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A Fourth Generation of Quarks Without Leptons

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Abstract

We discuss a class of models having the same general structure as the Standard Model. In particular we consider a model based on the gauge group $U(1) \otimes SU(2) \otimes SU(3) \otimes SU(5)$ supplemented with a natural generalisation of the Standard Model charge quantisation rule. The simplest solution of the gauge anomaly cancellation conditions, with mass protected fermions, involves a fourth generation of quarks without leptons, but having the anomalies cancelled by a generation of $SU(5)$ "quarks". We discuss the phenomenology of this solution and, in particular, show that the infrared quasifixed point values of the Yukawa coupling constants put upper limits on the new fermion masses close to the present experimental bounds.

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1 Introduction

In this paper we consider extensions of the Standard Model (SM) which have the same distinctive features as the SM itself. We are thereby led to consider gauge groups of the type

$$SMG_{2,3,N} \equiv U(1) \otimes SU(2) \otimes SU(3) \otimes SU(N)/D_N \qquad (1)$$

where N is greater than 3 and is not divisible by 2 or 3. The discrete group

$$D_{6N} \equiv \{(e^{i\pi/3N}, -I_2, e^{i2\pi/3}I_3, e^{i2\pi m_N/N}I_N)^r : r \in \mathcal{Z}_{6N}\} \qquad (2)$$

gives the charge quantisation rule (I_N is the identity of $SU(N)$):

$$\frac{y}{2} + \frac{1}{2}d + \frac{1}{3}t + \frac{m_N}{N}n \equiv 0 \pmod 1 \qquad (3)$$

where y is the conventional weak hypercharge, d is the duality, t is the triality and n is the N-ality of a representation; also m_N is an integer which is not a multiple of N. In fact we can obviously choose $0 \leq m_N \leq N-1$ since m_N is really only defined modulo N. The duality has value 1 if the representation is an $SU(2)$ doublet and 0 if it is an $SU(2)$ singlet. The triality has value 1 if the representation is an $SU(3)$ triplet, 0 if it is an $SU(3)$ singlet, and -1 if it is an $SU(3)$ anti-triplet. In general we can define N-ality to be the number of N-plet representations of $SU(N)$ which must be combined to give the representation of $SU(N)$. In particular N-ality has value 1 if a representation is an $SU(N)$ N-plet, 0 if it is an $SU(N)$ singlet, and -1 if it is an $SU(N)$ anti-N-plet. Note that in $SU(2)$ the $\overline{2}$ representation is equivalent to the 2 representation.

This $SMG_{2,3,N}$ class of groups is the minimal extension of the Standard Model Group [1] (SMG) [1, 2]

$$SMG \equiv S(U(2) \otimes U(3)) = U(1) \otimes SU(2) \otimes SU(3)/D_6 \qquad (4)$$

$$D_6 \equiv \{(e^{i2\pi/6}, -I_2, e^{i2\pi/3}I_3)^n : n \in \mathcal{Z}_6\} \qquad (5)$$

which has the same features of being essentially a cross product of several factors supplemented by a non-trivial charge quantisation rule eq.(3). We feel that the SMG charge quantisation rule

$$\frac{y}{2} + \frac{1}{2}\text{"duality"} + \frac{1}{3}\text{"triality"} \equiv 0 \pmod 1 \qquad (6)$$

[1] $S(U(2) \otimes U(3))$ is of course a proper subgroup of $SU(5)$

is a fundamental feature of the Standard Model (SM), giving a sophisticated quantisation in the sense that all 3 cross-product factors of SMG are involved in it and $y/2$ determines both the duality and triality. Another feature that we take over from the SM is the principle of using only small (fundamental or singlet) fermion representations; a feature not shared with supersymmetric models, where the gauginos are in adjoint repesentations.

We will consider the fundamental scale to be the Planck mass (M_{Planck}) and our models will be a full description of physics without gravity below this scale. By requiring an anomaly free theory and the absence of any Landau poles below M_{Planck}, we obtain an essentially unique extension of the SM with an extra generation of quarks, but with the extra generation of leptons replaced by fermions in the fundamental representations of the $SU(N)$ component of $SMG_{2,3,N}$.

We assume that the extra $SU(N)$ component of the gauge group is not spontaneously broken and therefore confines forming fermion condensates. As we already know from the SM, the $SU(3)$ component acts as a technicolour group [3] and gives a contribution to the $W^{\pm}$ and Z^0 masses. In the SM this contribution is very small but when confining groups with $N > 3$ are considered we must carefully consider the effect this will have. Since we are not wanting the complications of extended technicolour in order to generate quark and lepton masses, we assume that there is a Higgs doublet and that the masses of the weak gauge bosons are generated by a combination of the Higgs sector of the theory and the technicolour effects of the gauge groups. Thus the extra $SU(N)$ component of the gauge group acts only as a partial technicolour [4]. It follows that the $SU(N)$ confinement scale must be at the Tev scale or below.

In order to avoid conflict with precision electroweak data, it is not phenomenologically consistent to introduce a large number of extra $SU(2)_L$ doublets. We shall therefore concentrate on the phenomenology of the $SMG_{2,3,5}$ model which has the minimal value of $N = 5$.

2 Structure of the $SMG_{2,3,N}$ Model

The three SM generations form representations of $SMG_{2,3,N}$ and we now wish to consider a new "generation" of fermions coupling to the $SU(N)$ component of the group. We assume all the fermions get a mass via the SM Higgs mechanism and, by analogy with the SM quarks, we consider a generation of $SU(N)$ "quarks" made up of the left-handed fermion representation $(y, 2, N)$ together with the left-handed anti-fermion representations $(-[y+1], 1, \overline{N})$ and $(-[y-1], 1, \overline{N})$. The weak hypercharge y must of course satisfy the charge quantisation rule eq.(3). We now consider the cancellation of the anomalies generated by such a generation of $SU(N)$ "quarks" and the technicolour effects of vacuum condensates formed from them.

2.1 Anomaly Cancellation

The contributions to each type of anomaly from this generation of $SU(N)$ "quarks" are:

$$
\begin{array}{lcll}
[SU(N)]^3 & \rightarrow & & 0 \\
[SU(N)]^2 U(1) & \rightarrow & 2y - (y+1) - (y-1) & = 0 \\
[Grav]^2 U(1) & \rightarrow & 2y - (y+1) - (y-1) & = 0 \\
[U(1)]^3 & \rightarrow & N[2y^3 - (y+1)^3 - (y-1)^3] & = -6Ny \\
[SU(2)]^2 U(1) & \rightarrow & & Ny
\end{array}
$$

So we must cancel the resulting $[SU(N)]^2 U(1)$ and $[U(1)]^3$ anomalies against other fermion representations in order to obtain a consistent theory. We can in fact cancel the anomalies against a fourth generation of $SU(3)_C$ quarks, in just the same way that the quark anomalies cancel the lepton anomalies in the three SM generations [2]. Alternatively we could take the $SU(N)$ "quark" generation to belong to the conjugate of the above representations and cancel its anomalies against a fourth lepton generation. However this is not a phenomenologically viable alternative, since we know from the invisible partial width of the Z boson that there are only three massless (or light) neutrinos.

The non-zero anomalies due to a fourth generation of quarks are:

$$
\begin{array}{lcll}
[U(1)]^3 & \rightarrow & 3[2(\frac{1}{3})^3 - (\frac{4}{3})^3 + (\frac{2}{3})^3] & = -6 \\
[SU(2)]^2 U(1) & \rightarrow & 3 \cdot \frac{1}{3} & = 1
\end{array}
$$

[2] The resulting generation of $SU(3)$ quarks and $SU(5)$ "quarks" is the smallest mass-protected anomaly free representation which couples non-trivially to all the components of the gauge group $SMG_{2,3,N}$

Table 1: Fermions coupling to $SU(N)$ which would form an anomaly-free set of fermions together with a fourth generation of quarks.

Representation under $SU(2) \otimes SU(3) \otimes SU(N)$	$U(1)$ Representation $\frac{y}{2}$	Electric Charge Q
$2, 1, N$	$-\frac{1}{2N}$	$\begin{pmatrix} \frac{N-1}{2N} \\ -\frac{N+1}{2N} \end{pmatrix}$
$1, 1, \overline{N}$	$-\frac{N-1}{2N}$	$-\frac{N-1}{2N}$
$1, 1, \overline{N}$	$\frac{N+1}{2N}$	$\frac{N+1}{2N}$

Anomaly cancellation between a generation of $SU(N)$ "quarks" and a fourth generation of $SU(3)_C$ quarks gives the condition:

$$Ny + 1 = 0 \tag{7}$$

and hence

$$y = -\frac{1}{N} \tag{8}$$

However the weak hypercharge y of the $SU(N)$ "quarks" must satisfy the charge quantisation rule eq.(3), which takes the form:

$$y = 2J - 1 - \frac{2m_N}{N} \tag{9}$$

where J is an integer. So we have a solution with $J = 1$ and

$$m_N = \frac{1}{2}(N + 1) \tag{10}$$

Table 1 shows the properties of the left-handed fermions belonging to such a generation of $SU(N)$ "quarks". Note that this is a generalisation of the SM quarks, coupling to $SU(N)$ with the specific choice of $m_N = \frac{1}{2}(N+1)$. If we set $N = 3$ we would in fact get a generation of quarks with the opposite chirality to those in the SM. This is to be expected since we are using these fermions to cancel the anomaly contribution of a 4th generation of SM quarks (with the usual chirality).

Finally we remark that N is odd and hence Table 1 contains an odd number of SU(2) doublets. Combined with the three $SU(2)$ doublets in a fourth generation of quarks, this gives an even number of doublets and hence there is no Witten discrete $SU(2)$ anomaly [5].

2.2 Technicolour Contributions

We assume that the $SU(N)$ component of the gauge group confines and that the $SU(N)$ "quarks" form condensates. These condensates have the same quantum numbers as the SM Higgs boson and contribute to the $W^{\pm}$ and Z^0 masses, via the usual technicolour [3] mechanism.

We stress that we are not proposing a technicolour model as such, but simply taking into account the unavoidable effect that adding an $SU(N)$ group has. We are assuming that the Higgs sector of our models is the same

These upper limits are obtained by using quasi-fixed-point values for the Yukawa coupling constants, y_f, as determined from the renormalisation group equations (RGEs) assuming a desert above the TeV scale up to the Planck scale.

3 Phenomenology of the $SMG_{2,3,5}$ Model

We shall now discuss the minimal extension of the SM in our class of models. It is based on the gauge group $SMG_{2,3,5}$ and, in addition to the three SM generations, contains a fourth generation of quarks and a single generation of $SU(5)$ "quarks". The representations of the left-handed fermions which couple to the $SU(5)$ subgroup are shown in table 2. This is a generalisation of the quarks in the SM, coupling to $SU(5)$ rather than $SU(3)$.

Table 2: Left-handed fermions coupling to $SU(5)$. The electric charges are in units of $\frac{1}{5}$ due to the charge quantisation rule.

Representation under $SU(2) \times SU(3) \times SU(5)$	$U(1)$ Representation $\frac{y}{2}$	Electric Charge Q
$2, 1, 5$	$-\frac{1}{10}$	$\begin{pmatrix} \frac{2}{5} \\ -\frac{3}{5} \end{pmatrix}$
$1, 1, \overline{5}$	$-\frac{4}{10}$	$-\frac{2}{5}$
$1, 1, \overline{5}$	$\frac{6}{10}$	$\frac{3}{5}$

We will first consider the experimental constraints on the model and then discuss its predictions for the fine structure constants and new fermion masses in the model.

3.1 Experimental Constraints on Fermion Masses

The limits on the fermion masses are dependent on the type of particle and its decay modes. We will consider first the usual SU(3) quarks and then the $SU(5)$ "quarks".

The top quark has recently been observed by the CDF [6] and D0 [7] collaborations. The mass is in the range 150-220 GeV. We use the dilepton mode analyses of the CDF [8] and D0 [9] groups to place a lower limit on the possible masses of a fourth generation of t' and b' quarks. If the t' quark is lighter than the b' quark, it is expected to decay via the mode $t' \to bW^+$ and hence give a dilepton signal similar to the top quark. If the b' quark is lighter than the t' and top quarks, it is expected to decay via the mode $b' \to cW^-$ and again give a similar dilepton signal. So we take the limit on the masses of possible fourth generation quarks, t' and b', to be

$$m_{t'}, m_{b'} > 130 \text{ GeV}$$

from the dilepton analyses of the CDF [8] and D0 [9] groups (less restrictive limits apply if other decay modes are dominant).

The above experimental limits do not apply to possible $SU(5)$ "quarks". These fermions would be more difficult to produce and detect at hadron colliders. They would anyway be confined inside 'hadrons' with a confinement scale typically expected to be of order 200 Gev. So we conclude that the $SU(5)$ "quarks" would be unlikely to be detected with current accelerators.

3.2 Precision Electroweak Data

Measurements of electroweak interactions are now accurate enough to be sensitive to loop corrections to propagators and vertex corrections. These effects are model dependent and can be sensitive to the values of some parameters such as fermion and Higgs masses. So far there is no evidence for deviations from the three generation SM and the precision electroweak measurements can be used to set limits on possible new physics at the electroweak scale. For example, the closeness of the observed value of the ρ parameter ($\rho \equiv \frac{M_W^2}{M_Z^2 cos^2\theta_W}$) to unity [10] indicates that the mass squared differences within any new fermion $SU(2)$ doublets must be small ($\ll (100 \text{ GeV})^2$).

More generally any new physics which affects only the gauge boson self-energies can be parameterised by the S, T and U parameters [11] or some equivalent set of parameters [12, 13]. The T parameter is a measure of the loop corrections to the ρ parameter and is induced by mass splittings within new fermion $SU(2)$ multiplets. We shall assume that these mass splittings are small in our model. So the new physics contribution T_{new} to the T parameter in our model can be neglected. Also U_{new} can be neglected, as in most other extensions of the SM.

The S parameter provides a strong constraint on the number of new fermion doublets, since perturbatively a mass degenerate doublet contributes $\frac{1}{6\pi}$ to S. Analysis of the precision electroweak data gives [14]:

$$S_{new} = -0.21 \pm 0.24^{-0.08}_{+0.17} \tag{12}$$

where the second error is from the Higgs mass M_H. The central value is for $M_H = 300$ GeV, the upper second error for $M_H = 1000$ GeV and the lower one for $M_H = 60$ GeV. There are 8 new $SU(2)$ doublets in our $SMG_{2,3,5}$ model, made up of a fourth generation of quarks and a generation of $SU(5)$ "quarks". In a single SM generation, there are 4 doublets. So treating all the new fermions perturbatively [3], their contribution to S_{new} is equivalent to that of two SM generations: $S_{new} = \frac{4}{3\pi} \sim 0.42$. Thus for $M_H = 60$ GeV, our model deviates by ~ 2 standard deviations from the experimental value of S_{new}. Our model is therefore just consistent with the precision electroweak data.

3.3 Running Fine Structure Constants

Using the RGEs we now investigate how the gauge coupling constants vary with energy up to the Planck scale. Here we set the thresholds for all the unknown fermions (4th generation quarks and fermions coupling to $SU(5)$), as well as for the top quark and Higgs boson, to M_Z. The absence of Landau poles in this case will guarantee their absence if some of the thresholds are set higher than M_Z. From experimental limits we would expect that all these thresholds should be greater than M_Z.

There are four fine structure constants, which we shall label by α_1, α_2, α_3 and α_5, corresponding to the four gauge groups $U(1)$, $SU(2)$, $SU(3)$ and $SU(5)$ respectively. The fine structure constants, α_i, are related to the gauge coupling constants, g_i, by the relation $\alpha_i = \frac{g_i^2}{4\pi}$. The equations governing the running coupling constants to first order in perturbation theory [16, 17] (a good discussion of RGEs in the SM is given in [18]) can be integrated

[3]Non-perturbative corrections, estimated by analogy with QCD, tend to increase the $SU(5)$ fermion contribution to S [15]

analytically to give

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_1(\mu_0)} - \frac{1}{12\pi}\left(Y^2 + n_H\right) ln\left(\frac{\mu}{\mu_0}\right) \quad (13)$$

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_2(\mu_0)} + \frac{1}{12\pi}(44 - 2n_{2f} - n_H) ln\left(\frac{\mu}{\mu_0}\right) \quad (14)$$

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(\mu_0)} + \frac{1}{12\pi}(66 - 2n_{3f}) ln\left(\frac{\mu}{\mu_0}\right) \quad (15)$$

$$\frac{1}{\alpha_5(\mu)} = \frac{1}{\alpha_5(\mu_0)} + \frac{1}{12\pi}(110 - 2n_{5f}) ln\left(\frac{\mu}{\mu_0}\right) \quad (16)$$

where we calculate $\alpha_i(\mu)$ (the running coupling constants at the energy scale $\mu > \mu_0 = M_Z$) in terms of $\alpha_i(M_Z)$. $Y^2 \equiv \sum y^2$ is the sum of the weak hypercharges squared for all the fermions and n_{mf} are the number of fermion m and $\overline{m}$ representations of $SU(m)$. n_H is the number of Higgs doublets.

From [19] we find

$$\alpha_1^{-1}(M_Z) = 98.08 \pm 0.16 \quad (17)$$

$$\alpha_2^{-1}(M_Z) = 29.794 \pm 0.048 \quad (18)$$

$$\alpha_3^{-1}(M_Z) = 8.55 \pm 0.37 \quad (19)$$

We can now use the above equations to examine how the coupling constants behave up to the Planck scale in our model with 1 Higgs doublet, 3 SM generations, a fourth quark generation and a generation of $SU(5)$ "quarks". Since there is no experimental value for α_5 at any energy scale we shall, for definiteness, assume that $\alpha_5^{-1}(M_Z) = 2$ (the precise value is unimportant) so that the $SU(5)$ interaction is stronger than QCD at M_Z and confines at the electroweak scale. Fig. 1 shows what happens for each group. For the graphs we normalise the $U(1)$ gauge coupling as if the $U(1)$ group was embedded in a simple group. This essentially corresponds to redefinition of g_1.

$$(g_1^2)_{\rm GUT} \equiv \frac{5}{3}(g_1^2)_{\rm SM} \quad (20)$$

$$(\alpha_1^{-1})_{\rm GUT} \equiv \frac{3}{5}(\alpha_1^{-1})_{\rm SM} \quad (21)$$

So henceforth we use the standard GUT normalisation. Eqs. (13) and

(17) now become,

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_1(\mu_0)} - \frac{1}{20\pi}\left(Y^2 + n_H\right) ln\left(\frac{\mu}{\mu_0}\right) \quad (22)$$

$$\alpha_1^{-1}(M_Z) = 58.85 \pm 0.10 \quad (23)$$

We use eqs. (14)-(16) and (22) to run the gauge coupling constants up to the Planck scale as shown in fig. 1. We see that there are no problems with Landau poles below the Planck scale and our model is perturbatively consistent.

3.4 Upper Limits for Yukawa Couplings

As stated in section 3.1, we take the limits on the masses of a fourth generation of quarks to be $m_{b'} > 130$ GeV, $m_{t'} > 130$ GeV and the top quark mass to be $m_t \sim 170$ GeV. We can now use the RGEs equations to estimate upper limits on the values of the Yukawa couplings to the SM Higgs field of these fermions. This will lead to upper limits on the masses indicating that the t' and b' quarks would be almost within reach of present experiments.

Now we can choose initial values for the Yukawa couplings at the Planck scale and use the RGEs to see how they evolve, as they are run down to the electro-weak scale. Assuming no mixing for the quarks and neglecting the masses of all SM fermions except the top quark (a good approximation), the RGEs are, to one loop order in perturbation theory [17]:

$$\frac{dy_t}{dt} = y_t\frac{1}{16\pi^2}\left(\frac{3}{2}y_t^2 + Y_2(S) - G_{3u}\right) \quad (24)$$

$$\frac{dy_{t'}}{dt} = y_{t'}\frac{1}{16\pi^2}\left(\frac{3}{2}(y_{t'}^2 - y_{b'}^2) + Y_2(S) - G_{3u}\right) \quad (25)$$

$$\frac{dy_{b'}}{dt} = y_{b'}\frac{1}{16\pi^2}\left(\frac{3}{2}(y_{b'}^2 - y_{t'}^2) + Y_2(S) - G_{3d}\right) \quad (26)$$

$$\frac{dy_{5u}}{dt} = y_{5u}\frac{1}{16\pi^2}\left(\frac{3}{2}(y_{5u}^2 - y_{5d}^2) + Y_2(S) - G_{5u}\right) \quad (27)$$

$$\frac{dy_{5d}}{dt} = y_{5d}\frac{1}{16\pi^2}\left(\frac{3}{2}(y_{5d}^2 - y_{5u}^2) + Y_2(S) - G_{5d}\right) \quad (28)$$

where the $SU(5)$ fermions have been labelled 5u and 5d as generalisations of the naming of $SU(3)$ quarks. The other variables are defined as

$$Y_2(S) = 5y_{5u}^2 + 5y_{5d}^2 + 3y_{t'}^2 + 3y_{b'}^2 + 3y_t^2 \quad (29)$$

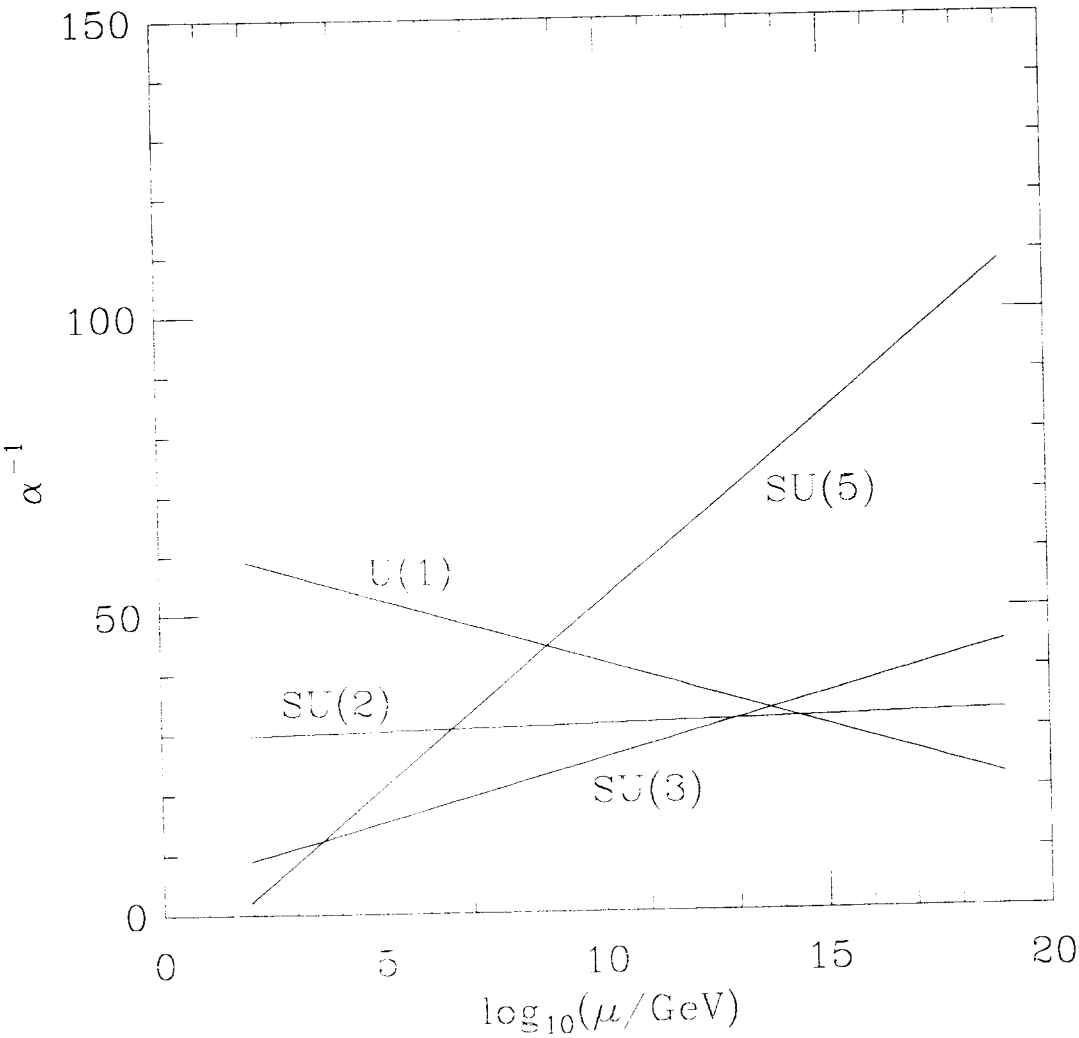

Figure 1: α^{-1} from M_Z to the Planck scale for each group. The initial value for $\alpha_5(M_Z)$ was chosen so that it would be confine at the electroweak scale. There are obviously no Landau poles so this model is self-consistent.

$$G_{3u} = \frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \tag{30}$$

$$G_{3d} = \frac{1}{4}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2 \tag{31}$$

$$G_{5u} = \frac{153}{500}g_1^2 + \frac{9}{4}g_2^2 + \frac{72}{5}g_5^2 \tag{32}$$

$$G_{5d} = \frac{333}{500}g_1^2 + \frac{9}{4}g_2^2 + \frac{72}{5}g_5^2 \tag{33}$$

Here $Y_2(S)$ is really $Tr(Y^\dagger Y)$ where Y is the Yukawa matrix for all the fermions.

We can choose values for the Yukawa couplings at the Planck scale and then use the RGEs to see what values the Yukawa couplings will have at any other scale. We have chosen the low energy scale to be M_Z as shown in fig. 2. We observe fixed points similar to the case for the top quark in the SM [18] and these will provide upper limits on the fermion masses. However, the Yukawa coupling for any fermion at M_Z depends on the Yukawa couplings of the other fermions. But there is an approximate infrared fixed point limit on $Y_2(S)$ and so one Yukawa coupling can be increased at the expense of the others. This limit is quite precise if there is only one strong interaction at low energies such as QCD in the SM [4]. We observe numerically that $Y_2(S) \approx 7.5 \pm 0.3$ provided the Yukawa couplings of the three heavy quarks are greater than 1 at the Planck scale and that the Yukawa couplings of the fermions coupling to the $SU(5)$ gauge group are less than the Yukawa couplings of the heavy quarks at the Planck scale.

The values chosen for fig. 2 have been chosen so that $m_t \approx 170$GeV and the fourth generation quark masses are above the current experimental limit of 130 GeV. Also $m_{b'} \sim m_{t'}$ and $m_{5u} \sim m_{5d}$ have been chosen so that there is only a small contribution to the ρ parameter described in section 3.2. Table 3 gives the value of the Yukawa couplings at M_Z and the corresponding masses neglecting the technicolour contribution to the VEV, $v = 246$GeV. Therefore these masses should be considered upper limits on the masses of the fermions for this particular choice of Yukawa couplings at the Planck scale. For other choices of Yukawa couplings at the Planck scale we could, for example, increase the mass of the fourth generation of quarks but this would have to be compensated for by a reduction in the mass of some of the

[4] Detailed results for a general number of heavy SM generations are derived in [20].

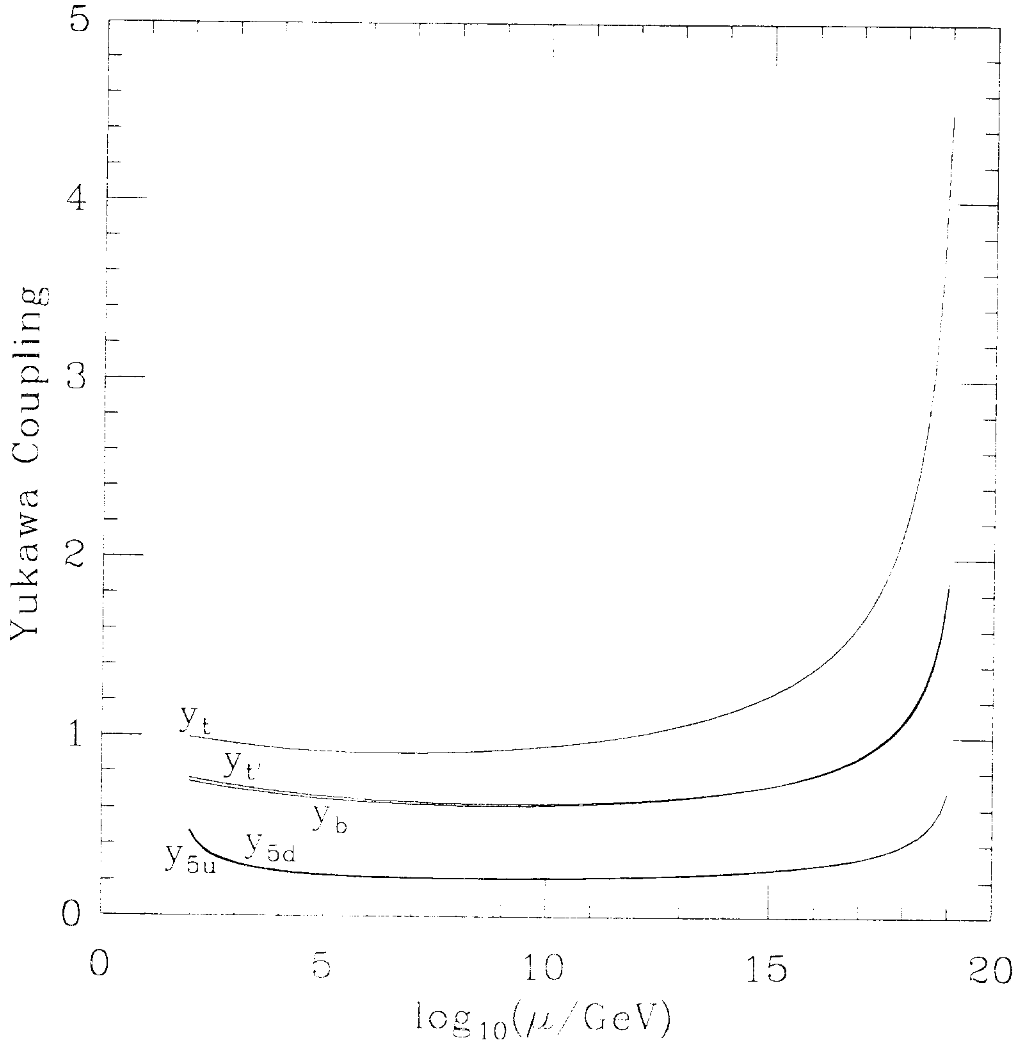

Figure 2: An example of running Yukawa couplings for all fermions with a mass the same order of magnitude as the electroweak scale. The values were chosen at the Planck scale and run down to M_Z so that all the fermions would have a mass allowed by current experimental limits.

Table 3: Infrared fixed point Yukawa couplings and corresponding maximum masses (corresponding to no $SU(5)$ condensate contribution to the VEV) for a particular choice of Yukawa couplings at the Planck scale.

Fermion	Yukawa Coupling	Maximum Mass (GeV)
y_t	0.99	172
$y_{t'}$	0.76	132
$y_{b'}$	0.75	131
y_{5u}	0.47	82
y_{5d}	0.49	85

other fermions.

These values for the masses are consistent with current experimental limits but are not so high that all the new fermions could remain undetected for long. In fact the quark masses may even be within the limits of current accelerators. However it is unlikely that the fermions coupling to $SU(5)$ could be observed: they would be confined inside $SU(5)$ "hadrons", with a confinement scale of order 200 Gev, and would have a small production cross section at hadron colliders. For this reason we consider the clearest evidence for this model would come from the detection of a fourth generation quark. The masses of some fermions could be increased, but not by much, since this would mean a reduction in the mass of other fermions. This means that this model is consistent and relatively easy to test.

4 Conclusion

We have described a class of extensions of the SM, having gauge groups with similar characteristic properties to those of the SMG. In particular we introduce generalised charge quantisation rules. The simplest extension of this type, based on the gauge group $SMG_{2,3,5}$, involves an $SU(5)$ technicolour interaction, but with the fermion masses generated by the usual SM Higgs mechanism. The smallest anomaly free representation of mass protected fermions, involving all components of $SMG_{2,3,5}$, contains a fourth generation of quaks without leptons together with a generation of $SU(5)$ "quarks". This $SMG_{2,3,5}$ model is consistent with experiment, predicting the existence of t' and b' quarks with masses at or below the top quark mass scale. So the model is relatively easy to test.

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