МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ ИНЖЕНЕРНО-ФИЗИЧЕСКИЙ ИНСТИТУТ (ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ)

S.R.Kelner, R.P.Kokoulin, A.A.Petrukhin

024-95

ABOUT CROSS SECTION FOR HIGH-ENERGY MUON BREMSSTRAHLUNG

Moscow 1995

State Committee of the Russian Federation
in Higher Education

Moscow State Engineering Physics Institute
(Technical University)

S.R.Kelner, R.P.Kokoulin, A.A.Petrukhin

ABOUT CROSS SECTION FOR HIGH-ENERGY MUON BREMSSTRAHLUNG

Preprint 024-95

Moscow 1995

Kelner S.R., Kokoulin R.P., Petrukhin A.A. About cross section for high-energy muon bremsstrahlung. Moscow:Preprint/MEPhI, 024-95, 1995.-32 p.

Various corrections to muon bremsstrahlung cross section are considered. Reasons of differences between the results calculations of nuclear formfactor influence by different authors are analysed. New results concerning the contribution of muon bremsstrahlung on atomic electrons are presented. Accuracy of calculations of muon energy loss for the bremsstrahlung is discussed.

ISBN 5-7262-0236-8

1. *INTRODUCTION*

Muon bremsstrahlung plays an important role in investigations of muon interactions and in calculations of muon transport in matter. To the first approximation, the cross section of the process may be obtained from the well-known Bethe-Heitler formulae for the electron bremsstrahlung [1,2] by a simple substitution of the electron mass by the muon one. Necessary for muon, account for the finite nuclear size was correctly performed in [3]; a simple approximate formula describing with a good accuracy the influence of both atomic and nuclear elastic formfactors was also obtained in the same paper. For almost 30 years since then, no serious grounds appeared to call the validity of this formula in question, and it was widely used for calculations of both interaction spectra and muon energy loss (e.g., [4]).

However, the increasing accuracy of measurements, transition to higher muon energies and greater depths make higher demands of the accuracy of calculations of muon electromagnetic interaction processes, the key position among which is occupied by the bremsstrahlung. In this connection, the attempts of various authors to obtain a more accurate bremsstrahlung cross section by means of a revision of some corrections calculated earlier or by introducing new corrections which had not been included previously because of their small contribution are quite understandable [5-8]. Therefore, it is expedient to review in brief recent results in order to fix well established positions and to separate problems which require the further investigation.

Historically, the cross sections for electron and muon bremsstrahlung were calculated first for the Coulomb center (heavy, point-like electrically charged target), and later various corrections to it were introduced. In a general case the cross section may be written in the form

$$\sigma = \sigma_0 - \Delta\sigma_a^{el} - \Delta\sigma_n^{el} + \sigma_a^{in} + \sigma_n^{in} + \dots , \qquad (1)$$

where σ_0 is the cross section for the Coulomb center; $\Delta\sigma_{a,n}^{el}$ - corrections accounting for alteration of the Coulomb field by the atomic electrons and nuclear charge distribution; $\sigma_{a,n}^{in}$ -contributions of additional processes, in which the bremsstrahlung is accompanied by the changing of electron and nuclear structure of the atom in the final state.

In a similar way, other corrections (for example, radiative ones, correction to the Born approximation, etc.) may be represented.

Completely differential cross section for the bremsstrahlung in the field of the Coulomb center depends on five independent variables. However, from the practical point of view, the most important is the cross section differential in the energy ω of the emitted photon (or its fraction of the total primary particle energy $v = \omega/E$). For this cross section, a different representation of Eq.1 is frequently used:

$$\sigma(E,v) = \frac{\alpha}{v}\left(2Z\frac{m}{\mu}r_e\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\times$$
$$\times\left[\Phi_0(\delta)-\Delta_a^{el}-\Delta_n^{el}+\frac{1}{Z}\left(f_a^{in}+f_n^{in}\right)\right], \qquad (2)$$

where the corrections are introduced to the main logarithmic factor

$$\Phi_0(\delta) = ln(\mu/\delta)-1/2, \qquad (3)$$

which describes the dependence of the bremsstrahlung cross section for the Coulomb center on the minimal momentum transfer δ.

Introduction of the corrections $\Delta\sigma_{a,n}^{el}$ in this manner seems to be quite justified since they are related with a modification of the electric field at distances of the order of atomic and nuclear size. The situation with the corrections $f_{a,n}^{in}$ is however different, since they correspond to some additional processes (described by their own diagrams), and the results are not obligatory similar to those for the bremsstrahlung on the Coulomb center. Therefore, the use of the parameterization in the form of Eq.1 seems to be more general.

2. *BREMSSTRAHLUNG ON THE COULOMB CENTER*

To take into account the influence of the atomic and nuclear formfactors, the cross section differential in the momentum transfer to the target is necessary. Unfortunately, it is difficult to obtain relatively simple formulae without some approximations. The most widely used is the approximation of ultrarelativistic projectile $(E,E' >> \mu)$. In this case, in accordance with Bether-Heitler results [1,2] the cross section may be written as

$$\sigma(E,v) = \frac{\alpha}{v}\left(2Z\frac{m}{\mu}r_e\right)^2\left[\left(2-2v+v^2\right)\Phi_1(\delta)-\frac{2}{3}(1-v)\Phi_2(\delta)\right], \quad (4)$$

where μ is the mass of the incident particle (muon), m is the electron mass, and

$$\Phi_{1,2}(\delta) = \int_{\delta}^{q_{max}} \psi_{1,2}(q,\delta)dq/q^3 . \quad (5)$$

Integration limits here are determined by the minimal and maximum momenta transferred to the target. For the Coulomb center (neglecting recoil energy)

$$q_{min} = p - p' - \omega ; \qquad q_{max} = p + p' + \omega . \quad (6)$$

For ultrarelativistic incident particle $(E, E' \approx p, p' >> \mu)$

$$q_{min} = \delta \simeq \frac{\mu^2 \omega}{2E(E-\omega)} ; \qquad q_{max} \simeq 2E . \quad (7)$$

In most cases, the infinite upper limit may be used in Eq.5.

In paper [2], simple analytical expressions for $\psi_{1,2}$ in overlapping (for $\delta <<< \mu$) regions $\delta \le q << \mu$ and $\delta << q \sim \mu$ were obtained. These expressions allow to demonstrate the main features of the cross section behaviour, and also to take into account the influence of atomic and nuclear formfactors. In the range $\delta << q << \mu$ the functions $\psi_{1,2}$ coincide and are equal to q^2 (see Fig.1); these functions do not strongly differ from each other at $q \sim \delta$ and $q \sim \mu$. Integrals (Eq.5) of these functions (the area under the corresponding curves in Fig.1) coincide and equal to $\Phi_o(\delta)$ defined by Eq.3.

Fig.1 illustrates also the combined influence of elastic atomic and nuclear formfactors (solid curve in the figure). For a heavy projectile

$(\mu \geq 1/R_n)$ and the most important high energy limit $(\delta << 1/R_a)$, the formfactors completely cut the regions where ψ_1 and ψ_2 are different. By this reason, the difference between these functions may be often neglected, and a single function (e.g., ψ_1) may be used.

Unfortunately, Bethe formulae [2] do not describe the $\psi_{1,2}$ dependence for $q_{min} \sim \mu$ (maximum value for the bremsstrahlung in the field of a heavy target). Therefore the use of these formulae for arbitrary relations between particle energies (E, E', ω) is not appropriate. Several attempts have been made to obtain analytical formulae for $\psi_{1,2}(q,\delta)$ in a whole range of q [3,9-10]. In order to estimate the accuracy of various parameterizations, here we use the accurate form of the bremsstahlung cross section $\sigma(E,\omega,q)$ for the Coulomb center in the Born approximation (see the Appendix). Integration over q was performed numerically. As the test of calculation procedure, we have used the comparison of the results of integration with formula (15) of [1] for $\sigma(E,\omega)$ valid for $\omega \leq E-\mu$ (without ultrarelativistic approximation); calculation results appeared identical. Therefore, most of the numerical results presented below have been obtained with the formulae given in the Appendix. In some cases, however, the approximate expressions have been used; the validity of these approximations has been checked by the comparison with the accurate cross section in the corresponding region of (q,δ).

3. *ELASTIC FORMFACTORS*

Influence of the elastic atomic and nuclear formfactors can be taken into account by means of the modification of Eq.5 in a following way:

$$\Phi_{1,2}(\delta) = \int_{\delta}^{q_{max}} \left[F_n(q) - F_a(q)\right]^2 \psi_{1,2}(q,\delta)\, dq/q^3 . \qquad (8)$$

Here normalisation of the formfactors is tne following: $F_a^{el}(0) = F_n^{el}(0) = 1$. The atomic formfactor limits the logarithmic increase of the cross section at $\delta \to 0$ and determines the value of the radiation logarithm, whereas the nuclear formfactor decreases the contribution of the region $q \gtrsim 1/R_n$. The influence of the atomic formfactor does not depend on the type of radiating particle (muon or electron). The nuclear formfactor is essential only for muon bremsstrahlung.

Since the formfactors are important in absolutely different regions ($q \sim 1/R_a$ and $q \sim 1/R_n$), their influence may be considered independently, and corresponding corrections may be calculated separately (indices 1,2 in functions ψ, Φ, Δ are omitted):

$$\Delta_a^{el} = \Phi_0 - \Phi_a = \int_\delta^{q_{max}} \left[1 - (1 - F_a)^2\right] \psi \, dq/q^3 , \tag{9}$$

$$\Delta_n^{el} = \Phi_0 - \Phi_n = \int_\delta^{q_{max}} \left[1 - F_n^2\right] \psi \, dq/q^3 \tag{10}$$

3.1. *Atomic formfactor (screening)*

Account for the atomic formfactor includes calculations of the constant B determining the value of the radiation logarithm, and of the functional dependence describing the transition from the limit of absence of screening $(\delta >> 1/R_a)$ to complete screening $(\delta << 1/R_a)$. The radiation logarithm is usually defined as

$$L_{rad} = \Phi_1(\delta = 0) = \int_0^{\mu} [1 - F_a(q)]^2 \, dq/q + 1 . \tag{11}$$

In the Thomas-Fermi model, which is usually applied for screening effect calculations,

$$L_{rad} = ln\left(B \frac{\mu}{m} Z^{-1/3}\right) . \tag{12}$$

The first solution of the problem within the frames of the Thomas-Fermi model of the atom was obtained by Bethe and Heitler [1] who calculated the value B=183 and tabulated $\Phi_{1,2}(\delta)$. Later, calculations of radiation logarithm were repeated with different approximate or numeric solutions $\varphi(x)$ of Thomas-Fermi equation (see Table 1).

TABLE 1. Constant B in the radiation logarithm for the Thomas-Fermi model.

Ref.	***B***	***Data used in calculations***
[1]	183	Tabulated $F_a(q)$
[6]	184.15	Molier approximation for $\varphi(x)$
[11]	189	Tietz approximation for $\varphi(x)$
[12]	191	Tabulated $F_a(q)$
present	182.7	Numerical solution for $\varphi(x)$

The variation of B from 183 to 191 results in a change of radiation logarithm about 1% for electron and only 0.5% for muon bremsstrahlung (due to higher absolute value of L_{rad} for muons, note μ/m ratio in Eq.12). The Thomas-Fermi model cannot serve a good approximation for a hydrogen atom (Z=1). However, in this case value B=202.4 may be easily found analytically [6].

Behaviour of the function Φ_1 in the intermediate region is well described by the parameterization [3]

$$\Phi_a(\delta) = ln \frac{B \frac{\mu}{m} Z^{-1/3}}{1 + \delta\sqrt{e} BZ^{-1/3}/m}, \tag{13}$$

where e = 2.718; the corresponding screening correction

$$\Delta_a^{el} = ln\left(1 + \frac{1}{\delta\sqrt{e}BZ^{-1/3}/m}\right). \tag{14}$$

Remarkably, the correction does not depend evidently on the mass of the projectile (electron mass in Eq.14 serves as the scale of the atomic radius). Comparison of Eq.14 with accurate calculations involving numeric results for the Thomas-Fermi formfactor shows that the error of this parameterization does not exceed 1% of the muon bremsstrahlung cross section for any degree of screening.

3.2. Nuclear formfactor

As we have point above, the influence of the nuclear size is negligibly small for electrons and always important for muon bremsstrahlung. Qualitatively, the dependence of the correction on the radius of the nucleus was estimated by Christy and Kusaka [13]; however, the first correct calculations of the correction were performed in [3]. Several later papers [5-8] cast doubts on these results, therefore we will consider the problem in more detail.

As one can see from Fig.1, the influence of the nuclear formfactor reduces the contribution of the region $q \gtrsim 1/R_n$. For muons, this reduction is essential for any δ, and correspondingly for any degree of screening. For small $\delta <<< \mu$, when functions $\psi_{1,2}$ do not depend on δ in the region $q \sim \mu$, the value of the correction does not depend on δ at all.

The value of the nuclear size correction is determined by Eq.10 and depends on the nuclear formfactor used in the calculations. Since in [3] some analytical approximation was used for the electric charge distribution in nuclei (approximate dependence of the half-density radius on Z), we have performed new calculations using the experimentally measured parameters of the formfactors for specific nuclei and the accurate formula for the cross section $\sigma(E,\omega,q)$. Exponential charge distribution for proton, Gaussian one for helium, and Fermi density distribution for heavier nuclei with the parameters given in [14] have been used.

Results of the present calculations of the correction Δ_n^{el} to the function Φ_1 are presented by dark squares in Fig.2. Various parameterizations of the correction often quoted in the literature are given by the curves. Parameterization [3] in the form $ln\left(1.5Z^{1/3}\right)$ well describes new results for medium and heavy nuclei, for light elements the agreement being somewhat worse (it could be clearly seen also from Fig.1 in [3]). The advantage of the approximation [3] was the possibility to express the main logarithmic factor (including both atomic and nuclear elastic formfactor corrections), in a very compact form. Present calculations may be approximated as $ln\left(1.75Z^{0.3}\right)$, or alternately, bearing in mind that nuclear radius depends mostly on the mass number A, as

$$\Delta_n^{el} = ln\left(1.54A^{0.27}\right). \tag{15}$$

Both parameterization [3] and Eq.15 give quite reasonable value of the correction to Φ_1 for hydrogen (the corresponding numerical result is $\Delta_n = 0.396$).

The use of other results for the nuclear size correction [8,13,15,16] presented in Fig.2 leads to the error in muon bremsstruhlung cross section about 10 - 15%. As it was pointed out in [3], a typical mistake in considerations of the nuclear formfactor influence is the replacement of the upper integration limit in Eq.10 by the muon mass. The source of this mistake lies in the formal use of Bethe formulae [2] for the atomic screening effect. Really, if we take the final Bethe result for $\Phi_1(\delta)$ for the screened point-like nucleus (formula (50) in [2], which later passed into some review papers)

$$\Phi_a(\delta) = \int_\delta^\mu (q-\delta)^2 \left[1 - F_a\right]^2 dq/q^3 + 1, \tag{16}$$

and substitute the square bracket with F_n^2 (which seems reasonable - see Eq.8 - but which cannot be done in Eq.16), we will get very simple (but twice erroneous - compare Eq.10) formula for the nuclear size correction:

$$\Delta_n^{el} = \Phi_o - \Phi_n = \int_\delta^\mu (q-\delta)^2 \left[1 - F_n^2\right] dq/q^3 \ (incorrect!). \tag{17}$$

The first error (less important) is the use for $\psi_l(q,\delta)$ of the expression which is valid only for $q << \mu$. The second error (decisive) is that the contribution of the interval $\mu < q < q_{max}$, where the influence of the nuclear size is the most important, completely drops out (in derivation of Eq.16, Bethe integrated the region of $q \sim \mu$ analytically for the Coulomb center; an attentive reader may find all necessary information on preceding pages of [2]).

For the comparison, we calculated Δ_n^{el} also with the same formfactors but using the incorrect Eq.17 (open circles in Fig.2). Results are in perfect agreement with the parameterization given in [8]. Though the authors do not publish details of the calculation procedure, the observed coincidence confirms indirectly the presence of the mistake discussed here in their consideration.

The next step in the refinement of the nuclear size correction is related with the introduction of its dependence on δ, which is not negligible for $\delta \sim \mu$. Fig.3 presents results of calculations of $\Delta_n^{el}(\delta)$ for carbon nucleus in the region of large δ. A simple formula may be used to approximate this dependence:

$$\Delta_n^{el}(\delta) = ln\frac{D_n}{1+\delta\left(D_n\sqrt{e}-2\right)/\mu}; \qquad D_n = 1.54A^{0.27}; \qquad (18)$$

δ is given by Eq.7. This formula not only describes well the results of accurate calculations for any δ, but also provides "zeroing" of the cross section near the kinematic limit $\omega = E - \mu$.

Let us summarise formulae for the cross section of muon bremsstrahlung on the screened nucleus, which is not accompanied by the changes of the target in a final state ("elastic" bremsstrahlung):

$$\sigma(E,v) = \frac{\alpha}{v}\left(2Z\frac{m}{\mu}r_e\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\Phi(\delta); \qquad (19)$$

$$\Phi(\delta) = ln\frac{B\frac{\mu}{m}Z^{-1/3}}{1+\delta\sqrt{e}BZ^{-1/3}/m} - \Delta_n^{el}(\delta); \qquad (20)$$

$\Delta_n^{el}(\delta)$ and δ are defined by Eq.18 and Eq.7, respectively; B =183; e =2.718. In this parameterization, we use a single function $\Phi(\delta) \simeq \Phi_1(\delta)$. For muon, $\Phi_1 = \Phi_2$ in the most important case of complete screening; in a non-screening limit, $\Phi_2 = \Phi_1 + 1/6$ (because of the finite nuclear size, these relations are different from those for Coulomb center and for electron bremsstrahlung). An overall check of Eqs.19, 20 (by means of the comparison with the numerical integration based on the accurate cross section and accurate formfactors) showed that the error of these formulae does not exceed about 1% for $E > 100$ GeV.

4. *INELASTIC FORMFACTORS*

While the influence of elastic formactors (atomic and nuclear) leads to some corrections to the cross section for the Coulomb center, the inelastic formfactor serve to describe additional processes which cannot be distinguished from the bremsstrahlung on a screened nucleus. Among these processes, the most important is bremsstrahlung on the electrons.

4.1. *Bremsstrahlung on atomic electrons*

For free electron, the process is described by 4 diagrams (Fig.4). The important feature of muon bremsstrahlung on the electron is that the target recoil cannot be neglected. In particular (unlike the bremsstrahlung on a heavy target) muon cannot transfer all its kinetic energy to the photon

$$\omega_{max} = \frac{m(E-\mu)}{E-p+m} \simeq \frac{E}{1+\mu^2/2mE}. \tag{21}$$

Notably, maximum photon energy (for ultrarelativistics muon) is close to maximum energy of the knock-on electron (in elastic $\mu - e$ scattering), and to maximum total energy loss $(E-E')_{max}$ of muon in a collision with an electron.

The cross section for muon bremsstrahlung on a free resting electron has been derived by Galitsky and Kelner [17]. Their results show that the behaviour of the cross section is rather different for the upper diagrams in Fig.4 (μ - diagrams, photon is emitted by the projectile) and the lower diagrams (e - diagrams, photon is emitted by the target electron). While the cross section for μ - diagrams exhibits a usual for bremsstrahlung dependence on the photon energy ($1/\omega$, see Fig.5), the e - diagrams give the cross section proportional to $1/\omega^2$ (similar to elastic $\mu - e$ scattering). By this reason, it is expedient to consider the contribution of e - diagrams as an addition to the cross section of the knock-on electron production (Sect. 6.1), while considering μ - diagrams as a correction to muon bremsstrahlung on nucleus. In this case, the neglection of target recoil (heavy target approximation) may be justified only in the extreme limits of very low momentum transfer $(q << m)$ or very high muon energies $E, E' >> \mu^2/2m$ (see Fig.6).

Since, in fact, muon bremsstrahlung occurs not on free electrons but on electrons bound in the atom, the inelastic atomic formfactor has to be taken into account. The required for the purpose differential cross section $\sigma(E, \omega, q)$ has not been given in [17] in the explicit form. However, bearing in mind that F_a^{in} differs from the unit only for $q << m$, the cross section of the bremsstrahlung on the electrons of the atom may be calculated as

$$\sigma_a^{in}(E, \omega) = Z\left(\sigma_{free}(E, \omega) - \int_{\delta}^{q_{max}} \left[1 - F_a^{in}(q)\right] \sigma_0(E, \omega, q) dq\right), \tag{22}$$

where σ_{free} is the cross section for free electron target (μ-diagrams, [17]), and $\sigma_0(E, \omega, q)$ is the differential cross section for the Coulomb center. The inelastic formfactor has been calculated for Thomas-Fermi model following the procedure described in [6] (formula (B28) of that paper). Results of calculations of the constant B' in the "inelastic" radiation logarithm

$$L'_{rad} = \int_0^{\mu} F_a^{in}\, dq/q + 1 = ln\left(B' \frac{\mu}{m} Z^{-2/3}\right) \tag{23}$$

are given in Table 2. For hydrogen atom (Z=1, exact solution) $B' = 446$ [6].

TABLE 2. Constant in "inelastic" radiation logarithm.

Ref.	***B'***	***Data used in calculated***
[18]	1440	Tabulated $\varphi(x)$
[6]	1274	Tabulated $F_a^{in}(q)$
[6]	1194	Moliere appr. for $\varphi(x)$
present	1429	Numerical solution for $\varphi(x)$

Usually, contribution of the bremsstrahlung on atomic electrons is taken into account by a simple substitution of Z^2 in the cross section on a screened nucleus with $Z(Z+\zeta)$, where $\zeta \approx 1$. In Fig.7, values of ζ calculated for various E and v are presented. One can see that ζ approaches a constant only at extremely high muon energies. Fig.8 gives energy dependence of the effective value of the parameter ζ for energy loss estimation (weighted with the bremsstrahlung spectrum). At very high energies, the parameter ζ approaches a limit

$$\zeta = \frac{ln\left(B' \frac{\mu}{m} Z^{-2/3}\right)}{ln\left(B \frac{\mu}{m} Z^{-1/3}\right) - \Delta_n^{el}}, \qquad (24)$$

Δ_n^{el} is given by Eq.15. Asymptotic values for hydrogen, standard rock and lead are 1.12, 1.30 and 1.35, respectively.

Bearing in mind that the bremsstrahlung on atomic electrons is only an addition to the bremsstrahlung on nucleus (about 10% for rock and less for heavier substances), for the practical purposes a simple approximate formula may be used for this process (μ - diagrams):

$$\sigma_a^{in}(E,v) \simeq \frac{\alpha Z}{v}\left(2\frac{m}{\mu}r_e\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\widetilde{\Phi}_a^{in}(\delta), \qquad (25)$$

$$\widetilde{\Phi}_a^{in}(\delta) = ln\frac{\mu/\delta}{\delta\mu/m^2+\sqrt{e}} - ln\left(1+\frac{1}{\delta\sqrt{e}B'Z^{-2/3}/m}\right). \qquad (26)$$

Fig.9 gives the comparison of this approximation (dots) with accurate calculations (Eq.22, solid curves in the figure). Note also a similarity of the binding correction (last term in Eq.26) to the screening correction for the elastic formfactor (Eq.14).

4.2. *Influence of the nucleus excitation*

The problem has been considered in [9,10]. Inelastic nuclear processes have been separated by the authors in two categories related respectively with the excitation of the nucleus, and with deep inelastic excitation of separate nucleons.

In the first case, excitation of the nucleus which is described by the inelastic nuclear formfactor is taken into account. Within the frames of the model adopted in [9,10] (the wave function of the nucleus is represented as a non-antisymmetrized product of the wave functions of individual protons), the inelastic formfactor is

$$F_n^{in} = 1-\left(F_n^{el}\right)^2, \qquad (27)$$

and the resulting (positive) correction to the main logarithm may be immediately written (see Eq.10) as

$$\Delta_n^{in} = \frac{1}{Z}\Delta_n^{el}; \quad (Z \neq 1). \qquad (28)$$

For the rock, additional contribution of this process is of the order of 10% of the correction for elastic formfactor (or about 1% of the total cross section), and is decreasing with the increase of Z. Note also, that the value of the correction Δ_n^{in} is of the same order of magnitude as the difference of Δ_n^{el} corrections for different formfactor parameterizations.

Obviously, correction for hydrogen equals to 0, since in this model the proton structure is not taken into account.

Quantitatively, the approximation given in [10] for the $\left(\Delta_n^{el} - \Delta_n^{in}\right) \approx ln\left(1.43Z^{1/3}\right)$ for $Z \geq 10$ practically coincides with the difference between the corrections given by Eq.15 and Eq.28.

Contribution of deep inelastic excitation of nucleons to muon bremsstrahlung has also been considered in [9,10], the corresponding correction being estimated as 0.5% at E = 100 GeV (for silicon), logarithmically increasing with muon energy. A serious problem of separation of different processes arises however in this case: for example, emission of 20 GeV photon by 100 GeV muon may be accompanied by the target excitation energy up to 80 GeV (see Fig.3 in[10]). Probably, it would be more appropriate to consider this process as a correction not to bremsstrahlung, but to deep inelastic scattering (simultaneously with all essential corrections of the order of α^3).

5. OTHER CORRECTIONS

5.1. Correction to the Born approximation (Coulomb correction)

For ultrarelativistic projectile, consideration of this correction to the cross section of the electron bremsstrahlung reduces to a subtraction of a universal function $f_c(\alpha Z)$ from the main logarithm

$$\Phi(\delta) = \Phi_0(\delta) - f_c(\alpha Z)\,; \quad f_c(y) = y^2 \sum_{n=1}^{\infty} \frac{1}{n\left(n^2 + y^2\right)}\,. \qquad (29)$$

For complete screening, the relative value of this correction to electron bremsstrahlung on lead is about 9%.

For muons (because of large value of Φ) correction is less, but still large for high Z (about 4.5% for lead). For iron (Z=26), correction to the Born approximation does not exceed 0.5% and is less for light nuclei.

It should be noted however, that staightfoward use of Eq.29 for muons is unlikely appropriate, since the electric field (espesially for heavy nuclei) is appreciably modified at distances of the order $1/\mu$ by the finite size of the nucleus. Strictly speaking, for a heavy projectile it is necessary to consider the Coulomb correction and nuclear size correction simultaneously.

5.2. *Radiative corrections*

Calculation of radiative corrections to the bremsstrahlung, which describe the processes with contribution of the order of α^4, is rather laborious problem being still not solved in a full volume. Available results of correction calculations deal only with the case of low energy of one of the emitted photons.

5.3. *Medium influence*

Two effects limiting the applicability of formulae derived for an isolated atom to the bremsstrahlung in a medium are known. First of them - multiple scattering of radiating particle (LPM effect) - puts the limitation from the side of high particle energies, and the second (medium polarization, Ter-Mikaelyan) at low relative energies of emitted photons. The curves in Fig.10 illustrate these limitations for electrons and muons in lead and standard rock. In the area above these curves, the usual formulae may be used. Bearing in mind that the most practically important ragion for the bremsstrahlung (contributing to interaction spectra and total energy loss) corresponds to $v \gtrsim 10^{-1}$, for muons the influence of these effects may be neglected at least up to 10^{20} eV.

6. *AVERAGE ENERGY LOSS FOR THE BREMSSTRAHLUNG*

Taking into account the corrections, considered in the present paper, we calculated the coefficient $b_\gamma(E)$ in the muon energy loss relation

$$b_\gamma = (N_0/A)\int_0^1 v\sigma(E,v)dv. \tag{30}$$

(Here N_0 is the Avogadro number). Results are presented in Fig.11 for 4 different substances. In Fig.12, the ratios of $b_\gamma(E)$ to the values calculated without newly introduced corrections are given (namely, $Z(Z+1)$; B = 189; $\Delta_n = ln(1.5Z^{1/3})$ were adopted in the "old" version). As it could be expected, the most appreciable changes (related with muon bremsstrahlung on atomic electrons) appear for light substances.

Since the problem with Coulomb correction for muon bremsstrahlung has not been settled yet, we have not introduce this correction in calculations for lead (for other materials presented in Fig.11 this correction is small).

6.1. Correction to ionisation energy loss.

As we have pointed above (Sect.4), contribution of e-diagrams for muon bremsstrahlung on electron target gives the inverse square dependence of the cross section on the photon energy and, correspondingly, logarithmic dependence of the average energy loss. Therefore, it seems expedient to consider this process together with the energy loss for knock - on electron production.

In these considerations, it is necessary to take into account that photon emission for e-diagram is accompanied by the production of an energetic recoil electron. By this reason, to estimate muon energy loss correctly, the cross section differential in total muon energy loss in the interaction $\varepsilon = E - E' = \omega + T$ (where T is the kinetic energy of the electron in the final state) is needed. Straightforward calculation of this cross section leads to infrared divergency, which is compensated however (as it should be) if the cross section is calculated simultaneously with the same order (α^3) radiative corrections to elastic $\mu - e$ scattering. Keeping in the final results only highest powers of large logarithms, one can get for the combined contribution of the muon bremsstrahlung (e-diagrams) and α^3-radiative corrections to the knock-on electron production [19]:

$$\Delta\sigma(E,\varepsilon) \simeq \frac{\alpha}{\pi}\Big[2ln\frac{2\varepsilon}{m}\cdot ln\frac{2\varepsilon(1-\varepsilon/\varepsilon_m)}{m(1-\varepsilon/E)} - \frac{1}{2}ln^2\frac{2\varepsilon(1-\varepsilon/\varepsilon_m)}{m(1-\varepsilon/E)} - \frac{1}{2}ln^2\frac{2\varepsilon}{m} + \frac{1}{2}ln\frac{2\varepsilon}{m}\cdot ln\frac{2E(E-\varepsilon)m}{\mu^2\varepsilon}\Big]\sigma_o(E,\varepsilon), \qquad (31)$$

where $\sigma_o(E,\varepsilon)$ is Bhabha cross section for elastic $\mu - e$ scattering:

$$\sigma_o(E,\varepsilon) = 2\pi m r_e^2 \left(1 - \varepsilon/\varepsilon_m + \varepsilon^2/2E^2\right)/\varepsilon^2 , \qquad (32)$$

and $\varepsilon_m = E/\left(1+\mu^2/2mE\right)$. Eq.31 is valid (with logarithmic accuracy) for $\varepsilon >> m$; at $\varepsilon \to 0$ the corresponding correction tends to 0. In Fig.13, this correction is compared with the cross section for usual knock-on electron production (Bhabha) and with the photon spectrum for *e*-diagram [17].

Integration of Eq.31 (multiplied by ε) gives the correction to the average muon energy loss rate (also with logarithmic accuracy):

$$\Delta\left|\frac{dE}{dx}\right| = \frac{NZ}{A} m \alpha r_e^2 \left(\ln\frac{2E}{\mu} + \frac{1}{3}\ln\frac{2\varepsilon_m}{m} \right) \ln^2\frac{2\varepsilon_m}{m} . \qquad (33)$$

This correction is represented in Fig.14 by the dotted curve. The main contribution (solid curve in the figure) is calculated following Bethe-Bloch-Sternheimer formula, the parameters for standard rock being taken according to [4]. An addition to the average energy loss for ionisation and energetic knock-on electron production amounts to about 3% at E = 10 GeV and 6 - 7% near 100 GeV muon energy.

7. *CONCLUSION*

Results of the present consideration show that Petrukhin-Shestakov formula [3] describes with the accuracy about 1% the high-energy muon bremsstrahlung on a screened nucleus, accounting for elastic atomic and nuclear formfactors. In calculations where large values of δ are important ($v \sim 1$; for example, spectra of bremsstrahlung photons generated by cosmic ray muons), a more accurate nuclear size correction given by Eq.18 may be recommended, the final expression for the cross section is given by Eqs. (18-20).

Corrections for inelastic atomic and nuclear processes, which had not been taken into account in parameterization [3], should be included as additional contributions. The substitution of Z^2 with $Z(Z+const)$ for muon bremsstrahlung on atomic electrons seems to be very rough approximation for light substances, and formulae accounting for electron recoil (Eq.22 or Eq.25) have to be used. Excitation of the nuclear target may be approximately included by means of the correction given by Eq.28; it contributes about 1% to the total cross section in rock.

Coulomb correction for muon bremsstrahlung on light and medium targets may be neglected. However, it may appear appreciable for $Z \geq 40-50$. An additional consideration accounting for modification of the Coulomb field at distances $\sim 1/\mu$ by the finite nuclear size is needed.

In accurate calculations of high-energy muon transport ($E \approx 10$ - 10000 GeV), an increase of the logarithmic term of the average muon energy loss arising from muon bremsstrahlung on the electrons (e-diagram) has to be taken into account. This contribution may appear also appreciable in calculations of spectra of high-energy muon interactions in the transferred energy range $m << \varepsilon \leq \mu^2/mZ$, especially when photon and knock-on electron are not distinguished in the experiment.

Appendix. ***ACCURATE FORMULA FOR THE DIFFERENTIAL CROSS SECTION FOR THE COULOMB CENTER.***

Integrating completely differential cross section for the bremsstrahlung on the Coulomb center (Born approximation, without ultrarelativism restriction), one can get the equation

$$d\sigma = \frac{\alpha r_\mu^2 Z^2}{2\pi} \cdot \frac{\mu^4}{pq^3} dq \frac{d^3 p'}{E'} \left\{ \left(\frac{1}{Q^{1/2}} - \frac{1}{Q'^{1/2}} \right) \times \right. \tag{A1}$$

$$\times \left[\frac{8EE'}{\xi} - \frac{2\xi}{\mu^2} + q^2 \left(\frac{2E^2 + 2E'^2 - q^2}{\mu^2 \xi} - \frac{2}{\xi} - \frac{2}{\mu^2} \right) \right] -$$

$$\left. -\left(4E'^2 - q^2\right) \frac{S}{Q^{3/2}} - \left(4E^2 - q^2\right) \frac{S'}{Q'^{3/2}} - \frac{4}{\mu^2 R^{1/2}} \right\}.$$

Here

$$Q = \xi^2 p'^2 - \xi q^2 \omega E' + \omega^2 q^2 \mu^2 + \omega^2 q^4/4 ,$$
$$Q' = \xi^2 p^2 + \xi q^2 \omega E + \omega^2 q^2 \mu^2 + \omega^2 q^4/4 ,$$
$$S = -\xi^2 + \xi\left(\omega E' - q^2/2\right) + q^2 E\omega - q^2 \omega^2/2 ,$$
$$S' = \xi^2 + \xi\left(\omega E + q^2/2\right) + q^2 E'\omega + q^2 \omega^2/2 ,$$
$$R = \omega^2 + q^2 + 2\xi ,$$

and $\xi = -\vec{q}\vec{k} = EE' - \vec{p}\vec{p}' \cos\theta - \mu^2 - q^2/2$; other notations are commonly accepted. Eq. A1 describes the distribution in q (momentum transferred to the target) and $\vec{p}'$ (momentum of the radiating particle in a final state). Using the relation $d^3 p'/E' = 2\pi d\omega d\xi/p$, one can perform elementary integration over angular variables, and get the cross section $\sigma(E,\omega,q)$ differential in photon energy ω and momentum transfer q. Integration limits are defined by

$$\xi_{min} = max\left(-\omega q,\ EE' - pp' - \mu^2 - q^2/2\right); \qquad \text{(A2)}$$
$$\xi_{max} = min\left(\omega q,\ EE' + pp' - \mu^2 - q^2/2\right);$$

and $p - p' - \omega \le q \le p + p' + \omega$.

We do not give here the corresponding formulae for the cross section because of their cumbersome form.

REFERENCES

1. H.A.Bethe, W.Heitler, Proc.Roy.Soc., **A146**, 83 (1934).
2. H.A.Bethe, Proc.Cambr.Phil.Soc., **30**, 524 (1934).
3. A.A.Petrukhin, V.V.Shestakov, Canad.J.Phys., **46**, S377 (1968).
4. W.Lohmann, R.Kopp, R.Voss, CERN Yellow Report 85-03 (1985).
5. E.V.Bugaev, Yu.D.Kotov, I.L.Rozental, "Cosmic muon and neutrinos", Atomizdat, Moscow, 1970 (in Russian).
6. Y.-S.Tsai, Rev.Mod.Phys., **46**, 815 (1974).
7. M.J.Tannenbaum, Preprint CERN PPE/91-134 (1991).
8. W.K.Sakumoto, e.a., Phys.Rev., **D45**, 3042 (1992).
9. Yu.M.Andreyev, E.V.Bugaev, Preprint INR P-0071, Moscow, 1978 (in Russian).
10. Yu.M.Andreyev, L.B.Bezrukov, E.V.Bugaev, Physics of Atomic Nuclei (Yad.Fiz.), **57**, 2066 (1994).
11. A.A.Petrukhin, V.V.Shestakov, in "Elementary particle physics", Atomizdat, Moscow, p.102, 1966 (in Russian).
12. O.I.Dovzhenko, A.A.Pomansky, Zh.E.T.F. (Sov.Phys.JETP), **45**, No.2(8), 268 (1963).
13. R.F.Christy, S.Kusaka, Phys.Rev., **59**, No.5, 405 (1941).
14. L.R.B.Elton, "Nuclear sizes", Oxford University Press, 1961.
15. I.L.Rozental, Usp.Fiz.Nauk (Sov.Phys.Uspekhi), **94**, 91 (1968).
16. A.D.Erlykin, Proc. 9th ICCR, London, **2**, 999 (1966).
17. V.M.Galitsky, S.R.Kelner, Zh.E.T.F. (Sov.Phys.JETP), **52**, 1427 (1967).
18. J.A.Wheeler, W.E.Lamb, Phys.Rev., **55**, 858 (1939).
19. S.R.Kelner, to be published.

FIGURE CAPTIONS

Fig.1. Functions describing the q-dependence of the cross section (Eq.5): ψ_1-dotted curve, ψ_2-dashed curve. Solid curve: the same multiplied by $\left[F_n^{el} - F_a^{el}\right]^2$, see Eq.8.

Fig.2. Correction for the elastic nuclear formfactor. Squares: present calculations (Eq.10). Circles: calculations with the incorrect formula (Eq.17). Diamond: calculation [16]. The curves correspond to different parameterizations of the corrections (p.w. - present work).

Fig.3. Dependence of Φ (upper solid curve) and nuclear size correction (lower solid curve) on δ. Dashed curve represents $\Phi(\delta)$ for the Coulomb center. Dots: approximation, Eq.18.

Fig.4. Diagrams describing muon bremsstrahlung on the electron.

Fig.5. Cross section for muon bremsstrahlung on free resting electron [17]. Solid curves: contribution of μ-diagrams; dotted curves: e-diagrams.

Fig.6. Comparison of muon bremsstrahlung cross section on the Coulomb center (solid curves) and on the electron (dashed curves, μ-diagrams).

Fig.7. Parameter ζ describing the contribution of muon bremsstrahlung on atomic electrons (see text) versus E, v. Standard rock (Z=11, A=22).

Fig.8. Effective values of ζ for estimation of average muon energy loss for the bremsstrahlung in various substances.

Fig.9. Cross section for muon bremsstrahlung on atomic electrons (μ-diagrams, Eq.22, solid curves) and the approximation (Eqs. 25-26, dots).

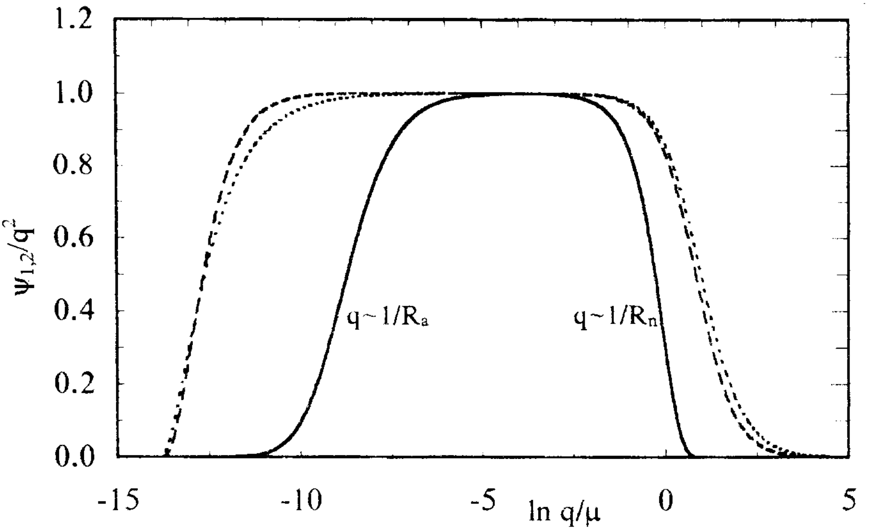
Fig.10. Regions of the validity of usual formulae for muon and electron bremsstrahlung in matter (limitations from LPM and Ter-Mikaelyan effects).

Fig.11. Coefficient $b_\gamma(E)$ in the average energy loss relation for muons (calculated with cross section given by Eqs. 18-20 and Eq.22 for atomic electron contribution).

Fig.12. Ratio of the present results for $b_\gamma(E)$ and the "old" version ([3] with $Z(Z+1)$ substitution, see f.c.[4]).

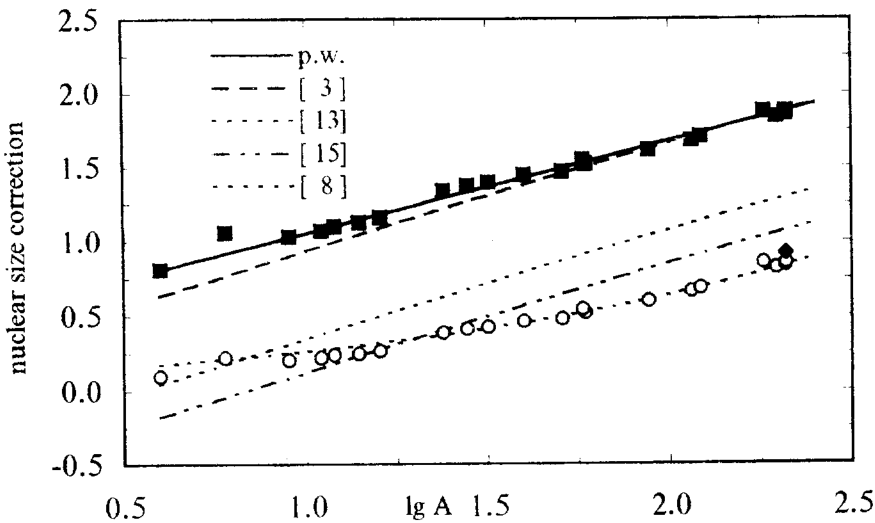
Fig.13. Distributions of energy lost by a muon in a collision with an electron. Solid curve: Bhabha cross section; dotted curve: contribution from e-diagrams for muon bremsstrahlung plus radiative corrections to elastic scattering (Eq.31); dashed curve: sum of two processes. Energy spectrum of photons (e-diagrams) is given for the comparison.

Fig.14. Contribution of muon bremsstrahlung on the electrons (e-diagrams) to muon energy loss (dots). Solid curve: Bethe-Bloch-Sternheimer formula. Dash-dotted curve: ionisation loss plus correction.



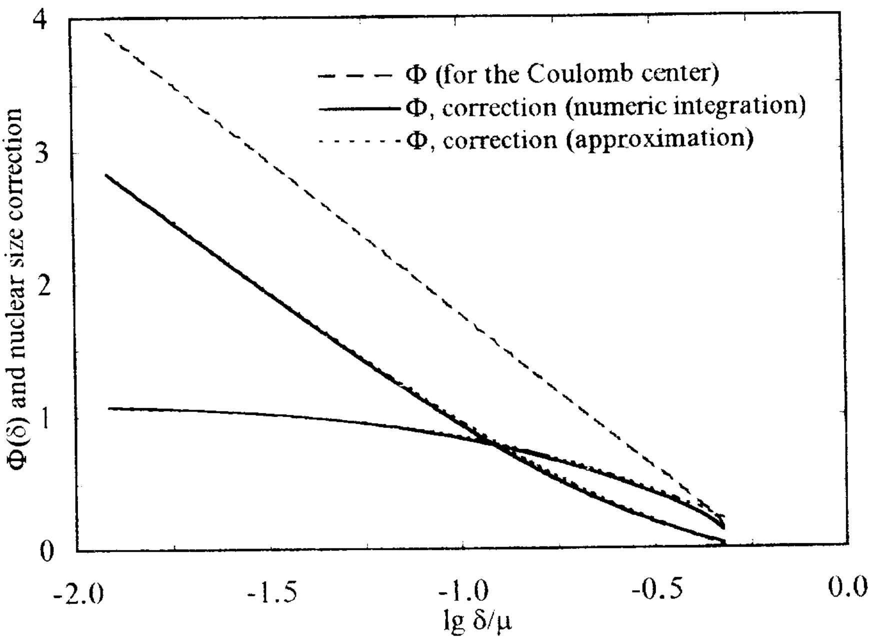
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\psi_{1,2}/q^2$
$q\sim1/R_a$
$q\sim1/R_n$
-15
-10
-5
ln q/μ
0
5

Fig.1



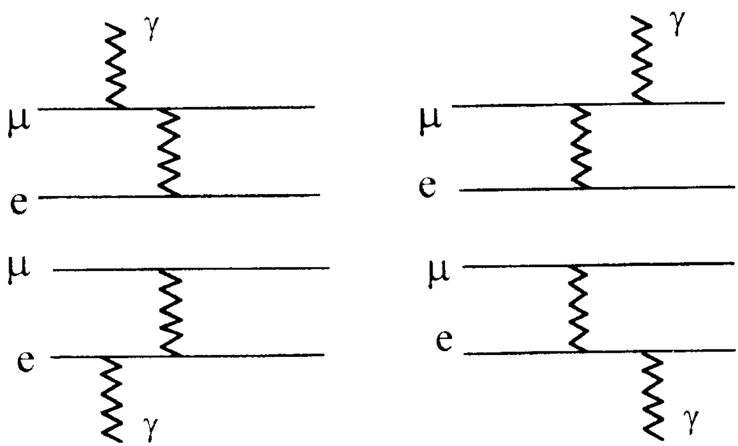
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
nuclear size correction
p.w.
[3]
[13]
[15]
[8]
0.5
1.0
lg A
1.5
2.0
2.5

Fig.2



Φ (for the Coulomb center)
Φ, correction (numeric integration)
Φ, correction (approximation)
Φ(δ) and nuclear size correction
lg δ/μ
4
3
2
1
0
-2.0
-1.5
-1.0
-0.5
0.0

Fig.3



γ
μ
e
γ
μ
e
μ
e
γ
μ
e
γ

Fig.4

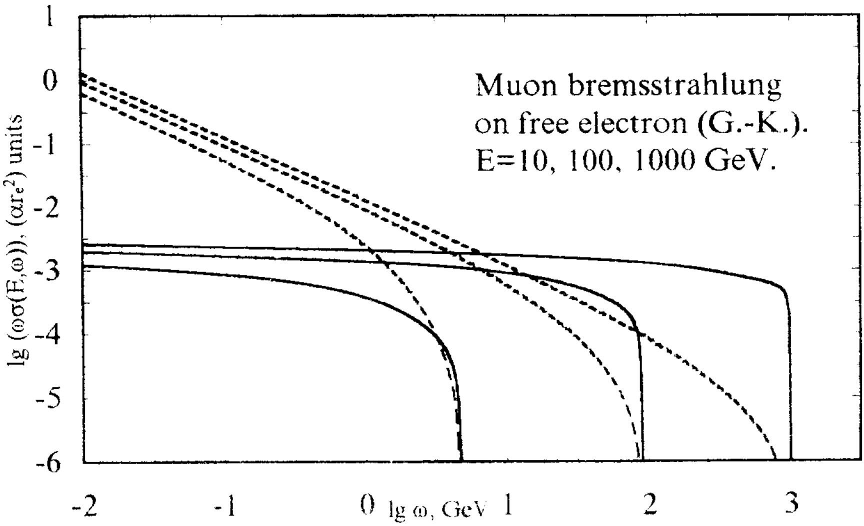

Muon bremsstrahlung
on free electron (G.-K.).
E=10, 100, 1000 GeV.
lg (ωσ(E,ω)), (αr$_e^2$) units
lg ω, GeV
1
0
-1
-2
-3
-4
-5
-6
-2
-1
0
1
2
3

Fig.5

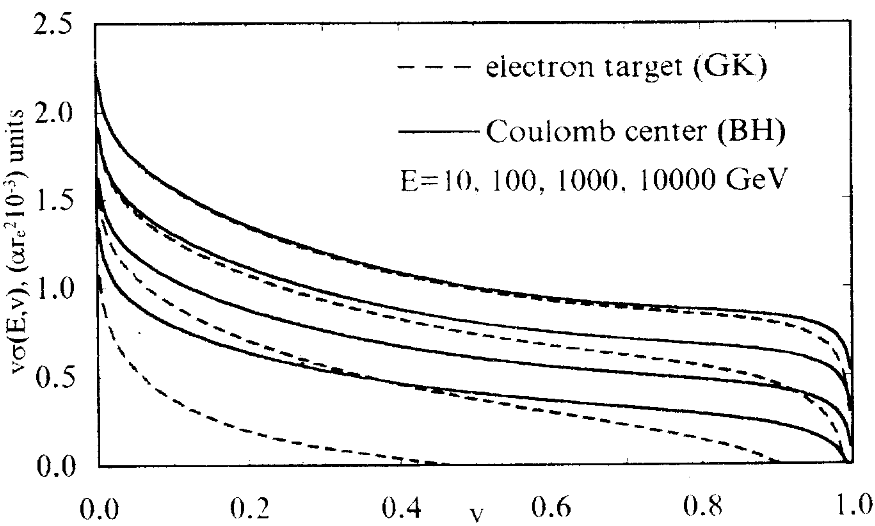
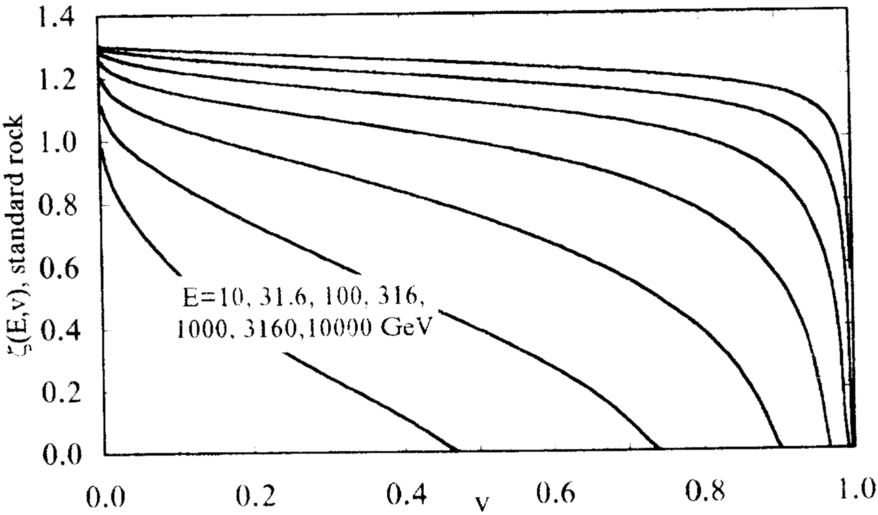
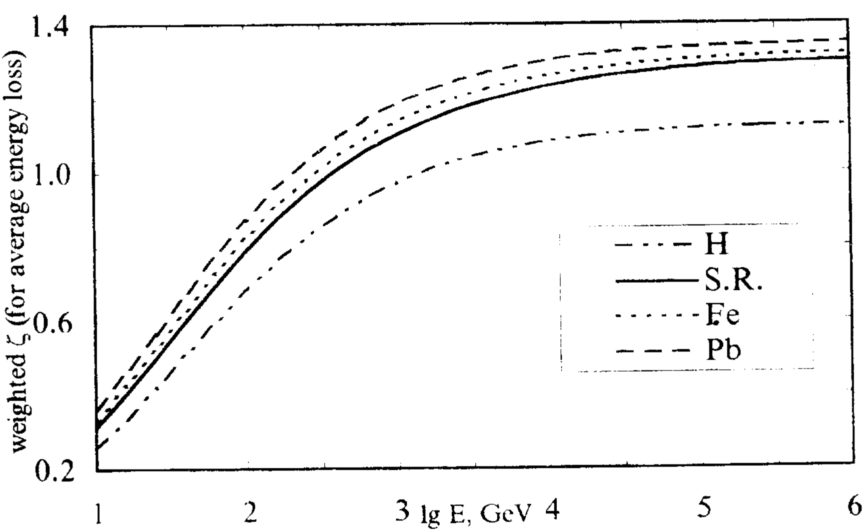

electron target (GK)
Coulomb center (BH)
E=10, 100, 1000, 10000 GeV
vσ(E,v), (αr$_e^2$10^{-3}) units
v
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Fig.6



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
ζ(E,v), standard rock
E=10, 31.6, 100, 316,
1000, 3160,10000 GeV
0.0
0.2
0.4
v
0.6
0.8
1.0

Fig.7



1.4
1.0
0.6
0.2
weighted ζ (for average energy loss)
H
S.R.
Fe
Pb
1
2
3 lg E, GeV 4
5
6

Fig.8

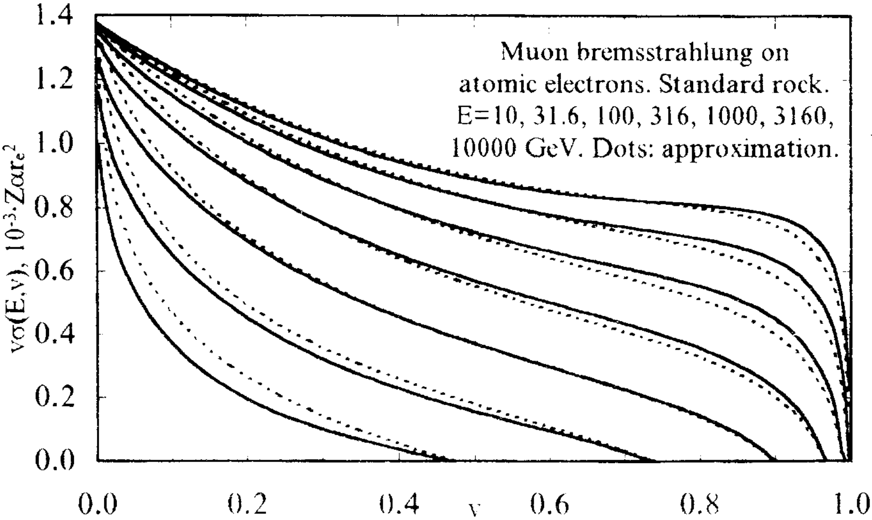

Muon bremsstrahlung on
atomic electrons. Standard rock.
E=10, 31.6, 100, 316, 1000, 3160,
10000 GeV. Dots: approximation.
vσ(E,v), 10^-3·Zαr_e^2
v
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Fig.9

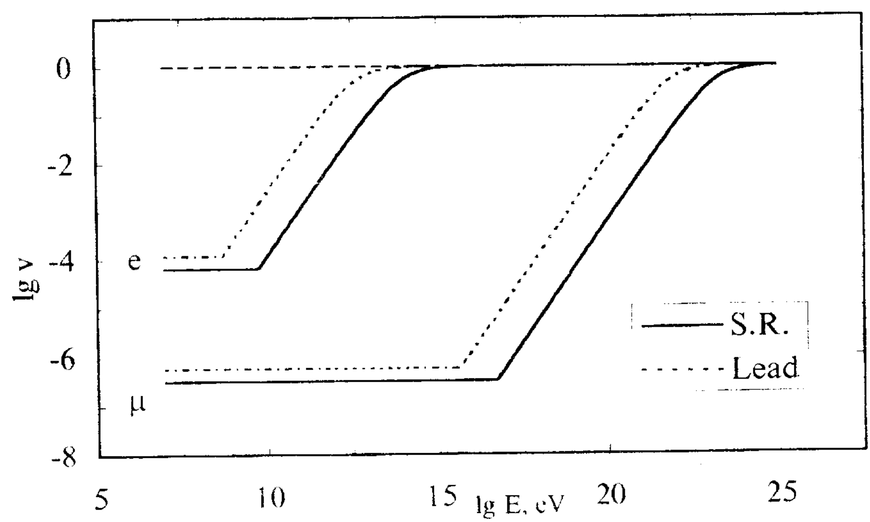
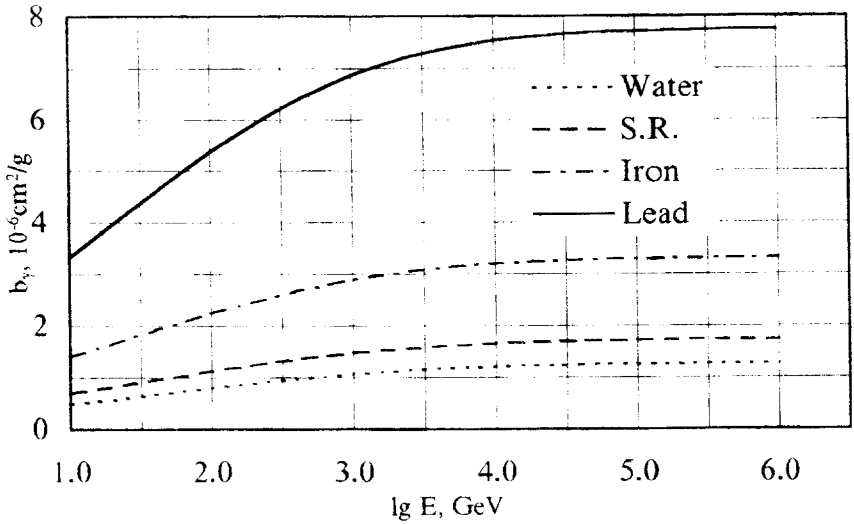
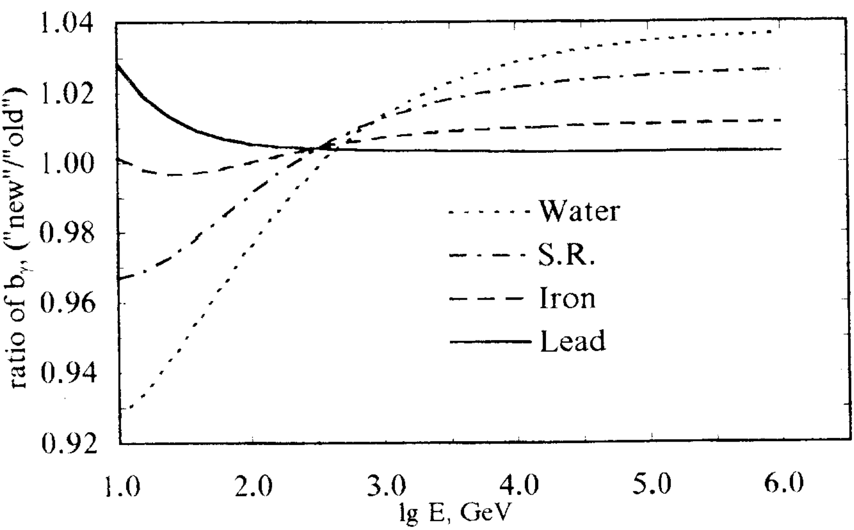

lg v
lg E, eV
e
μ
S.R.
Lead
0
-2
-4
-6
-8
5
10
15
20
25

Fig.10



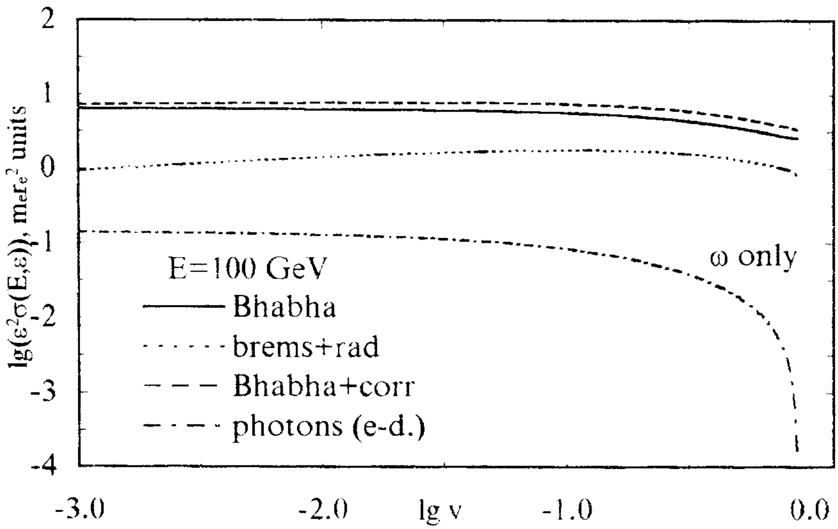
8
6
4
2
0
b$_\gamma$, 10^{-6}cm^2/g
Water
S.R.
Iron
Lead
1.0
2.0
3.0
4.0
5.0
6.0
lg E, GeV

Fig. 11



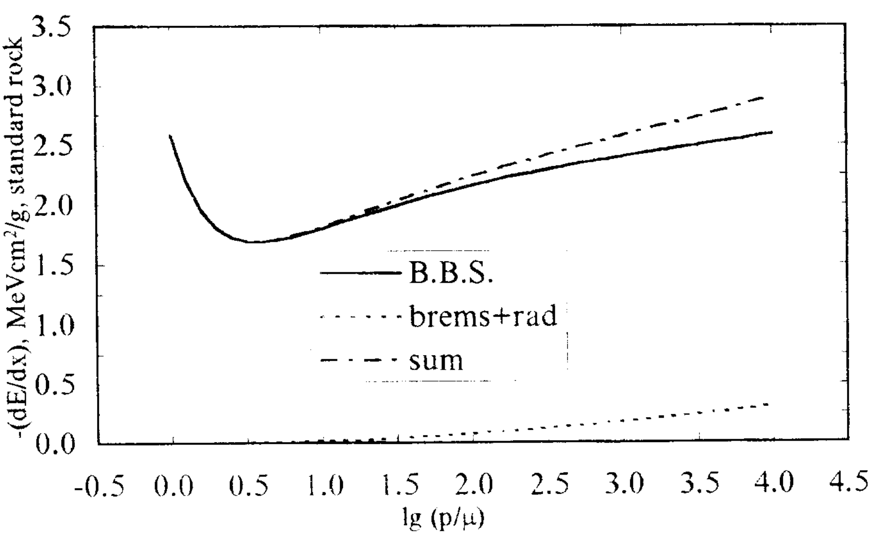
1.04
1.02
1.00
0.98
0.96
0.94
0.92
ratio of b$_\gamma$, ("new"/"old")
Water
S.R.
Iron
Lead
1.0
2.0
3.0
4.0
5.0
6.0
lg E, GeV

Fig. 12



2
1
0
-1
-2
-3
-4
lg(ε²σ(E,ε)), m_e r_e² units
E=100 GeV
Bhabha
brems+rad
Bhabha+corr
photons (e-d.)
ω only
-3.0
-2.0
lg v
-1.0
0.0

Fig.13



3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-(dE/dx), MeVcm²/g, standard rock
B.B.S.
brems+rad
sum
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
lg (p/μ)

Fig.14

Станислав Рихардович Кельнер
Ростислав Павлович Кокоулин
Анатолий Афанасьевич Петрухин

О СЕЧЕНИИ ТОРМОЗНОГО ИЗЛУЧЕНИЯ МЮОНОВ ВЫСОКИХ ЭНЕРГИЙ

Рукопись поступила в издательский отдел 23.08.95 г.

Редактор Е.Н.Кочубей

Ответственный за выпуск Р.П.Кокоулин

Лицензия ЛР № 020676 от 09.12.92 г.

Подписано в печать 24.08.95 Формат 60x84 1/16

Печ.л. 2,0 Уч.-изд.л. 2,0 Тираж 100 экз. Изд. № 024-95 Заказ 846

Московский государственный инженерно-физический институт (технический университет). Типография МИФИ.
115409, Москва, Каширское ш., 31