Graviton-Photon Conversion in Atoms and the Detection of Gravitons

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Abstract

We study graviton-photon conversion in ground-based experiments. From graviton to photon transition, we calculate the cross section of graviton-atom interaction in the presence of spherical atomic electric fields; the obtained results hold for graviton energy around $10^5 \sim 10^9$ eV and would be enhanced by crystal structures, thus it gives a chance to catch MeV level gravitons from the universe with current neutrino facilities. From photon to graviton transition, we propose an experiment using entangled photon pairs to count missing photons passing through transverse magnetic tunnel, which could be used to verify the energy quantization of gravitational field.

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I. INTRODUCTION

The direct detection of gravitational wave (GW) has led us to the era of gravitational wave astronomy [1]. It signifies the triumph of Einstein's theory of general relativity (GR) — the geometric description of classical gravity. Yet, the observed frequency band ranging from $10 \sim 10^4$ Hz are relatively much narrower than that of electromagnetic wave (EMW), which is generally above 10^3 Hz till up to 10^{26} Hz. To observe the quantum aspects of gravity, it is necessary to extend the ceiling of the range to much higher frequencies, preferably to that of visible light. Various of methods [2–4] have been proposed to detect high frequency GW, with working mechanisms different from that of interferometry. The graviton-photon conversion (GRAPH [5]) , or known as the "gravitational Hertz experiment"[6–8], is supposed to detect ultra-high frequency GW about $10^8 \sim 10^{12}$ Hz. The mechanism works when a background electromagnetic field is provided, with the converting direction double-sided: from graviton to photon (G→P) or from photon to graviton (P→G), thus "mixing" or "oscillation" is sometimes invoked to name it. Since it came to sight, GRAPH has been investigated in a large amount of literatures. Analytically, the conversion is solved in a background of simple static electromagnetic (EM) fields and readily generalized to cases with different EM backgrounds [9–11]. After that, it applies to real astronomical context to extract information on properties of relevant astro-objects [12, 13], the evolution of the universe [14, 15], and even the dark components [16]. Further, it was also studied in modified theories and models of gravity [17, 18], or with higher order corrections in GR [19–22], and via new mechanism as parametric resonance [23]. On the other side, the possible sources to generate GW with such high frequencies are also proposed [24–26], with the evaporating primordial black holes as one of the important candidate [27–30], and recently magnetospheres of a single supermassive black hole as a new origin [13].

In this paper, we mainly focus on GPAPH on ground-based experiments, and particularly we calculate the interaction of gravitons with earth matter through atomic electric field, the resulting G→P cross section holds for graviton energy around $10^5 \sim 10^9$ eV and would be enhanced by diffraction in crystals, thus giving a chance to catch MeV level gravitons from the universe with current neutrino facilities. Further, we discuss the reverse P→G process in magnetic tunnel between parallel plates and give hints to testify the energy quantization of gravitational field. The paper is organized in a corresponding manner. We will provide a general formalism of

GRAPH in this introduction, and in Section II we will firstly give a review on G→P in transverse electromagnetic field with showing the transition probability and its physical implications, and then calculate the process in atoms as an important example with drawing useful inference on ground-based detection from the results. In Section III, we discuss the possibility of testing the energy quantization of gravitational field on earth with a beam of entangled photons as a source of P→G process. Conclusions and prospects are presented in the last section. We will work in flat spacetime throughout this paper, and use geometrical units $c = G = \hbar = 1$ in analytical process but recover to SI units when applying results to phenomenology.

The lowest order GRAPH in a general spacetime is fully described by perturbations of the action

$$S = S_g + S_{\mathrm{EM}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{\kappa^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) \tag{1}$$

with $\kappa^2 \equiv 16\pi$, R the Ricci scalar, and $F_{\mu\nu}$ the electromagnetic tensor as

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2}$$

and indices are raised by the metric, $F^{\mu\nu} = g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}$. Since we're working in flat spacetime, GW is treated as a perturbation on Minkowski spacetime, the metric is taken to be

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{3}$$

Doing the metric expansion on the Einstein-Hilbert action $S_g = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}R$ of the purely gravitational part to the second order with respect to $h_{\mu\nu}$, would lead to a Lagrangian

$$\mathcal{L}_h = \frac{1}{2\kappa^2}\left(\nabla^\mu h^{\lambda\nu}\nabla_\lambda h_{\mu\nu} - \frac{1}{2}\nabla^\lambda h^{\mu\nu}\nabla_\lambda h_{\mu\nu} - \nabla^\rho h_{\lambda\rho}\nabla^\lambda h + \frac{1}{2}\nabla^\lambda h\nabla_\lambda h\right) \tag{4}$$

which further gives, after choosing the transverse traceless (TT) gauge, the equation of motion (EOM) of the propagating part of GW

$$\partial_\lambda\partial^\lambda h_{\mu\nu} = 0, \tag{5}$$

generally a wave from solution is given by

$$h_{\mu\nu} = e_{\mu\nu}\mathrm{e}^{\mathrm{i}kx} + e^*_{\mu\nu}\mathrm{e}^{-\mathrm{i}kx}, \tag{6}$$

with $k_\mu = (\omega, 0, 0, \omega)$, and

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & e_{11} & e_{12} & 0 \\ 0 & e_{21} & -e_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

The propagating part of EMW and the interactive GRAPH part is encoded in the electromagnetic action $S_{\rm EM} = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\cdot F_{\mu\nu}F^{\mu\nu}$, and we will see that the interaction part would give a source term both to the free GW and EMW equations.

Now we decompose the full EM fields into a background (with a bar on top) and a free part,

$$A_\mu \to \overline{A}_\mu + A_\mu, \quad F_{\mu\nu} \to \overline{F}_{\mu\nu} + f_{\mu\nu}. \tag{8}$$

Keeping terms containing both $f_{\mu\nu}$ and $h_{\mu\nu}$ up to the 2nd order, we expand the EM action to obtain

$$\delta S_{\rm EM} = \int \mathrm{d}^4x \left(-\frac{1}{4} f_{\mu\nu}f^{\mu\nu} + \overline{F}_{\lambda(\mu}f^{\lambda}{}_{\nu)}h^{\mu\nu} - \frac{1}{4}h^{\lambda}{}_{\lambda}\overline{F}^{\rho\sigma}f_{\rho\sigma} \right), \tag{9}$$

the corresponding Lagrangian is naturally composed of a free term and an interaction term,

$$\mathcal{L}_{\rm EM} = \mathcal{L}_f + \mathcal{L}_{\rm int} = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \frac{1}{2}\mathcal{T}^{\mu\nu}h_{\mu\nu} \tag{10}$$

with the "interactive tensor", we name it, governing the core of GRAPH, written as

$$\mathcal{T}^{\mu\nu} = \overline{F}^{\mu\lambda}f^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}\overline{F}_{\rho\sigma}f^{\rho\sigma}. \tag{11}$$

Piecing $\mathcal{L}_{\rm EM}$ and $\mathcal{L}_h$ together will give the full description of GRAPH, with the source term also obtained by choosing TT gauge.

For G→P conversion, $\mathcal{L}_{\rm EM}$ alone is enough, but in a more explicit form with a current J^μ, extracted as

$$J_\rho = -\partial^\sigma \left[\left(\overline{F}_{\mu\rho}\eta_{\nu\sigma} - \overline{F}_{\mu\sigma}\eta_{\nu\rho} - \cancel{\frac{1}{2}\eta_{\mu\nu}\overline{F}_{\sigma\rho}} \right) h^{\mu\nu} \right], \tag{12}$$

where the last term vanishes due to the TT gauge. This can be done from an integration by parts to the interactive term, $\int \mathrm{d}^4x\sqrt{-g}\ \mathcal{T}_{\mu\nu}h^{\mu\nu} = \int \mathrm{d}^4x\ J_\rho A^\rho$. Therefore, the EOM for G→P conversion is

$$\nabla_\mu f^{\mu\nu} = -J^\mu \tag{13}$$

and the retarded potential is

$$A^{\mu}(r,t) = \frac{1}{4\pi}\int \frac{J^{\mu}(r',t-|r-r'|)}{|r-r'|}\,\mathrm{d}V'. \tag{14}$$

For P→G conversion, $\mathcal{L}_{int}$ is pieced with $\mathcal{L}_h$, giving the source term on the right hand side of equation (5),

$$\partial_{\lambda}\partial^{\lambda}h_{\mu\nu} = -2\kappa^2\mathcal{T}_{\mu\nu} = -2\kappa^2\overline{F}_{\lambda(\mu}f^{\lambda}_{\ \nu)} \tag{15}$$

and the retarded solution is thus

$$h_{\mu\nu}(r,t) = \frac{\kappa^2}{2\pi}\int \frac{\mathcal{T}_{\mu\nu}(r',t-|r-r'|)}{|r-r'|}\,\mathrm{d}V'. \tag{16}$$

The above formalism suits in a general sense. We will quantitatively study G→P process in atoms, and qualitatively discuss P→G process and its physical implications in the following sections.

II. GRAVITON TO PHOTON CONVERSION IN TRANSVERSE ELECTRO-MAGNETIC FIELDS AND ATOMS

Among the earliest analytical solution of GRAPH, a static transverse EM field is provided to be the background [9], we will reorganize the procedure in our consistent treatment, and review some crucial properties and implications of the transition probability.

A. G→P conversion in parallel plates and the transition probability

Consider a pair of electrically charged parallel plates, in between there is a static electric field in the x direction, when a gravitational wave with plane wave form (6) passes through the space in between the plates in the z direction, space distortion will happen. The e_{11} mode will effectively cause the plates to oscillate up and down. One might expect the plates to emit dipole radiation, which means a graviton changes into a photon in the same frequency. Because a single photon with high enough frequency can be observed, at high frequency, this effect may be used to detect gravitons.

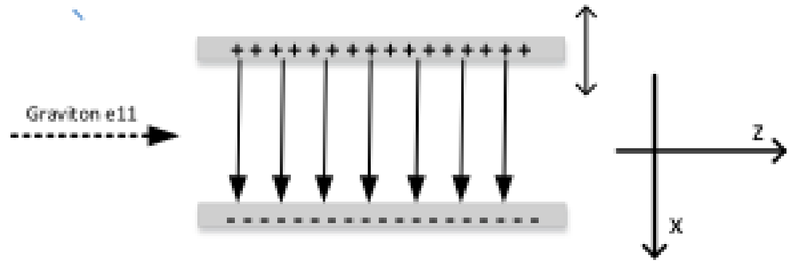

FIG. 1. G→P process in transverse electric field between parallel plates. The picture depicts that the excitation of photons is due to the space distortion of EM field caused by GW.

From the TT-gauged interactive Lagrangian $\mathcal{L}_{\text{int}} = \frac{1}{2}\mathcal{T}^{\mu\nu}h_{\mu\nu} = \frac{1}{2}\overline{F}^{\mu\lambda}f^{\nu}h_{\mu\nu}$ and its varied form $J_\rho A^\rho$, we see that background electric or magnetic field in the transverse direction can induce graviton-photon switching, while fixed charge on the plates cannot. If the back ground field is constant, and we replace the classical field with quantum field, $\mathcal{L}_{\text{int}}$ will give us a term that a graviton turns into a photon of the same frequency and direction, and another term for the reverse process. Note that we still have space time translational symmetry but no rotational symmetry, therefore energy and momentum is conserved but angular momentum is not.

To calculate the probability of a graviton turning into a photon, we go back to classical EM field theory. Let there be an incident gravitational wave on z direction $h_{\mu\nu} = 2\text{Re}\left[e_{\mu\nu}\text{e}^{\text{i}\omega(z-t)}\right]$ and a background electric field $\overline{E}$ in the x direction $\overline{E} = \overline{F}_{10} = -\overline{F}_{01}$, we get the Lagrangian as

$$\mathcal{L}(h, E) = -2\overline{E}\text{Re}\left[\left(E_1 e_{11} + E_2 e_{12}\right)\text{e}^{\text{i}\omega(z-t)}\right], \tag{17}$$

the corresponding electric current results as

$$j_i = 2\omega\overline{E}|e_{1i}|\cos\left[\omega(z-t) + \varphi_i - \frac{\pi}{2}\right], \tag{18}$$

where the phase $\varphi_i - \frac{\pi}{2}$ is irrelevant to our problem. We see that only j_x and j_y exist in our case.

The electric field is distributed in a space between parallel plates, let the height, width, length be H, W, L, and the origin be at the center of the box. We can use Green's function to get the EM field value at faraway point $x = (r, \hat{k})$, where $\hat{k}$ is unit vector representing a propagation direction. The x component of 4-potential is

$$A_x(r, \hat{k}, t) \approx \int \text{d}x' \frac{\omega\overline{E}|e_{11}|}{2\pi r}\cos\left[\omega(z' - t + r - \hat{k}\cdot\vec{r}')\right] \equiv \frac{\omega\overline{E}V|e_{11}}{2\pi r}\beta(\hat{k})\cos\left[\omega(r-t)\right], \tag{19}$$

with $\beta(\hat{k})$ as

$$\beta(\hat{k}) = \frac{1}{V}\int_V \mathrm{d}^3x' \;\cos\left[\omega(z' - \hat{k}\cdot\vec{r}')\right]. \tag{20}$$

The electric field follows straightforward as

$$E_x(r,\hat{k},t) = -\frac{\partial A_x}{\partial t} = \frac{\omega^2\overline{E}V|e_{11}|}{2\pi r}\beta(\hat{k})\sin\left[\omega(r-t)\right] \equiv E_0\cos\left[\omega(r-t)\right]. \tag{21}$$

The power radiated is calculated by integrating the average energy flux density over the whole spherical region:

$$P_{EM} = \int |\overline{S}|r^2\,\mathrm{d}\Omega = \frac{1}{2}\int E_0^2 r^2\,\mathrm{d}\Omega = \frac{1}{2}\left(\frac{\omega^2\overline{E}V|e_{11}}{2\pi r^2}\right)^2 r^2\int \beta^2(\hat{r})\,\mathrm{d}\Omega \equiv \frac{\omega^4(\overline{E}V|e_{11})^2}{8\pi^2}\alpha, \tag{22}$$

and α as

$$\alpha(\hat{k}) = \int\left[\beta(\hat{k})\right]^2 \mathrm{d}^2\hat{k} = \frac{4\pi^2}{\omega^2 HW}. \tag{23}$$

go back to common SI units, bring back c and $\overline{E}^2 \to \epsilon_0\overline{E}^2$, we have

$$P_{\mathrm{EM}} = \frac{1}{2}\omega^2\epsilon_0\overline{E}^2VL|e_{11}|^2/c \tag{24}$$

and we know the incoming gravitational radiation power is

$$P_{\mathrm{GW}} = \frac{\omega^2c^3|e_{11}|^2}{8\pi G}WH. \tag{25}$$

Therefore, the ration between EM radiation power and incoming power, i.e. the probability that a graviton turns into photon, is

$$\epsilon_{g\to\gamma} = P_{EM}/P_{\mathrm{GW}} = 4\pi G\epsilon_0\bar{E}^2L^2/c^4. \tag{26}$$

For e_{12} mode, it is the same conversion probability.

A background magnetic field will also produce EM radiation from gravitational radiation. The coupling strength is the same, only that when $\overline{B}$ in x direction, e_{11} produce EM polarization in y direction and e_{12} produce polarization in x direction. For a constant magnetic field background,

$$\epsilon_{G-EM} = P_{EM}/P_h = 4\pi G\bar{B}^2L^2/\mu_0c^4. \tag{27}$$

The picture becomes clear, when a graviton travels in a background electric or magnetic field, it can be turned into a photon. The switching amplitude grows as it travels, and it does not depend on the frequency of the graviton, only on the strength of the background field. Given a long enough travel distance, it can oscillate back and forth between the photon and graviton states. When the distance is not that long, the probability of switching grows with the square of the distance. For non-constant, but slowly varying background field, it can be generalized to

$$\epsilon_{G-EM} = \frac{4\pi G}{\mu_0 c^4}\left[\left(\int \mathrm{d}l\bar{B}_x\right)^2 + \left(\int \mathrm{d}l\bar{B}_y\right)^2\right], \tag{28}$$

where the graviton still travels in z direction, the $\bar{B}_x$ and $\overline{B}_y$ term corresponds to the probability of creating photons of different polarization.

The graviton-photon switching effect benefit from the fact that both particles are massless, the probability amplitudes adds up coherently along the path of the particle. It is almost a resonance; but the effect is very sensitive to phase changes. If there were even a tiny speed difference between the speed of the two particles, the coherence will break after a short distance, the probability will stop grow with the square of the distance. QED one-loop effect must be considered, background field changes the speed of light. We leave the QED correction for future works.

In lab environment and visible light frequency, photon and graviton can travel light years without losing coherence. However, in strong stellar magnetic fields such as on neutron stars and magnetars, coherence can break down quickly. Thus it's enough to use the lowest order results in ground experiment.

B. G→P conversion in Atomic electric field and the catch of gravitons

Graviton-photon conversion amplitude is proportional to the strength of background EM filed. Inside an atom, there is strong electric field, must stronger than EM field that can be created in a lab. The field strength near nuclei is particularly strong. The electric field is spherically symmetric around the nuclei and cancels out when the wave length is long. But when the wave length is shorter than atomic radius, atomic electric field can produce graviton-photon conversion. Here we consider an incoming high-energy graviton from the universe, we will use formula (17) and the Green's function approach to compute its interaction with earth matter through atomic electric field. Let the incoming graviton to have direction in z, polarization e_{12}

, and not considering the atomic magnetic field, (17) becomes

$$\mathcal{L}(e_{12}) = -2\mathrm{Re}\left[\left(\overline{E}_x E_y + \overline{E}_y E_x\right)\mathrm{e}^{\mathrm{i}\omega(z-t)}\right], \tag{29}$$

we can compute the effective current spacetime vector to be

$$\begin{cases} j_0^{\text{eff}} = -2\mathrm{Re}\left[\left(\partial_x \overline{E}_y + \partial_y \overline{E}_x\right) e_{12}\mathrm{e}^{\mathrm{i}\omega(z-t)}\right] \\ j_x^{\text{eff}} = -2\mathrm{Re}\left[\left(\mathrm{i}\omega \overline{E}_y\right) e_{12}\mathrm{e}^{\mathrm{i}\omega(z-t)}\right] \\ j_y^{\text{eff}} = -2\mathrm{Re}\left[\left(\mathrm{i}\omega \overline{E}_x\right) e_{12}\mathrm{e}^{\mathrm{i}\omega(z-t)}\right] \\ j_z^{\text{eff}} = 0 \end{cases} \tag{30}$$

We will see this effective current produces a quadrupole EM radiation. Using Green's function, at an infinitely faraway point:

$$A_\mu(r, \hat{k}, t) = \frac{1}{4\pi r}\int \mathrm{d}^3x'\; j_\mu^{\text{eff}}\left(x', t - r + \hat{k}\cdot\vec{x}'\right) \tag{31}$$

where the integration of x' is on the whole atom. We will compute the cross section of graviton-photon conversion through a spherical symmetric atom. We use spherical coordinates with axis on z. The atomic electric filed is:

$$\vec{E}(x) = \overline{E}(r)\hat{r} \tag{32}$$

with

$$\overline{E}(r) = \frac{Ze}{4\pi r^2}\, q\left(\frac{r}{r_A}\right). \tag{33}$$

The formula is in natural units where $\epsilon_0 = 1$, where Z is the atomic number (number of protons inside the nucleus), e is the unit electric charge, $\hat{r}$ is the unit vector in radial direction, and $q(r/r_A)$ is the fraction of total net charges (that of nucleus minus electrons) distributed inside this radius, r_A is the radius of the atom. In real matter, electric field is affected by molecular and crystal structures, but near the nuclei, it is always spherically symmetric. High energy gravitons can sense the electric field in the center.

From the above equations, we get

$$\begin{cases} A_x(r,\theta,\varphi,t) = \mathrm{Re}\left[\frac{e_{12}\mathrm{e}^{\mathrm{i}\omega(r-t)}Ze}{4\pi r} f(\theta)\sin\varphi\right] \\ A_y(r,\theta,\varphi,t) = \mathrm{Re}\left[\frac{e_{12}\mathrm{e}^{\mathrm{i}\omega(r-t)}Ze}{4\pi r} f(\theta)\cos\varphi\right] \end{cases} \tag{34}$$

with

$$f(\theta) = \int_0^{\omega r_A} \mathrm{d}\rho \; q\left(\frac{\rho}{\omega r_A}\right) \int_0^{\pi} \mathrm{d}\theta' \sin^2\theta' \cos\left[\rho(1-\cos\theta)\cos\theta'\right] J_1(\rho\sin\theta'\sin\theta), \tag{35}$$

where $J_1(x)$ is the 1st order Bessel function of the first kind.

To calculate radiation power for direction $\hat{k}$ at a faraway point, note that A_0 and longitudinal vector potential $A_{\hat{k}}$ cancels out in a gauge transformation, the radiation power is determined by the transverse vector potential:

$$\vec{A}_T = A_x\left[\hat{x} - \hat{k}(\hat{x}\cdot\hat{k})\right] + A_y\left[\hat{y} - \hat{k}(\hat{y}\cdot\hat{k})\right], \tag{36}$$

and hence

$$A_T^2 = A_x^2\left[1-(\hat{x}\cdot\hat{k})^2\right] + A_y^2\left[1-(\hat{y}\cdot\hat{k})^2\right] - 2A_xA_y(\hat{x}\cdot\hat{k})(\hat{y}\cdot\hat{k}). \tag{37}$$

Then we can get the angular EM radiation power distribution as

$$P(\theta,\varphi) = (\omega r)^2\overline{A}_T^2 = \frac{1}{32\pi^2}|e_{12}|^2(Ze\omega)^2 f^2(\theta)(1-\sin^2\theta\sin^2 2\varphi) \tag{38}$$

Its polar distribution is of quadrupole nature, with maximum at 4 directions of $\pm x$ and $\pm y$, and minimum at 4 direction of 45°. If the incoming graviton has polarization e_{11}, the EM radiation distribution is rotated 45°.

The total EM radiation power is integrated as

$$P_{\mathrm{EM}} = \int_0^{\pi}\int_{-\pi}^{\pi} \mathrm{d}\varphi \; P(\theta,\varphi) = \frac{|e_{12}|^2(Ze\omega)^2}{16\pi}\beta_A(\omega r_A) \tag{39}$$

with

$$\beta_A(\omega r_A) = \int_0^{\pi} \mathrm{d}\theta \; f^2(\theta)\left(1 - \frac{1}{2}\sin^2\theta\right). \tag{40}$$

Recovering SI units, we have

$$P_{\mathrm{EM}} = \frac{|e_{12}|^2(Ze\omega)^2}{16\pi\epsilon_0 c}\beta_A(\omega r_A) \tag{41}$$

And the cross section is obtained as

$$\sigma = \frac{P_{\mathrm{EM}}}{P_{\mathrm{GW}}} = \frac{G(Ze)^2}{2\epsilon_0 c^4}\beta_A(\omega r_A) = Z^2\beta_A(\omega r_A)\times 1.2\times 10^{-7} \quad \mathrm{m}^2. \tag{42}$$

Given the quantum atomic wave function, $f(\theta)$ and $\beta_A(\omega r_A)$ can be computed numerically. We will use a simple atom model, in which the negative charges (electron clouds) are uniformly distributed within a sphere of radius r_A, and positive charges are uniformly distributed within r_N. Note that r_N is not the radius of the nucleus but somewhat larger, in quantum mechanics, the center of the atom is the center of mass, the range of the nucleus is therefore determined by the mass ratio of electrons and the nuclei, in most atoms this ratio is around $1 : 4000$. We will take r_A/r_N=8000, considering average the inside and outside electrons.

$$q(r) = \begin{cases} 0, & r > r_A \\ 1 - \frac{r^3}{r_A^3}, & r_N < r < r_A \\ \frac{r^3}{r_N^3} - \frac{r^3}{r_A^3}, & r < r_N. \end{cases} \tag{43}$$

We use the above simple atom model (43) and MATLAB and obtained numerical results. Figure 2 shows $f(\theta)$ which determines the outgoing EM wave azimuthal distribution. The curve on the left is for $\omega r_A = 1$, the outgoing EM wave peaks at 90° to the incoming graviton direction; the curve on the right is for $\omega r_A = 10$, which peaks at a very forward direction. When the incoming graviton's energy is higher, the outgoing photons will be peak more and more in line with the incoming photon. Despite that $f(0) = 0$, high energy gravitons will convert into photons almost in the same direction.

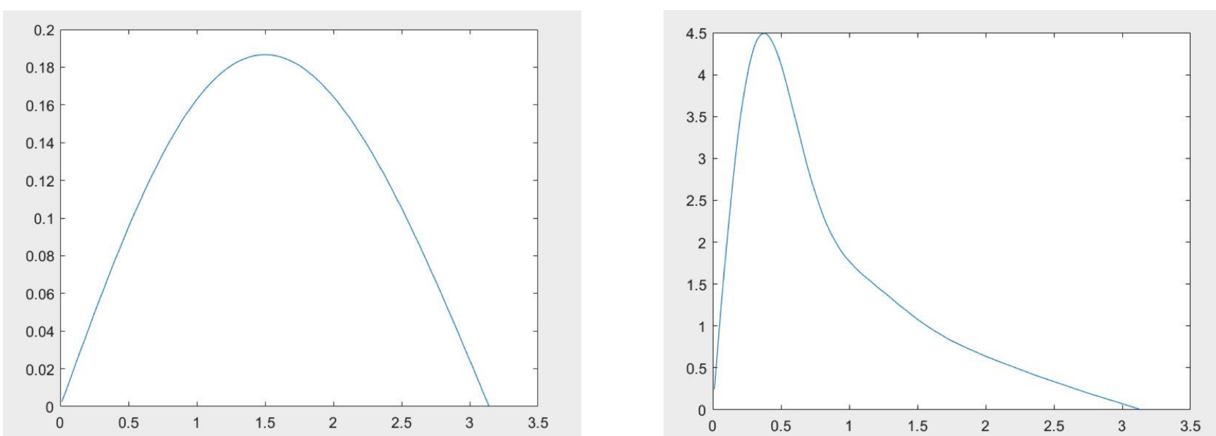

FIG. 2. $f(\theta)$ distribution for $0 \leqslant \theta \leqslant \pi$ when $\omega r_A = 1$ (left) and $\omega r_A = 10$ (right). The horizontal axis is range of θ, and the vertical axis is the numerical value of $f(\theta)$. It is seen that as ωr_A increases, the peak of $f(\theta)$ distribution moves to a small angle, shrinking the emitting EMW in almost the same direction with that of incident GW.

Further, $\beta_A(\omega r_A)$ is also computed with MATLAB. At $\omega r_A = 1$, $\beta_A = 0.01$, but it rises very quickly. When $\omega r_A = 100$, and at least till $\omega r_A = 10^6$, $\beta_A(\omega r_A) \approx \frac{1}{2}\omega r_A$. Therefore the cross section (42) becomes

$$\sigma = \omega r_A Z^2 \times 6 \times 10^{-72}\mathrm{m}^2, \quad \text{with} \quad 100 \leqslant \omega r_A \leqslant 10^6. \tag{44}$$

This formula holds for graviton energy from around 100 keV to 1 GeV, when energy is higher, recoil effect, or phonon excitation, has to be taken into account. At MeV energy level, for a medium sized atom, this cross section is about 17 orders of magnitude smaller than neutrino cross section with atoms. It is still much larger than one would naively expect from $M_{weak}/M_{Planck} = 10^{-34}$.

In crystals, the cross section shall be enhanced by an effect similar to X-ray diffraction: on some particular frequencies and directions, outgoing EM wave adds up coherently, the cross section gets amplified by a factor that equals to the number of atoms on the path. In a poly-crystal such as ice, many frequency/direction combinations will be amplified easily by a factor of $6 \sim 7$ order of magnitude.

This makes it feasible to try to capture high energy gravitons from the universe using the current neutrino experiment facilities, or some upgraded version of it. It needs to be done deep underground, when the energy is higher than a few MeV, there is pretty much no radio activity background, the only background is neutrino. It can generate photons through higher order weak interactions, proper detections need to rule out neutrinos.

A known source of high energy gravitons is from primordial black holes, in its final moment of Hawking evaporation [27–30]. Such kind of black holes has recently been brought up as candidate to dark matter again. If there is one that is evaporating away not too far away, it may be observed as gamma photon events.

III. PHOTON TO GRAVITON CONVERSION AND THE ENERGY QUANTIZATION OF GRAVITATIONAL FIELDS

On the other side of GRAPH, a consequence of P→G switching is to produce small number of gravitons in the universe, with a spectrum matching that of photons, because magnetic field is everywhere. The transition probability is about $\epsilon \simeq 8.2 \times 10^{-38} \left(BL/T \cdot m\right)^2$, which is a very small effect. Neutron stars and magnetars have very strong magnetic field, but unfortunately QED effects mentioned above breaks coherence. Moderate magnetic field in large space can convert photons to gravitons, a typical galaxy has a size of 100000 light years ($\sim 10^{21}$ m) and an average magnetic field of 10^{-9} T, and most of the field are not turbulent, the conversion ratio can be on the order of 10^{-14}. The graviton spectrum matches that of the photons, except for radio frequency of which the speed is affected by interstellar dust. Although this ratio is still small, it is much larger than you would naively expect by compare the strength of gravitational interaction with that of EM interaction. Primordial magnetic field in the universe is also a subject of interests in recent years, reference [15] studied conversion of graviton to photons in early universe magnetic field. In today's universe, magnetic field can convert photons to gravitons.

For ground experiments, reference [9] suggested use this effect to generate gravitational wave from EM wave, then regenerate EM wave from gravitational wave. However, if we make a graviton detector with 30T magnetic field and 10km length, it gives $\epsilon = 7.2 \times 10^{-27}$, which is a very small efficiency; for 1W wave of $\omega = 10^{14}$ Hz, we will get about 2 events a month. EM-GW-EM process will square this efficiency, hopelessly small.

A more feasible experiment is to have a photon beam, entangled with one another for comparison, goes through the long magnetic tunnel, and to count the missing photons. Recent studies on axions pointed out there are also photon-axion conversion given a background EM field, but with different polarizations; in transverse EM field, photons will be converted to gravitons. We hereby point out this is an experiment that can test the quantum feature of gravity.

An example experimental set up can be as shown below: creating a pair of entangled γ photons from electron-positron annihilation, try to capture the event that one of them going through a long magnetic channel, count the photons on the 2 detectors to find the missing photons.

The electron and positron beams need to be properly cooled to make sure the total transverse momentum is 0, therefore the opposite going photon event can be used to tag the photon going through a long magnetic tunnel that might be converted to a graviton. The photons of interest are in a particular direction and energy, this will help to eliminate background from environmental radio activities.

Here one may ask, is it possible that gravitational field remains classical while other fields is quantized? How to prove gravitational field must be quantized? One can argue that classical gravitational theory will encounter blackbody radiation trouble similar to EM field. But this argument is weak, in any practical system, gravitational field will never have thermal equilibrium.

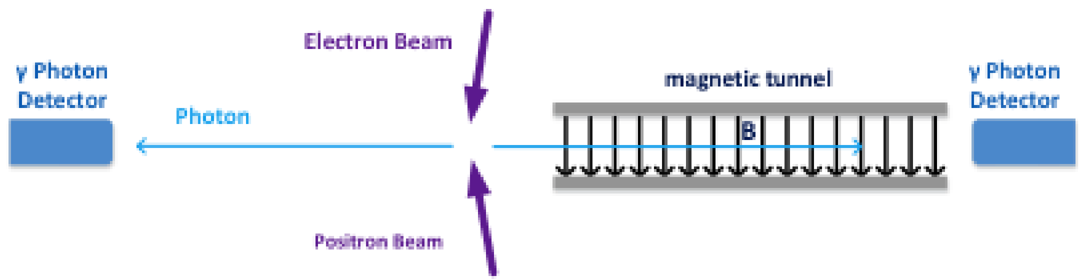

FIG. 3. Experimental setup to count the missing photon and to testify the energy quantization of gravity. Entangled photon pairs are created from positron-electron beams, with one going left directly to the γ photon detector, and the other going right through a magnetic tunnel before running into the detector.

Instead, missing photons after going through magnetic tunnel shall prove that the energy of gravitational field is quantized.

Given the fact that EM field is quantized, and GR predicts gravitational radiation when EM wave goes through magnetic tunnel, if gravitational field were not quantized, then each photon must radiate gravitational wave and lose some energy, and be red-shifted. However, general relativity predicts that EM wave lose some intensity without changing frequency, gravitational wave generates EM wave in the reverse process, cause original EM wave to lose intensity. It is only consistent to re-interpret the gravitational wave as a quantum probability wave. If gravitational field is not quantized, general relativity needs modification.

Photon magnetic tunnel experiment can demonstrate or rule out the quantum feature of gravity, the reverse process, gravitational wave generate photons, if detected, does not prove gravity is quantized. If energy quantization of gravity is indeed verified, the gravitational constant at the given frequency can be measured. Plausibly it is the same as low frequency, but it will be nice to check.

IV. CONCLUSIONS AND PROSPECTS

In this work, we studied the GRAPH in two aspects. In the G→P process, we calculated the transition probability in transverse EM fields with classical Green's function approach, and obtained the cross section of graviton-atom interaction in presence of atomic electric field. The results hold for graviton energy from around 100 keV to 1 GeV, and would be amplified by the crystal structure, thus making it feasible to capture high energy gravitons from the universe using the current neutrino experiment facilities underground; the only factor that should be ruled out is neutrino background. The relevant sources are guaranteed in relevant literatures. In the G→P process, we illustrated in detail the possibility of testing the quantum feature of gravity, and proposed an experiment using entangled photon pairs to count missing photon passing through transverse magnetic tunnel. We pointed out this could be a criteria to judge the energy quantization of gravitational field, and a positive results (if it is) would suggest that one is able to study quantum gravity without going to the Plank energy.

Although being studied along the history, GRAPH research is still far from being fully investigated. Analytically, GRAPH in curved spacetime, e.g, at photon sphere of a charged black hole, or in a wide class of modified gravity, would be interesting topics to explore, and useful physical insights and implications are expected. Phenomenologically, GRAPH in different EM background, either in various astronomical environments, or in ground-based and man-made facilities, would give us crucial information on the basic properties of interactions between gravity and other components of the universe. We will report our research on these parallel lines in future.

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Appendix A: Some detail computations

1. Calculations of the β and α integrals in (20) and (23)

The $\beta(\hat{k})$ is integrated as

$$\begin{aligned}\beta(\hat{k}) &= \frac{1}{V}\int_V \mathrm{d}^3x' \ \cos\left[\omega(z' - \hat{k}\cdot\vec{r}')\right] \\ &= \frac{1}{V}\int_{-L/2}^{L/2} \cos\left[\omega z'(1-k_z)\right] \, \mathrm{d}z' \int_{-H/2}^{H/2} \cos(\omega k_x x')\,\mathrm{d}x' \int_{-W/2}^{W/2} \cos(\omega k_y y')\,\mathrm{d}y' \\ &= \int_{-1/2}^{1/2} \cos\left[\omega L z(1-k_z)\right] \, \mathrm{d}z \int_{-1/2}^{1/2} \cos(\omega H k_x x)\,\mathrm{d}x \int_{-1/2}^{1/2} \cos(\omega W k_y y)\,\mathrm{d}y \\ &= \frac{2\sin\left[\frac{\omega L(1-k_z)}{2}\right]}{\omega L(1-k_z)} \frac{2\sin\left(\frac{\omega H k_x}{2}\right)}{\omega H k_x} \frac{2\sin\left(\frac{\omega W k_y}{2}\right)}{\omega W k_y},\end{aligned} \tag{A1}$$

since there's a resonance along the z-axis, we have

$$\begin{aligned} k_z &= \sqrt{1-(k_x^2+k_y^2)} \approx 1 - \frac{1}{2}(k_x^2+k_y^2) \\ \Longrightarrow \quad \omega L(1-k_z) &\approx \frac{1}{2}\omega L(k_x^2+k_y^2) \ll \omega H k_x, \quad \omega W k_y,\end{aligned} \tag{A2}$$

so we can approximate the k_z term as $\frac{2\sin\left[\frac{\omega L}{2}(1-k_z)\right]}{\omega L(1-k_z)} \approx 1$, while the other two terms keep the original form, thus the integral becomes

$$\beta(\hat{k}) \approx \frac{\sin\left(\frac{\omega H k_x}{2}\right)}{\frac{\omega H k_x}{2}} \frac{\sin\left(\frac{\omega W k_y}{2}\right)}{\frac{\omega W k_y}{2}}. \tag{A3}$$

Then it is easy to compute the $\alpha(\hat{k})$ as

$$\begin{aligned}\alpha(\hat{k}) &= \int \left[\beta(\hat{k})\right]^2 \mathrm{d}^2\hat{k} \\ &= \int_{-1}^{1} \left[\frac{\sin\left(\frac{\omega H k_x}{2}\right)}{\frac{\omega H k_x}{2}}\right]^2 \mathrm{d}k_x \cdot \int_{-1}^{1} \left[\frac{\sin\left(\frac{\omega W k_y}{2}\right)}{\frac{\omega W k_y}{2}}\right]^2 \mathrm{d}k_y \\ &= \frac{4}{\omega^2 H W}\left[\int_{-\infty}^{\infty}\left(\frac{\sin^2 u}{u^2}\right)\mathrm{d}u\right]^2 = \frac{4\pi^2}{\omega^2 H W},\end{aligned} \tag{A4}$$

where we have taken the limit of $\omega H, \omega W \gg 1$.

2. Derivation of $A_x(r,\hat{k},t)$ in equation (34)

From (31),(32),(33) we get:

$$A_x(r,\hat{k},t) = 2\mathrm{Re}\left\{\frac{\mathrm{i}\omega e_{12}}{4\pi r}\int \sin\theta' r'^2\,\mathrm{d}r'\,\mathrm{d}\theta'\,\mathrm{d}\varphi'\overline{E}(r')\sin\theta'\sin\varphi'\ \mathrm{e}^{\mathrm{i}\omega\left(r'\cos\theta'-t+r-\hat{k}\cdot\vec{x}'\right)}\right\} \tag{A5}$$

and hence:

$$\begin{aligned}A_x(r,\hat{k},t) =& 2\mathrm{Re}\left\{\frac{\mathrm{i}Ze\omega e_{12}\mathrm{e}^{\mathrm{i}\omega(r-t)}}{4\pi\cdot 4\pi r}\int_0^{r_A}\mathrm{d}r'\int_0^{\pi}\mathrm{d}\theta'\int_{-\pi}^{\pi}\mathrm{d}\varphi'\right.\\ &\left. q\left(\frac{r'}{r'_A}\right)\sin^2\theta'\sin\varphi'\ \mathrm{e}^{\mathrm{i}\omega r'(\cos\theta'-\cos\theta'\cos\theta-\sin\theta'\sin\theta\cos(\varphi'-\varphi)}\right\}\\ =& \mathrm{Re}\left\{\frac{\mathrm{i}(Ze)e_{12}\mathrm{e}^{\mathrm{i}\omega(r-t)}}{8\pi^2 r}\int_0^{\omega r_A}\mathrm{d}\rho\ q\left(\frac{r'}{r'_A}\right)\int_0^{\pi}\mathrm{d}\theta'\sin^2\theta'\right.\\ &\left.\int_{-\pi}^{\pi}\mathrm{d}\varphi'\sin\varphi'\ \mathrm{e}^{\mathrm{i}\rho(\cos\theta'-\cos\theta'\cos\theta-\sin\theta'\sin\theta\cos(\varphi'-\varphi)}\right\},\end{aligned} \tag{A6}$$

where in the second equator we substituted $\rho=\omega r'$. The integration over ϕ' can be done analytically

$$\begin{aligned}&\int_{-\pi}^{\pi}\mathrm{d}\varphi'\sin\varphi'\ \mathrm{e}^{-\mathrm{i}\rho\sin\theta'\sin\theta\cos(\varphi'-\varphi)}\\ =&\int_{-\pi}^{\pi}\mathrm{d}\varphi'\sin(\varphi'+\varphi)\ \mathrm{e}^{-\mathrm{i}\rho\sin\theta'\sin\theta\cos\varphi'}\\ =&\sin\varphi\int_{-\pi}^{\pi}\mathrm{d}\varphi'\cos\varphi'\ \mathrm{e}^{-\mathrm{i}\rho\sin\theta'\sin\theta\cos\varphi'}\\ =&-\mathrm{i}\sin\varphi\int_{-\pi}^{\pi}\mathrm{d}\varphi'\cos\varphi'\sin(-\mathrm{i}\rho\sin\theta'\sin\theta\cos\varphi')\\ =&-2\pi\mathrm{i}J_1(\rho\sin\theta'\sin\theta)\sin\varphi\end{aligned} \tag{A7}$$

with $J_1(x)$ the 1st order Bessel function of the first kind. Joining (A6) and (A7) we have the 1st line of (34); the derivation of the 2nd line is similar.

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