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BCCNT 95/071/248

Magnitude of Chiral Fields in Nuclei:
Meson-Nucleon Vertex Corrections for Composite Nucleons

Shun-fu Gao, C.M. Shakin, Wei-Dong Sun, and J. Szweda

Department of Physics and Center for Nuclear Theory

Brooklyn College of the City University of New York

Brooklyn, New York 11210

(July, 1995)

Submitted to Physical Review C (Subnucleon Aspects of Nuclei/Physics of Hadrons)

Abstract

In this work we suggest a scheme for the calculation of vertex corrections for mesons coupled to composite nucleons and make application to the estimation of the magnitude of the chiral (scalar) field in nuclear matter. Previously, we have considered vertex corrections for the coupling of mesons to quarks. However, replacing the quarks by (composite) nucleons creates a number of problems for the analysis. We follow the suggestion that the excitation of antinucleon states is very strongly suppressed at meson-nucleon vertices and, therefore, we evaluate our diagrams using on-mass-shell nucleons. With that approximation, we may use a form for the pion-nucleon vertex function that was recently obtained in a lattice simulation of QCD. Quite reasonable results are obtained for most diagrams; however, the method fails when we attempt to calculate the part of the nucleon self-energy due to emission and absorption of pions. That leads to the necessity of treating the nucleon wave function renormalization constant in a phenomenological manner. We find that our results are generally satisfactory when we calculate the sigma-nucleon coupling constant, $G_{\sigma NN}$, which has an empirical value of about 9.5 when m_σ = 550 MeV. (A simple estimate based upon the constituent quark model yields $G_{\sigma NN} \leq 7.7$.) If we accept the larger value of $G_{\sigma NN}$, we can suggest that the entire scalar field in nuclei can be identified as a chiral field in our model. That result is consistent with a particular treatment of QCD sum rules in matter in which the four-quark condensates are kept at their vacuum value. (Support for the last approximation has been presented in the literature.) When we calculate the meson-nucleon vertex corrections, the results for the pion-nucleon sigma term, σ_N, and for $G_{\sigma NN}$ depend upon a parameter, α, that characterizes the quark wave function of the nucleon. If we use the value of that parameter found in our earlier work on a covariant

soliton model of the nucleon, we find that σ_N = 53.1 MeV and $G_{\sigma NN}$ = 9.75. (These values correspond to the use of a Euclidean momentum-space cutoff of Λ_E = 0.90 GeV in the Nambu−Jona-Lasinio model. That model provides the basis for our analysis.) While our results are somewhat model-dependent, we attempt to remove some of the model dependence by using the Feynman-Hellman theorem in conjunction with a sigma-dominance model to calculate σ_N and $G_{\sigma NN}$. The results obtained in this manner are generally consistent with the results obtained in more detailed calculations.

I. Introduction

There are several reasons to believe that some portion of the large scalar fields in nuclei found in Dirac phenomenology [1], in the Walecka model [2] and in Relativistic-Brueckner-Hartree-Fock (RBHF) theory [3] are chiral fields. For example, there is the model-independent relation that relates the $\bar{q}q$ condensate in matter to that in vacuum. To first order in the density, ρ_N, we have [4]

$$\frac{<\bar{q}q>_\rho}{<\bar{q}q>_{vac}} = \left[1 - \frac{\sigma_N \rho_N}{f_\pi^2 m_\pi^2}\right] , \qquad (1.1)$$

where σ_N is the pion-nucleon sigma term ($\sigma_N = 45 \pm 8$ MeV) and f_π is the pion decay constant ($f_\pi = 93$ MeV). Equation (1.1) implies a reduction of the condensate value of about 34% in nuclear matter. If one performs a bosonization of the Nambu–Jona-Lasinio (NJL) model one obtains a relation between the scalar field so introduced and the value of $\bar{q}(x)q(x)$ [5],

$$\sigma(x) = -\frac{G_S}{g_{\sigma qq}}\bar{q}(x)q(x) . \qquad (1.2)$$

Here, G_S is the coupling constant of the NJL model and $g_{\sigma qq}$ is the coupling constant of the scalar field to the quarks. (In the chiral limit $g_{\pi qq} = g_{\sigma qq} = g$.) Note that in the vacuum $\sigma_{vac} = f_\pi$, so that, upon using the Goldberger-Treiman relation, $m_q = gf_\pi$, we see that Eq. (1.2) relates the value of the constituent quark mass to the vacuum condensate,

$$m_q = -G_S<\bar{q}q>_{vac} , \qquad (1.3)$$

where $<\bar{q}q>_{vac} = <\bar{u}u + \bar{d}d>_{vac}$.

Using Eq. (1.2) and Eq. (1.1) we have

$$\frac{\sigma}{\sigma_{vac}} = \left[1 - \frac{\sigma_N \rho_N}{f_\pi^2 m_\pi^2} \right] \quad . \tag{1.4}$$

We may write $\sigma = f_\pi + \tilde{\sigma}$, where $\tilde{\sigma}$ represents the fluctuation away from the vacuum value.

Thus

$$\tilde{\sigma} = - \frac{\sigma_N \rho_N}{f_\pi m_\pi^2} \quad , \tag{1.5}$$

which yields $\tilde{\sigma} \simeq -36$ MeV in nuclear matter if $\sigma_N = 50$ MeV. If we choose the sigma coupling to the nucleon to be $G_{\sigma NN} = 10$, as is typical of various boson-exchange models of the nuclear force [6], one has for the scalar potential in matter, $U_S \simeq -360$ MeV which is of the order of magnitude of the scalar fields in the Walecka model [2] or in RBHF theory [3]. From the study of the bosonization procedure it is clear that the sigma field is the chiral partner of the pion [7].

The above discussion is suggestive. However, we wish to introduce more quantitative considerations and study, with the context of a specific model, how much of the scalar field found to be present in nuclei is a chiral field. To discuss this matter, we find it quite useful to use a somewhat extended version of the NJL model.

At the outset, we remark that there is no low-mass (physical) sigma meson. It is necessary to understand how that can be true, since a (spacelike) sigma meson plays an important role in nuclear physics. In a previous work we have discussed correlated two-pion exchange in the nucleon-nucleon interaction [8]. It is well known that, for spacelike momentum exchange ($t = q^2 < 0$), correlated two-pion exchange may be represented by an effective sigma meson of low mass ($m_\sigma \sim 550$ MeV) [9]. In our analysis of the quark-quark T matrices of the NJL model we saw how the introduction of confinement moved the low-mass sigma meson of that model to high energy ($m_\sigma > 900$ MeV) [10]. However, for spacelike momenta of the sigma, the T matrices and other amplitudes behave as if a sigma meson with $m_\sigma = 540$ MeV was present. In our analysis the exchanged system for $q^2 < 0$ is predominantly of $q\bar{q}$ character and may be identified with the chiral partner of the pion [10]. With that interpretation, the mean scalar field in nuclei generated by spacelike sigma exchange is a chiral mean field. The question that then arises as to the magnitude of this chiral field. For example, we have asked how much of the mean scalar field in the Walecka model or in Relativistic-Brueckner-Hartree-Fock theory is a chiral field. One way to answer that question is to calculate the value of the sigma-nucleon coupling constant, $G_{\sigma NN}$, using a model with chiral symmetry such as the NJL model. In the following we will suggest a method for the calculation of $G_{\sigma NN}$ making use of the NJL model. To that end we study the matrix element $<N|\bar{q}(0)q(0)|N>_0$, where $\bar{q}(0)$ and $q(0)$ are quark operators and $|N>$ is a state of a nucleon. (See Fig. 1.) It is important that we distinguish $<N|\bar{q}(0)q(0)|N>_0$ from the complete matrix element $<N|\bar{q}(0)q(0)|N>$ [11]. Here $<N|\bar{q}(0)q(0)|N>_0$ is defined so as not to contain the bubble string that represents the sigma. [See Fig. 1.] Thus, in the Nambu–Jona-Lasinio model the value for $<N|\bar{q}(0)q(0)|N>$ would be equal to $(1 - G_S J_S(0))^{-1} <N|\bar{q}(0)q(0)|N>_0$ where $(1 - G_S J_S(0))^{-1} \simeq 2.9$ represents the enhancement of the matrix element found in a sigma-dominance model [11]. For example, in Fig. 1a we show a leading contribution to $<N|\bar{q}(0)q(0)|N>_0$ calculated in terms of the wave functions of a constituent quark model. (There, the filled circle denotes the operator $\bar{q}(0)q(0)$.) In Fig. 1b we show corrections to the matrix element $<N|\bar{q}(0)q(0)|N>_0$ due to the insertion of a string of the quark loops of the NJL model. In this manner we define $<N|\bar{q}(0)q(0)|N>$, which has the value given above. In this work we will calculate corrections to what we have called $<N|\bar{q}(0)q(0)|N>_0$, but we will not include the factor $(1 - G_S J_S(0))^{-1}$ at first. Therefore, we will be calculating better approximations for $<N|\bar{q}q|N>_0$, exclusive of the diagrams that form $<N|\bar{q}q|N>$. If we know $<N|\bar{q}q|N>_0$, we can calculate the sigma-nucleon coupling constant as $G_{\sigma NN} = g_{\sigma qq} <N|\bar{q}(0)q(0)|N>_0$, where $g_{\sigma qq}$ is the sigma-quark coupling constant. (In our earlier work using the Nambu–Jona-Lasinio model we found $g_{\sigma qq} = 2.58$ and $g_{\pi qq} = 2.68$ in one particular study [11].) An elementary estimate of $<N|\bar{q}(0)q(0)|N>_0$ in a constituent quark model would give a number somewhat less than 3. We will write the leading contribution to $<N|\bar{q}(0)q(0)|N>_0$ as $3(1 - \alpha)$, where α is a correction due to the presence of small components in the quark wave function. (For example, in a quark-diquark model of the nucleon $\alpha = 2\kappa$, where κ is the fraction of the wave function normalization integral that has its origin in the small components of the wave function of relative motion of the quark and the diquark.) If we use the estimate of $3(1-\alpha)$ for $<N|\bar{q}(0)q(0)|N>_0$, we obtain $G_{\sigma NN} \leq 7.7$, which is a good bit smaller than the empirical value of $G_{\sigma NN} \sim 10$ [6]. (We may be able to account for this difference by including diagrams that involve the excitation of the delta. However, in this work we consider only nucleon degrees of freedom, since there are a number of uncertainties in the treatment of the delta. We hope to return to effects due to delta excitation in a future work.)

The organization of our work is as follows. In Section II we describe the various diagrams that represent vertex corrections in the calculation $<N|\bar{q}(0)q(0)|N>_0$. In Section III we provide analytic expressions for these diagrams and in Section IV we discuss the numerical results. In Section V we consider the calculation of the pion-nucleon sigma term, σ_N, and attempt to achieve consistency with our calculation of $G_{\sigma NN}$. In Section VI we use the Feynman-Hellman theorem and a sigma-dominance model to calculate both σ_N and $G_{\sigma NN}$. Section VII contains some further discussion and conclusions. The Appendix contains some further refinements of our calculations and what we believe to be our most accurate results.

II. Diagrammatic Analysis

The various comments made in the introduction are clarified if we consider a diagrammatic analysis such as that of Fig. 2. These diagrams appearing there are similar to those calculated in Ref. [11]; however, they now contain form factors for nucleons considered as composite systems. In the upper part of Fig. 2 we show the matrix element $<N|\bar{q}(0)q(0)|N>_0$. Recall that this element does not contain the enhancement factor, $\left(1 - G_S J_S(0)\right)^{-1}$, of the sigma-dominance model. This factor is shown in Fig. 3 as a double line. It represents the sum of loops in the NJL model, as may be seen by expanding $\left(1 - G_S J_S(0)\right)^{-1}$ in a power series. (See Ref. [11].)

Returning to Fig. 2, we see in Fig. 2a a schematic representation of the calculation of $<N|\bar{q}q\;|N>_0$ in a constituent quark model. As noted before, we will denote the value of this diagram as $3(1 - \alpha)$, where α accounts for the relativistic nature of the quark wave function of the nucleon. (Sometimes it will be useful to write this factor as $F(0)\;(1 - \alpha)$ with $F(0) = 3$.)

In Fig. 2b we have two wavy lines that represent pions. There is a quark loop at the top of the diagram and at the bottom we see two pion-nucleon vertex form factors. These form factors will be written as

$$F_\pi(k^2) = \frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2} \quad , \tag{2.1}$$

so that the entire vertex is $i g_{\pi NN} \gamma_5 \vec{\tau} F_\pi(k^2)$ for on-mass-shell nucleons. In keeping with the comments made above, we put the intermediate nucleon on-mass-shell. Since the external nucleons are also on mass shell, the form factor is the one usually defined. (From a recent lattice simulation of QCD, we have $\lambda_\pi = 0.75 \pm 0.14$ [12].)

In Fig. 2c, we have another correction which contains a factor of $3(1 - \alpha)$. [See Fig. 1a.] The diagram of Fig. 2c may be calculated by forming a derivative, as we will discuss shortly. Alternatively, we may consider finite momentum transfer and place either intermediate nucleons on mass shell, thus, generating two diagrams. Then the $q \to 0$ limit may be taken for the sum of the two diagrams.

The result obtained for the diagram in Fig. 2d also contains a factor of $3(1 - \alpha)$. The rest of the diagram may be calculated as in Ref. [11]. Here the wavy lines are pions and the double line is a sum of quark loops. It is often useful to introduce a sigma propagator. The following relation that appears in a momentum-space bosonization of the NJL model [5],

$$-\frac{G_S}{1 - G_S J_S(q^2)} = \frac{g_{\sigma qq}^2(q^2)}{q^2 - m_\sigma^2(q^2)} \quad , \tag{2.2}$$

is quite useful in passing between diagrams written in terms of the quark loops and those containing the sigma propagator. For $q^2 \simeq 0$ we may write

$$-\frac{G_S}{1 - G_S J_S(q^2)} = \frac{g_{\sigma qq}^2}{q^2 - m_\sigma^2} , \tag{2.3}$$

where $m_\sigma = 0.540$ GeV, $g_{\sigma qq} = 2.58$, $G_S = 7.91$ GeV^{-2} and $J_S(0) = 0.0826$ GeV^2 [13]. The value of the diagram of Fig. 2d is $3(1-\alpha)K_S(0)g_{\sigma qq}^2/m_\sigma^2$, where $K_S(0)$ is the diagrammatic element appearing between the filled circle at the upper part of the figure and the sigma propagator represented here by a double line. From a previous analysis [13] we have $K_S(0) \simeq 0.0088$ GeV^2, so that the result for the diagram in Fig. 2d is $3(1 - \alpha)(0.20)$. The calculation of the diagram of Fig. 2d is the most reliable of our results, since we do not have to decide how to treat an intermediate nucleon that could, in principle, go off mass shell.

The two diagrams of Fig. 2e may be called wave function renormalization diagrams in analogy to the corresponding analysis of vertex corrections in QED. Here the perturbative analysis fails since, if we write the nucleon self-energy as $\Sigma(p) = \tilde{A}(p^2) + B(p^2)(\not{p} - m_N)$, we calculate $B(m_N^2) \sim -2.7$ which is at least an order-of-magnitude too large. For example, the strength at the nucleon pole is $Z = (1-B)^{-1}$, so that we expect the value of B to be in the range -0.1 to -0.2. The larger value of B obtained here represents a defect of the model. However, the other diagrams yield quite sensible results and we present those results here, while using a phenomenological value for $B(m_N^2)$.

To simplify our discussion, we will now write $F(0)(1-\alpha)$ instead of $3(1 - \alpha)$ and define a number of integrals. The evaluation of the diagram of Fig. 2b will yield J_b, the evaluation of Fig. 2c will yield $F(0)(1-\alpha)J_c$, and the evaluation of Fig. 2d will yield $F(0)(1-\alpha)J_d$. The contribution of Fig. 2e will be denoted $F(0)(1-\alpha)B$ and we will put $B = -0.2$, since we have not been able to calculate a sensible value for B using perturbation theory. Note that the corresponding calculation for the quark self-energy yields a useful result, $B \sim -0.15$ [11]. However, in that case the pion-quark coupling constant ($g_{\pi qq} = 2.68$) is about four times smaller than the pion-nucleon coupling constant. That feature leads to reasonable results for the quark self-energy. In the case of the nucleon, it appears that we obtain sensible results for three-point functions, such as form factors, while the results are unsatisfactory when we calculate a two-point function, such as the self-energy, in perturbation theory. The large value obtained for B (and for $\tilde{A}$) suggests that the various calculations might be more correct if performed at the quark level. That is a quite formidable task and we do not attempt such a calculation in this work.

III. Calculation of Various Diagrams

In this section we provide expressions for the diagrams of Fig. 2. First we consider Fig. 2b. The integral J_b is defined to be

$$J_b = I_b g_{\pi NN}^2 i^3 \int \frac{d^4k}{(2\pi)^4} H(k^2) \frac{1}{(k^2 - m_\pi^2 + i\epsilon)^2} \left[\frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2} \right]^2 \times \frac{\mathrm{Tr}}{2} \left[\gamma_5 \frac{1}{\not{p} - \not{k} - m_N + i\epsilon} \gamma_5 \left[\frac{\not{p} + m_N}{2m_N} \right] \right] , \tag{3.1}$$

where we have included a factor, $H(k^2)$, originally defined in Ref. [11]. That factor represents the quark loop integral at the top of the diagram. We had

$$H(k^2) = -2i^3 g_{\pi qq}^2 \int \frac{d^4 l}{(2\pi)^4} \mathrm{Tr}[\gamma_5 S(l) S(l) \gamma_5 S(l-k)] \quad . \tag{3.2}$$

Further, $I_b = 6$ is an isospin factor and $(\lambda_\pi^2 - m_\pi^2)/(\lambda_\pi^2 - k^2)$ is the pion-nucleon form factor with $\lambda_\pi = 0.8$ GeV. Note that the factor $(\not{p} + m_N)/2m_N$ arises from averaging over the nucleon spin. (The external nucleon is on mass-shell so that $p^2 = m_N^2$.)

In evaluating Eq. (3.1) we use $g_{\pi NN} = 12.7$. We take the intermediate nucleon of momentum $p - k$ to be on mass shell. That restriction is achieved by the replacement

$$\frac{1}{\not{p} - \not{k} - m_N} = \frac{\not{p} - \not{k} + m_N}{(p-k)^2 - m_N^2 + i\epsilon} \quad , \tag{3.3}$$

$$\rightarrow -(2\pi i) \frac{\delta^{(+)}\left(p^0 - k^0 - E_N(\vec{p} - \vec{k})\right)}{2E_N(\vec{p} - \vec{k})} (\not{p} - \not{k} + m_N) \quad , \tag{3.4}$$

or

$$\rightarrow -(2\pi i) \frac{m_N}{E_N(\vec{p} - \vec{k})} \delta^{(+)}\left(p^0 - k^0 - E_N(\vec{p} - \vec{k})\right) \left[\frac{\not{p} - \not{k} + m_N}{2m_N}\right] \quad . \tag{3.5}$$

We now turn to a calculation of the diagram of Fig. 2c. In this case we write the result as $F(0)(1-\alpha)J_c$, where we recall that $F(0) = 3$. We have

$$J_c = I_c i^3 g_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2 + i\epsilon} \left(\frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2} \right)^2$$
$$\times \frac{\mathrm{Tr}}{2} \left[\gamma_5 \frac{1}{\not p - \not k - m_N + i\epsilon} \frac{1}{\not p - \not k - m_N + i\epsilon} \gamma_5 \left(\frac{\not p + m_N}{2m_N} \right) \right] \quad , \tag{3.6}$$

where $I_c = 3$ is an isospin factor. One way to evaluate the integral is to note that

$$\frac{\partial}{\partial \beta} \frac{1}{\not p - \not k - \beta + i\epsilon} = \frac{1}{(\not p - \not k - \beta + i\epsilon)^2} \quad . \tag{3.7}$$

Then

$$J_c = I_c g_{\pi NN}^2 \frac{\partial}{\partial \beta} i^3 \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}}{2} \left[\gamma_5 \left(\frac{1}{\not p - \not k - \beta + i\epsilon} \right) \gamma_5 \left(\frac{\not p + m_N)}{2m_N} \right) \right]$$
$$\times \left(\frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2} \right)^2 \frac{1}{k^2 - m_\pi^2 + i\epsilon} \quad . \tag{3.8}$$

Note that

$$\mathrm{Tr}[\gamma_5 (\not p - \not k + \lambda) \gamma_5 (\not p + m_N)] = -4m_N^2 + 4k \cdot p + 4\lambda m_N \quad . \tag{3.9}$$

At this point we use Eq. (3.5) and also make the replacement

$$\frac{1}{k^2 - m_\pi^2 + i\epsilon} \rightarrow \frac{1}{2\omega(\vec{k})} \frac{1}{k^0 - \omega(\vec{k}) + i\epsilon} \quad . \tag{3.10}$$

The motivation for the last step lies in the fact that the nucleon goes forward in time when it is on mass shell and the structure of the diagram leads to the observation that the pion also goes forward in time. Therefore, we choose the appropriate part of the pion propagator in Eq. (3.10). Finally, we obtain

$$J_c = -g_{\pi NN}^2 I_c \frac{\partial}{\partial\beta}\int \frac{d^3k}{(2\pi)^3}\frac{m_N}{E_\beta(\vec{p}-\vec{k})}\left(\frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2}\right)^2$$

$$\times \frac{1}{2\omega(\vec{k})}\left[\frac{1}{k^0 - \omega(\vec{k})}\right]\left[\frac{-4m_N^2 + 4k\cdot p + 4\beta m_N}{8m_N^2}\right] \tag{3.11}$$

where $E_\beta(\vec{p}-\vec{k}) \equiv [(\vec{p}-\vec{k})^2 + \beta^2]^{1/2}$ and $k^0 = E_N(\vec{p}) - E_\beta(\vec{p}-\vec{k})$. (It is particularly easy to evaluate the integral in the nucleon rest frame where $\vec{p} = 0$.) As a final step one takes the limit $\beta \to m_N$.

We now consider Fig. 2d. The value of this diagram is defined to be $F(0)(1-\alpha)J_d$ where

$$J_d = K_S(0)\frac{G_S}{1 - G_S J_S(0)}\ , \tag{3.12}$$

$$= K_S(0)\left(\frac{g_{\sigma qq}^2}{m_\sigma^2}\right)\ . \tag{3.13}$$

As noted above, the amplitude $K_S(0)$ has been obtained in previous work. We had $K_S(0) = 0.0088\ \text{GeV}^2$ [13]. Thus, we find $J_d = 0.20$, as was noted previously.

Finally, we turn to the calculation of Fig. 2e. To that end we write the nucleon self-energy as

$$\Sigma(p) = \tilde{A}(p^2) + B(p^2)(\not{p} - m_N) \quad . \tag{3.14}$$

For our analysis we need $B(m_N^2)$, since the contribution of the two diagrams in Fig. 2e is $F(0)(1-\alpha)B(m_N^2)$. In order to calculate B, we write

$$B = \frac{\mathrm{Tr}[\not{p}\Sigma(p)]}{4m_N^2} = I_B \frac{1}{4m_N^2}(-i)g_{\pi NN}^2 \int \frac{d^4k}{2\pi)^4} \frac{1}{k^2 - m_\pi^2 + i\epsilon} \left(\frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2} \right)^2$$
$$\times \mathrm{Tr}\left[\not{p}\gamma_5 \frac{1}{\not{p} - \not{k} - m_N + i\epsilon} \gamma_5 \right] \quad , \tag{3.15}$$

where $I_B = 3$ is an isospin factor. Note that

$$\mathrm{Tr}[\not{p}(-\not{p} + \not{k} + m_N)] = -4m_N^2 + 4p \cdot k \quad . \tag{3.16}$$

Again, placing the intermediate nucleon on mass shell, we have

$$B = \frac{I_B(-i)}{4m_N^2} g_{\pi NN}^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{2\omega(\vec{k})} \left[\frac{1}{k^0 - \omega(\vec{k}) + i\epsilon} \right]$$
$$\times \left(\frac{\lambda_\pi^2 - m_\pi^2}{\lambda_\pi^2 - k^2} \right)^2 [-4m_N^2 + 4p \cdot k](-2\pi i) \frac{1}{2E_N(\vec{p} - \vec{k})} \delta^{(+)}[p^0 - k^0 - E_N(\vec{p} - \vec{k})] \quad .$$

(3.17)

In the nucleon rest frame, we may write

$$B = -I_B \frac{g_{\pi NN}^2}{8\pi^2} \frac{(\lambda_\pi^2 - m_\pi^2)^2}{m_N^2} \int |\vec{k}|^2 d|\vec{k}| \frac{1}{\omega(\vec{k}) E_N(\vec{k})}$$

$$\times \frac{1}{k^0 - \omega(\vec{k}) + i\epsilon} \left[\frac{1}{\lambda_\pi^2 - k^2} \right]^2 \left[k^0 m_N - m_N^2 \right] , \tag{3.18}$$

where $k^0 = m_N - E_N(\vec{k})$. Evaluation of this expression yields $B \simeq -2.7$ which indicates a breakdown of perturbation theory in the calculation of the nucleon self-energy. Because of this we choose a phenomenological value for B of $B = -0.2$.

IV. Numerical Results

In the boson-exchange model the phenomenological value of the sigma-nucleon coupling constant, $g_{\sigma NN}^2/4\pi$, varies from 8.07 to 8.80 [6]. We note, however, that at the sigma-nucleon vertices one includes a form factor, $(\Lambda_\sigma^2 - m_\sigma^2)/(\Lambda_\sigma^2 - q^2)$, in the boson-exchange model. Therefore, it is useful to define the effective value of the coupling constant at $q^2 = 0$ using the relation

$$\frac{G_{\sigma NN}^2}{4\pi} = \frac{g_{\sigma NN}^2}{4\pi} \left[\frac{\Lambda_\sigma^2 - m_\sigma^2}{\Lambda_\sigma^2} \right]^2 . \tag{4.1}$$

Equation (4.1) yields values of $G_{\sigma NN}$ in the range $9.31 \leq G_{\sigma NN} \leq 9.73$, if $\Lambda_\sigma = 2.0$ GeV and $m_\sigma = 550$ MeV [6]. We choose $G_{\sigma NN} = 9.50$ as the empirical value of the sigma-nucleon coupling constant, keeping in mind the uncertainty in that value.

Gathering up the various corrections in the calculation of $< N|\bar{q}q|N>_0$, we have

$$< N|\bar{q}q|N>_0 = F(0)(1-\alpha)[\,1 + \frac{J_b}{F(0)(1-\alpha)} + J_c + J_d + B\,] \quad . \tag{4.2}$$

Inserting the calculated values, $J_b = 0.378$, $J_c = 0.525$, $J_d = 0.20$ and taking $B = -0.20$, we have

$$< N|\bar{q}q|N>_0 = F(0)(1-\alpha)[\,1 + \frac{0.378}{F(0)(1-\alpha)} + 0.525\,] \quad , \tag{4.3}$$

since our choice for B cancels J_d. Now, if we put $\alpha = 0.1$, we have $< N|\bar{q}q|N>_0 = 4.50$ and with $\alpha = 0.2$, we have $< N|\bar{q}q|N>_0 = 4.04$. Thus, with $g_{\sigma qq} = 2.58$ [11], $G_{\sigma NN} = (2.58) < N|\bar{q}q|N>_0 = 11.6$ for $\alpha = 0.1$ and $G_{\sigma NN} = 10.4$ for $\alpha = 0.2$. [See Table 1.] For the boson-exchange model of nuclear forces, we have taken $G_{\sigma NN} = 9.50$ as an average value [6]. Since we use $m_\sigma = 0.540$ GeV, while $m_\sigma = 0.550$ is used in the boson-exchange model, we infer that, in this case, the empirical value of the coupling constant adjusted for the slightly different sigma mass, is $G_{\sigma NN} = 9.30$. Our value, when $\alpha = 0.25$, is $G_{\sigma NN} = 9.83$, which is close to the empirical value noted above. If we take this result at face value, we would conclude that the entire scalar field in nuclei is a chiral field. Note that the scalar potential in nuclei is then

$$U_S = -\frac{G^2_{\sigma NN}}{m^2_\sigma}\rho_N \quad , \tag{4.4}$$

$$= -373\,\mathrm{MeV} \tag{4.5}$$

in a Hartree approximation, if $G_{\sigma NN} = 9.30$, $m_\sigma = 0.540$ GeV and $\rho_N = (0.108\ \mathrm{GeV})^3$. The value $U_S \simeq -373$ MeV is fairly close to the empirical value of the (Lorentz) scalar potential in nuclei [1,2] and the theoretical value obtained for nuclear matter using RBHF theory [3].

(In the calculations reported in this section we used the parameters of the NJL model that are calculated with a Euclidean momentum-space cutoff of $\Lambda_E = 1.0$ GeV.)

V. The Pion-Nucleon Sigma Term

In this section we investigate whether we can achieve consistency in our calculations of $G_{\sigma NN}$ with the value of the pion-nucleon sigma term, σ_N. We recall that

$$\sigma_N = m_q^0 < N|\bar{q}q|N> \quad , \tag{5.1}$$

where $< N|\bar{q}q|N> \; = \; < N|\bar{u}u + \bar{d}d|N>$ in our notation. Here, m_q^0 is the average current quark mass. Theoretical analysis yields $\sigma_N = 45 \pm 8$ MeV [14].

In the last section we used $\Lambda_E = 1.0$ GeV. We also find it useful to use consider a somewhat smaller value for Λ_E. We now use the results given in Table 1 of Ref. [11] for $\Lambda_E = 0.90$ GeV: $m_q^0 = 6.6$ MeV, $g_{\sigma qq} = 2.97$, $m_q = 295$ MeV, $g_{\pi qq} = 3.10$, and $[1 - G_S J_S(0)]^{-1} = 2.38$. [See Table 3.] If we recalculate J_b with the new parameters, we find $J_b = 0.452$. Note that J_c does not change when we change Λ_E. Further, we will assume that J_d is again cancelled by B. Thus, our new result for $\Lambda_E = 0.90$ GeV is

$$< N|\bar{q}q|N>_0 \; = 3(1-\alpha)\left[\, 1 + \frac{0.452}{3(1-\alpha)} + 0.525 \,\right] \quad . \tag{5.2}$$

Now, if $\alpha = 0.4$, we find $< N|\bar{q}q|N>_0 = 3.20$. [See Table 2.] With $< N|\bar{q}q|N> \; = [1 - G_S J_S(0)]^{-1} < N|\bar{q}q|N>_0$, we have $< N|\bar{q}q|N> \; = 7.62$ and also

$G_{\sigma NN} = 9.50$, which is close to the empirical value. Further, Eq. (2.1) yields $\sigma_N = 50.3$ MeV. if we use the parameters for $\Lambda_E = 0.90$ GeV.

Our calculation for $\Lambda = 900$ MeV gives satisfactory values for σ_N and $G_{\sigma NN}$; however, our value of $\alpha = 0.4$ appears to be rather large. (It corresponds to a 20% contribution to the quark wave function normalization from small components or other relativistic effects.) Therefore, we discuss another approach to this problem that is less dependent upon our calculation of vertex corrections. For example, let us again consider the two equations,

$$\sigma_N = m_q^0 < N | \bar{q} q | N > = m_q^0 [1 - G_S J_S(0)]^{-1} < N | \bar{q} q | N >_0$$

and $G_{\sigma NN} = g_{\sigma qq} < N | \bar{q} q | N >_0$. These relations allow us to write $G_{\sigma NN}$ in terms of σ_N in our sigma-dominance model,

$$G_{\sigma NN} = g_{\sigma qq} \frac{\sigma_N}{m_q^0} [1 - G_S J_S(0)] \quad . \tag{5.3}$$

If we again use the values for Λ_E 900 MeV in Table 3 and put $\sigma_N = 50$ MeV, we have $G_{\sigma NN} = 9.45$ and $< N | \bar{q} q | N >_0 = 3.18$. Note that this analysis is independent of the choice of α and also suggests that the value of $< N | \bar{q} q | N >_0$ is rather close to the nonrelativistic value of $< N | \bar{q} q | N >_0 = 3$.

VI. Use of the Feynman-Hellman Theorem in the Calculation of σ_N and $G_{\sigma NN}$

If the Lagrangian contains a term of the form $- m_q^0 (\bar{u}(x) u(x) + \bar{d}(x) d(x))$, we may obtain a value for $< N | \bar{q} q | N > = < N | \bar{u} u + \bar{d} d | N >$ by the following scheme. We write m_q for the constituent quark mass, so that the gap equation is

$$m_q = m_q^0 + iG_S n_f n_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} \frac{1}{\not{k} - m_q + i\epsilon} \quad . \tag{6.1}$$

We next assume that, to a good approximation, $m_N = 3m_q$. (For example, with Λ = 900 MeV, we have m_q = 295 MeV from Table 3.) Now $m_N = \langle N|H|N\rangle$, so that

$$\frac{\partial m_N}{\partial m_q^0} \simeq 3 \frac{\partial m_q}{\partial m_q^0} \quad , \tag{6.2}$$

$$= \langle N|\bar{q}q|N\rangle \quad , \tag{6.3}$$

where we have made use of the Feynmann-Hellman theorem [15]. From Eq. (6.1) we have

$$\frac{\partial m_q}{\partial m_q^0} = 1 + 4iG_S n_f n_c \left[\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_q^2} + \int \frac{d^4k}{(2\pi)^4} \frac{2m_q^2}{(k^2 - m_q^2)^2} \right] \left(\frac{\partial m_q}{\partial m_q^0} \right) \quad . \tag{6.4}$$

This leads to the result

$$\langle N|\bar{q}q|N\rangle = 3 \left\{ 1 - \frac{3G_S}{2\pi^2} \left[\Lambda_E^2 \left(\frac{\Lambda_E^2 + 3m_q^2}{\Lambda_E^2 + m_q^2} \right) - 3m_q^2 \ln \left(\frac{\Lambda_E^2 + m_q^2}{m_q^2} \right) \right] \right\}^{-1} \quad . \tag{6.5}$$

From Table 3 we see that we may use Λ_E = 900 MeV, m_q^0 = 6.6 MeV and G_S = 10.6 GeV^{-2}. With those parameters, we have $\langle N|\bar{q}q|N\rangle$ = 7.10. Thus, $\sigma_N = m_q^0 \langle N|\bar{q}q|N\rangle$ = 46.9 MeV. Now we use the relation $\langle N|\bar{q}q|N\rangle = [1 - G_S J_S(0)]^{-1} \langle N|\bar{q}q|N\rangle_0$ to obtain $\langle N|\bar{q}q|N\rangle_0 = 3.00$, which in turn yields $G_{\sigma NN} = g_{\sigma qq} \langle N|\bar{q}q|N\rangle_0 = 8.91$.

A similar analysis for $\Lambda_E = 1.0$ GeV yields $\sigma_N = 51.8$ MeV and $G_{\sigma NN} = 8.38$. To obtain these values we have used $m_q^0 = 5.5$ MeV, $m_q = 0.260$ GeV and $[1 - G_S J_S(0)] = 0.346$ and, as an intermediate step, we found that $<N|\bar{q}q|N>_0 = 3.25$.

We note that the values obtained when $\Lambda_E = 900$ MeV may be preferred, since the relation $m_N = 3m_q$ is more closely satisfied than when $\Lambda_E = 1.0$ GeV. The values obtained in this section ($\sigma_N = 46.9$ MeV and $G_{\sigma NN} = 8.91$, for example) are sufficiently close to the values obtained by our other methods so as to provide increased confidence in the entire procedure. (See Tables 3 and 4.)

It is of interest to note that Eq. (6.5) may also be written as $<N|\bar{q}q|N> = 3[1 - G_S J_S(0)]^{-1}$. This result has a simple interpretation. The value of $<N|\bar{q}q|N>$ is given as three times the single-quark value. The factor of $[1 - G_S J_S(0)]^{-1}$ provides the enhancement of the single-quark value which would otherwise be 1. The simple relation, $<N|\bar{q}q|N> = 3[1 - G_S J_S(0)]^{-1}$, is consistent with the values given in Table 4 for $\Lambda_E = 900$ MeV. However, the value of $1 - G_S J_S(0)$ given for $\Lambda_E = 1.0$ GeV in Table 3 has been modified slightly from the value originally listed in Table 1 of Ref. [11]. Here, the value given in Table 3 should more properly be written as the value of $1 - G_S \hat{J}_S(0)$, where $\hat{J}_S(0)$ includes some features of confinement in its evaluation [13]. Therefore, one cannot check the calculation of $<N|\bar{q}q|N>$ made at $\Lambda_E = 1.0$ GeV by using the formula $<N|\bar{q}q|N> = 3[1 - G_S J_S(0)]^{-1}$ and the values given in Table 3 for $\Lambda_E = 1.0$ GeV.

From our discussion we see that the use of Eqs. (6.2) and (6.3) corresponds to a simplified calculation that does not contain the full range of effects considered in this work.

Therefore, we place more reliance upon our detailed calculation of vertex corrections. [See the Appendix.]

VII. Discussion

In our earlier discussion we remarked that the value of $\alpha = 0.4$ appeared somewhat large, since it gave $\kappa = 0.2$ as the fraction of the normalization integral due to small components of the quark wave function of the nucleon. However, we have made some calculations of nucleon properties using a covariant solition model some year ago [16]. There, we found that $\kappa = 0.18$, which corresponds to $\alpha = 0.36$. Further, in our model the axial coupling constant had the value

$$g_A = \frac{5}{3}(1 - \frac{4}{3}\kappa) \quad , \tag{7.1}$$

so that with $\kappa = 0.18$, we found $g_A = 1.27$, which is very close to the experimental value of $g_A = 1.25$. In addition, our wave functions gave very good values for the neutron and proton magnetic moments and the electromagnetic form factors [16]. Therefore, we will take $\alpha = 0.36$ as the preferred value of that parameter. We record the results obtained when $\alpha = 0.36$ in Table 3. For example, we find $<N|\bar{q}q|N>_0 = 3.38$, $<N|\bar{q}q|N> = 8.04$, $\sigma_N = 53.1$ MeV, and $G_{\sigma NN} = 10.0$, when $\Lambda_E = 0.9$ GeV. It is of some interest to note that the use of the value of α found previously [16] yields a value of $G_{\sigma NN}$ that is close to the phenomenological value, $G_{\sigma NN} = 9.5$. (Further refinements of these calculations are presented in the Appendix.)

In this work we have attempted to calculate the sigma-nucleon coupling constant, $G_{\sigma NN}$, using the NJL model. We found that a reasonable result could be obtained if we used a phenomenological value for the constant B that appears in the expression for the nucleon self-energy. We have also seen that our results for $G_{\sigma NN}$ could be made consistent with a successful calculation of the pion-nucleon sigma term. [See Table 3 and the Appendix.] We were motivated in this effort by the observation that $G_{\sigma NN}$ was somewhat less than 7.7 in the simplest analysis. We thought that larger values of $G_{\sigma NN}$ might be found if we considered a number of corrections to the sigma-nucleon vertex. Aside from the problems associated with the calculation of B, our results indicated that the empirical value of $G_{\sigma NN} \simeq 9.5$ could be obtained. (See the Appendix.) That is consistent with the identification of the entire scalar field in nuclei (or nuclear matter) as a chiral field.

That result is also consistent with recent work on QCD sum rules in matter [17]. If these sum rules are used to calculate the nucleon self-energy in matter, the simplest version of the formalism yields results in accord with Dirac phenomenology if the four-quark condensates remain close to their value in vacuum [18]. (Some justification for keeping the four quark condensates at their vacuum value may be found in Ref. [19].) The result of the most elementary analysis for the nucleon self-energy is given in terms of the Borel mass, M_B, and the $\bar{q}q$ and $q^\dagger q$ condensates. The (Lorentz) vector part of the self-energy was found to be [18]

$$\Sigma_V = \frac{64\pi^2}{3M_B^2} < q^\dagger q >_\rho \quad , \tag{7.2}$$

$$= \frac{32\pi^2}{M_B^2}\rho_N \quad , \tag{7.3}$$

where we have used $<q^\dagger q>_\rho = (3/2)\rho_N$. (Here the notation is such that $<q^\dagger q>_\rho$ is the average value of $u^\dagger u$ or $d^\dagger d$ in nuclear matter and ρ_N is the baryon density of nuclear matter.) For typical values of $M_B (M_B \sim 1 \text{ GeV})$, Σ_V is seen to be quite large and positive. The scalar self-energy, Σ_S, is added to the nucleon mass, m_N, to yield the mass in matter, m_N^*, where

$$m_N^* = -\frac{8\pi^2}{M_B^2} <\bar{q}q>_\rho \quad . \tag{7.4}$$

(Here $<\bar{q}q>_\rho$ is equal to either $<\bar{u}u>_\rho$ or $<\bar{d}d>_\rho$.) Further, one has

$$\Sigma_S = -\frac{4\pi^2}{M_B^2}\frac{\sigma_N \rho_N}{m_q^0} \tag{7.5}$$

and

$$\frac{\Sigma_S}{\Sigma_V} = -\frac{\sigma_N}{8m_q^0} \quad , \tag{7.6}$$

$$\simeq -1 \quad . \tag{7.7}$$

(We recall that m_q^0 is the average current quark mass.) From Eq. (7.4), we have $\Sigma_S = -406$ MeV, if $\sigma_N = 45$ MeV, $M_B = 1$ GeV, $m_q^0 = 5.5$ MeV and $\rho_N = (0.108 \text{ GeV})^3$.

It is seen from this elementary analysis that the scalar self-energy is directly related to the change of the condensate $<\bar{q}q>$ from the vacuum value, $<\bar{q}q>_{vac}$, to its value in matter, $<\bar{q}q>_\rho$. Proceeding in this manner, one obtains

$$\frac{m_N^*}{m_N} = \left[1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} \right] \quad , \tag{7.8}$$

which is analogous to the equation quoted earlier,

$$\frac{\sigma}{\sigma_{vac}} = \left[1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} \right] \quad . \tag{7.9}$$

If this simple analysis is correct, we may infer that the entire scalar field in nuclei is related to $\tilde{\sigma}$, which is an order parameter describing the deviation of $<\bar{q}q>_\rho$ from the vacuum value,

$$\tilde{\sigma} = -\frac{G_S}{g}[<\bar{q}q>_\rho - <\bar{q}q>_{vac}] \quad . \tag{7.10}$$

We have reviewed these elementary results, since they indicate that the identification of the entire scalar field in nuclei and in nuclear matter as a chiral field is not unreasonable. It is worth noting that the simple relations presented here for Σ_S and Σ_V are largely unchanged if condensates of higher dimension are included in the analysis, as long as the four-quark condensates remain near their vacuum value [17,19].

In summary, we note that the analysis of QCD sum rules in matter suggests that the scalar self-energy of the nucleon represents a chiral field related to the partial restoration of chiral symmetry in matter [7]. While there are a number of uncertainties in our analysis, such as the neglect of effects of the excitation of the delta and the use of a phenomenological value for the wave function renormalization constant, the various calculations we have made of

$G_{\sigma NN}$ go in the direction of supporting the identification of the scalar field in nuclei with a chiral field. (Some refinements of our calculations are given in the Appendix.) Further confidence in the overall picture is achieved by application of the Feynmann-Hellman theorem. (See Table 4.) The result based upon this theorem is given in Eq. (6.5) and implies a calculation which is similar to that given in Ref. [11], except that the various corrections to meson-quark vertices considered in that reference do not appear in the analysis presented in Section VI.

Appendix

In this appendix we consider some refinements of the bosonization procedure for the NJL model. For example, at one-loop order one has [5]

$$- \frac{G_S}{1 - G_S J_S(q^2)} = \frac{g_{\sigma qq}^2}{q^2 - m_\sigma^2} \quad , \tag{A1}$$

which is written here for $q^2 = 0$, so that the q^2 dependence of $g_{\sigma qq}$ and m_σ may be neglected. Introduction of coupling to the two-pion continuum modifies that relation somewhat [13]:

$$- \frac{G_S}{1 - G_S J_S(q^2) - G_S K_S(q^2)} = \frac{\tilde{g}_{\sigma qq}^2}{q^2 - m_\sigma^2} \quad . \tag{A2}$$

(More generally, we should also replace m_σ by $\tilde{m}_\sigma$ in Eq. (A2). However, to avoid rescaling the sigma mass in our calculations, we ascribe the entire effect of the introduction of $K_S(q^2)$ to an increase in $g_{\sigma qq}$.) Here $K_S(q^2)$ is the diagrammatic element that appears in Fig. 2d. We can calculate $\tilde{g}_{\sigma qq}$ in the limit $q^2 \to 0$. Thus, we have

$$- \frac{G_S}{1 - G_S J_S(0) - G_S K_S(0)} = \frac{\tilde{g}_{\sigma qq}^2}{m_\sigma^2} \quad . \tag{A3}$$

Note that $\tilde{g}_{\sigma qq}$ is greater than $g_{\sigma qq}$ since $J_S(0)$ and $K_S(0)$ are both positive. The enhanced value of $\tilde{g}_{\sigma qq}$ relative to $g_{\sigma qq}$ represents a small enhancement of the sigma field due to (virtual) coupling to the two-pion continuum.

Consider the situation for $\Lambda_E = 1.0$ GeV, where $J_S(0) = 0.0826$ GeV^2 and $K_S(0) = 0.0088$ GeV^2. [See Table 3.] We find $\tilde{g}_{\sigma qq} = 2.89$ (instead of $g_{\sigma qq} = 2.58$) upon using Eq. (A3) with m_σ = 540 MeV. If we then write $\tilde{G}_{\sigma NN} = \tilde{g}_{\sigma qq} < N | \bar{q} q | N >_0$, we have $\tilde{G}_{\sigma NN}$ = 9.57, when we use the preferred value of α = 0.36. (See Section VII.)

The same procedure may be used for Λ_E = 0.90 GeV, where G_S = 10.6 GeV^{-2}, $K_S(0)$ = 0.0097 GeV^2 and $J_S(0)$ = 0.0547 GeV^2. In this case we find $\tilde{g}_{\sigma qq}$ = 3.41 and infer $\tilde{G}_{\sigma NN}$ = 11.48. However, for comparison to the phenomenological value we should scale our result with m_σ. Thus, $G_{\sigma NN}^{eff}$ = (11.48)(0.550/0.592) = 10.7, where we have used the phenomenological value of m_σ = 550 MeV [6] and the value of m_σ given in Table 3 for Λ_E = 0.90 GeV. Scaling the value calculated above for Λ_E = 1.0 GeV, we have $G_{\sigma NN}^{eff}$ = (9.57)(0.550/0.540) = 9.75. Thus, we have the results

$$\left.\begin{array}{l} G_{\sigma NN}^{eff} = 10.7 \\ \sigma_N = 52.4 \text{ MeV} \end{array}\right\} \quad \text{at } \Lambda_E = 1.0 \text{ GeV}$$

and

$$\left.\begin{array}{l} G_{\sigma NN}^{eff} = 9.75 \\ \sigma_N = 53.1 \text{ MeV} \end{array}\right\} \quad \text{at } \Lambda_E = 0.90 \text{ GeV} \ .$$

These results are fairly close to the empirical value of $G_{\sigma NN} \simeq 9.5$ that corresponds to the use of m_σ = 550 MeV in applications of the boson-exchange model [6]. (Recall that the empirical values of $G_{\sigma NN}$ lie in the range: $9.32 \le G_{\sigma NN} \le 9.73$. See Eq. (4.1) and Ref. [6].)

Acknowledgment

This work was supported in part by a grant from the National Science Foundation and by the PSC-CUNY Faculty Award Program.

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Table 1. Values of $< N | \bar{q}q | N >_0$ and $G_{\sigma NN}$ calculated for various values of α are shown. Here $g_{\sigma qq}$ = 2.58 and Λ_E = 1.0 GeV [11]. (See Eq. (4.3) and Table 3.)

α	$< N \vert \bar{q}q \vert N >_0$	$G_{\sigma NN}$
0.0	4.95	12.8
0.1	4.50	11.6
0.2	4.04	10.4
0.3	3.58	9.23
0.4	3.12	8.05

Table 2. Values of $< N | \bar{q}q | N >_0$ and $G_{\sigma NN}$ calculated for various values of α are shown. Here $g_{\sigma qq}$ = 2.97 and Λ_E = 0.9 GeV [11]. (See Eq. (5.2) and Table 3.)

α	$< N \vert \bar{q}q \vert N >_0$	$G_{\sigma NN}$
0.0	5.03	14.9
0.1	4.57	13.57
0.2	3.93	11.7
0.3	3.65	10.9
0.4	3.20	9.50

Table 3. Parameters of the NJL model for two values of Λ_E [11]. Also shown are various integrals defined in the text as well as values of σ_N and $G_{\sigma NN}$. (The values given for Λ = 1000 MeV differ somewhat from those in Ref. [11].) Further refinements of these calculations are given in the Appendix.

Λ_E	900 MeV	1000 MeV
m_q^0	6.6 MeV	5.5 MeV
G_S	10.6 GeV^{-2}	7.91 GeV^{-2}
$g_{\sigma qq}$	2.97	2.58
$g_{\pi qq}$	3.10	2.68
$[1 - G_S J_S(0)]^{-1}$	2.38	2.88
m_q	295 MeV	260 MeV
m_σ	592 MeV	540 MeV
J_b	0.452	0.378
J_c	0.525	0.525
J_d	0.24	0.20
$K_S(0)$	0.0097 GeV^2	0.0088 GeV^2
B	- 0.24	- 0.20
σ_N	53.1 MeV (α = 0.36)	56.7 MeV (α = 0.30)
	50.3 MeV (α = 0.40)	52.4 Mev (α = 0.36)
$G_{\sigma NN}$	10.0 (α = 0.36)	9.23 (α = 0.30)
	9.50 (α = 0.40)	8.54 (α = 0.36)
$\langle N \vert \bar{q}q \vert N \rangle$	8.04 (α = 0.36)	10.3 (α = 0.30)
	7.62 (α = 0.40)	9.53 (α = 0.36)

Table 4. Values of σ_N, $G_{\sigma NN}$, $<N|\bar{q}q|N>_0$ and $<N|\bar{q}q|N>$ obtained by use of the Feynman-Hellman theorem.

	Λ_E = 900 MeV	Λ_E = 1000 MeV
σ_N	46.9 MeV	51.7 Mev
$G_{\sigma NN}$	8.91	8.38
$<N\|\bar{q}q\|N>_0$	3.00	3.25
$<N\|\bar{q}q\|N>$	7.10	9.39

Figure Captions

Fig. 1 (a) Here the triple lines denote on-mass-shell nucleons, the single lines denote quarks and the filled circle denotes the action of the operator $\bar{q}q$. The open circles represent the vertex amplitude for the nucleon to go into three constituent quarks.

(b) Here we consider corrections to the process in (a). The $\bar{q}q$ operator is now measured at the end of a string of quark loops, yielding an enhancement factor for the diagram in (a) of $\left(1 - G_S J_S(0)\right)^{-1} \simeq 2.88$, if the Euclidean cutoff for the momentum-space integrals is $\Lambda_E = 1.0$ GeV. [See Table 3.]

Fig. 2 Here the uppermost figure serves to define $< N|\bar{q}q\,|N>_0$ and the remaining figures describe a number of corrections that supplement the contribution of Fig. 2(a). The open circles at the end of the various nucleon lines are pion-nucleon vertex functions and the wavy lines represent pions. The nucleons in these diagrams are on-mass-shell.

(a) This diagram is given a value of $F(0)(1 - \alpha)$ with $F(0) = 3$.

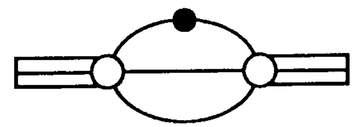
(b) This figure represents the integral J_b. The quark loop that forms the upper part of this figure gives rise to an integral which was calculated in Ref. [11].

(c) This diagram contributes $F(0)(1 - \alpha)J_c$ to the value of $< N|\bar{q}q\,|N>_0$.

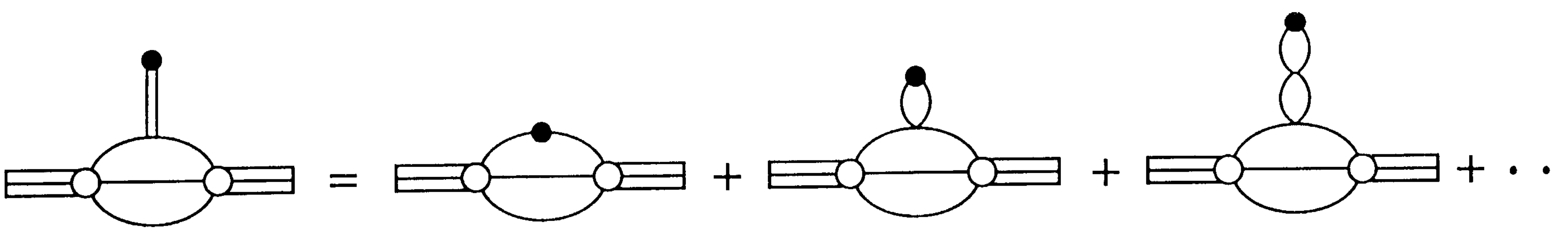
(d) This diagram contributes $F(0)(1 - \alpha)J_d$ to the value of $< N|\bar{q}q\,|N>_0$. The upper part of the figure defines $K_S(0)$ which was calculated in Ref. [13]. The double line may be taken to be either a sigma propagator or a sum of quark loops. One may use either representation, since $G_S/[1 - G_S J_S(0)] = g^2_{\sigma qq}/m^2_\sigma$ is a relation obtained in a bosonization procedure.

(e) The sum of the two diagrams shown contributes $BF(0)(1 - \alpha)$ to the calculation of $<N|\bar{q}q|N>_0$. In our work B is taken to be a phenomenological parameter.

Fig. 3 The matrix element $<N|\bar{q}q|N>$ is related to $<N|\bar{q}q|N>_0$ in a sigma-dominance model. Here the double line denotes a series of quark loops and the operator $\bar{q}q$ (filled circle) is measured at the end of the loop string, as shown in the figure. The resulting relation is $<N|\bar{q}q|N> = [1 - G_S J_S(0)]^{-1} <N|\bar{q}q|N>_0$. (See caption of Fig. 2d.)



(a)



(b)

FIG. 1

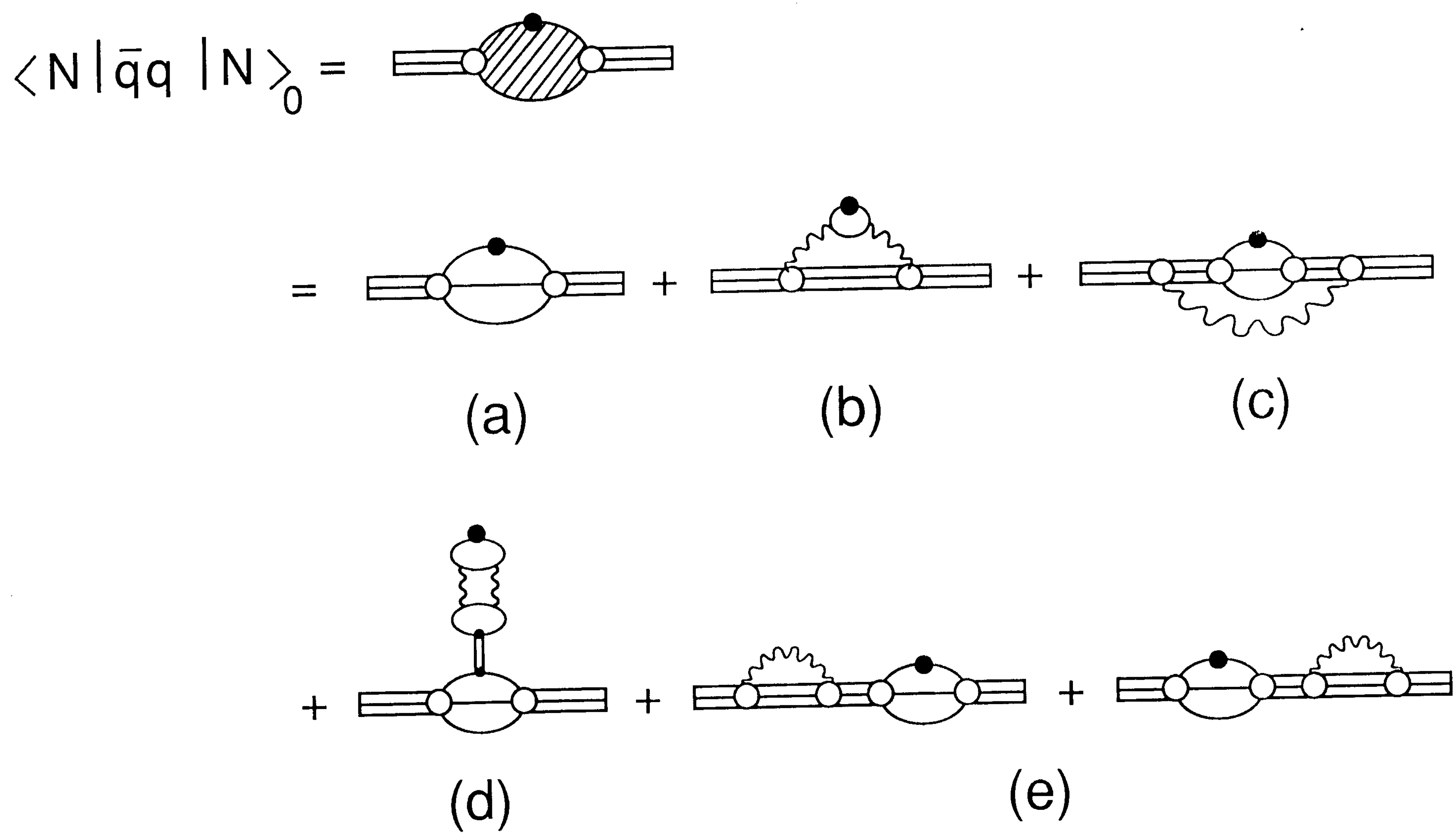


FIG. 2

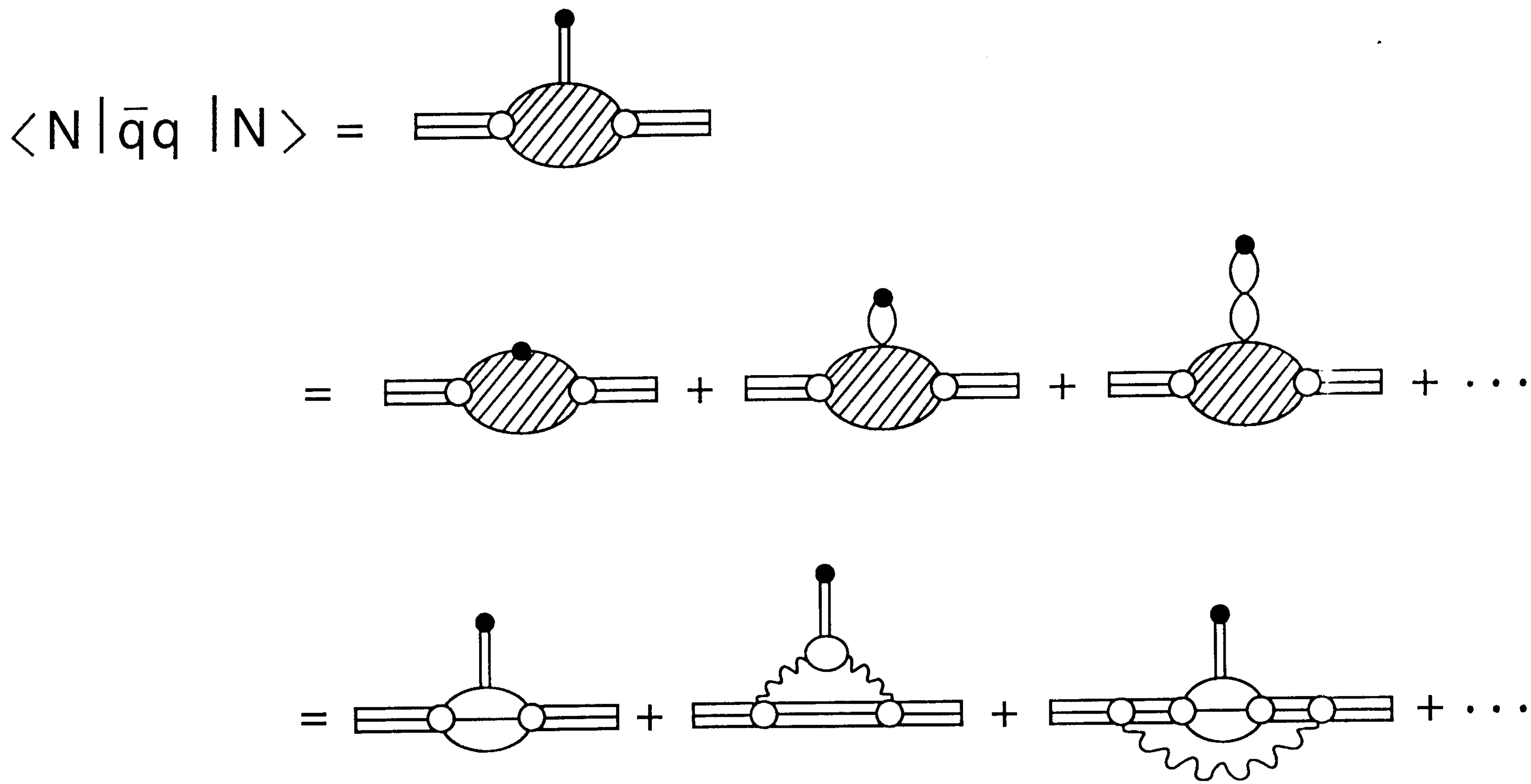


FIG. 3