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INTENSITY LIMITATIONS BY TRANSVERSE SPACE-CHARGE EFFECTS IN PRESENT AND PLANNED CERN MACHINES

The effect of transverse space charge on the vertical stability of particles in a circular accelerator has been thoroughly discussed in a recent report by J. Laslett[1], where the limitations imposed by the need for stability of individual particle motion and stability with respect to coherent motion of the beam are both considered, taking into account the effect of image forces arising from the presence of a vacuum chamber with conducting walls and of the magnet poles. The influence of space-charge neutralization, however, is purposely omitted by Laslett, who refers especially to the situation of injection into an accelerator and assumes that little neutralization may occur in the time interval considered. If, on the contrary, the injected beam has to be stored for an appreciable time as in the case of the proposed storage rings for the CERN-PS and also, possibly, in the case of injection of successive pulses from a booster synchrotron into a large machine, the effect of partial neutralization has to be taken into account.

The cloud of electrons captured in the positive beam is supposed to follow the coherent transverse displacements of the beam, so that the electric fields produced by an unbunched beam and by its images are everywhere reduced in the proportion of the partial neutralization.

If the beam is bunched, the distribution of the neutralizing electrons is assumed to remain practically uniform and their density to be obtained by applying the neutralization coefficient to the average space-charge density.

Therefore in the expression of the electric field produced by a beam having "bunching factor" B, defined as the ratio of the average to the maximum linear particle density, the cofficient $\frac{1}{B}$, which is applied to the electric field produced by a uniform beam to take account of the particle concentration

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1) L.J. Laslett, Proceedings of the 1963 Summer Study on Storage Rings, Accelerators and Experimentation at Super-High Energies, BNL 7534, in print.

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in the bunches, must be replaced by $(\frac{1}{B} - \nu)$, where ν is the "neutralization factor" defined as the fraction of the average charge density that is neutralized.

The presence of the neutralizing electrons, which are practically stationary, does not modify the magnetic field produced by the beam current, and the expression of this field, given by Laslett, apply also to the neutralized beam.

The space-charge limits, deducted by means of the modified field expressions, are given by the following formulae:

a) For single particle stability in the vertical plane:

$$N = \frac{\pi}{2} \frac{h^2}{r_p R} \gamma \frac{- 2 Q_{vo} \quad \Delta Q_{vS}}{\varepsilon_1 \left(1 + \frac{B^{-1} - \nu \gamma^2}{\gamma^2 \beta^2}\right) + \varepsilon_2 \frac{h^2}{g^2} + \frac{h^2}{b(a+b)} \quad \frac{B^{-1} - \nu \gamma^2}{\gamma^2 \beta^2}}$$

where N is the total number of particles in the beam,

r is the "classical radius" for the particles and may be taken as 1.536×10^{-16} cm for a proton,

R is the average orbit radius,

a and b are the radial and vertical semi-axis of the beam cross section (assumed elliptical),

h is the half height of the vacuum chamber

g is the half height of the magnet gap

β and γ are the usual relativistic coefficients

ε_1 and ε_2 are the image-force coefficients which are a function of the particular geometric configuration of the vacuum chamber and of the magnet gap respectively.

Q_{vo} is the number of betatron oscillations per revolution for a low intensity beam and ΔQ_{vS} is the permitted variation of Q_v by space-charge forces for single particle stability. The minus sign is due to the fact that when the denominator is positive, ΔQ_v must be taken negative, because then Q_v

is reduced by space-charge forces, while the contrary happens when the denominator is negative.

In assuming a numerical value for ΔQ_v in this formula, one should take into account Lloyd Smith's remark[2)] that in practice a large blow-up of the beam should occur only when the computed Q-variation by space-charge forces is double the distance from the next integral or half-integral stopband. In fact, a tendency of the beam to increase in diameter is inhibited by the change in internal frequency because of the non-linear character of space-charge forces with respect to beam diameter.

b) For stability with respect to a coherent vertical displacement it must be considered that magnetostatic images will refer to the average position of the beam in the chamber, while the variable components of the magnetic field resulting from the coherent oscillations of the beam will be subject to the boundary conditions imposed by the vacuum chamber.

Therefore the space-charge limit will be

$$N = \frac{\pi}{2} \frac{h^2}{r_p R} \gamma \frac{-2 Q_{vo} \Delta Q_{vc}}{\xi_1 \frac{B^{-1} - \nu \gamma^2}{\gamma^2 \beta^2} + \varepsilon_2 \frac{h^2}{g^2}}$$

If the stopband approached is integral, the vertical displacement of the beam appearing at a given azimuth from revolution to revolution will become slower and slower and the penetration of the variable magnetic field into the chamber wall will increase, thus adding a positive term to the denominator. If the denominator is already positive, the space-charge limit will be decreased; if it is negative, it will be increased.

2) L.R.L. int. rep. UCID - 1879, LS - 7. April 1963.

In the first case the limit tends to

$$N = \frac{\pi}{2} \frac{h^2}{r_p R} \gamma \frac{-2 Q_{vo} \Delta Q_{vc}}{\xi_1 \left(1 + \frac{B^{-1} - \nu \gamma^2}{\gamma^2 \beta^2}\right) + \xi_2 \frac{h^2}{g^2}}$$

while for the second case it is only possible to say that it will be larger than given by the first formula .

ξ_1 and ξ_2 are image force coefficients, different from ε_1 and ε_2, which are also a function of the particular geometric configuration of the vacuum chamber and of the magnet gap respectively .

Expressions for the coefficients are given by Laslett for several cases of interest and in particular for a vacuum chamber of elliptical cross section and for a wedge-shaped magnet gap. Some of them are reproduced in the appendix.

The above modified formulae have been applied to the cases of the Storage Rings for the CERN P.S. and of the 300 GeV machine, using present tentative parameters. As a comparison, similar computations have been carried out for the present P.S. with different injection energies.

The later results of Laslett, Neil and Sessler[3)] on the transverse instabilities associated with the energy dissipation in a resistive vacuum tank are not used; the possible consequences of internal energy dissipation in the partially neutralized beam are not considered either. Therefore the figures obtained can only be regarded as upper limits.

Transverse space charge limit in the I.S.R. for the CERN-P.S.

The stored beam is not bunched, and therefore B = 1. On the other hand, neutralization may play an important role : we shall compute the extreme cases $\nu = 0$ and $\nu = 1$.

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3) L.J. Laslett, V.K. Neil and A.M. Sessler, UCRL - 11090, October 1963.

We assume the following values:

Average radius $R = 150$ m

Number of betatron oscillations $Q_o = 8.75$

Nominal momentum $p = 25$ GeV/c

Average half width of the beam: $a = 3$ cm

Average half height of the beam: $b = 0.5$ cm

Half height of the vacuum chamber: $h = 2.6$ cm

Half width of the vacuum chamber: $W = 8.0$ cm

H alf height of the magnet gap: $g = 5.0$ cm

Profile parameters $n/\rho = 4\ m^{-1}$

$\Delta Q_{vs} = \pm 0.5$

$\Delta Q_{vc} = \pm 0.25$

The limit currents with regard to coherent vertical displacements of the beam are:

for $\nu = 0$ $\qquad I_{co} = 370$ A

for $\nu = 1$ $\qquad I_{c1} > 53$ A

and with regard to single particle stability in vertical oscillations:

for $\nu = 0$ $\qquad I_{So} = 200$ A

for $\nu = 1$ $\qquad I_{S1} = 14$ A

We can see that in this case image effects reduce drastically the conventional space charge limit for a non neutralized beam, which would have been as high as 10 000 A, but they do not make it lower than that of a neutralized beam.

Transverse space-charge limit in the 300 GeV machine at 8 GeV injection energy.

If it is assumed to transfer the bunches from the booster into RF buckets in the main machine, the bunch half width may reach $\varphi = 20^o$. This would correspond to a bunching factor about $\frac{1}{10}$ if the particles were uniformly distributed in the bunch. Taking into account the higher particle density near the center of the bunch, an assumption of $B = \frac{1}{20}$ will give a more realistic estimate.

We assume the following data:

Average radius	$R = 1200$ m
Number of betatron oscillation	$Q = 28{,}75$
Average half width of the beam	$a = 1{,}0$ cm
Average half height of the beam	$b = 1{,}0$ cm
Half height of the vacuum chamber	$h = 2{,}7$ cm
Half height of the magnet gap	$g = 3{,}5$ cm
Half width of the vacuum chamber	$W = 4{,}5$ cm
Profile parameter	$n/\rho = 6$ cm^{-1}

$\Delta Q_{vs} = \pm 0.5$

$\Delta Q_{vc} = \pm 0.25$

The limit number of particles in the beam with respect to coherent vertical displacements of the beam is:

for $\nu = 0$ $\quad N_{co} = 2{,}85 \cdot 10^{14}$ part/pulse

for $\nu = 1$ $\quad N_{cl} > 3{,}05 \cdot 10^{14}$ part/pulse

and with respect to single particle stability in vertical oscillations:

for $\nu = 0$ $\quad N_{so} = 1{,}5 \cdot 10^{14}$ part/pulse

for $\nu = 1$ $\quad N_{sl} = 6{,}4 \cdot 10^{13}$ part/pulse

Transverse space charge limit in the CERN PS at 50 MeV injection energy.

The bunch half-width is almost 90°. This would correspond to a bunching factor of about $\frac{3}{8}$ if the particles were uniformly distributed in the bunch. Taking into account the higher particle density near the center of the bunch, an assumption of $B = \frac{1}{4}$ will give a more realistic estimate.

The relevant P.S. data are:

Average radius	$R = 100$ m
Number of betatron oscillations	$Q = 6,25$
Average half width of the beam	$a = 4$ cm
Average half height of the beam	$b = 2,5$ cm
Half height of the vacuum chamber	$h = 3,7$ cm
Half width of the vacuum chamber	$W = 7,5$ cm
Half height of the magnet gap	$g = 5,0$ cm
Profile parameter	$n.\rho \simeq 4$ m^{-1}

Here Lloyd Smith's factor of 2 in ΔQ_{vs} is not really available, because the beam almost fills the vacuum chamber. Therefore we shall assume also $\Delta Q_{vs} = \pm 0,25$.

The limit number of particles in the beam with respect to coherent vertical displacements of the beam is:

for $\nu = 0$	$N_{co} = 1,85 \cdot 10^{12}$ part/pulse
for $\nu = 1$	$N_{c1} = 2,4 \cdot 10^{12}$ part/pulse

and with respect to single particle stability in vertical oscillations

for $\nu = 0$	$N_{so} = 1,1 \cdot 10^{12}$ part/pulse
for $\nu = 1$	$N_{s1} = 1,4 \cdot 10^{12}$ part/pulse

Therefore, in the C.P.S. the limitations due to collective phenomena and those due to single particles instability are of the same order and neutralization would not change the situation appreciably.

If the injection energy were raised to 200 MeV in the C.P.S., without altering the above parameters, and in particular the bunching factor, the calculated space charge limits would be raised to the following values:

For coherent vertical displacements

for $\nu = 0$ $\quad N_{co} = 8{,}9 \cdot 10^{12}$ part/pulse

for $\nu = 1$ $\quad N_{c1} = 1{,}3 \cdot 10^{13}$ part/pulse

for single particle stability in vertical oscillations

for $\nu = 0$ $\quad N_{so} = 6 \cdot 10^{12}$ part/pulse

for $\nu = 1$ $\quad N_{s1} = 9 \cdot 10^{12}$ part/pulse.

Appendix.

Image coefficients computed by J. Laslett.

a) Electrostatic image coefficients.

The values of these coefficients, ε_1, and ξ_1, are given in diagram 1 as a function of the ratio of the semi-axès for a vacuum chamber of ellyptical cross section.

b) Magnetostatic image coefficients.

For a strong focussing magnet, these coefficients are computed by considering a wedge-shaped gap having the right slope at the position of the equilibrium orbit. Their expressions, as a function of the gap half height g and of the profile parameter n/ρ are:

$$\varepsilon_2 = \frac{\pi^2}{24}\left[1 - \frac{6}{\pi}\,\frac{g}{\rho/n} + \left(\frac{2}{3} + \frac{5}{\pi^2}\right)\left(\frac{g}{\rho/n}\right)^2\right]$$

$$\xi_2 = \frac{\pi^2}{16}\left[1 - \frac{4}{\pi}\,\frac{g}{\rho/n} + 2\left(\frac{1}{3} + \frac{2}{\pi^2}\right)\left(\frac{g}{\rho/n}\right)^2\right]$$

where g and ρ/n are in the same units.

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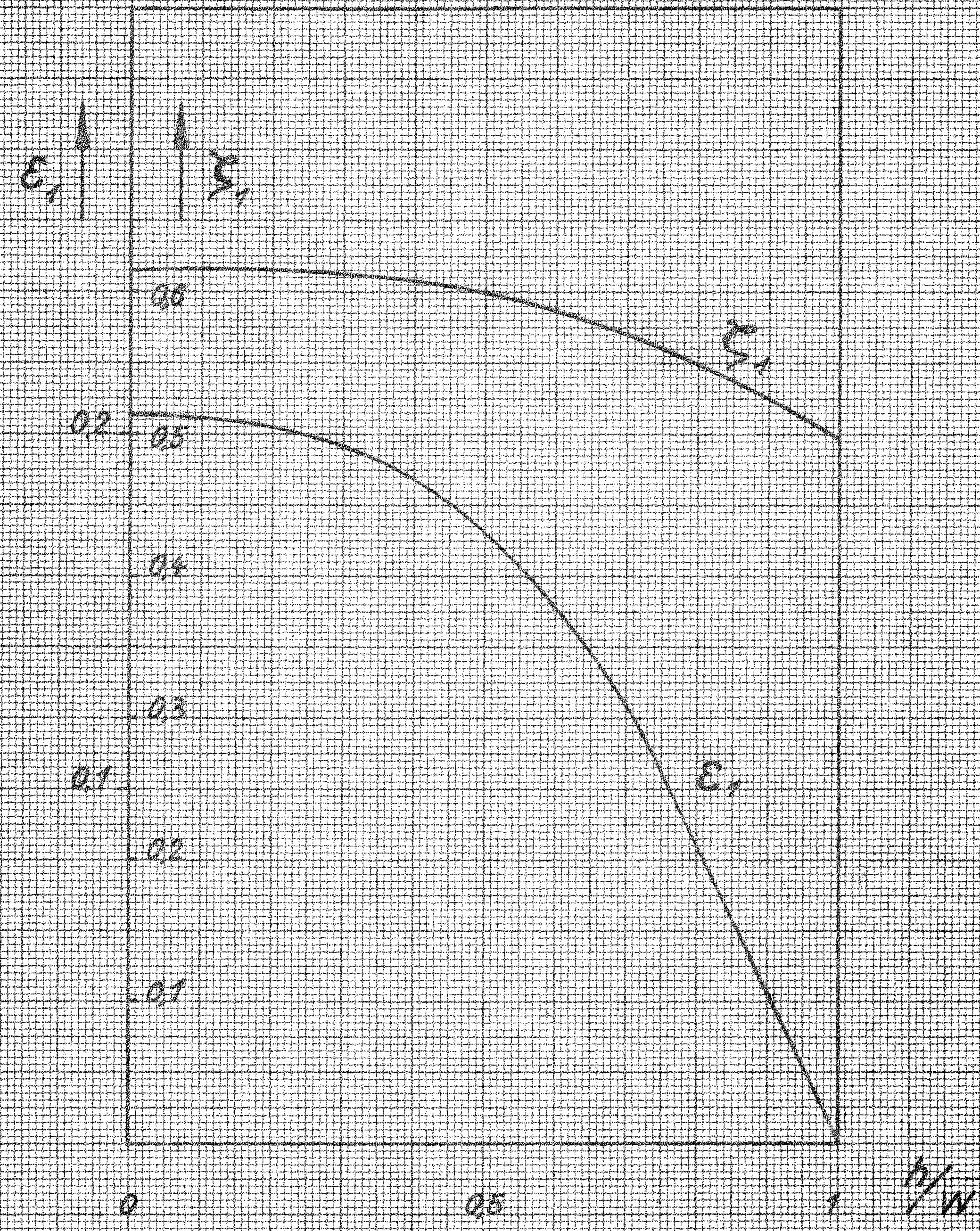

Electrostatic image cofficients
for a cylinder of ellyptical cross section

(from Laslett)