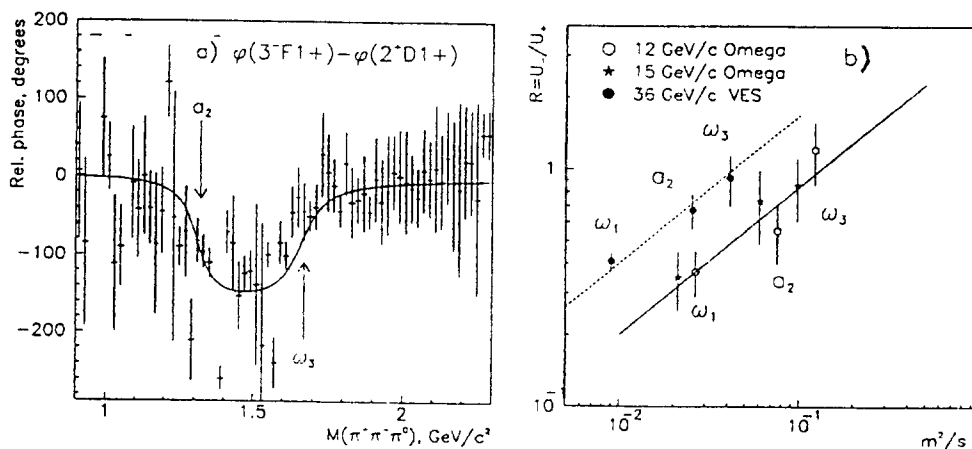


Fig. 3. a) Relative phase of  $3^-F1+(0, \rho\pi)$  and  $2^+D1+(1, \rho\pi)$  waves, b) The ratio of unnatural to natural exchange contributions  $R = U_- / U_+$  as a function of  $m^2/s$ .

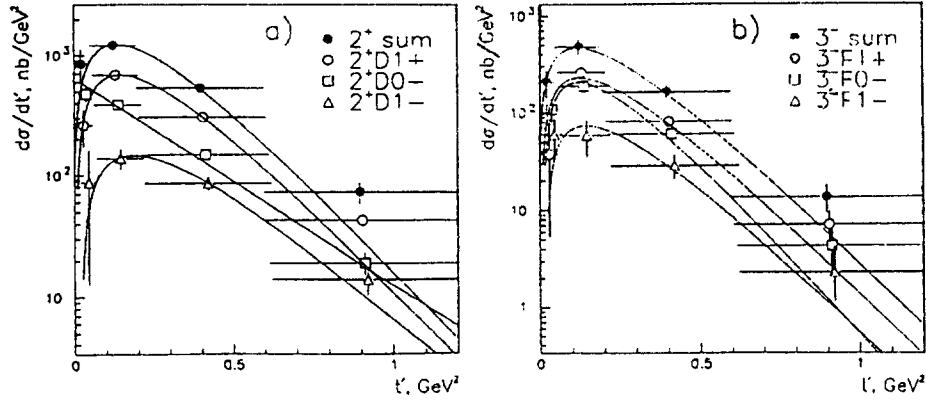


Fig. 4. The  $t'$ -dependencies of the separate partial waves of  $a_2$  (a) and  $\omega_3$  (b) resonances.

### 3. The $t'$ -dependencies of partial waves

The  $t$ -dependencies of  $a_2$  and  $\omega_3$  partial waves are presented in Figs. 4a and 4b. We observe usual dips at  $t \sim 0$  for waves with  $J_z \neq 0$  (however, the presence of a dip in  $2^+D1-$  is questionable) and typical exponential dependence for  $2^+D0-$  wave. There is the evidence for a dip at  $t=0$  for  $3^-F0-$  wave, which may reflect some peculiarity of the  $\omega_3$  production mechanism. Parameters of fits are presented in Table 2.

Table 2. Slope parameters of  $t$ -dependencies for waves of Figs.4a and 4b. All the waves (except  $2^+D0-$ ) were fitted by the function  $t' e^{-Bt'}$ , and  $2^+D0-$  was fitted by  $e^{-Bt'}$ .

Wave	B, $\text{GeV}^{-2}$	Wave	B, $\text{GeV}^{-2}$
$2^+$ sum	-	$3^-$ sum	$8.6 \pm 0.8$
$2^+D1+$	$8.4 \pm 0.6$	$3^-F1+$	$8.1 \pm 0.3$
$2^+D0-$	$(4.1 \pm 0.3)^*$	$3^-F0-$	$9.0 \pm 0.8$
$2^+D1-$	$5.6 \pm 1.7$	$3^-F1-$	$7.8 \pm 2.7$

These results can be compared with  $t'$ -distributions of the  $\omega(783)$  meson, which should be produced via the same  $\rho$  and  $b_1$  exchanges, as  $a_2$  and  $\omega_3$  [12]. On Fig.5 the intensities of  $\omega(783)$  partial waves, plotted with  $0.02 \text{ GeV}^{-2}$  interval, are presented. They exhibit usual dips at  $t' \sim 0$  for projectional waves. Fit results are presented in Table 3.



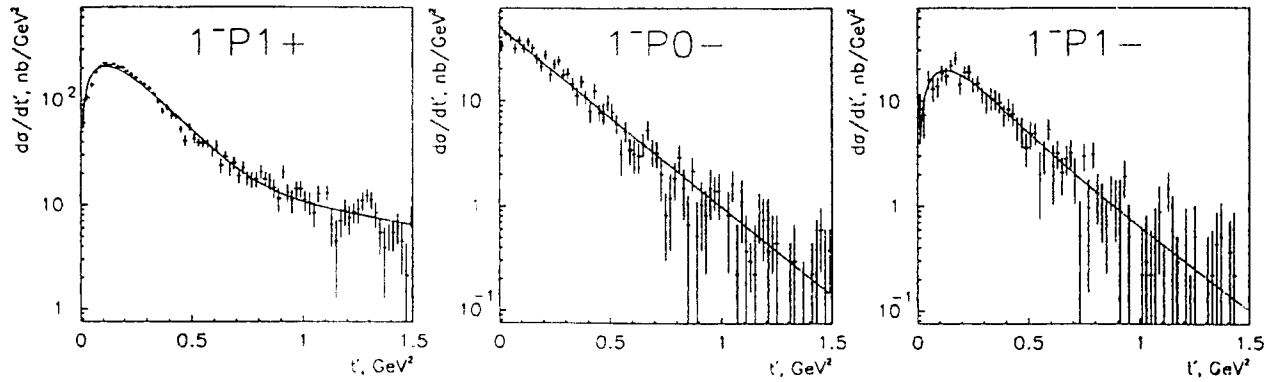


Fig. 5. The  $t'$ -dependencies of different partial waves in  $\omega(783)$  mass region.

**Table 3.** Parameters of  $\omega(783)$  fits. The  $t'$  distributions, presented on Fig. 4, were fitted with the functions  $F_0 = e^{-b_1 t'}$ ,  $F_1 = b_1^2 p t' e^{-b_1 t'} + b_2^2 (1-p) t' e^{-b_2 t'}$ .

Wave	Function	Parameter values		
		$b_1$	$b_2$	p
$1^-P1+$	$F_1$	$8.9 \pm 0.06$	$1.8 \pm 0.2$	$0.91 \pm 0.003$
$1^-P1-$	$F_1$	$8.7 \pm 0.4$	$4.3 \pm 0.4$	$0.68 \pm 0.08$
$1^-P0-$	$F_0$	$4.02 \pm 0.05$		

The ratios of unnatural to natural parity exchange contributions are  $U_{-/+} = 0.45 \pm 0.03$  for  $\omega_1(783)$ ,  $U_{-/+} = 0.77 \pm 0.11$  for the  $a_2(1320)$  and  $U_{-/+} = 0.93 \pm 0.22$  for  $\omega_3(1670)$ . In [13] Hoyer, Roberts and Roy derived the mass and energy dependence for this ratio

$$R = \frac{U_-}{U_+} = \left( \frac{m^2}{s} \right)^{2(\alpha_N(t) - \alpha_U(t))}, \quad (2)$$

where  $m$  is a mass of decaying resonance,  $\alpha_N(t)$  and  $\alpha_U(t)$  – natural and unnatural exchange trajectories. In [14] was found  $\alpha_N - \alpha_U = 0.31 \pm 0.08$  at  $|t| \approx 0.45 \text{ GeV}^{-2}$ . A comparison of these predictions with our results is presented on Fig.3b. The same  $m^2$  dependence as that in [14] is observed, however, the  $s$ -dependence of formula (2) has not confirmed. Nevertheless, note that all these results were obtained on the nucleon target.

#### 4. Search for other resonances

Table 4 contains limits for the production cross sections of some other resonances [9], which may be observed in  $\pi^+\pi^-\pi^0$  mode. These limits do not contradict the expected cross sections, provided that the  $\rho$ -exchange is the main production mechanism.

For  $\omega_1(1420)$  and  $\omega_1(1600)$  the  $\rho$ -exchange diagrams were computed explicitly, and the predicted cross sections are compatible with the experimental limits.

Table 4. Estimated production rates at  $P_{\pi^-} = 36$  GeV/c. The limits correspond to the  $1\sigma$  confidence level, resonance parameters were taken from PGD [9].

Name	M, MeV/c <sup>2</sup>	$\Gamma$ , MeV	$\sigma \times \text{Br}$ , nb
$\pi_0(1300)$	$1300 \pm 100$	200.600	< 100
$\pi_2(1670)$	$1670 \pm 20$	$240 \pm 15$	< 15
$\omega_1(1420)$	$1419 \pm 31$	$174 \pm 60$	< 3.5
$\omega_1(1600)$	$1662 \pm 13$	$280 \pm 24$	< 8

## Conclusion

The systematic study of  $a_2(1320)$  and  $\omega_3(1670)$  resonances by the PWA technique has been performed. The cross section of  $\pi^-p \rightarrow a_2n$  reaction at  $P_{\pi^-} = 36$  GeV/c is found to be  $\sigma(a_2) = 7.7 \pm 0.9 \mu\text{b}$ . We present the most accurate measurement of the  $\omega_3$  meson parameters:  $M(\omega_3) = 1665.3 \pm 5.2(\text{stat}) \pm 4.5(\text{syst})$  MeV/c<sup>2</sup>,  $\Gamma_0(\omega_3) = 149 \pm 19(\text{stat}) \pm 7(\text{syst})$  MeV. Clear evidence for the resonance phase behaviour of  $3^-F$  waves was also obtained.

The measured ratios of unnatural to natural exchange contributions do not confirm the predictions of the naive Regge model [13].

Limits for the production cross sections of  $\pi_0(1300)$ ,  $\pi_2(1670)$ ,  $\omega_1(1420)$  and  $\omega_1(1600)$  were obtained. In the case of  $\omega_1$  mesons these limits are compatible with the expected cross sections.

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Д.В.Амелин и др.

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ГНЦ РФ Институт физики высоких энергий  
142284, Протвино Московской обл.

