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THE MEASURE OF THE INDEX OF REFRACTION OF A GAS WITH THE REFRACTOMETER

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The principle of operation of our refractometer (Rayleigh type) is shown in Fig. 1.

The cell of length X_c is pneumatically separated from the rest, so that gases at different pressures can fill the regions of space denoted by (1) and (2).

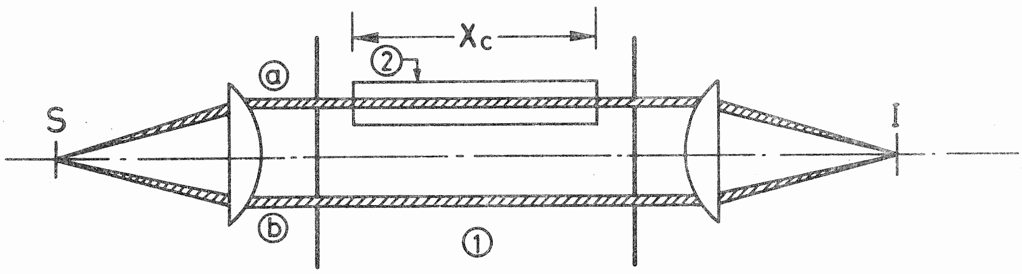

Fig. 1

The light intensity at the image point I can be written as :

$$I = I_o \cos \omega\left(t - \frac{X_{a,b}}{v_{1,2}}\right) = I_o \cos 2\pi \left(\nu t - \frac{X_{a,b}}{\lambda_{1,2}}\right) \qquad (1)$$

where $X_{a,b}$ are the path lengths between S and I along the trajectories ⓐ and ⓑ, and $\lambda_{1,2}$ the wave-lengths of the light of frequency ν in the two media ① and ②

$$\lambda_{1,2} = \lambda_v / n_{1,2} \qquad (2)$$

here $n_{1,2}$ are the absolute refractive indexes in ① and ② and λ_v the wave-length in the vacuum for the used source.

The phase difference in I between the light paths ① and ② is, from (1) and (2) :

$$\Delta\phi = \frac{2\pi}{\lambda_v} \left\{ \int_b n_i \, ds - \int_a n_i \, ds \right\} = 2\pi N_f \qquad (3)$$

where N_f is the number of fringes.

Denoting by X_a and X_b the lengths of the paths ⓐ and ⓑ between S and I, and by X a possible difference between them, due to some misalignment of the optics,

$$X = X_b - X_a \ ,$$

the phase difference (3) gives

$$\lambda_v N_f = \left\{ X_b n_1 - \left[(X_a - X_c) n_1 + X_c n_2 \right] \right\} = X_c (n_1 - n_2) + X \cdot n_1 \ . \qquad (3')$$

The information from the refractometer is in terms of 16th's of fringe

$$N = 16 N_f \qquad (4)$$

and it can be affected by an error, N_D, due to a misalignment of the diodes by which the fringes are detected, so that the number which is really measured is :

$$N = \frac{16}{\lambda_v} \left\{ X_c (n_1 - n_2) + n_1 \cdot \delta x \right\} + N_D \ . \qquad (4')$$

Four types of measure are possible

1. $n_1 = n_2 = 1 \rightarrow N^{(1)} = \frac{16}{\lambda_v} \cdot X + N_D$ vacuum everywhere

2. $n_1^{(2)} = n_2^{(2)} > 1 \rightarrow N^{(2)} = \frac{16}{\lambda_v} n_1^{(2)} \cdot X + N_D$ gas everywhere

3. (*) $n_1 = 1,\ n_2^{(3)} > 1 \rightarrow N^{(3)} = \frac{16}{\lambda_v} \left\{ X_c (1 - n_2^{(3)}) + X \right\} + N_D$ vacuum in the tank

4. $n_1^{(4)} > 1,\ n_2 = 1 \rightarrow N^{(4)} = \frac{16}{\lambda_v} \left\{ X_c (n_1^{(4)} - 1) + n_1^{(4)} \cdot X \right\} + N_D$ vacuum in the cell

From these one can derive

$$\left.\begin{aligned}
N_D &= N^{(1)} - \frac{16}{\lambda_v} \cdot X \\
X &= \frac{\lambda_v}{16(n_1^{(2)}-1)} \left(N^{(2)} - N^{(1)}\right) \\
1 - n_2^{(3)} &= \frac{\lambda_v}{16\ X_c} \left(N^{(3)} - N^{(1)}\right) \\
n_1^{(4)} - 1 &= \frac{\lambda_v \left(N^{(4)} - N^{(1)}\right)}{16\ X_c + \dfrac{\lambda_v \left(N^{(2)} - N^{(1)}\right)}{(n_1^{(2)} - 1)}}
\end{aligned}\right\} \quad (5)$$

If the same gas under identical thermodynamic conditions (temperature and pressure) is used in the measures 2 and 3, then

$$n_1^{(2)} = n_2^{(3)} \qquad (6)$$

and by using $(5)_3$, one finds finally :

(*) In the measure 3 the pressure in the cell must not exceed 1 bar.

$$\left.\begin{aligned} X &= -X_c \frac{N^{(2)} - N^{(1)}}{N^{(3)} - N^{(1)}} \\ N_D &= N^{(1)} + \frac{16\,X_c}{\lambda_v} \frac{N^{(2)} - N^{(1)}}{N^{(3)} - N^{(1)}} \end{aligned}\right\} \quad (7)$$

$$\boxed{n_l^{(4)} - 1 = \frac{\lambda_v}{16\,X_c} \frac{(N^{(4)} - N^{(1)})\ (N^{(3)} - N^{(1)})}{N^{(3)} - N^{(2)}} = (n-1)^{*} \cdot \varepsilon(X)} \quad (8)$$

$$\varepsilon(X) = \frac{N^{(3)} - N^{(1)}}{N^{(3)} - N^{(2)}} = \text{correction factor for } X\,. \quad (9)$$

If, in particular, $\underline{X = 0}$ (no difference between the paths ⓐ and ⓑ), then $\underline{\varepsilon = 1}$ and from (8) one gets the known relation :

$$(n-1)^{*} = \frac{\lambda_v}{16\,X_c} (N^{(4)} - N^{(1)}) \quad (8')$$

It should be stressed that while the value of $N^{(4)}$ depends on the gas, on its pressure and temperature, this is not true for the other measures of fringes, which depend essentially on the temperature of the refractometer body, so that for a fixed temperature of operation of this instrument the determination of the correction factor $\varepsilon(X)$ can be done once for ever.

By introducing the technical characteristics of the refractometer :

He-Ne laser wave-length in vacuum λ_v^{laser} = 632.816 mm
cell length X_c = (801.917 ± .006) mm ,

the complete relationship between the index of refraction and the measured number of fringes for a gas at pressure p, absolute temperature T and for $\lambda = \lambda_v^{laser}$ is :

$$\boxed{(n-1)_r = 4.93206 \cdot 10^{-8} * (N^{(4)} - N^{(1)}) * \varepsilon(X)}\,. \quad (8'')$$

This experimental value can be checked with what can be derived from literature [1)] assumed a scaling law of the type

$$(n - 1)_{lit} = k_{gas}(\lambda) * \frac{p}{T} \tag{10}$$

provided the gas pressure p can be measured in a proper way (i.e. with an instrument which has been calibrated for the gas of interest).

In this case the values of the coefficient k_{gas} are, for $\lambda = \lambda_v^{laser}$ and for He and N_2 gases :

$$k_{gas} (\lambda = \lambda_v^{laser}) = \begin{cases} 9.40138 \ 10^{-3} \ k/bar & (He) \\ 8.05121 \ 10^{-2} \ k/bar & (N_2) \end{cases} \tag{11}$$

REFERENCES

1) Landolt-Börnstein, 6. Auflage, II Band, 8 Teil. Springer-Verlag 1962.