AU

BCCNT 95/082/250R1
Revised

Many-Body Theory of ρ-ω Mixing

CERN LIBRARIES, GENEVA
SCAN-9510040

Shun-fu Gao, C.M. Shakin, and Wei-Dong Sun

Department of Physics and Center for Nuclear Theory

Brooklyn College of the City University of New York

Brooklyn, New York 11210

SW 9541

(August, 1995)

Submitted to Physical Review C (Subnucleon Aspects of Nuclei/Physics of Hadrons)

Abstract

We calculate the tensor describing ρ-ω mixing making use of an extended Nambu–Jona-Lasinio (NJL) model that we have developed in recent years. We use the definition of the rho and omega fields that arises upon a momentum-space bosonization of the extended NJL model. A quantity of interest is the on-shell $(q^2 = m_\omega^2)$ matrix element that describes the coupling of the rho and omega fields, $<\rho\,|\,H_{SB}\,|\,\omega>$. That quantity has been determined to be $<\rho\,|\,H_{SB}\,|\,\omega> = -(4520 \pm 600)$ MeV^2 in a study of the two-pion decay of the omega meson. Our calculation of $<\rho\,|\,H_{SB}\,|\,\omega>$ is sensitive to the difference of the current quark masses, $m_d^0 - m_u^0$. Our analysis was first made for $m_d^0 - m_u^0 = 2.0$ MeV. However, our results may be put into agreement with the data, if we use $m_d^0 - m_u^0 = 2.7 \pm 0.3$ MeV. The momentum-space bosonization procedure naturally leads to momentum-dependent coupling constants, $g_{\omega qq}(q^2)$ and $g_{\rho qq}(q^2)$. The value of these constants increases by about a factor of $\sqrt{2}$, when one goes from $q^2 = m_\omega^2$ (or $q^2 = m_\rho^2$) to $q^2 = 0$. The values at $q^2 = m_\omega^2$ and $q^2 = m_\rho^2$ are here shown to be consistent with known values of the meson decay constants, while the values at $q^2 = 0$ reproduce the strength of the relevant components of the nucleon-nucleon interaction at small momentum transfer, as was demonstrated in an earlier work.

I. Introduction

In this work we will use a generalized Nambu–Jona-Lasinio (NJL) model [1-3] to calculate the polarization tensor that describes rho-omega mixing. That quantity has been of some interest in recent years, since the value of the tensor at $q^2 = m_\omega^2$ was used to estimate one form of charge symmetry breaking (CSB) in the nucleon-nucleon force. However, it was pointed out by several authors that for the calculation of the nucleon-nucleon force one needs the tensor for spacelike q^2, that is $q^2 \leq 0$ [4]. It was found that the relevant matrix element is small at $q^2 = 0$ and, therefore, ρ-ω mixing was seen to be unimportant in the calculation of CSB. Therefore, the motivation for studying ρ-ω mixing is diminished. However, there are a number of interesting theoretical issues associated with the calculation of the mixing that have not been fully resolved. One problem that arises is the ambiguity associated with the definition of the rho and omega interpolating fields. For example, Cohen and Miller [5] point out that it is possible to shift CSB effects from the mixed ρ-ω propagator to the meson-nucleon vertex functions. This leads, of course, to significant ambiguities. To a large degree these ambiguities can be avoided by working at the quark level and calculating the current correlator of isoscalar and isovector vector currents. That program has been carried out in Ref. [6] using QCD sum-rule techniques.

We may also use quark degrees of freedom to perform a calculation of the current correlator of isovector and isoscalar currents using the NJL model. It should be clear, however, that if we use the NJL model, we need a model for confinement. We have extended the NJL model to include a description of confinement [2,3] and will use that model in this work. The Lagrangian of our model is

$$
\begin{aligned}
\mathcal{L}(x) = & \bar{q}(i\partial\!\!\!/ - m_q^0)q + \frac{G_S}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] \\
& - \frac{G_\rho}{2}[(\bar{q}\gamma^\mu\vec{\tau}q)^2 + (\bar{q}\gamma_5\gamma_\mu\vec{\tau}q)^2] \\
& - \frac{G_\omega}{2}(\bar{q}\gamma^\mu q)^2 + \mathcal{L}_{conf}(x) \quad ,
\end{aligned}
\tag{1.1}
$$

which we present here in order to define the coupling constants, G_ρ and G_ω, which appear in our discussion. (Work somewhat related to ours has been reported in Ref. [7], where confinement is implemented in a Euclidean-momentum-space analysis of a global color model.)

The organization of our work is as follows. In Section II we introduce various current correlators and the associated vector currents. We also relate the omega and rho fields to these currents. In Section III we discuss the momentum-space bosonization of the extended NJL model and calculate the fundamental matrix element that parametrizes the on-shell $(q^2 = m_\omega^2)$ ρ-ω mixing. In Section IV we attempt to introduce a greater degree of consistency in our calculation, relating the ρ and ω decay constants to the rho-quark and omega-quark coupling constants that we have calculated in an earlier work. In Section V we describe a subtracted current correlation function that vanishes at $q^2 = 0$. Finally, Section VI contains some further discussion and conclusions.

II. Calculation of Current-Current Correlation Functions

We find it useful to start our discussion with the definition made by Maltman [8]. He defines a mixed propagator for rho and omega fields in vacuum,

$$\Pi_{\mu\nu}^{(\rho\omega)}(q^2) = i\int d^4x e^{iq\cdot x} < T(\rho_\mu(x)\,\omega_\nu(0)) > \quad , \tag{2.1}$$

$$= \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] \frac{\theta(q^2)}{(q^2 - m_\rho^2 + i\epsilon)(q^2 - m_\omega^2 + i\epsilon)} \quad , \tag{2.2}$$

where $\theta(q^2)$ is the function to be determined. At $q^2 = m_\omega^2$, $\theta(q^2)$ is proportional to the matrix element $<\rho\,|\,H_{SB}\,|\,\omega>$ [9]. In this work we will exhibit the relation between $\theta(m_\omega^2)$ and $<\rho\,|\,H_{SB}\,|\,\omega>$ for the choice of omega and rho fields that we will use here. (In some studies the definition of the fields is such that $\theta(m_\omega^2)$ may be taken equal to $<\rho\,|\,H_{SB}\,|\,\omega>$ [6].)

It has been pointed out that the widths of the rho and omega mesons must be included, if we wish to obtain the correct q^2 dependence of $\theta(q^2)$ [10]. Therefore, we use the definition of $\theta_3(q^2)$ of Ref. [10]. That is, Eq. (2.2) is modified to read

$$\Pi_{\mu\nu}^{(\rho\omega)}(q) = \hat{g}_{\mu\nu}(q)\frac{\theta_3(q^2)}{\left[q^2 - \left(m_\rho - \frac{i\Gamma_\rho}{2}\right)^2\right]\left[q^2 - \left(m_\omega - \frac{i\Gamma_\omega}{2}\right)^2\right]} \quad , \tag{2.3}$$

with

$$\hat{g}_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2 \quad . \tag{2.4}$$

The functions Γ_ρ and Γ_ω are q^2-dependent with the values $\Gamma_\omega(m_\omega^2) = 8.4$ MeV and $\Gamma_\rho(m_\rho^2) = 151.5$ MeV. (The omega width is small, since two-pion decay of the omega violates G parity.)

As mentioned in the Introduction, some of the recent literature deals with the fact that the interpolating fields, $\rho_\mu(x)$ and $\omega_\mu(x)$, are not fixed, but may be transformed in many ways without changing the values of the S matrices of the theory [5,8]. Because of that ambiguity, it is found that CSB effects may be transferred from propagators to vertex functions and visa versa. Therefore, it is important to define the omega and rho fields in a definite scheme and relate that definition to the underlying quark degrees of freedom. The most natural choice is to define these fields via a momentum-space bosonization procedure [11]. In that case, the fields are proportional to the isoscalar and isovector electromagnetic currents [8],

$$j_\mu^S(x) = \frac{1}{6}\bar{q}(x)\gamma_\mu q(x) \quad , \tag{2.5}$$

and

$$j_\mu^V(x) = \frac{1}{2}\bar{q}(x)\gamma_\mu \tau_3 q(x) \quad . \tag{2.6}$$

Therefore, instead of working with Eq. (2.1), we define

$$\hat{\Pi}_{\mu\nu}^{(\rho\omega)}(q) = i\int d^4x e^{iq\cdot x}\left\langle T\left(j_\mu^V(x) j_\nu^S(0)\right)\right\rangle \quad . \tag{2.7}$$

We also introduce currents

$$J_\mu^S(x) = \bar{q}(x)\gamma_\mu q(x) \quad , \tag{2.8}$$

and

$$J_\mu^V(x) = \bar{q}(x)\gamma_\mu \tau_3 q(x) \quad . \tag{2.9}$$

We then define

$$\tilde{\Pi}^{(\rho\omega)}_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\left\langle T\left(J^V_\mu(x) J^S_\nu(0)\right)\right\rangle \quad , \tag{2.10}$$

where

$$\tilde{\Pi}^{(\rho\omega)}_{\mu\nu}(q) = 12\,\hat{\Pi}^{(\rho\omega)}_{\mu\nu}(q) \quad . \tag{2.11}$$

The reason for introducing $\tilde{\Pi}^{(\rho\omega)}_{\mu\nu}(q)$ is that it lends itself to a more transparent diagrammatic analysis. For completeness, we also define

$$\hat{\Pi}^{(\rho)}_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\left\langle T\left(j^V_\mu(x) j^V_\nu(0)\right)\right\rangle \quad , \tag{2.12}$$

$$= -\hat{g}_{\mu\nu}(q)\,\hat{\Pi}^{(\rho)}(q^2) \quad , \tag{2.13}$$

and

$$\hat{\Pi}^{(\omega)}_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\left\langle T\left(j^S_\mu(x) j^S_\nu(0)\right)\right\rangle \quad , \tag{2.14}$$

$$= -\hat{g}_{\mu\nu}(q)\,\hat{\Pi}^{(\omega)}(q^2) \quad . \tag{2.15}$$

Let us write

$$\tilde{\Pi}^{(\rho\omega)}_{\mu\nu}(q) = \hat{g}_{\mu\nu}(q)\frac{\tilde{\theta}_3(q^2)}{\left[q^2 - \left(m_\rho - \frac{i\Gamma_\rho}{2}\right)^2\right]\left[q^2 - \left(m_\omega - \frac{i\Gamma_\omega}{2}\right)^2\right]} \quad , \tag{2.16}$$

so that we now need to specify the relation between $\theta_3(q^2)$ of Eq. (2.5) and $\tilde{\theta}_3(q^2)$ of Eq. (2.16). A simple way to do that is to relate the ρ and ω fields to the isoscalar and isovector currents as in Ref. [8],

$$\rho^{\mu}(x) = \frac{g^{\rho}}{m_{\rho}^{2}} j_{V}^{\mu}(x) \tag{2.17}$$

and

$$\omega^{\mu}(x) = \frac{g^{\omega}}{m_{\omega}^{2}} j_{S}^{\mu}(x) \quad . \tag{2.18}$$

Here, g^{ρ} and g^{ω} are rho and omega decay constants, defined such that the matrix elements of the current between the vacuum and the vector meson states are

$$\left\langle 0 \,|\, j_{V}^{\mu} \,|\, \rho(q, \epsilon_{\lambda}) \right\rangle = \frac{m_{\rho}^{2}}{g^{\rho}} \epsilon_{\lambda}^{\mu}(q) \quad , \tag{2.19}$$

and

$$\left\langle 0 \,|\, j_{S}^{\mu} \,|\, \omega(q, \epsilon_{\lambda}) \right\rangle = \frac{m_{\omega}^{2}}{g^{\omega}} \epsilon_{\lambda}^{\mu}(q) \quad . \tag{2.20}$$

Here, $\epsilon_{\lambda}^{\mu}(q)$ is a polarization four vector. (At this point, we note the difference between g^{ρ} and g^{ω} and the meson-quark coupling constants, such as $g_{\omega qq}$ and $g_{\rho qq}$, that describe omega-quark and rho-quark coupling, respectively. See Section IV.)

It follows that

$$\theta_{3}(q^{2}) = \frac{g^{\rho} g^{\omega}}{12 m_{\rho}^{2} m_{\omega}^{2}} \tilde{\theta}_{3}(q^{2}) \quad , \tag{2.21}$$

if we assume that the isoscalar and isovector currents couple predominantly to the low-lying omega and rho fields. Note that, when we use Eqs. (2.19) and (2.20), the rho and omega mesons are on-mass-shell. Therefore, while $\tilde{\theta}_{3}(q^{2})$ is well-defined for all q^{2}, the off-mass-shell behavior of $\theta_{3}(q^{2})$ is somewhat arbitrary. If so desired, it is possible to carry out the entire analysis for $\tilde{\theta}_{3}(q^{2})$ only.

III. Bosonization for Omega and Rho Fields

Consider the calculation of $\tilde{\Pi}_{\mu\nu}^{(\rho\omega)}(q)$. That may be done in terms of fundamental quark-loop integrals of the NJL model. We define tensors [3]

$$\hat{J}_{(\rho)}^{\mu\nu}(q) = -\hat{g}^{\mu\nu}(q)\hat{J}_{(\rho)}(q^2) \quad , \tag{3.1}$$

and

$$\hat{J}_{(\omega)}^{\mu\nu}(q) = -\hat{g}^{\mu\nu}(q)\hat{J}_{(\omega)}(q^2) \quad . \tag{3.2}$$

The caret over the symbols in Eqs. (3.1) and (3.2) indicates that we have implemented a model of confinement. For example, in the absence of confinement, $J_{(\omega)}(q^2)$ and $J_{(\rho)}(q^2)$ would be obtained in a calculation of the diagram in Fig. 1a. Figure 1b shows the addition of a ladder of confinement interactions described by a linear potential, $V^C(r) = \kappa r \exp[-\mu r]$. The parameter μ is included to soften the momentum-space singularities of the Fourier transform of $V^C(r)$. (We have used μ = 50 MeV and κ = 0.22 GeV^2 in our calculations.) The ladder of interactions may be summed to define a vertex function [3,12]. (See the shaded area of Fig. 1b.) The equation for the vertex is shown in Fig. 1c. The solution of that equation and the calculation of $\hat{J}_{(\rho)}(q^2)$ and $\hat{J}_{(\omega)}(q^2)$ have been discussed at length in our earlier work [2,3]. (Note that, in the absence of isospin symmetry breaking in the Lagrangian, $\hat{J}_{(\omega)}(q^2) = \hat{J}_{(\rho)}(q^2)$.)

In Fig. 2 we show $\hat{J}_{(\rho)}(q^2)$ for both timelike and spacelike values of q^2. The calculation is made in the timelike region using the methods outlined in Ref. [3]. A cutoff of $\Lambda_3 = 0.702$ GeV is used and the confinement vertex is included with a string tension κ = 0.22 GeV^2. The calculation for the spacelike region is made in a Euclidean momentum space with cutoff Λ_E = 1.0 GeV. Confinement is neglected for the spacelike region. We find

$J_{(\rho)}(0) = 0.0944\ \mathrm{GeV}^2$ for the calculation made in the spacelike region and $\hat{J}_{(\rho)}(0) = 0.0860\ \mathrm{GeV}^2$ for the calculation made in the timelike domain. Ideally, the results of the two calculations should overlap near $q^2 = 0$. In Fig. 2 we have introduced a dotted curve that interpolates between the spacelike and timelike regions.

In the case of the rho meson, we also define a tensor [3]

$$\hat{K}^{\mu\nu}_{(\rho)}(q) = -\hat{g}^{\mu\nu}(q)\hat{K}_{(\rho)}(q^2) \quad , \tag{3.3}$$

where $\hat{K}^{\mu\nu}_{(\rho)}(q)$ is obtained by evaluating the diagram of Fig. 3b. The imaginary part of $\hat{K}^{\mu\nu}_{(\rho)}(q)$ arises when both pions go on mass shell, since the introduction of vertex functions associated with the confining potential removes the (unphysical) $q\bar{q}$ cuts that appear in $K^{\mu\nu}_{(\rho)}(q)$.

The calculation of ρ-ω mixing requires a small modification of the calculation already made to obtain $\hat{J}^{\mu\nu}_{(\rho)}(q)$ [3]. We had

$$-i\hat{J}^{\mu\nu}_{(\rho)}(q) = (-1)n_c n_f \int \frac{d^4k}{(2\pi)^3} \mathrm{Tr}\left[iS(q/2+k)\Gamma^\mu(q,k)iS(-q/2+k)\hat{\gamma}^\nu\right] \tag{3.4}$$

where $\Gamma^\mu(q,k)$ is the confining vertex and $\hat{\gamma}^\nu \equiv \gamma^\nu - \not{q}q^\nu/q^2$. Here, $n_f = 2$ is the number of flavors and $n_c = 3$ is the number of colors. We may define a tensor $\hat{J}^{\mu\nu}_{(\rho\omega)}(q)$ by introducing one factor of τ_3 in Eq. (3.4) and removing the factor of n_f. More precisely, we can define

$$\hat{J}^{\mu\nu}_{(\rho\omega)}(q) = \frac{1}{n_f}\left[\hat{J}^{\mu\nu}_{(\rho)}(q,m_u) - J^{\mu\nu}_{(\rho)}(q,m_d)\right] \quad , \tag{3.5}$$

where m_u and m_d are the constituent masses of the up and down quarks. (Note that $\hat{J}^{\mu\nu}_{(\rho)}(q) = \hat{J}^{\mu\nu}_{(\omega)}(q)$, if $m_u = m_d$.)

Thus, we see that we may use the original calculation made for $\hat{J}^{\mu\nu}_{(\rho)}(q)$ and only consider the variation of that quantity with the constituent quark mass, as in Eq. (3.5). [See Fig. 4.] Indeed, the ρ subscripts on the right-hand side of Eq. (3.5) could be changed to ω subscripts with any change in $\hat{J}^{\mu\nu}_{(\rho\omega)}(q)$, since we are actually using Eq. (3.5) to obtain the difference of an integral involving the up quark and one involving the down quark.

By studying the gap equation of the NJL model, one finds that $m_u - m_d = m_u^0 - m_d^0$, where m_u^0 and m_d^0 are the current quark masses that appear in the Lagrangian. It is again useful to write

$$\hat{J}^{\mu\nu}_{(\rho\omega)}(q) = -\tilde{g}^{\mu\nu}(q)\hat{J}_{(\rho\omega)}(q^2) \quad , \tag{3.6}$$

and, using Eq. (3.5), we see that

$$\hat{J}_{(\rho\omega)}(q^2) = \frac{1}{n_f}\left[\hat{J}_{(\rho)}(q^2, m_u) - \hat{J}_{(\rho)}(q^2, m_d)\right] \quad . \tag{3.7}$$

Note that $\hat{J}_{(\rho\omega)}(q^2)$ is positive for $q^2 > 0$, since $\hat{J}_{(\rho)}(q^2, m)$ is a decreasing function of m and $m_d > m_u$. (For example, we show $\hat{J}_{(\rho)}(q^2, m)$ as a function of m in Fig. 4 for several values of q^2.) Note that, since $\hat{K}_{(\rho)}(q^2)$ is small, we do not concern ourselves with isospin symmetry violations ($m_d \neq m_u$) in the calculation of $\hat{K}_{(\rho)}(q^2)$.

At this point we may proceed with a diagrammatic analysis. For example, in Fig. 5a we show $\hat{J}_{(\rho\omega)}(q^2)$, where the cross-hatching reminds us that this diagram is nonzero due to the isospin violation. The calculation of the current correlator requires that we sum the additional diagrams shown in Fig. 5b, where we have included only a single factor of $\hat{J}_{(\rho\omega)}(q^2)$ in each diagram, since that quantity is quite small. In evaluating these diagrams it is useful to note that

$$\hat{g}^{\mu\alpha}(q)\,\hat{g}_{\alpha}{}^{\nu}(q) = \hat{g}^{\mu\nu}(q) \quad . \tag{3.8}$$

The result for the correlator may be improve : upon by including factors of $\hat{K}_{(\rho)}(q^2)$ and $\hat{K}_{(\omega)}(q^2)$. We find that the diagrammatic analysis leads to the relation

$$\frac{\tilde{\theta}_3(q^2)}{\left[q^2 - \left(m_\rho - \frac{i\Gamma_\rho}{2}\right)^2\right]\left[q^2 - \left(m_\omega - \frac{i\Gamma_\omega}{2}\right)^2\right]}$$
$$= -\frac{1}{1 - G_\rho\left[\hat{J}_{(\rho)}(q^2) + \hat{K}_{(\rho)}(q^2)\right]}\hat{J}_{(\rho\omega)}(q^2)\frac{1}{1 - G_\omega\left[\hat{J}_{(\omega)}(q^2) + \hat{K}_{(\omega)}(q^2)\right]} \tag{3.9}$$

We have noted that $\hat{J}_{(\rho\omega)}(q^2)$ is positive for $q^2 > 0$, so that $\tilde{\theta}_3(q^2)$ is negative in that region.

In order to extract an expression for $\tilde{\theta}_3(q^2)$ from Eq. (3.9), it is useful to use a momentum-space bosonization procedure with the aim of exhibiting the complex zeroes of the denominators on the right-hand side of Eq. (3.9) [11]. As a first step, we separate $\hat{K}_{(\rho)}(q^2)$ and $\hat{K}_{(\omega)}(q^2)$ into real and imaginary parts. We have seen in a recent work [12] that the following representation is useful for $q^2 > 0$,

$$\hat{J}_{(\rho)}(q^2) + \mathrm{Re}\,\hat{K}_{(\rho)}(q^2) = r_1 - \frac{r_2}{q^2 - \tilde{m}_\rho^2} \quad , \tag{3.10}$$

where r_1, r_2, and $\tilde{m}_\rho$ are parameters. We will provide a similar representation for the isoscalar channel, but we will drop Re $\hat{K}_{(\omega)}(q^2)$, since it is very small. Thus, we put

$$\hat{J}_{(\omega)}(q^2) = v_1 - \frac{v_2}{q^2 - \tilde{m}_\omega^2} \quad . \tag{3.11}$$

For example, in the absence of $\hat{K}_{(\omega)}(q^2)$, we find

$$\frac{1}{1 - G_\omega \hat{J}_{(\omega)}(q^2)} = - \frac{g^2_{\omega qq}(q^2)}{q^2 - m^2_\omega} \frac{1}{G_\omega} \quad , \tag{3.12}$$

with

$$m^2_\omega = \tilde{m}^2_\omega - \frac{v_2}{G^{-1}_\omega - v_1} \quad , \tag{3.13}$$

and

$$g^2_{\omega qq}(q^2) = \frac{\tilde{m}^2_\omega - q^2}{G^{-1}_\omega - v_1} \quad . \tag{3.14}$$

[See Table 1.] The expressions for m^2_ρ and $g^2_{\rho qq}(q^2)$ are analogous to those in Eqs. (3.13) and (3.14), with G_ω, v_1 and v_2 replaced by G_ρ, r_1 and r_2. Note that $\tilde{m}^2_\omega > q^2$ in this representation.

The momentum-dependent coupling constant, $g^2_{\omega qq}(q^2)$, appears naturally in this analysis. For nuclear structure studies the relevant value of the coupling constant is $g^2_{\omega qq}(0)$, since such studies are performed for relatively small, <u>spacelike</u> values of q^2. (The representation given in Eqs. (3.10) and (3.11) may also be used for <u>small</u> spacelike values of q^2. For timelike values, we have the restriction $q^2 < \tilde{m}^2_\omega$ or $q^2 < \tilde{m}^2_\rho$.)

In the case of the rho, we keep Re $\hat{K}_{(\rho)}(q^2)$ and Im $\hat{K}_{(\rho)}(q^2)$ and find, upon use of Eq. (3.10), that

$$\frac{1}{1 - G_\rho \left\{ [\hat{J}_{(\rho)}(q^2) + \mathrm{Re}\,\hat{K}_{(\rho)}(q^2)] + i\,\mathrm{Im}\,\hat{K}_{(\rho)}(q^2) \right\}}$$

$$= - \frac{g^2_{\rho qq}(q^2)\, G_\rho^{-1}}{q^2 - m_\rho^2 - i \dfrac{(q^2 - \tilde{m}_\rho^2)}{G_\rho^{-1} - r_1} \mathrm{Im}\,\hat{K}_{(\rho)}(q^2)} \tag{3.15}$$

$$\simeq - \frac{g^2_{\rho qq}(q^2)\, G_\rho^{-1}}{q^2 - \left[m_\rho - i \dfrac{\Gamma_\rho(q^2)}{2} \right]^2} \quad . \tag{3.16}$$

We may obtain an expression for $\Gamma_\rho(q^2)$ by comparing Eqs. (3.16) and (3.15), if we neglect $\Gamma_\rho^2(q^2)$. Alternatively we may use Eq. (3.15), which provides more accurate representation of the result of our analysis.

Finally, we have, upon use of Eq. (3.9),

$$\tilde{\theta}_3(q^2) = - g^2_{\rho qq}(q^2) \frac{\hat{J}_{(\rho\omega)}(q^2)}{G_\rho G_\omega} g^2_{\omega qq}(q^2) \quad , \tag{3.17}$$

and

$$\theta_3(q^2) = - \frac{g^\omega g^\rho}{12 m_\omega^2 m_\rho^2} g^2_{\rho qq}(q^2) \frac{\hat{J}_{(\rho\omega)}(q^2)}{G_\rho G_\omega} g^2_{\omega qq}(q^2) \quad . \tag{3.18}$$

To carry out the calculation, we need the following parameters [12]:

$G_\rho = 7.12\ \mathrm{GeV}^{-2}$, $r_1 = 0.0304\ \mathrm{GeV}^2$, $r_2 = 0.0968\ \mathrm{GeV}^4$, $\tilde{m}_\rho^2 = \tilde{m}_\omega^2 = 1.476\ \mathrm{GeV}^2$,

$G_\omega = 7.86\ \mathrm{GeV}^{-2}$, $v_1 = 0.0284\ \mathrm{GeV}^2$, and $v_2 = 0.0850\ \mathrm{GeV}^4$.

We also use the empirical values, $g^\omega = 15.2$ and $g^\rho = 5.3$ [13], while Ref. [6] has $g^\omega = 3 g^\rho$ and $(g^\rho)^2/4\pi = 2.4$.

Considering the on-shell value, $q^2 = m_\omega^2$, we find $\hat{J}_{(\rho\omega)}(m_\omega^2) = 4.03 \times 10^{-4}\ \text{GeV}^2$, if $m_d = 262$ MeV and $m_u = 260$ MeV. (See Fig. 6.) Using the various parameters given above, we then calculate $\tilde{\theta}_3(m_\omega^2) = -0.493 \times 10^{-3}\ \text{GeV}^6$ and, from Eq. (3.18), we find $\theta_3(m_\omega^2) = (18.47\ \text{GeV}^{-4})\ \tilde{\theta}(m_\omega^2) = -9106\ \text{MeV}^2$. Note that, if $m_d^0 - m_u^0 = 3.0$ MeV, we have $\theta_3(m_\omega^2) = -13{,}660\ \text{MeV}^2$.

It is of interest to ask for the relation between $\theta_3(m_\omega^2)$ and $<\rho\,|\,H_{SB}\,|\,\omega>$. In the third reference listed in Ref. [9], $<\rho\,|\,H_{SB}\,|\,\omega>$ was defined in terms of the ratio of the width for two-pion decay of the omega to the width of the rho:

$$\frac{\Gamma_{\omega\to 2\pi}}{\Gamma_\rho} = \left[\frac{<\rho\,|\,H_{SB}\,|\,\omega>}{m_\rho \Gamma_\rho}\right]^2 \quad . \tag{3.19}$$

From this relation, it was found that $<\rho\,|\,H_{SB}\,|\,\omega> = -(4520 \pm 600)\ \text{MeV}^2$, a value quoted in the recent literature.

Now, we note that, from Eqs. (3.15) and the analog of Eq. (3.14) for the rho meson, we have

$$\Gamma_\rho = \frac{g_{\rho qq}^2(m_\rho^2)}{m_\rho}\ \text{Im}\ \hat{K}_{(\rho)}(m_\rho^2) \quad . \tag{3.20}$$

In a similar fashion, we find

$$\Gamma_{\omega\to 2\pi} = \frac{g_{\omega qq}^2(m_\omega^2)}{m_\omega}\ \text{Im}\ \hat{K}_{(\omega)}(m_\omega^2) \quad . \tag{3.21}$$

Further, we see that Im $\hat{K}_{(\omega)}(m_\omega^2)$ contains Im $\hat{K}_{(\rho)}(m_\omega^2)$ as a factor in a ρ-ω mixing calculation of the width of the omega. We find

$$\frac{\Gamma_{\omega\to 2\pi}}{\Gamma_\rho} = \left[\frac{g^2_{\rho qq}(m^2_\omega)J_{(\rho\omega)}(m^2_\omega)}{m_\rho\,\Gamma_\rho}\left[\frac{g_{\omega qq}(m^2_\omega)}{g_{\rho qq}(m^2_\rho)}\right]\right]^2\left[\frac{m_\omega}{m_\rho}\right] \tag{3.22}$$

where we have used the fact that a consideration of the relevant phase space gives $\mathrm{Im}\,\hat{K}_{(\rho)}(m^2_\omega)/\mathrm{Im}\hat{K}_{(\rho)}(m^2_\rho) \simeq (m^2_\omega/m^2_\rho)$. In writing Eq. (3.22), we have neglected direct $\omega\to\pi^+ + \pi^-$ decay and taken the decay to proceed via ω-ρ mixing, as noted above.

We see upon using Eq. (3.19), that

$$<\rho\,|\,H_{SB}\,|\,\omega> \;=\; -\,g^2_{\rho qq}(m^2_\omega)\hat{J}_{(\rho\,\omega)}(m^2_\omega)\left[\frac{g_{\omega qq}(m^2_\omega)}{g_{\rho qq}(m^2_\rho)}\right] \quad , \tag{3.23}$$

where we have replaced $(m_\omega/m_\rho)^{1/2} \simeq 1.01$ by 1. The last factor in Eq. (3.23) is equal to 1.056, while $g_{\rho qq}(m^2_\omega) = 2.81$. We recall that $\hat{J}_{(\rho\,\omega)}(q^2)$ is proportional to $m^0_d - m^0_u$. If we put $m^0_d - m^0_u = 2.69$, we have $\hat{J}_{(\rho\,\omega)}(m^2_\omega) = 5.42\times 10^{-4}$ GeV^2 and, therefore $<\rho\,|\,H_{SB}\,|\,\omega> \;= -4520$ MeV^2.

Returning to Eq. (3.18), we see that, with our treatment of the ρ and ω fields,

$$\theta_3(m^2_\omega) = \frac{g^\omega g^\rho}{12 m^2_\rho m^2_\omega}\,\frac{g_{\omega qq}(m^2_\omega)\,g_{\rho qq}(m^2_\rho)}{G_\rho\,G_\omega} <\rho\,|\,H_{SB}\,|\,\omega> \tag{3.24}$$

$$\simeq 2.74 <\rho\,|\,H_{SB}\,|\,\omega> \tag{3.25}$$

where we have used $G_\rho = 7.12$ GeV^{-2}, $G_\omega = 7.86$ GeV^{-2}, $g^\rho = 5.3$, $g^\omega = 15.2$, $g_{\omega qq}(m^2_\omega) = 2.95$ and $g_{\rho qq}(m^2_\rho) = 2.81$. (See Table 1.) Our result for $\theta_3(m^2_\omega)$ is based upon the use of Eq. (3.19) to define $<\rho\,|\,H_{SB}\,|\,\omega>$, as in the third reference of Ref. [9]. It is in that reference that the value $<\rho\,|\,H_{SB}\,|\,\omega> \;= -(4520 \pm 600)$ MeV^2 is extracted from the data.

In this section we have used the empirical values for g^{ρ} and g^{ω}. In the next section we present our result for the case in which we calculate g^{ρ} and g^{ω} in our extended NJL model.

IV. Relation of Meson Decay Constants and the Bosonization Scheme

In the simplest bosonization scheme [11], the source of the rho and the omega fields are vector currents introduced previously,

$$(q^2 - m_\omega^2)\,\omega^\mu(x) = -\,g_{\omega qq}\bar{q}(x)\gamma^\mu q(x) \tag{4.1}$$

and

$$(q^2 - m_\rho^2)\,\rho^\mu(x) = -\,g_{\rho qq}\bar{q}(x)\gamma^\mu \tau_3\, q(x) \quad . \tag{4.2}$$

It is then useful to introduce the currents of Eqs. (2.5) and (2.6), so that Eqs. (4.1) and (4.2) become

$$(q^2 - m_\omega^2)\,\omega^\mu(x) = -\,6g_{\omega qq}\,j_S^\mu(x) \tag{4.3}$$

and

$$(q^2 - m_\rho^2)\,\rho^\mu(x) = -\,2g_{\omega qq}\,j_V^\mu(x) \quad . \tag{4.4}$$

Since the decay constants are calculated for on-mass-shell mesons, we have

$$g^\omega = 6g_{\omega qq}(m_\omega^2) \quad , \tag{4.5}$$

$$g^\rho = 2g_{\rho qq}(m_\omega^2) \quad , \tag{4.6}$$

where we expect that $g^\rho \simeq 3g^\omega$. Equations (4.5) and (4.6) are mean-field results for a theory without confinement. We will not make use of Eqs. (4.5) and (4.6) but will use the values of g^ρ and g^ω that are obtained by studying the structure of the isovector and isoscalar current correlation functions using our extended NJL model [3]. We find that

$$g^{\rho} = \frac{3m_{\rho}^{2}}{g_{\rho qq}(m_{\rho}^{2})\,[\hat{J}_{(\rho)}(m_{\rho}^{2}) + \mathrm{Re}\,\hat{K}_{(\rho)}(m_{\rho}^{2})]} \quad , \tag{4.7}$$

$$= \frac{3m_{\rho}^{2}G_{\rho}}{g_{\rho qq}(m_{\rho}^{2})} \quad , \tag{4.8}$$

and

$$g^{\omega} = \frac{9m_{\omega}^{2}}{g_{\omega qq}(m_{\omega}^{2})\,\hat{J}_{(\omega)}(m_{\omega}^{2})} \quad , \tag{4.9}$$

$$= \frac{9m_{\omega}^{2}G_{\omega}}{g_{\omega qq}(m_{\omega}^{2})} \quad . \tag{4.10}$$

Using $g_{\rho qq}(m_{\rho}^{2}) = 2.81$, $g_{\omega qq}(m_{\omega}^{2}) = 2.95$ (see Table 1), $G_{\omega} = 7.86$ GeV^{-2}, and $\hat{J}_{(\rho)}(m_{\rho}^{2}) + \mathrm{Re}\hat{K}_{(\rho)}(m_{\rho}^{2}) = 0.140$ GeV^{2}, we find $g^{\rho} = 4.52$ and $g^{\omega} = 14.7$. These numbers may be compared to the empirical values, $g^{\rho} = 5.3$ and $g^{\omega} = 15.2$, used in the previous section. (It is interesting to note that the calculations reported in Ref. [14] gave $g^{\rho} = 7.0$ and $g^{\omega} = 24.0$, which are larger than the empirical values quoted above. It is possible that our treatment of confinement in our extended NJL model leads to improved values for these decay constants relative to those given in Ref. [14]. Note that in Ref. [14] the notation for the decay constants differs from that used here.) If we use the calculated values of g^{ρ} and g^{ω} [Eqs. (4.7) - (4.10)], the value of $\theta_3(m_{\omega}^{2})$ is reduced from -13,659 MeV^2 to -11,268 MeV^2. We note that to obtain values of $\theta_3(q^2)$ for spacelike q^2, it is best to return to Eq. (3.9) and to calculate the various functions that appear on the right-hand side. To obtain $J_{(\rho)}(q^2)$ and $J_{(\omega)}(q^2)$, we may use the expressions given in the Appendix.

One often defines a rho decay constant called f_ρ [3]. The relation to g^ρ is

$$f_\rho = \frac{3m_\rho}{g^\rho} \quad . \tag{4.11}$$

If we use our calculated value of $g^\rho = 14.7$, we find $f_\rho = 0.157$ GeV, which is quite close to the empirical value $f_\rho \simeq 0.152$ GeV. (The calculations of isovector current correlators reported in Ref. [3] used rather different values of m_q and G_ρ than those used in the present work. In Ref. [3] we found $f_\rho = 0.166$ GeV.)

We may note that the values we have found in Ref. [12], $g_{\omega qq}(0) = 3.86$ and $g_{\rho qq}(0) = 3.66$, lead to the following consequences, when we consider the one boson-exchange (OBE) model of the nucleon-nucleon interaction [15]. For omega exchange, $g_{\omega qq}(0) = 3.86$ leads to $g^2_{\omega NN}/4\pi = 20.2$, while the phenomenological value is $g^2_{\omega NN}/4\pi = 20.0$ [15]. In the model of Ref. [12], the vector and tensor parts of the rho-nucleon interaction are related by $f_\rho / g_\rho = 3.70$, instead of the phenomenological value of $f_\rho / g_\rho = 6.1$. (The f_ρ of this paragraph should not be confused with f_ρ of Eq. (4.11).) However, with $g_{\rho qq}(0) = 3.66$, we found $f_\rho = 20.6$, while $f_\rho^{OBE} = 21.5$ [15]. The fact that we fit the meson decay constants, defined for $q^2 = m_\rho^2$ and $q^2 = m_\omega^2$, and the strength of the potential in the ω and ρ channels, calculated at $q^2 = 0$, lends some support to our model of momentum-dependent coupling constants. (See Table 1.)

V. Subtractions for the Correlator

One can argue that, if $\Pi^{(\rho\omega)}(q^2)$, $\Pi^{(\rho)}(q^2)$ and $\Pi^{(\omega)}(q^2)$ are unequal to zero for $q^2 = 0$, one would generate a mass for the photon. Actually, the condition $\Pi^{(\rho\omega)}(0) = 0$ arises on more general grounds [8,16]. To implement that constraint, we define a subtracted tensor

$$\bar{\Pi}^{(\rho\omega)}_{\mu\nu}(q) = \hat{g}_{\mu\nu}(q) \frac{\theta_3(q^2)}{\left[q^2 - \left(m_\rho - \frac{i\Gamma_\rho}{2}\right)^2\right]\left[q^2 - \left(m_\omega - \frac{i\Gamma_\omega}{2}\right)^2\right]} - \frac{\theta_3(0)}{m_\rho^2 m_\omega^2} , \tag{5.1}$$

$$= \hat{g}_{\mu\nu}(q) \frac{\bar{\theta}_3(q^2)}{\left[q^2 - \left(m_\rho - \frac{i\Gamma_\rho}{2}\right)^2\right]\left[q^2 - \left(m_\omega - \frac{i\Gamma_\omega}{2}\right)^2\right]} , \tag{5.2}$$

so that

$$\bar{\theta}_3(q^2) = \theta_3(q^2) - \left[q^2 - \left(m_\rho - \frac{i\Gamma_\rho}{2}\right)^2\right]\left[q^2 - \left(m_\omega - \frac{i\Gamma_\omega}{2}\right)^2\right] \frac{\theta_3(0)}{m_\rho^2 m_\omega^2} . \tag{5.3}$$

Note that $\bar{\theta}_3(0) = 0$, as desired. We remark that, if we evaluate $\bar{\theta}_3(q^2)$ at $q^2 = (m_\omega - i\Gamma_\omega/2)^2$, we find

$$\bar{\theta}_3(m_\omega^2) = \theta_3(m_\omega^2) . \tag{5.4}$$

Therefore, our results for $\theta_3(m_\omega^2)$ pertain to $\bar{\theta}_3(m_\omega^2)$ as well. (See Fig. 8.)

VI. Discussion

In this work we have shown how the NJL model, extended to include a description of confinement [2,3], may be used to calculate the mixed-current correlator, which is nonzero due to explicit isospin symmetry breaking in the Lagrangian of the model. (In previous works, we have shown how our methods may be used to calculate correlators of the scalar-isoscalar current, $j_s(x) = \bar{q}(x)q(x)$ [2] and the correlator of the vector-isovector current [3].) In the present study, we obtain the experimental value of $<\rho\,|\,H_{SB}\,|\,\omega> = -(4520 \pm 600)$ MeV^2, if $m_d^0 - m_u^0 = 2.69$ MeV. Taking the errors for $<\rho\,|\,H_{SB}\,|\,\omega>$ into account, we suggest that $m_d^0 - m_u^0 = 2.7 \pm 0.3$ MeV. From Ref. [17] we have $m_u^0 = 5 \pm 2$ MeV and $m_d^0 = 9 \pm 3$ MeV. These values are consistent with $m_{avg}^0 = 5.5$ MeV, $m_d = 6.85$ MeV, and $m_u = 4.15$ MeV. For that choice, $m_d^0 - m_u^0 = 2.70$ MeV, which was the value given above. However, values of $m_d^0 - m_u^0$ anywhere in the range of 1 to 5 MeV are also consistent with the uncertainties in the values of m_u^0 and m_d^0. (We remark that $m_{avg}^0 = 5.5$ MeV yields the correct pion mass, when $\Lambda_E = 1.0$ GeV [18].) Clearly, more information concerning the value of $m_d^0 - m_u^0$ would be quite useful in evaluating the success of our calculation.

Appendix

In this appendix we discuss the calculation of $J_{(\rho)}(q^2) = J_{(\omega)}(q^2)$ for spacelike q^2, or small timelike q^2. We neglect confinement in this discussion and start with

$$J^{\mu\nu}_{(\rho)}(q) = i n_c n_f \,\mathrm{Tr} \int \frac{d^4p}{(2\pi)^4} \left\{ \hat{\gamma}^{\mu} \frac{(\not{p} + \not{q}/2 + m)}{\left(p + \frac{q}{2}\right)^2 - m^2} \, \hat{\gamma}^{\nu} \frac{(\not{p} - \not{q}/2 + m)}{\left(p - \frac{q}{2}\right)^2 - m^2} \right\} . \tag{A1}$$

Here, m is the constituent quark mass and $\hat{\gamma}^{\mu}$ was defined after Eq. (3.4). We find

$$J^{\mu\nu}_{(\rho)}(q) = i n_c n_f \hat{g}^{\mu\nu}(q) \int \frac{d^4p}{(2\pi)^4} \frac{4\left[\frac{2}{3}\hat{p}^2 - \left[p^2 - \frac{q^2}{4}\right] + m^2\right]}{\left[\left(p + \frac{q}{2}\right)^2 - m^2\right]\left[\left(p - \frac{q}{2}\right)^2 - m^2\right]} , \tag{A2}$$

$$= - \hat{g}^{\mu\nu}(q)\, J_{(\rho)}(q^2) . \tag{A3}$$

In Eq. (A2), $\hat{p}^{\mu} = p^{\mu} - (p \cdot q) q^{\mu}/q^2$.

The result for $J_{(\rho)}(q^2)$ may be expressed in terms of two integrals that were defined in Ref. [14],

$$I_1(m) = 8 n_c i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2} , \tag{A4}$$

and

$$I_2(m, q^2) = 4 n_c i \int \frac{d^4p}{(2\pi)^4} \frac{1}{\left[\left(p + \frac{q}{2}\right)^2 - m^2\right]\left[\left(p - \frac{q}{2}\right)^2 - m^2\right]} . \tag{A5}$$

Analytic expressions for $I_1(m)$ and $I_2(m, q^2)$ are given in Ref. [14] in terms of a Euclidean momentum-space cutoff Λ_E. We have

$$I_1(m) = \frac{n_c}{2\pi^2}\left[\Lambda_E^2 - m^2 \ln\left(1 + \frac{\Lambda_E^2}{m^2}\right)\right] \quad , \tag{A6}$$

and

$$I_2(m, q^2) = \frac{n_c}{4\pi^2}\int_0^1 dx \left[\frac{\Lambda_E^2}{\Lambda_E^2 + y} + \ln\left(\frac{y}{\Lambda_E^2 + y}\right)\right] \quad , \tag{A7}$$

where $y(x, q^2) = q^2(x^2 - x) + m^2$. After some algebra, we obtain

$$J_{(\rho)}(q^2) = \frac{2}{3} I_1(m) - \left[\frac{4}{3}m^2 + \frac{2}{3}q^2\right] I_2(m, q^2) \quad . \tag{A8}$$

Note that

$$J_{(\rho)}(0) = \frac{2}{3} I_1(m) - \frac{4}{3}m^2 I_2(m, 0) \quad . \tag{A9}$$

With $m = 0.260$ GeV and $\Lambda_E = 1.0$ GeV, we have $I_1(m) = 0.123\ \text{GeV}^2$ and $m^2 I_2(m, 0) = -0.0093\ \text{GeV}^2$. From these values, we find $J_{(\rho)}(0) = 0.0944\ \text{GeV}^2$ in correspondence to the $q^2 = 0$ value for the curve shown in Fig. 2 that was calculated for spacelike q^2.

We may make contact with the result given in Ref. [14] if we perform a subtraction, so that $J_{(\rho)}(0) = 0$. (We must also change the sign, if we are to compare to $J_{VV}(q^2)$ of Ref. [14]. That reference has the opposite sign to ours for the coupling constants in the Lagrangian.) Thus

$$-\left[J_{(\rho)}(q^2) - J_{(\rho)}(0)\right] = \left[\frac{4}{3}m^2 + \frac{2}{3}q^2\right] I_2(m, q^2) - \frac{4}{3}m^2 I_2(m, 0) \quad , \tag{A10}$$

which agrees with the function $J_{VV}(q^2)$ of Ref. [14],

$$J_{VV}(q^2) = \frac{2}{3}\Big[(2m^2 + q^2) I_2(m, q^2) - 2m^2 I_2(m, 0)\Big] \quad . \tag{A11}$$

We have seen in our work that the subtraction should properly be made for the current correlator, rather than for the leading perturbative approximation to the correlator, $J_{(\rho)}(q^2)$ or $J_{(\omega)}(q^2)$.

Acknowledgement

This work was supported in part by a grant from the National Science Foundation and by the PSC-CUNY Faculty Research Award Program.

References

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961).

[2] L.S. Celenza, C.M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Intl. J. Mod. Phys. E2, 603 (1993).

[3] L.S. Celenza, C.M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Ann. Phys. (N.Y.) 241, 1 (1995).
The construction of vertex operators for the confinement potential is discussed at length in L.S. Celenza, C.M. Shakin, Wei-Dong Sun, J. Szweda, and Xiquan Zhu, Phys. Rev. D51, 3638 (1995).

[4] T. Goldman, J.A. Henderson, and A.W. Thomas, Few-Body System 12, 123 (1992).

[5] T.D. Cohen and G.A. Miller, Univ. of Washington preprint (1995).

[6] T. Hatsuda, E.M. Henley, Th. Meissner, and G. Krein, Phys. Rev. C49, 452 (1994). M.J. Iqbal, Xuemin Jin, and Derek B. Leinweber, TRIUMF preprint: TRI-PP-94-47 (1995). Different results for $\theta(q^2)$ are reported in these two references, since only the second reference takes into account the widths of the ρ and ω mesons. (The importance of that feature is discussed in Ref. [10].)

[7] K.L. Mitchell, P.C. Tandy, C.D. Roberts, and R.T. Cahill, Phys. Lett. B335, 282 (1994).

[8] Kim Maltman, preprint (York University, Dept. of Mathematics, North York, Ontario, Canada, 1995).

[9] P.C. McNamee, M.D. Scadron, and S.A. Coon, Nucl. Phys. A249, 483 (1975); S.A. Coon, M.D. Scadron, and P.C. McNamee, Nucl. Phys. A287, 381 (1977); S.A. Coon, and R.G. Barrett, Phys. Rev. C36, 2189 (1987).

[10] M.J. Iqbal, Xuemin Jin, and Derek B. Leinweber, TRIUMF preprint: TRI-PP-95-14 (1995).

[11] V. Bernard, A.A. Osipov, and Ulf-G. Meissner, Phys. Lett. B285, 119 (1992).

[12] Shun-fu Gao, L.S. Celenza, C.M. Shakin, Wei-Dong Sun, and J. Szweda, Brooklyn College Report: BCCNY 95/081/249 (1995) – submitted for publication in Physical Review C.

[13] D. Dumbrais, et al., Nucl. Phys. B216, 277 (1983).

[14] S. Klimt, M. Lutz, U. Vogl and W. Weise, Nucl. Phys. A516, 429 (1990).

[15] R. Machleidt, in Advances in Nuclear Physics, Vol. 19, eds. J.W. Negele and E. Vogt (Plenum, New York, 1989).

[16] H.B. O'Connell, B.C. Pearce, A.W. Thomas, and A.G. Williams, Phys. Lett. B336, 1 (1994). This work contains a proof that $\Pi^{(\rho)}(q^2)$, $\Pi^{(\omega)}(q^2)$ and $\Pi^{(\omega\rho)}(q^2)$ all require subtractions, so that these functions will be equal to zero, when $q^2 = 0$. See also J. Piekarawicz and A.G. Williams, Phys. Rev. C47, R2462 (1993).

[17] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982).

[18] Nan-Wei Cao, C.M. Shakin, and Wei-Dong Sun, Phys. Rev. C46, 2535 (1992).

Table 1. Values of $g_{\rho qq}(q^2)$ and $g_{\omega qq}(q^2)$ are presented. Here $g^2_{\rho qq}(q^2) = (\tilde{m}^2_\rho - q^2)/(G^{-1}_\rho - r_1)$ and $g^2_{\omega qq}(q^2) = (\tilde{m}^2_\omega - q^2)/(G^{-1}_\omega - v_1)$, with $G_\rho = 7.12$ GeV^{-2}, $G_\omega = 7.86$ GeV^{-2}, $\tilde{m}^2_\rho = \tilde{m}^2_\omega = 1.476$ GeV^2, $r_1 = 0.0304$ GeV^2, and $v_1 = 0.0284$ GeV^2 [12]. Note that $g_{\rho qq}(m^2_\omega) = 2.80$ and $g_{\omega qq}(m^2_\omega) = 2.95$.

q^2(GeV^2)	$g_{\rho qq}(q^2)$	$g_{\omega qq}(q^2)$
0.0	3.66	3.86
0.1	3.54	3.73
0.2	3.41	3.59
0.3	3.27	3.45
0.4	3.13	3.30
0.5	2.98	3.14
0.6	2.82	2.98
0.7	2.66	2.80

Figure Captions

Fig. 1. a) The basic quark-loop integral for the NJL model is used to define the tensor $J^{\mu\nu}_{(\rho)}(q) = -\tilde{g}^{\mu\nu} J_{(\rho)}(q^2)$. More precisely, the evaluation of the diagram using Feynman rules yields $-iJ^{\mu\nu}_{(\rho)}(q)$.

b) A summation of a ladder of confinement interactions (a linear potential) serves to define a confining vertex, shown as a shaded area. The diagram on the left serves to define $\hat{J}^{\mu\nu}_{(\rho)}(q) = -\hat{g}^{\mu\nu} \hat{J}_{(\rho)}(q^2)$.

c) The equation whose solution yields the confining vertex. (Solutions of this equation are described in detail in Ref. [3].) Note that $\hat{J}_{(\rho)}(q^2) = \hat{J}_{(\omega)}(q^2)$, if $m_d = m_u$.

Fig. 2. Values of $\hat{J}_{(\rho)}(q^2)$ are shown for spacelike and timelike values of q^2. The calculation in the timelike region is done in Minkowski space with a cutoff on all three-momenta of $\Lambda_3 = 0.702$ GeV [3]. The spacelike values are calculated in Euclidean momentum space with a cutoff $\Lambda_E = 1.0$ GeV. Confinement is included in the calculation made for $q^2 > 0$ and we find $\hat{J}_{(\rho)}(0) = 0.0860$ GeV^2. The Euclidean-space calculation made for $q^2 < 0$ yields $J_{(\rho)}(0) = 0.0944$ GeV^2. The dotted curve is used to interpolate between the two calculations. Note that $\hat{J}_{(\rho)}(q^2) = \hat{J}_{(\omega)}(q^2)$, if $m_d = m_u$. The quark mass in these calculations is $m_q = 262$ MeV.

Fig. 3. a) The diagram shown is used to define the tensor $\hat{K}^{\mu\nu}_{(\rho)}(q) = -\hat{g}^{\mu\nu}(q) K_{(\rho)}(q^2)$. Here the wavy lines represent pions. (In analogy to the comment made in the caption to Fig. 1, we note that the

evaluation of such diagrams using Feynman rules yields $-iK^{\mu\nu}_{(\rho)}(q)$.)

b) Introduction of the confining vertex of Fig. 1c serves to define the tensor $\hat{K}^{\mu\nu}_{(\rho)}(q) = -\hat{g}^{\mu\nu}\hat{K}_{(\rho)}(q^2)$. Note that $\hat{K}_{(\rho)}(q^2) \gg \hat{K}_{(\omega)}(q^2)$, since the two-pion decay of the omega violates G-parity.

Fig. 4. Values of $\hat{J}_{(\omega)}(q^2)$ are presented for several values of q^2 and for a range of values of the constituent quark mass.

a) $q^2 = 0.0 \text{ GeV}^2$

b) $q^2 = 0.10 \text{ GeV}^2$

c) $q^2 = 0.20 \text{ GeV}^2$

d) $q^2 = 0.60 \text{ GeV}^2$

Here $\kappa = 0.22 \text{ GeV}^2$ and $\Lambda_3 = 0.702$ GeV. Note that $\hat{J}_{(\omega)}(q^2) = \hat{J}_{(\rho)}(q^2)$, if $m_d = m_u$. Further, $\hat{J}_{(\omega)}(q^2)$ decreases when the constituent mass is increased. Therefore, $\left[\hat{J}_{(\omega)}(q^2, m_u) - \hat{J}_{(\omega)}(q^2, m_d)\right] > 0$, since $m_d > m_u$.

Fig. 5. a) The diagrammatic element that serves to define $\hat{J}^{\mu\nu}_{(\rho\omega)}(q) = -\hat{g}^{\mu\nu}\hat{J}_{(\rho\omega)}(q^2)$ is shown. The shaded triangular area represents the confining vertex of Fig. 1c. The small open circles denote the coupling constants, G_ρ or G_ω, of the NJL model. (See Eq. (1.1.)

b) The calculation of the mixed correlation function $\hat{\Pi}^{\mu\nu}_{(\rho\omega)}(q) = -\hat{g}^{\mu\nu}(q)\hat{\Pi}_{(\rho\omega)}(q^2)$ is shown. Each diagram contains a single factor of $\hat{J}_{(\rho\omega)}(q^2)$ and a varying number of factors of $\hat{J}_{(\rho)}(q^2)$ and $\hat{J}_{(\omega)}(q^2)$, with the $\hat{J}_{(\rho)}(q^2)$ factors to the left of $\hat{J}_{(\rho\omega)}(q^2)$ and the $\hat{J}_{(\omega)}(q^2)$ factors to the right of $\hat{J}_{(\rho\omega)}(q^2)$ in the diagram.

Fig. 6. Values of $\hat{J}_{(\rho\omega)}(q^2)$ are shown. Here $m_d = 262$ MeV, $m_u = 260$ MeV, $\kappa = 0.22$ GeV^2 and $\Lambda_3 = 0.702$ GeV. Note that κ is the string tension and Λ_3 is the cutoff on the magnitude of all three-momenta in a Minkowski-space calculation of the quark-loop integrals. At the origin, we have $\hat{J}_{(\rho\omega)}(0) = 0.628 \times 10^{-4}$ GeV^2. Note that $J_{(\rho)}(q^2) = 0$ for $q^2 = -0.45$ GeV^2. Values for spacelike q^2 were obtained using the formalism presented in the Appendix. (Values for $m_d^0 - m_u^0 = 2.69$ MeV are obtained by multiplying the values in the figure by 2.69/2.)

Fig. 7. Values of $-\theta_3(q^2)$ are shown as a solid line. [See Eq. (3.18).] Here $m_d^0 - m_u^0 = 2.0$ MeV. (Values for $m_d^0 - m_u^0 = 2.69$ MeV may be obtained by multiplying the values in the figure by 2.69/2.)

Fig. 8. Values of $-\bar{\theta}_3(q^2)$ are shown as a dashed line. [See Eq. (5.3). Here, the meson widths appearing in Eq. (5.3) were neglected.] We recall that $\theta_3(m_\omega^2) = \bar{\theta}_3(m_\omega^2)$. Here $m_d^0 - m_u^0 = 2.0$ MeV. (See the caption of Fig. 7.)

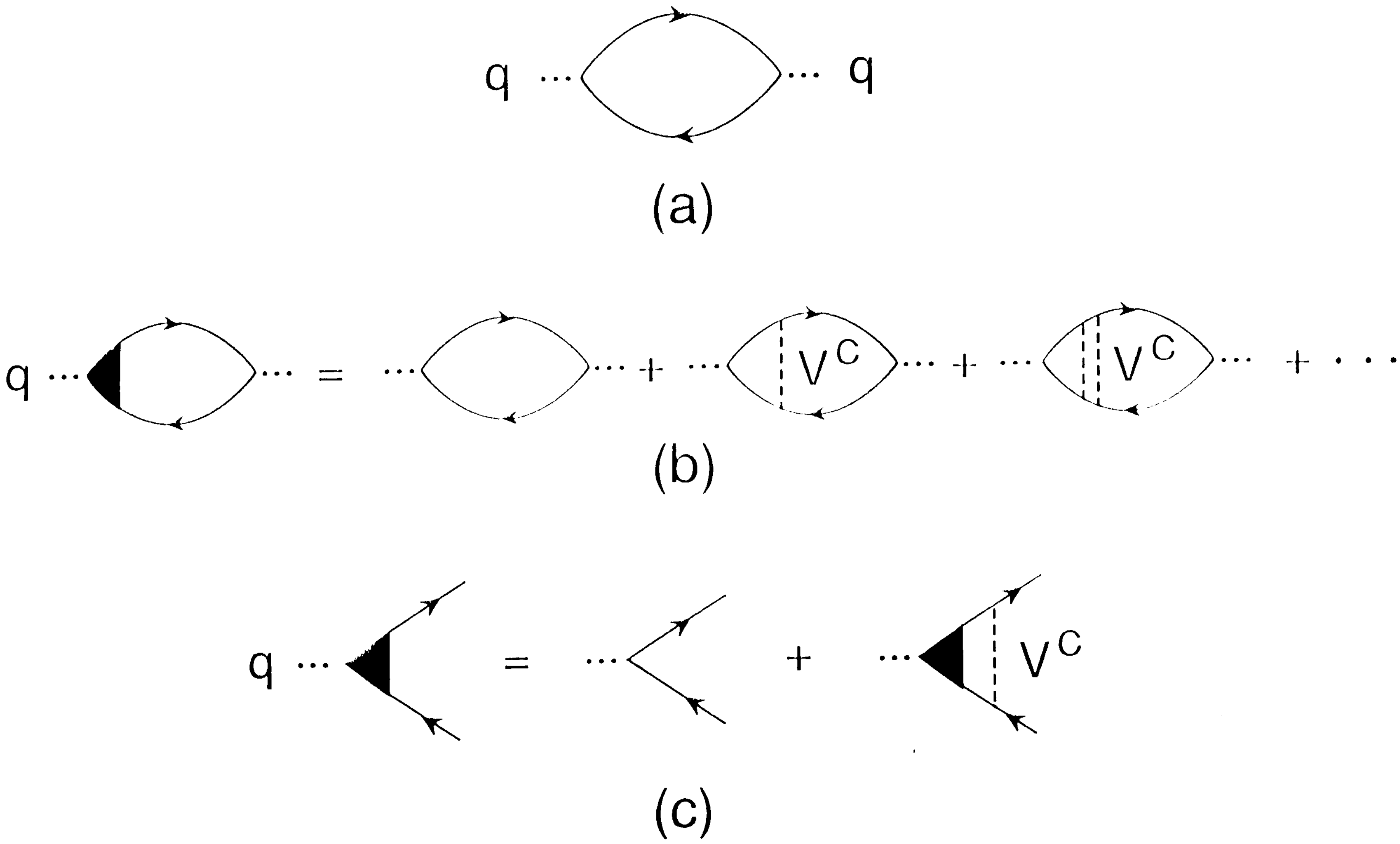


FIG. 1

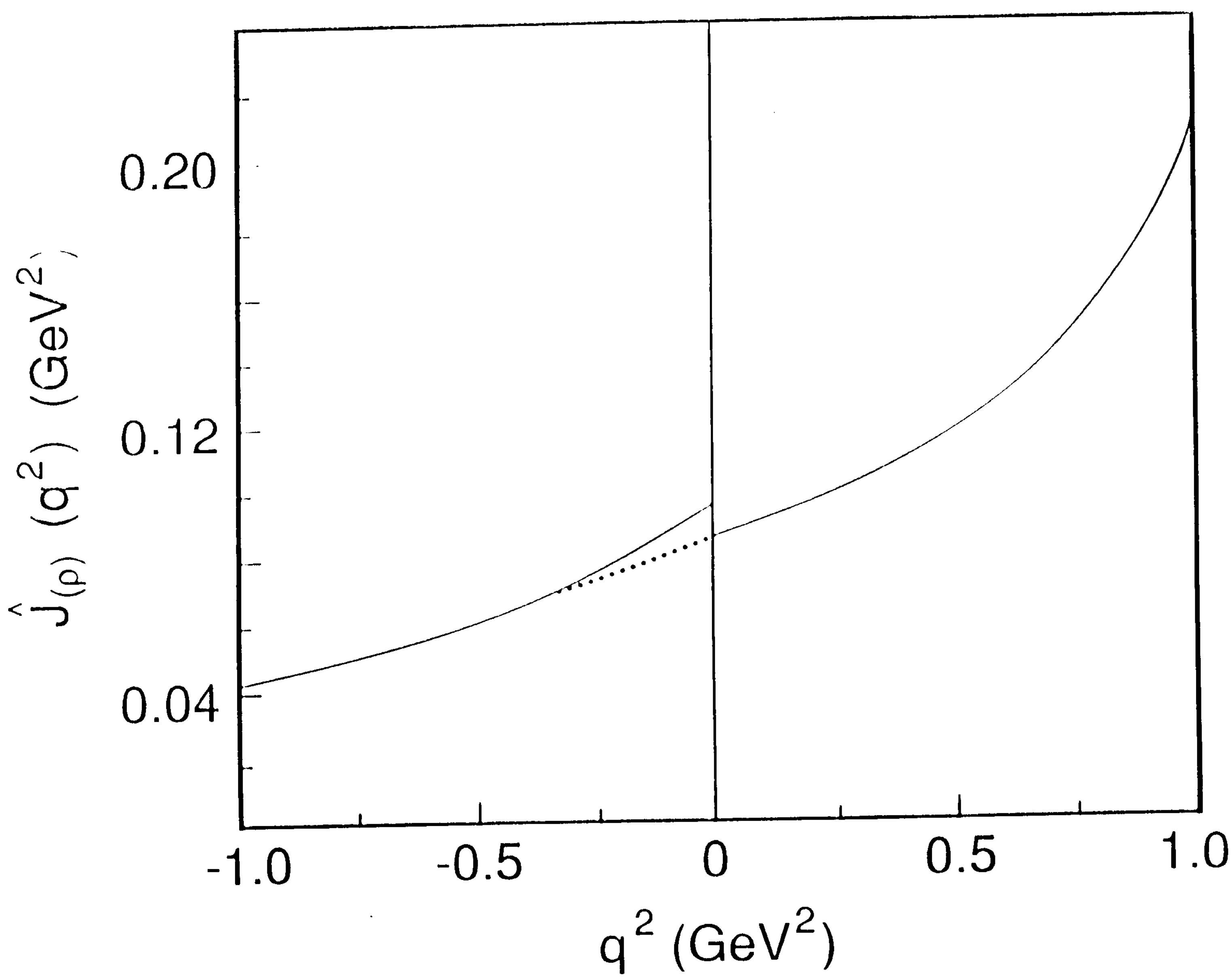


FIG. 2

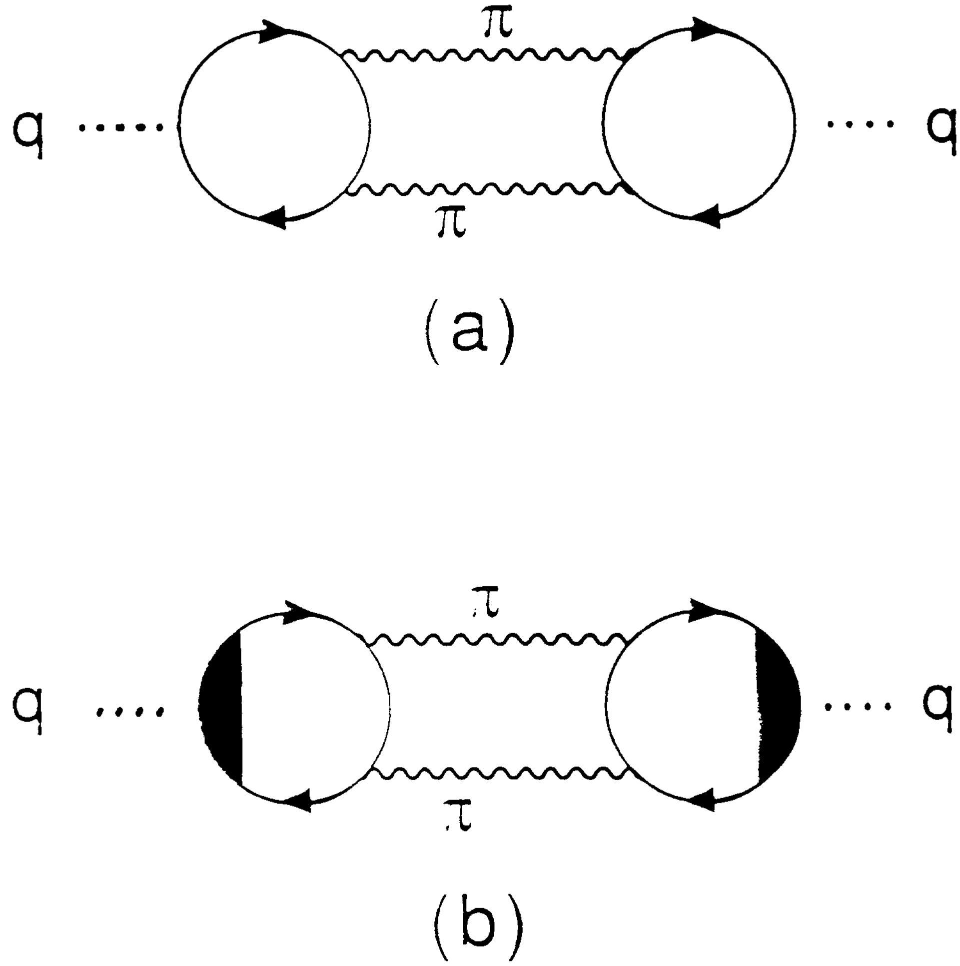


FIG. 3

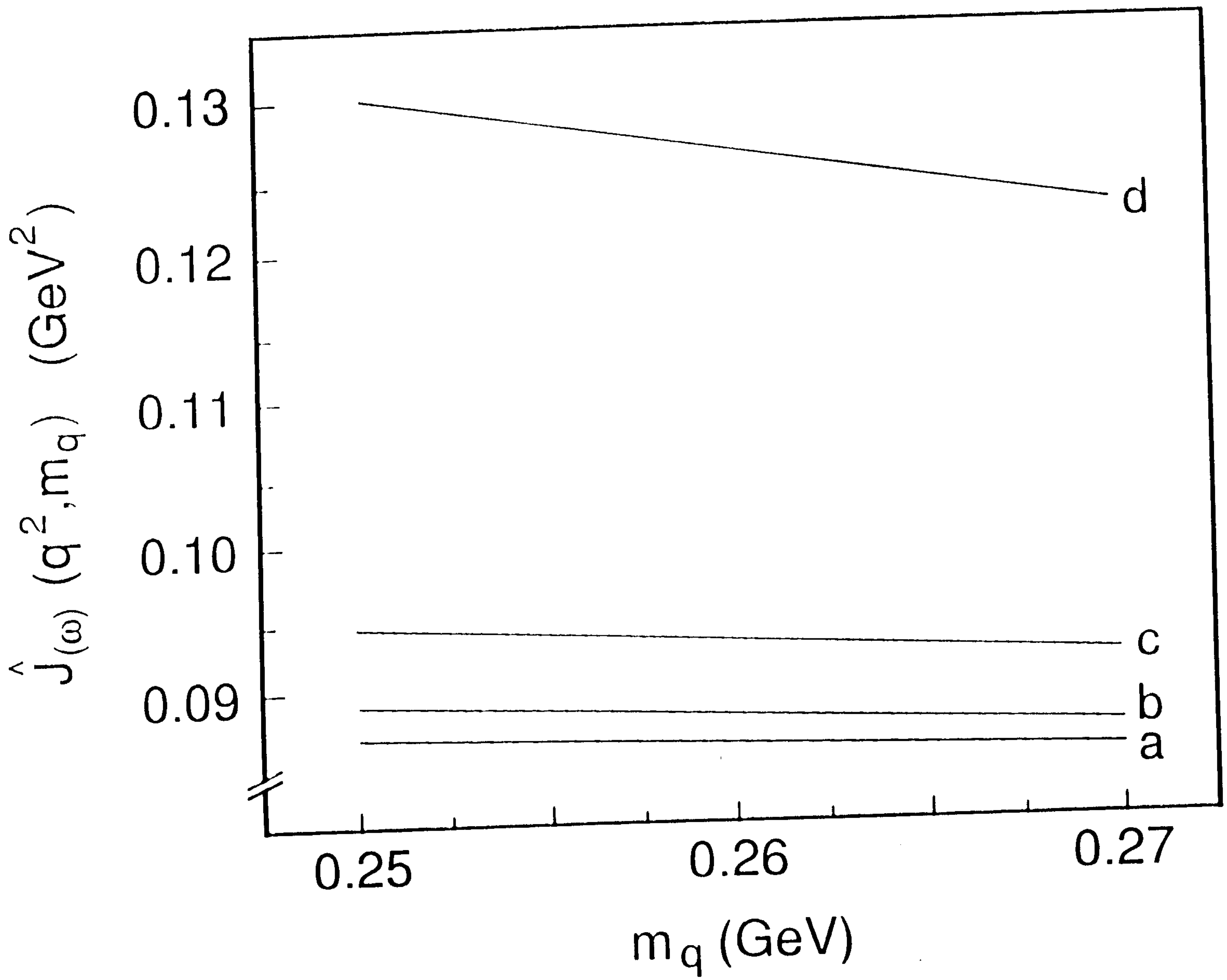
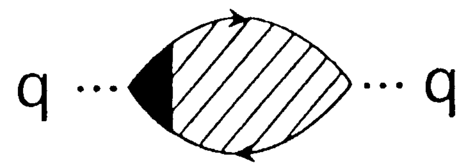
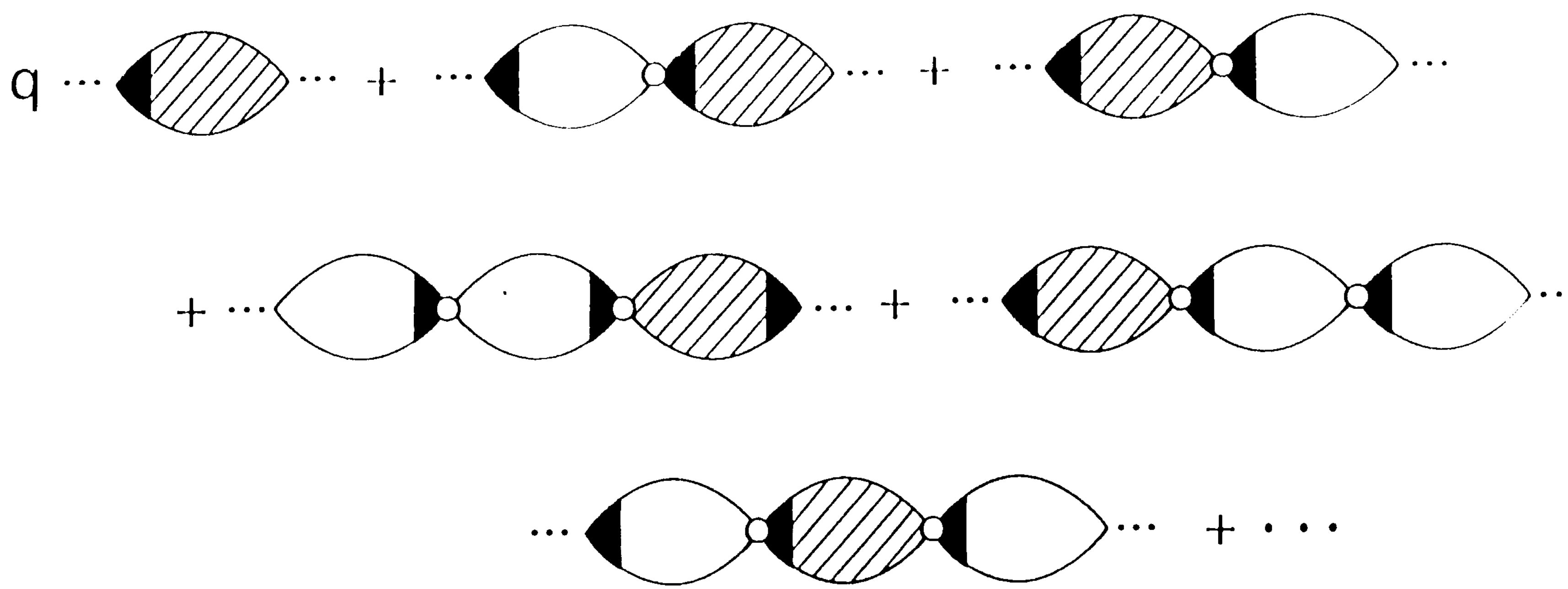

0.13
0.12
0.11
0.10
0.09
d
c
b
a
0.25
0.26
0.27
m_q (GeV)
$\hat{J}_{(\omega)}$ (q^2,m_q) (GeV^2)

FIG. 4



(a)



(b)

FIG. 5

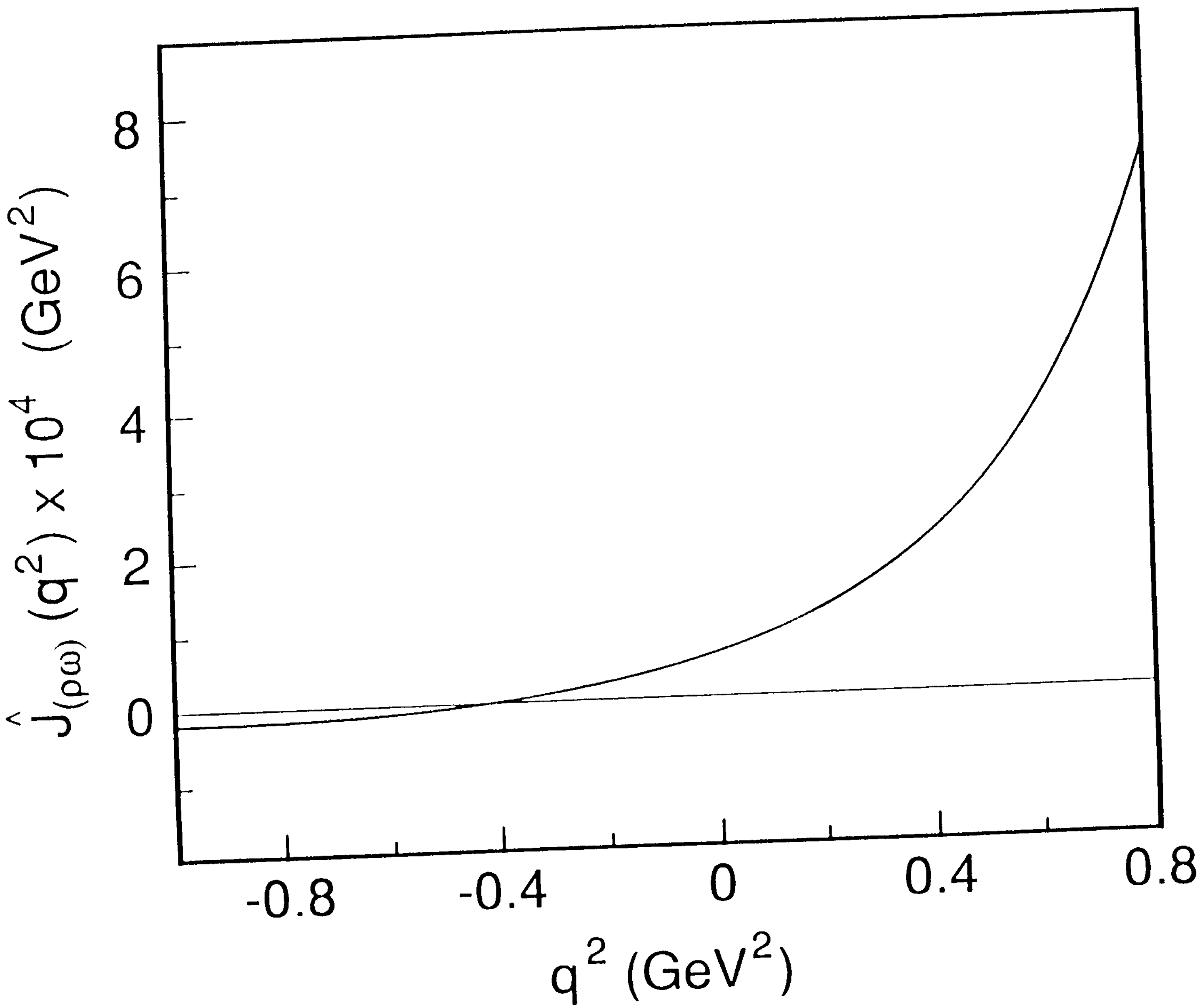


FIG. 6

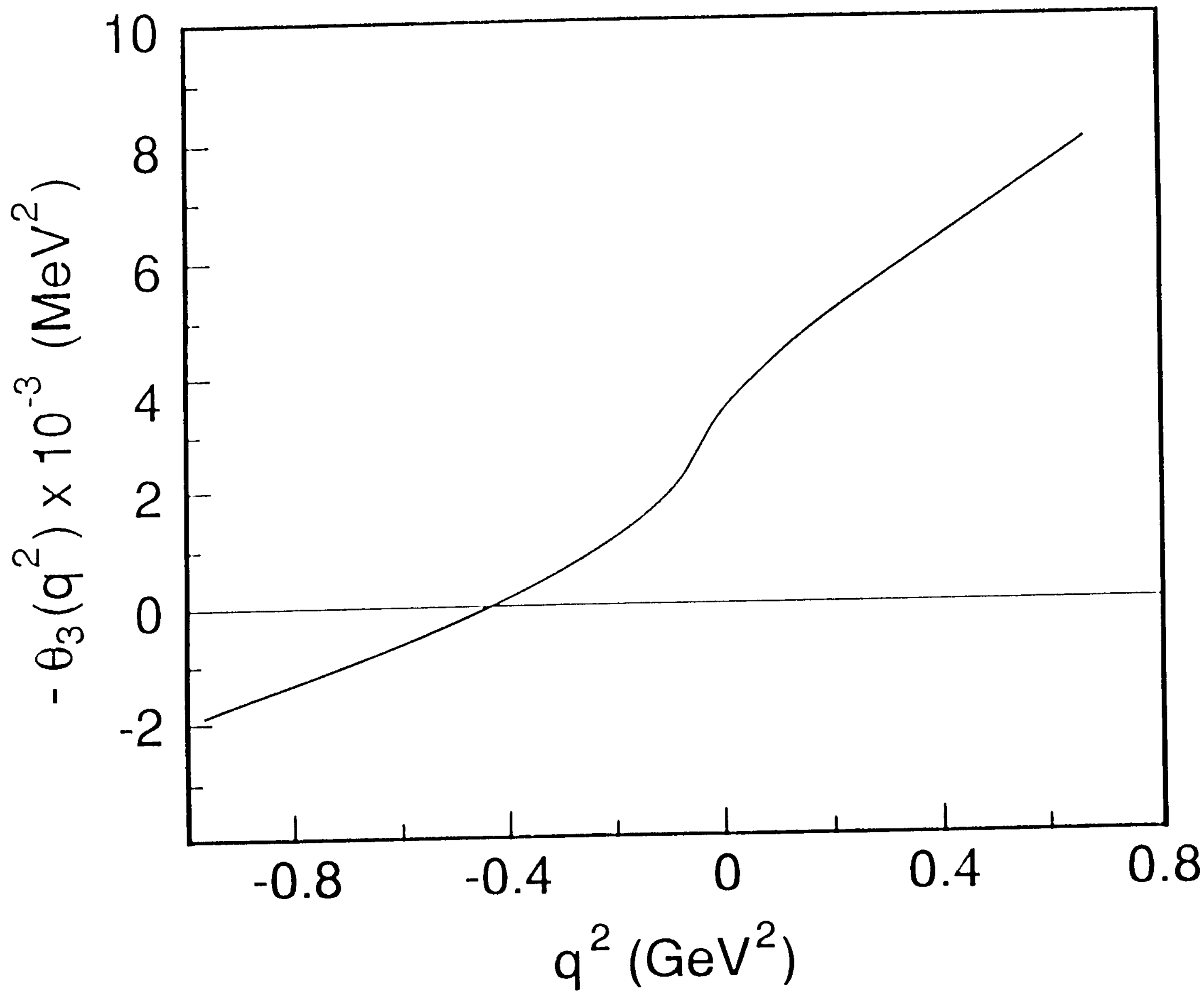


FIG. 7

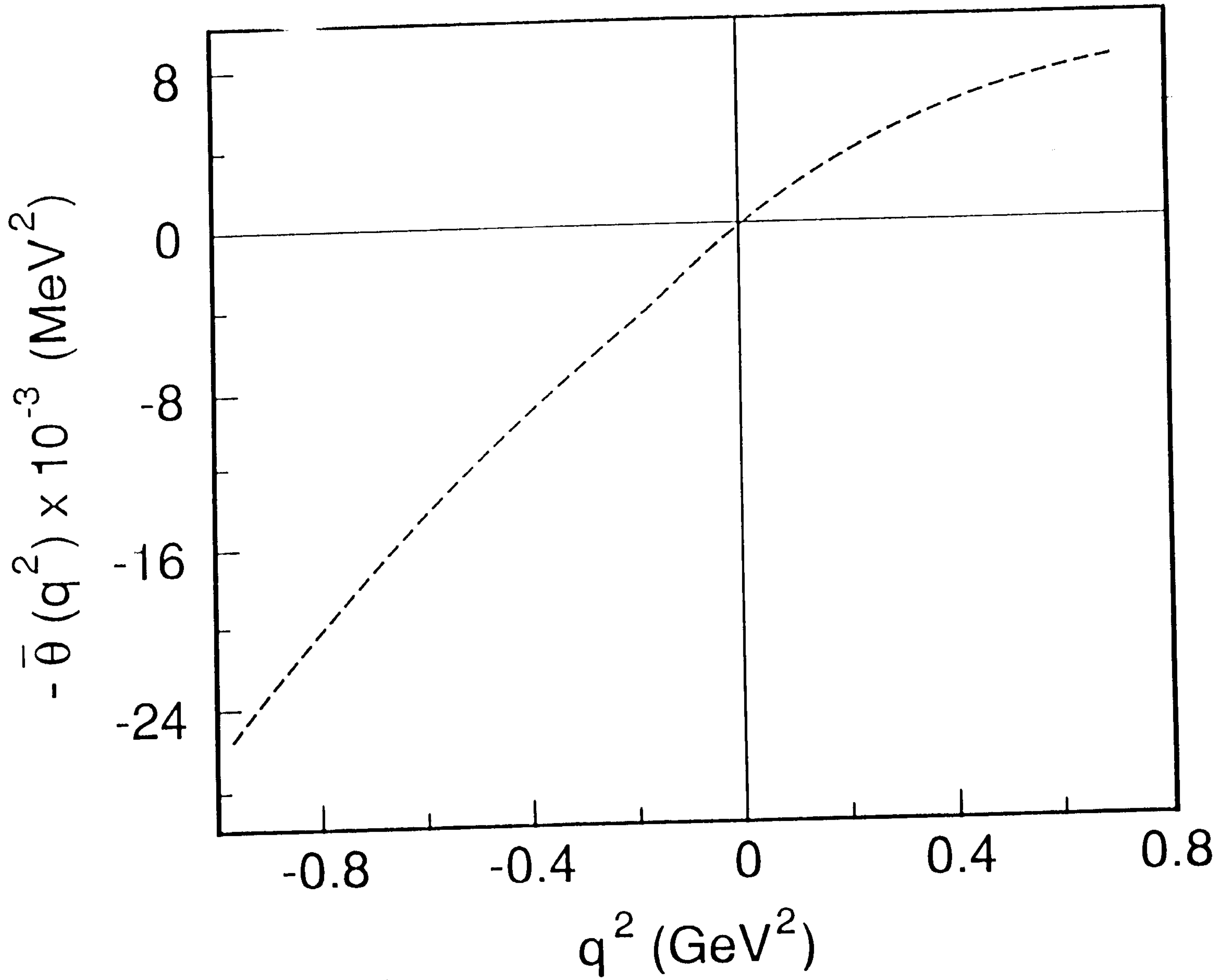


FIG. 8