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ABSTRACT

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The production of ϕ mesons from central Si + Au collisions has recently been measured at the AGS [1] [2] [3]. In the discussion of the experimental results, it was suggested that coalescence of $K\bar{K}$ pairs during the heavy-ion reaction might be one of the mechanisms for the ϕ production [2]. Encouraged by the success of the phase space coalescence model of nuclear cluster (e.g. deuteron, triton, helium) production [4] in heavy-ion reactions, we have decided to investigate kaon coalescence as mechanism of ϕ production in some detail.

The first thing that strikes one in consideration of kaon coalescence as a mechanism for ϕ production is that the coalescence cannot be as simple as the deuteron case, where the proton and neutron are seen as simply joining together without changing their internal structure. The spin one deuteron is constructed as a triplet state of the spin one half proton and neutron. The ϕ, on the other hand, is a spin one, angular momentum zero, particle that must be constructed from two spin zero kaons in a coalescence picture. But two kaons cannot simply stick together and even have the correct spin of a ϕ. Furthermore, the ϕ is composed of a strange, anti-strange pair in the quark model, while kaons are composed of pairs of an anti-strange plus up or down quark (and likewise, mutatis mutandis, for anti-kaons). It is clear that a physical model of kaon coalescence must occur at the quark level and must involve annihilation of non-strange quark pairs.

The "naive" quark-pair-creation model of strong-interaction vertices [5] has had a measure of success in describing widths for processes such as the decay of a ϕ into kaons [6]. (For consistency, we follow the notation and normalizations of Ref. [6].) In this model, mesons are described as bound quark-antiquark states in an $SU(8)$ (4 flavors: u, d, s, c $\times$ spin) wavefunction scheme. The process of ϕ decay into kaons can be represented by the diagram of Fig. 1. The strange, anti-strange quarks of the ϕ act as spectators while a non-strange quark-antiquark pair is created out of the vacuum. In the model, one assumes that the created quark-antiquark pair carries the quantum numbers of the vacuum. This means that the created pair is in a 0^{++} ($^3P_0$) state, that is, spin zero and orbital angular momentum one. The transition matrix element is of the form

$$< K^- K^+ |\hat{O}|\phi >= \gamma \sum_{l_z s_z} < 1 l_z 1 s_z | 00 >< \Phi_{K^-} \Phi_{K^+} | \Phi_\phi \Phi_0^{s_z} > I^{l_z}_{K^-,K^+;\phi}, \tag{1}$$

where $< \Phi_{K^-}\Phi_{K^+}|\Phi_\phi\Phi_0^{s_z} >$ is the SU(8) wave function overlap, and the integral for the spatial overlap in momentum space

$$I^{l_z}_{K^-,K^+;\phi}(\mathbf{k}_{K^-}) = \delta(\mathbf{k}_{K^-} + \mathbf{k}_{K^+})\frac{1}{8}\int d\mathbf{k} Y_1^{l_z}(\mathbf{k}_{K^-} - \mathbf{k})\tilde{\Psi}^*_{K^-}(-\mathbf{k})\tilde{\Psi}^*_{K^+}(\mathbf{k})\tilde{\Psi}_\phi(\mathbf{k}+\mathbf{k}_{K^-}) \tag{2}$$

may be written in coordinate space, and after some angular integrations becomes

$$I^{l_z}_{K^-,K^+;\phi}(\mathbf{k}_{K^-}) = \delta(\mathbf{k}_{K^-} + \mathbf{k}_{K^+}) 8 (3/2)^{1/2}(4\pi)^{-3/2}(\boldsymbol{\epsilon}(l_z)\cdot\mathbf{k}_{K^-})J_0(|\mathbf{k}_{K^-}|). \tag{3}$$

The model wave functions for the process are contained in J_0:

$$J_0(k) = \int_0^\infty dr_1 \int_0^\infty dr_2 \int_{-1}^{+1} dz r_2^2 \psi^*_{K^-}(r_2)\psi^*_{K^+}((r_1^2+r_2^2-2r_1r_2z)^{1/2}) \\ \times\left\{\left[\frac{2}{k^3}\sin\frac{kr_1}{2} - \frac{r_1}{k^2}\cos\frac{kr_1}{2}\right]\frac{1}{\partial r_1}\psi_\phi(r_1) + \frac{r_1}{k}\sin\frac{kr_1}{2}\psi_\phi(r_1)\right\} \tag{4}$$

where k the relative momentum of the kaons, equal in magnitude to k_{K^-} for this case of equal mass outgoing particles.

The decay width for a vector particle into two pseudoscalars may be expressed [6]

$$\Gamma(\phi \to K^- K^+) = f^2_{\phi K^- K^+}\frac{k^3}{6\pi m_\phi^2}, \tag{5}$$

where k is the relative momentum of the outgoing pair and m_ϕ is the mass of the decaying ϕ. There is a factor of 1/3 ($=< \epsilon(l_z)\cdot\mathbf{k}_{K^-} >$) in the right hand side of Eq. 1 to account for the polarization of the ϕ. The hadronic coupling constant, $f_{\phi K^- K^+}$, may be expressed in the quark-pair-creation model in terms of one dimensionless coefficient, γ, representing the strength of the vacuum pair creation or annihilation, and the integral, $J_0(k)$, over the wave functions of the diagram of Figure 1

$$f_{\phi K^- K^+} = \gamma 2((2\pi)^3 m_\phi E_{K^-} E_{K^+})^{1/2} J_0(k). \tag{6}$$

We will assume the time-reversed process, in which the non-strange 0^{++} pair is annihilated, as the mechanism of kaon coalescence.

If one assumes harmonic oscillator wave functions for the bound quark-antiquark pairs of the ϕ and the kaons

$$\psi(r) = \frac{1}{(\pi R^2)^{3/4}} \exp\Big(-\frac{r^2}{2R^2}\Big), \tag{7}$$

then the integral $J_0(k)$ may be carried out analytically [5] [6] to obtain

$$J_0(k) = \frac{2^{1/2}}{16\pi^{5/4}} \exp\Big[-\frac{(k_{K^-}^2) R_\phi^2 (R_{K^-}^2 + R_{K^+}^2)}{8(R_\phi^2 + R_{K^-}^2 + R_{K^+}^2)}\Big] \times \Big(\frac{2R_\phi R_{K^-} R_{K^+}}{R_\phi^2 + R_{K^-}^2 + R_{K^+}^2}\Big)^{3/2} \frac{(2R_\phi^2 + R_{K^-}^2 + R_{K^+}^2)}{(R_\phi^2 + R_{K^-}^2 + R_{K^+}^2)}. \tag{8}$$

For more realistic charmonium potential wave functions Chaichian and Kögerler [6] find that if they take $\gamma = 3.85$, they calculate $f_{\phi K^- K^+} = -4.325$, in agreement with with ϕ decay and close in magnitude to γ.

We may now state our ansatz for kaon coalescence. We will construct an appropriate phase space density function (Wigner function) for kaons to form the ϕ. Such a Wigner function approach has previously been used for deuteron coalescence at lower energies [7] and in consideration of the coalescence of hypothetical "pineuts" at the AGS [8]. To conserve total angular momentum, it is necessary for the Wigner function of the kaons have orbital angular momentum one. We have previously presented the expression for the angular momentum one harmonic oscillator Wigner function [8]

$$f^{L=1}(\mathbf{p},\mathbf{r}) = \left[\frac{16}{3}\frac{r^2}{b^2} - 8 + \frac{16}{3}b^2k^2\right]\exp\left(-\frac{r^2}{b^2} - b^2k^2\right) \tag{9}$$

where b is the harmonic oscillator wave function size parameter, i.e. R in Eq.(7). The derivation of Eq.(9) is presented in the Appendix. Figure 2 shows the dependence of $f^{L=1}(\mathbf{p},\mathbf{r})$ on r and k. The contribution of the negative density small r and k region is relatively unimportant due to the r^2 and k^2 radial scaling. Integration of this function over coordinate or momentum space provides the appropriate (everywhere positive) momentum or coordinate density [9].

We use the Wigner function as the phase space density for kaons to form the ϕ in a heavy ion reaction. The phase space of kaons in a heavy-ion reaction is simulated using the cascade code ARC [10] [11]. The spatial and momentum differences of all possible candidate kaon pairs are evaluated at the later of their last interaction times and then used as the values of k and r in Eq.(9) to evaluate the probability of making a ϕ.

In practice we make use of the 0^{++} $^3P_0$ annihilation explored above in only two aspects of our calculations: (1) determining the size parameter of the Wigner function, b; and (2) considering possible enhancement to the normalization of the Wigner function from the vertex of Fig. (1) by a factor of $\gamma^2/3$.

Let us consider size. For our Wigner function the relevant spatial and momentum coordinates are those of the kaons relative to each other. To investigate this point we can express Eq.(4) in an alternate set of spatial coordinates. Instead of the coordinate, r_1, of the strange quark-antiquark pair we will use the relative coordinate, r, of the two kaons. If we utilize harmonic oscillator wave functions, but only perform the intgrals over z and r_2 we obtain a form

$$J_0(k) \sim \int_0^\infty r^2 dr \exp\left[-\frac{(1+m_u/m_s)^2r^2}{2R_\phi^2}\right]\exp\left[-\frac{(1+m_u/m_s)^2r^2}{2(R_{K^-}^2+R_{K^+}^2)}\right] \tag{10}$$

The sizes of the ϕ, R_ϕ, and and of the kaons, R_{K^-}, R_{K^+} come in through their wave functions in the matrix element. Although there is no direct experimental data on the size of the ϕ, the mean square charge radius of the kaon has been measured by scattering of negative

one. The transition matrix element is of the form

$$< K^-K^+|\hat{O}|\phi >= \gamma\sum_{l_zs_z} < 1l_z1s_z|00 >< \Phi_{K^-}\Phi_{K^+}|\Phi_\phi\Phi_0^{s_z} > I^{l_z}_{K^-,K^+;\phi}, \tag{1}$$

kaons with electrons [12]. We use this information to determine R_{K^-} and R_{K^+}, and assume a similar value for R_ϕ.

In the quark model the mean square charge radius of the kaon may be expressed in terms of the mean square charge radii of the strange quark and the anti-up quark relative to the center of mass,

$$e < \mathbf{r}_K^2 >= \frac{e}{3} < \mathbf{r}_s^2 > + \frac{2e}{3} < \mathbf{r}_u^2 > . \tag{11}$$

But the relevant quark wave function coordinate is the distance between the quarks, $\mathbf{r}$

$$\begin{aligned} \mathbf{r}_s &= -\frac{m_u}{m_s + m_u}\mathbf{r} \\ \mathbf{r}_u &= \frac{m_s}{m_s + m_u}\mathbf{r}, \end{aligned} \tag{12}$$

and we can express the kaon quark wave function mean square radius in terms of the mean square charge radius of the kaon

$$< \mathbf{r}_K^2 >= \frac{m_u^2 + 2m_s^2}{3(m_s + m_u)^2} < \mathbf{r}^2 > . \tag{13}$$

For a harmonic oscillator quark wave function the mean square radius is related to the oscillator parameter

$$< \mathbf{r}^2 >= \frac{3}{2}b^2, \tag{14}$$

and we obtain

$$b^2 = \frac{(1 + m_u/m_s)^2}{1 + m_u^2/2m_s^2} < \mathbf{r}_K^2 > . \tag{15}$$

In the K^-K^+ coordinate (with $b = R_\phi = R_K$) the relevant size parameter, b', is seen to be given by

$$b'^2 = \frac{b^2}{(1 + m_u/m_s)^2} = \frac{< \mathbf{r}_K^2 >}{(1 + m_u^2/2m_s^2)} \tag{16}$$

if we consider only the effect of the ϕ wave function in Eq.(10), and by

$$b'^2 = \frac{2b^2}{3(1 + m_u/m_s)^2} = \frac{2 < \mathbf{r}_K^2 >}{3(1 + m_u^2/2m_s^2)} \tag{17}$$

if we also include the effect of the kaon wave functions. For the masses we assume constituent quarks and set m_u to 300 MeV and m_s to 500 MeV. The measured mean square charge radius of the kaon $< \mathbf{r}_K^2 >$ is .34 fm^2 [12]. Thus we have $b' = 0.44$–0.54 fm for the Wigner function $f^{L=1}(\mathbf{p}, \mathbf{r})$.

Figure 3 shows how the production of ϕ particles depends on the Wigner function size parameter b'. Note the excellent agreement of the kaon rapididy distributions from ARC with the E859 preliminary Si + Au data [3]. This is a measure of the reliability of the main imput to the coalescence calculations, the ARC phase space distribution of kaons. The ϕ data were taken over a narrower angular range than the kaons, and the difference is reflected in our acceptance of kaons and ϕ's seen in the plot.

In Table I we present the number of ϕ's produced in the calculations as a function b' integrated over rapidity. The calculations include coalescence from both K^-K^+ and $K^0\bar{K}^0$ pairs but reflect the fact that the experiment only detects the K^-K^+ decay channel. The additional possible enhancement from the vertex was not included in the tabulated numbers. For our preferred values of $b' = 0.44$–0.54 fm, we find .0025–.0051 ϕ's per central event. In contrast, the E859 Collaboration measures $0.09 \pm 0.04 \phi$'s per central event [3]. Thus the coalescence calculation underpredicts ϕ production by a factor of 18–36. Maximizing the coalescence predictions by the inclusion of the enhancemant factor $\gamma^2/3$ we find coalescence underpredicts the cross section by a factor of 4–8.

If we were to make the physically unjustified assumption that the ϕ is an $L = 1$, $K\bar{K}$ molecule of relatively larger size, then the production of ϕ's certainly goes up. At the largest values of b' the coalescence calculation produces about 1/2 of the observed rate.

In summary, for the small value of b'=0.44–0.54 fm consistent with the required quark model, we find a relatively small rate of ϕ production. The coalescence that does occur may be thought of as having a small cross section with comparatively high relative momentum pairs contributing. Of course the very small size of the production cross section, related to the small size of the ϕ itself, makes a model of coalescence at last interaction necessarily less complete than in the deuteron case. Large size deuterons produced before a final interac have a large probability of being broken up; small size ϕ's produced before effective freeze out have a higher probability of escaping unscathed. Inclusion of such earlier coalesced ϕ's might possibly increase the predicted rate.

The fact that we calculate a relatively small rate of ϕ production via coalescence is not inconsistent with recent analysis [3] of the data. It is pointed out that although data on direct production of ϕ's in $p+p$ reactions is only available at higher energies, the ϕ/K^- ratio is basically independent of $\sqrt{s}$ in $p+p$. Since the $p+p$ ratio $\phi/K^- \simeq 10\%$ agrees well with the central Si + Au value of $9 \pm 4\%$, the authors state that "This comparison indicates that there is no significant increase in the ϕ yield in Si-Au collisions beyond that already seen in the K$^-$ yields." [3]

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APPENDIX: Wigner Function for the $L = 1$ harmonic oscillator

Define the ground state and first excited state density matrix components in one dimension,

$$\rho_0(x, x') = \phi_0(x)\phi_0^*(x')$$
$$\rho_1(x, x') = \phi_1(x)\phi_1^*(x').$$

for the appropriate ground and first excited harmonic oscillator states

$$\phi_0 = \frac{1}{\sqrt{b\sqrt{\pi}}}\exp[-\frac{x^2}{2b^2}]$$
$$\phi_1 = \sqrt{\frac{2}{b\sqrt{\pi}}}\frac{x}{b}\exp[-\frac{x^2}{2b^2}]$$

where $b^2 = \hbar/(m\omega)$. The Wigner function is given by the expression [9]

$$f_i(p_x, x) = \int \rho_i(x + \frac{\eta}{2}, x - \frac{\eta}{2})\exp[ik_x\eta]d\eta$$

where $k_x = p_x/\hbar$.

We will construct the three dimensional $L = 1$ harmonic oscillator Wigner function in Cartesian coordinates from one dimensional Wigner functions f_0 and f_1. First construct $f_0(p_x, x)$ (problem, p.59, Feynman [9])

$$\begin{aligned}
f_0(p_x,x) &= \int_{-\infty}^{\infty} \rho_0(x+\frac{\eta}{2}, x-\frac{\eta}{2})\exp[ik_x\eta]d\eta \\
&= \frac{2}{b\sqrt{\pi}}\exp[-\frac{x^2}{b^2}]\int_0^{\infty}\exp[-\frac{\eta^2}{4b^2}]\cos k_x\eta d\eta \\
f_0(p_x,x) &= 2\exp[-\frac{x^2}{b^2} - b^2k_x^2].
\end{aligned}$$

Next construct $f_1(p_x, x)$

$$\begin{aligned}
f_1(p_x,x) &= \int_{-\infty}^{\infty} \rho_1(x+\frac{\eta}{2}, x-\frac{\eta}{2})\exp[ik_x\eta]d\eta \\
&= \frac{4}{b^3\sqrt{\pi}}\int_0^{\infty}(x^2-\frac{\eta^2}{4})\exp[-\frac{1}{b^2}(x^2+\frac{\eta^2}{4})]\cos k_x\eta \, d\eta \\
f_1(p_x,x) &= (4\frac{x^2}{b^2} - 2 + 4b^2k_x^2)\exp[-\frac{x^2}{b^2} - b^2k_x^2]
\end{aligned}$$

We immediately obtain the well known expression for the L=0 Wigner function

$$\begin{aligned}
\rho^{L=0}(\mathbf{r},\mathbf{r}') &= \phi_0(x)\phi_0(y)\phi_0(z)\phi_0^*(x')\phi_0^*(y')\phi_0^*(z') \\
f^{L=0}(\mathbf{p},\mathbf{r}) &= \int \rho^{L=0}(\mathbf{r},\mathbf{r}')\exp[i\mathbf{k}\cdot\boldsymbol{\eta}]d\boldsymbol{\eta} \\
&= 8\exp[-\frac{r^2}{b^2} - b^2k^2].
\end{aligned}$$

For L $= 1$ we may construct the M state wave functions in Cartesian coordinates

$$\begin{aligned}
\phi_{M=1}^{L=1}(\mathbf{r}) &= \frac{1}{\sqrt{2}}[(\phi_1(x)\phi_0(y) - i\phi_0(x)\phi_1(y)]\phi_0(z) \\
\phi_{M=-1}^{L=1}(\mathbf{r}) &= \phi_{M=1}^{L=1}(\mathbf{r})^* \\
\phi_{M=0}^{L=1}(\mathbf{r}) &= \phi_0(x)\phi_0(y)\phi_1(z).
\end{aligned}$$

Let us define the density matrix for $L = 1$ averaged over M states

$$\rho^{L=1}(\mathbf{r},\mathbf{r}') = \frac{1}{3}[\phi_{M=1}^{L=1}(\mathbf{r})\phi_{M=1}^{L=1}(\mathbf{r}')^* + \phi_{M=-1}^{L=1}(\mathbf{r})\phi_{M=-1}^{L=1}(\mathbf{r}')^* + \phi_{M=0}^{L=1}(\mathbf{r})\phi_{M=0}^{L=1}(\mathbf{r}')^*]$$

$$= \frac{1}{3}[\phi_1(x)\phi_1(x')\phi_0(y)\phi_0(y')\phi_0(z)\phi_0(z')$$
$$+ \phi_0(x)\phi_0(x')\phi_1(y)\phi_1(y')\phi_0(z)\phi_0(z')$$
$$+ \phi_0(x)\phi_0(x')\phi_0(y)\phi_0(y')\phi_1(z)\phi_1(z')].$$

We finally obtain the M-averaged $L = 1$ harmonic oscillator Wigner function,

$$f^{L=1}(\mathbf{p},\mathbf{r}) = \int \rho^{L=1}(\mathbf{r},\mathbf{r}') \exp[i\mathbf{k}\cdot\boldsymbol{\eta}]d\boldsymbol{\eta}$$
$$= \frac{1}{3}[f_1(p_x,x)f_0(p_y,y)f_0(p_z,z) + f_0(p_x,x)f_1(p_y,y)f_0(p_z,z)$$
$$+ f_0(p_x,x)f_0(p_y,y)f_1(p_z,z)]$$
$$f^{L=1}(\mathbf{p},\mathbf{r}) = [\frac{16}{3}\frac{r^2}{b^2} - 8 + \frac{16}{3}b^2k^2] \exp[-\frac{r^2}{b^2} - b^2k^2].$$

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FIGURES

FIG. 1. Quark lines for the decay of a vector meson ϕ into two pseudoscalar meson kaons.

FIG. 2. Harmonic oscillator Wigner function averaged over magnetic quantum numbers for angular momentum one.

FIG. 3. Multiplicity of kaons and ϕ's produced in 14.6 A·GeV/c Si + Au central collisions as a function of rapidity. Preliminary data for K^+ (triangles), K^- (squares), and ϕ (circles) are from Ref. [3].

TABLES

TABLE I. Number of ϕ's produced per central event in Si + Au from coalescence of $K^- K^+$ and $K^0 \bar{K}^0$ pairs as a function of the Wigner function size parameters b'.

b' (fm)	$1/b'$ (GeV/c)	No of ϕ's
0.2	0.99	0.000065
0.3	0.66	0.00049
0.44	0.45	0.0025
0.54	0.37	0.0051
0.7	0.28	0.012
1.0	0.197	0.028
1.5	0.132	0.048
2.3	0.086	0.051

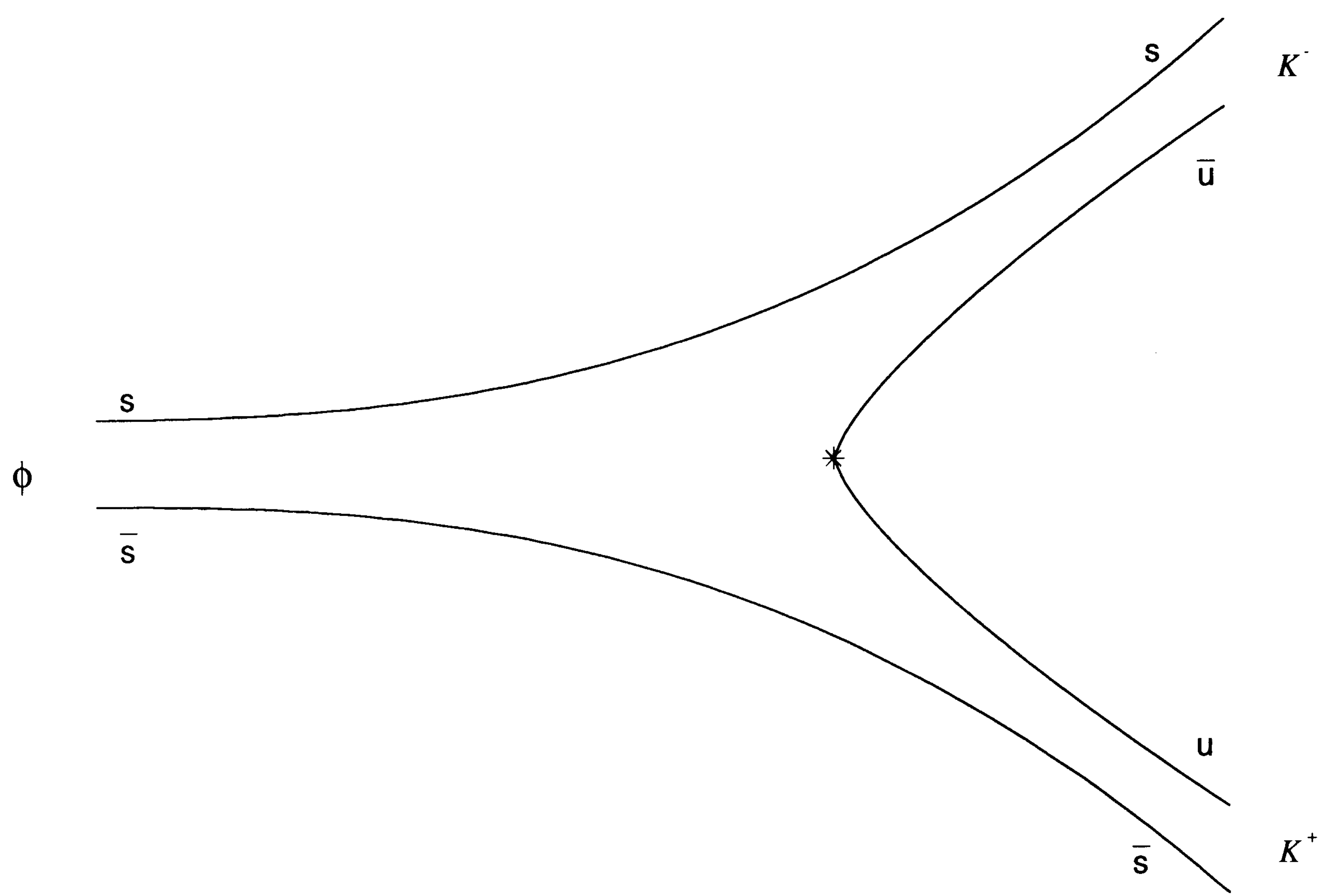


Fig. 1

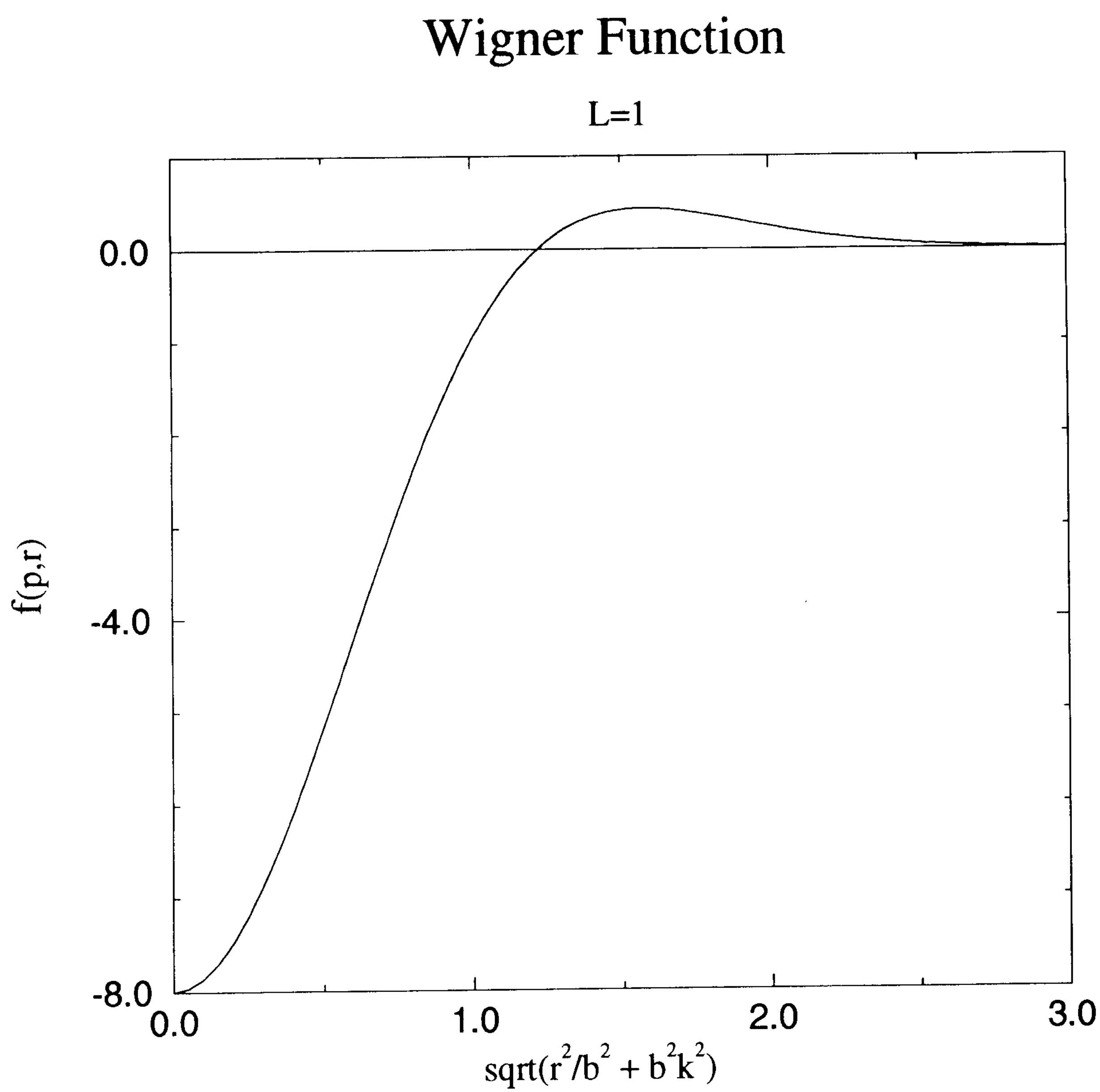
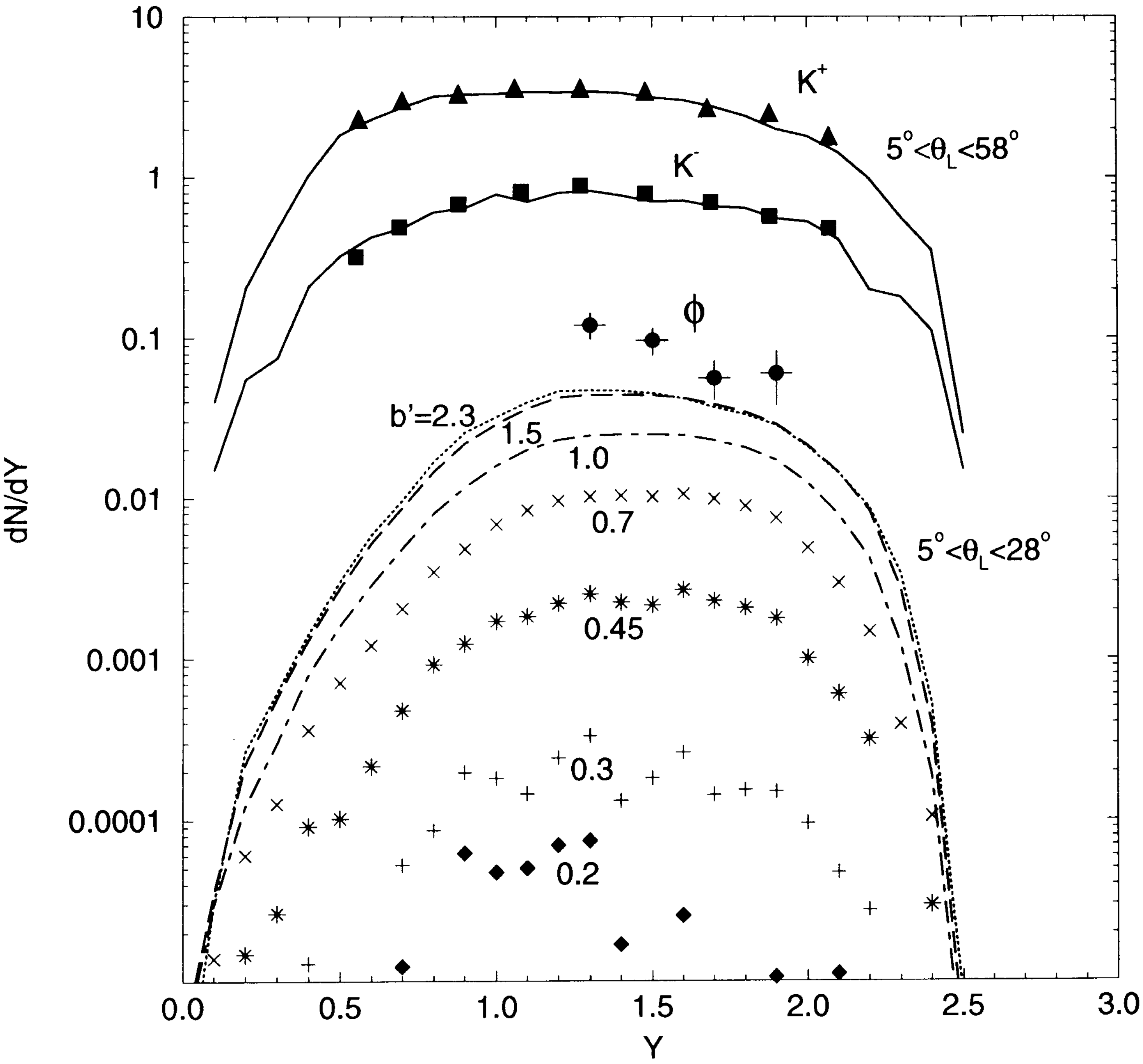


Fig. 2

Fig. 3



K+
5°<θL<58°
K-
φ
b'=2.3
1.5
1.0
0.7
0.45
0.3
0.2
5°<θL<28°
dN/dY
Y
10
1
0.1
0.01
0.001
0.0001
0.0
0.5
1.0
1.5
2.0
2.5
3.0