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Challenges in Lightcone Field Quantization

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Abstract

The most challenging two fundamental issues in the lightcone field quantization are the vacuum structure and the rotational symmetry on the lightcone. The rational relativistic energy-momentum relation on the lightcone indicates that the lightcone vacuum has a rather simple structure. We discuss on the issue how the novel phenomena such as the spontaneous symmetry breaking, Higgs mechanism, the chiral symmetry breaking, *etc.* which were known as the direct consequences of the nontrivial vacuum condensation can be realized from the trivial vacuum on the lightcone. Also, the rotational invariance is violated in the lightcone quantization method when the Fock space is truncated for practical calculations. To what extent the rotation symmetry is broken in the lightcone quantization approach can be quantified by calculating the explicit rotation dependence of the two-body scattering phase shifts. We analyze the scattering phase shifts in a simple scalar field model, extending the lightcone ladder approximation to the lowest order lightcone Tamm-Dancoff approximation in which the self-energy corrections are incorporated. We find that the self-energy effects significantly restore the rotation symmetry. These effects make the phase shifts stabilize as the coupling constant grows which is in a good agreement with the previous bound state results that the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects.

1. Introduction

Even though the time, t, is the ordinary choice for the variable to describe the evolution of the physical systems, the covariance of the special relativity offers other choices. The lightcone time, $\tau = t + z/c$, can be chosen as the evolution variable and the choices of t and τ in the quatization of field theories yield the equal-t and equal-τ quatization schemes, respectively [1]. In this paper, we will first discuss the consequences in the vacuum structure in contrast to the case of ordinary t. Suppose that a particle has the mass m and the four-momentum $k = (k^0, k^1, k^2, k^3)$, then the relativistic energy-momentum relation of the particle at equal-τ is given by

$$k^- = \frac{\vec{k}_\perp^2 + m^2}{k^+}, \qquad (1)$$

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where the lightcone energy conjugate to τ is given by $k^- = k^0 - k^3$ and the lightcone momenta $k^+ = k^0 + k^3$ and $\vec{k}_\perp = (k^1, k^2)$ are orthogonal to k^- and form the lightcone three-momentum $\underline{k} = (k^+, \vec{k}_\perp)$. The Eq. (1) provides the rational relation which is in a drastic contrast to the irrational energy-momentum relation at equal-t given by

$$k^0 = \sqrt{\vec{k}^2 + m^2}, \qquad (2)$$

where the energy k^0 is conjugate to t and the three-momentum vector $\vec{k}$ is given by $\vec{k} = (k^1, k^2, k^3)$. The main point is that the signs of k^+ and k^- are correlated and thus the momentum k^+ is always positive because only the positive energy k^- makes the system evolve to the future direction (*i.e.* positive τ), while at equal-t the signs of k^0 and $\vec{k}$ are not correlated and thus the momentum k^3 corresponding to k^+ of equal-τ can be either positive or negative. This provides a remarkable feature to the lightcone vacuum; *i.e.*, the trivial vacuum of the free lightcone theory is an eigenstate of the full Hamiltonian, *viz.*, the true vacuum [2, 3]. This can be proved by showing that the full lightcone Hamiltonian annihilates the trivial perturbative vacuum [4]. For example, in QED, the application of the interaction $H^I_{LC} = \int d^3\underline{x}\bar{\psi}\gamma^\mu\psi A_\mu$ to the perturbative vacuum $|0\rangle$ results in a sum of terms $b^\dagger(\underline{k}_1)a^\dagger(\underline{k}_2)d^\dagger(\underline{k}_3)|0\rangle$. While the conservation of the lightcone momentum requires $\sum_{i=1}^3 k_i^+ = 0$, the massive fermions with finite k_i^- cannot have $k_i^+ = 0$ due to Eq.(1). Thus, H^I_{LC} annihilates the trivial vacuum $|0\rangle$ and so does the full Hamiltonian $H_{LC} = H^I_{LC} + H^0_{LC}$ since $|0\rangle$ is annihilated by the free Hamiltonian H^0_{LC} by definition. This feature is drastically different from the equal-t quantization where the state $H|0\rangle$ is a highly complex composite of pair fluctuations.

However, the apparent simplicity of the lightcone vacuum yields a problem to understand the novel phenomena such as the spontaneous symmetry breaking, Higgs mechanism, chiral symmetry breaking,axial anomaly, θ-vacuua, *etc.*, because these were known as the direct consequencies of the nontrivial vacuum structures of various field theories. Thus, the question of how one can realize these nontrivial vacuum phenomena from the trivial lightcone vacuum arises [5] and an attempt to answer this question is made in the next section.

In this paper, we will also discuss the issue of the rotation symmetry in the lightcone field quantization. For an explicit illustration of the rotation dependence in the two-body scattering phase shifts [6], let's consider a scalar field model [7, 8] which describes the interaction between two scalar particles $\phi, \bar{\phi}$ with equal mass m exchanging a scalar particle χ with mass λ. This model with $\lambda = 0$ is known as the Wick- Cutkosky model [9] and the interaction Lagrangian is given by

$$\mathcal{L} = g\phi^2\chi \, . \qquad (3)$$

Because the transverse components of the angular momentum (J_x and J_y) in the lightcone Poincare algebra [10] contain interactions changing particle numbers in equal τ, the calculated scattering amplitude in the truncated Fock space is not rotationally invariant. The degree of the rotation symmetry breaking was quantified by our recent work of calculating the two-body scattering phase shifts [6]. The numerical results showed that the rotation symmetry is broken more severely as the coupling constant of the model gets larger. More recently, we extended the lightcone ladder approximation to the lowest order lightcone Tamm-Dancoff approximation in the same model [11] and investigated the effects of the self-energy corrections and counter-terms to the rotation problem [12]. We found astonishingly a siginificant restoration of the rotation symmetry by this extension. Also, we observed that the self-energy effects stabilized the phase shifts as the coupling constant grows. Even though the rotation problem is unavoidable in the lightcone quantization method with the Fock-space truncation, this calculation indicates that the rotation symmetry can be dynamically restored by adding the interactions which were neglected before. In Sections 3 and 4, we present our scattering formulation and numerical results, and discuss implications of our computation in the lightcone quantization scheme. Summary and Conclusions are followed in Section 5.

2. Nontrivial Vacuum Phenomena on the Lightcone

Since the novel phenomena are known to be realized from the nontrivial vacuum in the ordinary equal-t quantization as mentioned above, one can propose to interpolate the time axis between t and τ in order to trace the fate of the nontrivial vacuum and the vacuum expectation values in the limit to τ. The interpolation between t and τ can be given by an improper $SO(2)$ rotation matrix

$$\begin{pmatrix} x^+ \\ x^- \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1+C}{2}} & \sqrt{\frac{1-C}{2}} \\ \sqrt{\frac{1-C}{2}} & -\sqrt{\frac{1+C}{2}} \end{pmatrix} \begin{pmatrix} x^0 \\ x^3 \end{pmatrix}, \qquad (4)$$

where $x^0 = ct$, $x^3 = z$ and the interpolating parameter $C = -cos\theta$ when the angle between the ordinary time axis of x^0 and the interpolated time axis of x^+ is given by $\frac{\pi-\theta}{2}$. In the limit of $C = 0$ and 1, $x^+ = \frac{c\tau}{\sqrt{2}}$ and ct, respectively.

In order to show the main idea, let's consider a simple scalar field theory in 1+1 dimension with the Lagrangian density given by

$$\mathcal{L} = \mathcal{L}_0 - m^2 v\phi, \qquad (5)$$

where v is a constant field and the free Lagrangian density $\mathcal{L}_0$ is given by

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2. \qquad (6)$$

While $\mathcal{L}_0$ is invariant under the reflection symmetry $\phi \to -\phi$. $\mathcal{L}$ isn't because of the coupling of the scalar field ϕ to the constant source v as as one can see easily from Eqs.(5)

and (6). If we discretize the momentum of the quanta of ϕ field by imposing a periodic boundary condition in a line of length L and solve the Euler-Lagrange equation of $\mathcal{L}$, then the plane wave solution of the field operator ϕ is given by

$$\phi(x^+ = 0, x^-) = \sum_n^{integer} \frac{1}{\sqrt{4\pi\omega_n}}[a_n \exp(-i(\frac{n\pi}{L})x^-) + a_n^\dagger \exp(i(\frac{n\pi}{L})x^-)], \qquad (7)$$

where $a_n^\dagger$ and a_n are the creation and annihilation operators of the n'th mode quanta, $\omega_n = \sqrt{n^2 + C\hat{m}^2}$ and $\hat{m} = \frac{mL}{\pi}$. Using this solution, one can construct the Hamiltonian P_+ after normal ordering given by,

$$P_+ = (\frac{\pi}{L})[\sum_n(\frac{\omega_n - Sn}{C})a_n^\dagger a_n + \frac{\hat{m}^{3/2}\pi^{1/2}v}{C^{1/4}}(a_0 + a_0^\dagger)], \qquad (8)$$

where $S = \sin\theta$. The gound state $|\Omega\rangle$ and its energy E_Ω should satisfy the eigenvalue equation $P_+|\Omega\rangle = E_\Omega|\Omega\rangle$ and we obtain

$$|\Omega\rangle = \exp[-(C^{1/2}\pi\hat{m})\frac{v^2}{2}]\exp[-(C^{1/2}\pi\hat{m})^{1/2}va_0^\dagger]|0\rangle, \qquad (9)$$

where $|0\rangle$ is the perturbative vacuum and

$$E_\Omega = \int_{-L}^{L}(-\frac{1}{2}m^2v^2)dx^-. \qquad (10)$$

Thus, the ground state $|\Omega\rangle$ is the true vacuum and represents the coherent state of the zero momentum ($n = 0$) scalar quanta,*viz.*, zero modes. This shows that the zero modes of the scalar field condensate due to the coupling with the constant source v and the symmetry of the vacuum under $\phi \to -\phi$ is then broken. This is true as long as $C \neq 0$. However, in the lightcone limit ($C = 0$), $|\Omega\rangle = |0\rangle$ confirming the vacuum triviality on the lightcone as discussed in the previous section. Does that mean the nontrivial vacuum phenomena cannot occur on the lightcone? The answer is no! The reason is because the vacuum expectation value $\langle\Omega|\phi(x)|\Omega\rangle = -v$ and the ground state energy E_Ω given by Eq.(10) are independent from the interpolating parameter C. How can this happen? This happens due to the singular behavior of the field operator ϕ in the lightcone limit. As one can see in Eq.(7), the coefficient of $a_0^\dagger$ and a_0 diverges because $\omega_0 = 0$ in the limit of $C = 0$. Thus, the complication is transferred from the vacuum to the operator in lightcone limit. The similar phenomena can be seen in the axial vector current of the fermion field [5] and the axial anomaly [13] in the Schwinger model. Presumably, all the nontrivial vacuum phenomena may occur in the similar fashion on the lightcone.

3. Scattering Formalism

In order to discuss the lightcone quantization more physically, we consider the c.m. system of two particles where the initial and final momenta of the first (second) particle are $\boldsymbol{k}(-\boldsymbol{k})$and $\boldsymbol{l}(-\boldsymbol{l})$, respectively, and define the lightcone time τ as $\tau = t + \hat{\boldsymbol{n}}\cdot\boldsymbol{r}/c$ by introducing a unit vector $\hat{\boldsymbol{n}}$ on the lightcone surface (i.e. if $\tau = t + z/c$, then $\hat{\boldsymbol{n}} = \hat{z}$). In this reference frame, the lightcone two-body wavefunction, $\Psi(\boldsymbol{k},\hat{\boldsymbol{n}})$ satisfies the following equation in the lowest order Tamm-Dancoff approximation including the self-energy corrections and counter-terms [15] under the truncation of the Fock-space up to the three-body [11]:

$$(\boldsymbol{k}^2 - \boldsymbol{q}_{\rm in}^2)\Psi(\boldsymbol{k},\hat{\boldsymbol{n}}) = -\int \frac{d^3\boldsymbol{l}}{(2\pi)^3}\frac{m^2}{\epsilon(\boldsymbol{l})}V(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})\Psi(\boldsymbol{l},\hat{\boldsymbol{n}}), \tag{11}$$

where $\epsilon(\boldsymbol{l}) = \sqrt{\boldsymbol{l}^2 + m^2}$, $\boldsymbol{q}_{\rm in}^2 = s/4 - m^2$ (s is the square of the total c.m. energy), and

$$V(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) = \frac{4\pi\alpha V_{LA}(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})}{1 + (\alpha m^2/4\pi)g(\boldsymbol{l}^2, \hat{\boldsymbol{n}}\cdot\boldsymbol{l})}.$$

Here, the dimensionless coupling constant, α, is given by $\alpha = g^2/16\pi m^2$, the kernel in the ladder approximation, $V_{LA}(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})$, is given by

$$V_{LA} = -\left[(\boldsymbol{k}-\boldsymbol{l})^2 + \lambda^2 - (\hat{\boldsymbol{n}}\cdot\boldsymbol{k})(\hat{\boldsymbol{n}}\cdot\boldsymbol{l})\frac{(\epsilon(\boldsymbol{k})-\epsilon(\boldsymbol{l}))^2}{\epsilon(\boldsymbol{k})\epsilon(\boldsymbol{l})} + \left(\epsilon^2(\boldsymbol{k}) + \epsilon^2(\boldsymbol{l}) - \frac{s}{2}\right)\left|\frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{k}}{\epsilon(\boldsymbol{k})} - \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{l}}{\epsilon(\boldsymbol{l})}\right|\right]^{-1}, \tag{12}$$

and the self-energy corrections and counter-terms are summarized by

$$\begin{aligned} g(\boldsymbol{k}^2, \hat{\boldsymbol{n}}\cdot\boldsymbol{k}) &= \frac{4}{a(\boldsymbol{k}^2,\hat{\boldsymbol{n}}\cdot\boldsymbol{k})}\int_0^1 dz\log\left(1 + \frac{a(\boldsymbol{k}^2,\hat{\boldsymbol{n}}\cdot\boldsymbol{k})(z-z^2)}{\lambda^2 z + m^2(1-z)^2}\right) \\ &+ \frac{4}{b(\boldsymbol{k}^2,\hat{\boldsymbol{n}}\cdot\boldsymbol{k})}\int_0^1 dz\log\left(1 + \frac{b(\boldsymbol{k}^2,\hat{\boldsymbol{n}}\cdot\boldsymbol{k})(z-z^2)}{\lambda^2 z + m^2(1-z)^2}\right) \end{aligned} \tag{13}$$

with

$$\begin{aligned} a(\boldsymbol{k}^2,\hat{\boldsymbol{n}}\cdot\boldsymbol{k}) &= 2(\boldsymbol{k}^2 - \boldsymbol{q}_{\rm in}^2)\left(1 + \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{k}}{\epsilon(\boldsymbol{k})}\right), \\ b(\boldsymbol{k}^2,\hat{\boldsymbol{n}}\cdot\boldsymbol{k}) &= 2(\boldsymbol{k}^2 - \boldsymbol{q}_{\rm in}^2)\left(1 - \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{k}}{\epsilon(\boldsymbol{k})}\right). \end{aligned}$$

The conventional method to solve Eq.(11) is to set up an equivalent Lippman-Schwinger equation [16] which is given by

$$T(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) = V(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) - \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{m^2}{\epsilon(\boldsymbol{q})}\frac{V(\boldsymbol{k},\boldsymbol{q},\hat{\boldsymbol{n}})T(\boldsymbol{q},\boldsymbol{l},\hat{\boldsymbol{n}})}{\boldsymbol{q}^2 - \boldsymbol{q}_{\rm in}^2 + i\epsilon}. \tag{14}$$

Using the partial wave expansion of the scattering amplitude $T(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})$ given by

$$T_j(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) = \int d\Omega T(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})P_j(cos\theta), \tag{15}$$

where θ is the angle between $\boldsymbol{k}$ and $\boldsymbol{l}$, and similarly defining $V_j(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})$ as

$$V_j(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) = \int d\Omega V(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}})P_j(cos\theta), \tag{16}$$

we obtain

$$\begin{aligned} T_j(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) &= V_j(\boldsymbol{k},\boldsymbol{l},\hat{\boldsymbol{n}}) \\ &- \int \frac{|\boldsymbol{q}|^2 d|\boldsymbol{q}|}{(2\pi)^3}\frac{m^2}{\epsilon(\boldsymbol{q})}\frac{V_j(\boldsymbol{k},\boldsymbol{q},\hat{\boldsymbol{n}})T_j(\boldsymbol{q},\boldsymbol{l},\hat{\boldsymbol{n}})}{\boldsymbol{q}^2 - \boldsymbol{q}_{\rm in}^2 + i\epsilon}. \end{aligned} \tag{17}$$

The $\hat{\boldsymbol{n}}$-dependence in Eqs.(11)-(17) indicates the violation of the rotation invariance. However, we made a connection between our analysis and the bipolar harmonics formalism presented by Fuda [17] and found [6] that the physical amplitude suggested by Fuda is the rotational average of our $T_j(\boldsymbol{q}_{\rm in},\boldsymbol{q}_{\rm in},\hat{\boldsymbol{n}})$ over $\hat{\boldsymbol{n}}$-direction and does not carry $\hat{\boldsymbol{n}}$-dependence. Such procedure of integrating out the quantization axis dependence to obtain a physical amplitude is not possible in the ordinary equal t-quantization because the space for the boost operation is not compact. Integrating out the quantization axis dependence in equal t-quantization would necessarily require to include the lightcone surface. Therefore, the lightcone quantization method appears to be the most efficient way of solving quantum field theories. In order to quantify the dependence of the phase shift on the direction $\hat{\boldsymbol{n}}$, we fix the scattering plane as the plane made by $\hat{y}$ and $\hat{z}$ and the direction of initial momentum $\boldsymbol{k}$ as $\hat{z}$ and then vary the direction $\hat{\boldsymbol{n}}$. The effect of rotating the direction $\hat{\boldsymbol{n}}$ in a given scattering plane defined by its perpendicular direction $\boldsymbol{k}\times\boldsymbol{l}$ is equivalent to the effect of rotating $\boldsymbol{k}\times\boldsymbol{l}$ in a given direction of the lightcone time evolution, e.g., $\tau = t + z/c$. In any case, the focus of study is the dynamics dependent on the relative angle between $\hat{\boldsymbol{n}}$ and $\boldsymbol{k}\times\boldsymbol{l}$.

4. Numerical Results

We calculated both S-wave ($j = 0$) and P-wave ($j = 1$) phase shifts for various coupling constants ($\beta = \frac{\alpha}{\pi}$) and c.m. momenta. Since the detailed numerical results were presented in our recent papers [6, 12], we discuss only the main features of the numerical results. For the small β (*e.g.*, $\beta = 0.1$), the light-cone results for $\hat{\boldsymbol{n}} = \hat{x},\hat{y}$ and $\hat{z}$ are almost same whether the self energy corrections are included or not. As β gets larger(*e.g.* $\beta > 0.3$), however, one can see that the three lightcone results for $\hat{\boldsymbol{n}} = \hat{x},\hat{y}$, and $\hat{z}$ deviate. The results including the self-energy corrections are consistently lower than the ones without them, indicating that the self-energy effects are repulsive. Also, we observe that the phase shifts with the self-energy correction do not change much as the coupling constant grows. This is in a good agreement with the previous bound state results that the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects, and make the binding energy be frozen as the coupling constant increases [11]. The similar results were obtained in the generalized theory of the Wick-Cutkosky model using discretized lightcone quantization [18] and in the Yukawa model [19]. Furthermore, as we can observe from the numerical results, the deviations among $\hat{\boldsymbol{n}} = \hat{x}, \hat{y}$, and $\hat{z}$ are smaller after the self-energy corrections are included. Such reduction of the $\hat{\boldsymbol{n}}$-dependence is more dramatic in the P-wave analyses. Especially, as shown in Figure 1 attached at the end of this paper, the dramatic falloff [6] of the phase shift with $\hat{\boldsymbol{n}} = \hat{z}$ in the large c.m. momentum region ($\boldsymbol{k}^2/m^2 > 1$) shown for $\beta = 20$ disappears completely by the self-energy corrections. This indicates a significant restoration of the rotational invariance in the scattering kernel by adding the self-energy interactions. It shows an example that the rotation symmetry in the lightcone quantization can be dynamically restored.

5. Summary and Conclusions

In this paper, we discussed the most challenging two fundamental issues in the lightcone field quantization; the nontrivial vacuum structure and the restoration of the rotational symmetry. First, using the interpolation of the time axis between t and τ we showed that the vacuum becomes simple in the lightcone limit ,however, the realization of the nontrivial vacuum is not lost in this limit. This is possible due to the generation of the singular part of some field operators. Thus, it brings a caution in handling the field operators on the lightcone and one needs to distinguish the singular operators from the regular operators. If the reference frame is chosen to make the singular operators irrelevant, the phenomenology with the lightcone quantization would be extremely useful [14].

Second, we presented an explicit illustration of the rotation dependence in the two-body scattering phase shifts. Practical computations using the light-cone quantization method require, in general, the truncation of the higher Fock states. As a consequence, the calculated scattering amplitude in the truncated Fock-space is not rotationally invariant because the transverse angular momentum operator whose direction is perpendicular to the direction of the quantization axis in the light-cone quantization method involves the interaction that changes the particle number. However, in view of the rotational compactness, the lightcone quantization appears to be most efficient in solving quantum field theories. The extent of the rotation symmetry breaking can be quantified by the explicit rotation dependence of the two-body scattering phase shifts. In a recent work [12], we investigated the scattering problem in the light-cone formalism using a simple scalar field model by extending the lightcone ladder approximation to the lowest order lightcone Tamm-Dancoff approximation which includes the self-energy corrections and counter-terms. We found that the self-energy interactions significantly restore the rotation symmetry and remove the dramatic falloff of the phase shifts observed [6] in the P-wave analysis with the large coupling and the large momentum. It shows an example that the rotation symmetry in the lightcone quantization can be dynamically restored. Also, we observe that the self-energy effects make the phase shifts frozen as the coupling constant is increased. This is in a good agreement with the previous bound state results that the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects.

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Figure 1: P-wave phase shifts with the coupling constant $\beta = \alpha/\pi = 20.0$. The solid, long-dashed and short-dashed curves are the light-cone scattering results with $\hat{n} = \hat{x}, \hat{y}$, and $\hat{z}$, respectively, in the lightcone ladder approximation. The dotted, long-dash-dot and short-dash-dot curves are the corresponding results with $\hat{n} = \hat{x}, \hat{y}$, and $\hat{z}$, respectively, in the lowest order lightcone Tamm-Dancoff approximation.