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CHIRAL SUSCEPTIBILITIES IN NONCOMPACT QED: A NEW
DETERMINATION OF THE γ EXPONENT AND THE CRITICAL COUPLINGS

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ABSTRACT

We report the results of a measurement of susceptibilities in noncompact QED_4 in $8^4, 10^4$ and 12^4 lattices. Due to the potentialities of the *MFA* approach, we have done simulations in the chiral limit which are therefore free from arbitrary mass extrapolations. Our results in the Coulomb phase show unambiguously that the susceptibility critical exponent $\gamma = 1$ independently of the flavour symmetry group. The critical couplings extracted from these calculations are in perfect agreement with previous determinations based on the fermion effective action and plaquette energy, and outside the predictions of a logarithmically improved scalar mean field theory by eight standard deviations.

Non compact Electrodynamics in four dimensions (QED_4) has a strongly coupled phase where chiral symmetry is spontaneously broken. The strong coupling phase is connected to the weak coupling phase through a continuous phase transition characterized by a critical point where a continuum limit can be defined. The possible existence of a non trivial continuum limit for this model is an appealing question which has attracted much attention in the last years and whose answer is not well known yet.

The main difficulty in establishing the nature of the continuum limit of this model comes from the fact that it can not be investigated in perturbation theory. The possible existence of a non gaussian fixed point implies that operators of dimension higher than four could become nonperturbatively renormalizable [1] and therefore dimensional analysis does not help us to establish what the relevant couplings are.

Due to the high complexity of this problem, the first approaches have been concentrated in the determination of thermodynamical quantities, critical exponents and their comparison with the predictions of a mean field theory plus logarithmic corrections. Several numerical methods have been used by different groups to simulate dynamical fermions. In refs. [2], [3] the hybrid Monte Carlo algorithm was used whereas in our previous work [4] the *MFA* approach [5] was employed. Even if the numerical results obtained by all these groups are in reasonable agreement, their physical interpretations are in conflict.

In [3] the value of the critical coupling is left as a free parameter in a fit of the numerical results with a logarithmically improved scalar mean field theory. The five parameters fit accommodates quite well the experimental data and the prediction of the fit for the critical coupling β_c in the four flavour theory is $\beta_c = 0.186(1)$. These results have been interpreted by the authors of [3] as strong evidence of a trivial continuum limit for this model. Notice however that if as recently argued [6], triviality manifests in a different way in scalar and fermionic models, the logarithmically improved mean field test of [3] would not be justified.

Conversely, the results reported in [2] and [4] for the critical couplings, both of them in very good agreement, show a value for β_c larger than 0.186. The excellent agreement between the results of [2] and [4] should be of great physical relevance since both simulations are perfectly uncorrelated. In fact these two groups use different approaches to simulate dynamical fermions, different lattice sizes, different operators and different bare fermion masses (the simulations in [4] were done in the chiral limit).

Since the numerical determination of critical exponents is very sensitive to

the value of the critical coupling β_c and that values of $\beta_c > 0.186$ could produce critical exponents outside of the mean field range, it is very important to have more independent measurements of β_c .

We report here the results of a measurement of the longitudinal, transverse and non linear susceptibilities in QED_4 with 0, 2 and 4 flavours. This is the first time that susceptibilities in noncompact QED are calculated in the chiral limit (Coulomb phase) and the results are certainly encouraging. In fact the values obtained for the critical couplings are in perfect agreement with previous determinations based on measurements of the fermionic effective action and plaquette energy [4] and therefore we confirm again from these results that β_c in the four flavour model is definitely larger than 0.186. These measurements show also unambiguously that the susceptibility critical exponent γ is 1, independently of the flavour symmetry group. Furthermore the value of β_c in the quenched model extracted from the susceptibility is also in perfect agreement with previous determinations of it from the computation of the chiral condensate [7] and larger than the value $\beta_c = 0.244$ in the zero flavour model extracted from the singular behavior of the fermionic action [4], the last being also in perfect agreement with the critical β obtained from the monopole percolation transition [2].

We have done calculations in $8^4, 10^4$ and 12^4 lattices and dynamical fermions are simulated by mean of the *MFA* method, the only available numerical approach which allows to perform realistic numerical simulations in the chiral limit. *MFA* (Microcanonical Fermionic Average) has been extensively discussed [5] and tested [8] in the literature. Let us however remember what are the main ingredients of the *MFA* approach.

The basic idea in *MFA* is the exploitation of the physical equivalence between the canonical and the microcanonical formalism, in our case the introduction of an explicit dependence on the energy in the computation of the partition function. Indeed it can be written as follows:

$$Z(\beta, m) = \int dE n(E) e^{6V\beta E} \overline{\det\Delta}(E, m) \quad (1)$$

where

$$n(E) = \int [dA_\mu] \delta(6VE - S_G[A_\mu]) \quad (2)$$

is the density of states at fixed energy E and

$$\overline{\det\Delta}(E, m) = \frac{\int [dA_\mu] \delta(6VE - S_G[A_\mu]) \det\Delta[A_\mu, m]}{n(E)} \quad (3)$$

is the fermionic determinant averaged over field configurations of fixed energy E . Note that $\overline{\det}\Delta(E, m)$ does not depend on β . The fermionic determinant for a single configuration is a polynomial in the mass:

$$\det \Delta[A_\mu, m] = \sum_n C_n[A_\mu] m^n \quad (4)$$

A modified Lanczos algorithm [9] is used in order to obtain the complete set of eigenvalues of the massless fermion matrix, and then we can reconstruct the fermionic determinant at any value of the fermion mass m .

The most general expression for the vacuum expectation value of any operator O , after integration of the Grassmann variables, can be written as

$$\langle O \rangle = \frac{\int dEn(E) \frac{\overline{O \det}\Delta}{\det\Delta} e^{-6V\beta E - S_{eff}^F(E, m)}}{\int dEn(E) e^{-6V\beta E - S_{eff}^F(E, m)}} \quad (5)$$

where $S_{eff}^F(E, m)$ in (5) is the fermion effective action defined as

$$S_{eff}^F(E, m) = -\log \overline{\det}\Delta(E, m) \quad (6)$$

and $\overline{O \det}\Delta$ means the mean value of the product of the operator O times the fermionic determinant, computed over gauge field configurations at fixed energy E .

Since we are interested here in the computation of susceptibilities in the chiral limit, we will use the previous expression for the particular cases in which O is the longitudinal, transverse and non linear susceptibility.

In the Coulomb phase, characterized by a ground state invariant under chiral transformations in the chiral limit, the longitudinal and transverse susceptibilities are equal except a sign. They can be computed by taking for the operator O the expression

$$O = \frac{2}{V} \sum_i \frac{1}{\lambda_i^2} \quad (7)$$

where the sum in (7) runs over all positive eigenvalues of the massless Dirac operator.

The transverse susceptibility χ_T in the broken phase diverges always because of the Goldstone boson. The longitudinal susceptibility χ_L on the other hand, can not be computed in the broken phase using equation (7) since in this phase the ground state is not invariant under chiral transformations. In fact if we work directly in the chiral limit, we average over all ground states. In other words there are extra contributions to χ_L in the massive case, which

vanish in the chiral limit in the Coulomb phase because of the realization of chiral symmetry, but giving an important contribution to χ_L in the broken phase. The standard procedure in the broken phase is then to compute the susceptibility at non zero fermion mass and to extract its value in the chiral limit by fermion mass extrapolations.

Since the susceptibilities in the symmetric phase are free from mass extrapolations, we will devote the largest part of this paper to the analysis of our results in the Coulomb phase which are the more relevant from a physical point of view.

The results in the 8^4 lattice are the best from a statistical point of view since in this lattice we have diagonalized 3000 completely uncorrelated gauge configurations for each value of the energy and repeated this process for 17 different energies between 0.5 and 1.2. In the 10^4 lattice, 300 – 400 gauge configurations were diagonalized at each energy, whereas our results in the 12^4 lattice are statistically the poorest.

In Fig. 1 we have plotted the inverse susceptibility in the chiral limit against β for the four-flavour model in a 10^4 lattice. The continuous line is a fit of all the points for $\beta > 0.210$ with a function

$$\chi^{-1} = \frac{c}{(\beta_c + r)} \frac{(\beta_c - \beta)}{(\beta + r)} \quad (8)$$

The use of this fitting function is strongly suggested by the results of the inverse susceptibility as a function of the energy (Fig. 2), which are extremely well fitted in the Coulomb phase by a straight line, and by the relation between the mean plaquette energy and β in the massless case [4] which is extremely well reproduced by the function

$$E = \frac{1}{4(\beta + r)} \quad (9)$$

the value of r depending on the flavour number.

Fig. 2 contains the results for the inverse susceptibility χ^{-1} against the energy E in the four and zero flavour models. Again in these cases, all the points in the Coulomb phase are very well fitted by a straight line, which implies $\gamma = 1$ independently of the flavour number.

The results of these fits for the critical coupling in the four flavours model are $\beta_c = 0.202(3)(8^4)$, $\beta_c = 0.2026(21)(10^4)$. The $\frac{\chi^2}{d.o.f.}$ are 2.8, 0.56 respectively, which show the high reliability of these fits as well as of the value $\gamma = 1$ for the susceptibility critical exponent.

The previous values of β_c in the 8^4 and 10^4 lattices are on the other hand in perfect agreement with the corresponding values extracted, following the method reported in [4], from the singular behavior of the fermionic effective action (see also Table I). As anticipated in the introduction, these values are out of the value 0.186(1) reported in [3] by eight standard deviations.

The values of the critical couplings and energies for different lattice sizes and flavour number are reported in Table I, where we include also the critical values obtained from the singular behavior of the fermion effective action. As can be seen in this Table, the results for the critical couplings extracted from the susceptibility and fermion effective action in the two flavour model are also in very good agreement, but they are incompatible in the quenched case. Notice however that the quenched value of β_c obtained from the susceptibility is the same as the one obtained in an independent calculation of the chiral order parameter in 24^4 lattices [7] whereas the β_c value obtained from the anomalous behavior of the fermionic action agrees very well with the critical coupling for the monopole percolation transition in the quenched model [10].

It is not clear to us the physical origin of the discrepancy between the critical couplings obtained by measuring different operators in the zero flavour model. However the agreement between critical couplings for monopole percolation and singular behavior of the fermionic action is, we believe, a clear signal of strong correlation between these two phenomena.

We have also done measurements of the longitudinal susceptibility χ_L for massive fermions as well as of the non linear susceptibility χ_{nl} , which is related to the four point function and therefore to the low energy renormalized coupling. A detailed analysis of these calculations will be reported in a longer publication. However we report in Fig. 3 some results obtained for χ_L^{-1} as a function of the gauge coupling β at two values of the fermion mass $m = 0.00625, 0.01$. The value of the dynamical fermion mass in this figure (the mass in the integration measure) has been taken equal to zero. The inverse chiral susceptibility shows a minimum pointing to the critical β previously obtained from the fit of the results for the massless susceptibility in the Coulomb phase.

Concerning the non linear susceptibility χ_{nl} , we would like to anticipate that the results for χ_{nl} suffer, as expected, from strong finite size effects [11]. One of the contributions to the massless non linear susceptibility operator can be defined as

$$O_{nl} = \frac{2}{V} \sum_i \frac{1}{\lambda_i^4} \quad (10)$$

where again in this case the sum runs over all positive eigenvalues of the massless fermion matrix.

The expression given by equation (10) diverges in the thermodynamical limit due to the presence of zero modes both in the Coulomb and broken phases. However the divergency in the broken phase is much faster because chiral symmetry is spontaneously broken in this phase. Then a rough estimate of the critical coupling can be obtained from the results for the *v.e.v.* of the inverse of this operator, by defining it as the point where the slope of this *v.e.v.* as a function of the gauge coupling β (see Fig. 4) takes its maximum value. This criterion gives for β_c in the quenched model a value between 0.25 – 0.26 in good agreement with the results reported in Table I.

In conclusion we have shown how the analysis of the massless chiral susceptibility in the Coulomb phase of noncompact *QED*, even in rather small lattices as ours, can be very useful to get precise determinations of the critical couplings. Indeed what gives high reliability to the susceptibility fits we have done in the Coulomb phase is the fact that all the points, except those very near to β_c which could be affected by finite size effects, are very well fitted by a single power law term without higher power corrections, even far from β_c .

The numerical simulations quoted above have been done using the Transputer Networks of the Theoretical Group of the Frascati National Laboratories and the Reconfigurable Transputer Network (RTN), a 64 Transputers array, of the University of Zaragoza.

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Figure captions

Fig. 1. Inverse susceptibility in the chiral limit against β for the four flavour model, 10^4 lattice.

Fig. 2. Inverse susceptibility as a function of the energy E in a 10^4 lattice for zero and four dynamical flavours.

Fig. 3. Inverse longitudinal susceptibility for $m = 0.01$ and $m = 0.00625$ against β in the quenched (a) and four flavours model (b).

Fig. 4. *v.e.v.* of the inverse operator given by eq. (10) against β in the quenched case, 8^4 , 10^4 and 12^4 lattices.

Table caption

Table I. Critical couplings and energies for different lattice sizes and flavour numbers. The superscripts χ and S denote values extracted from the chiral susceptibility and effective fermion action respectively.

lattice	10^4	10^4	10^4	8^4	8^4	8^4
# flav.	4	2	0	4	2	0
E_c^x	1.028(8)	1.027(7)	0.977(12)	1.034(11)	1.038(10)	0.974(8)
β_c^x	0.2026(21)	0.2231(18)	0.256(3)	0.202(3)	0.221(2)	0.257(2)
E_c^S	1.02(2)	1.030(15)	1.030(10)			
β_c^S	0.205(5)	0.222(3)	0.243(2)			

Table I

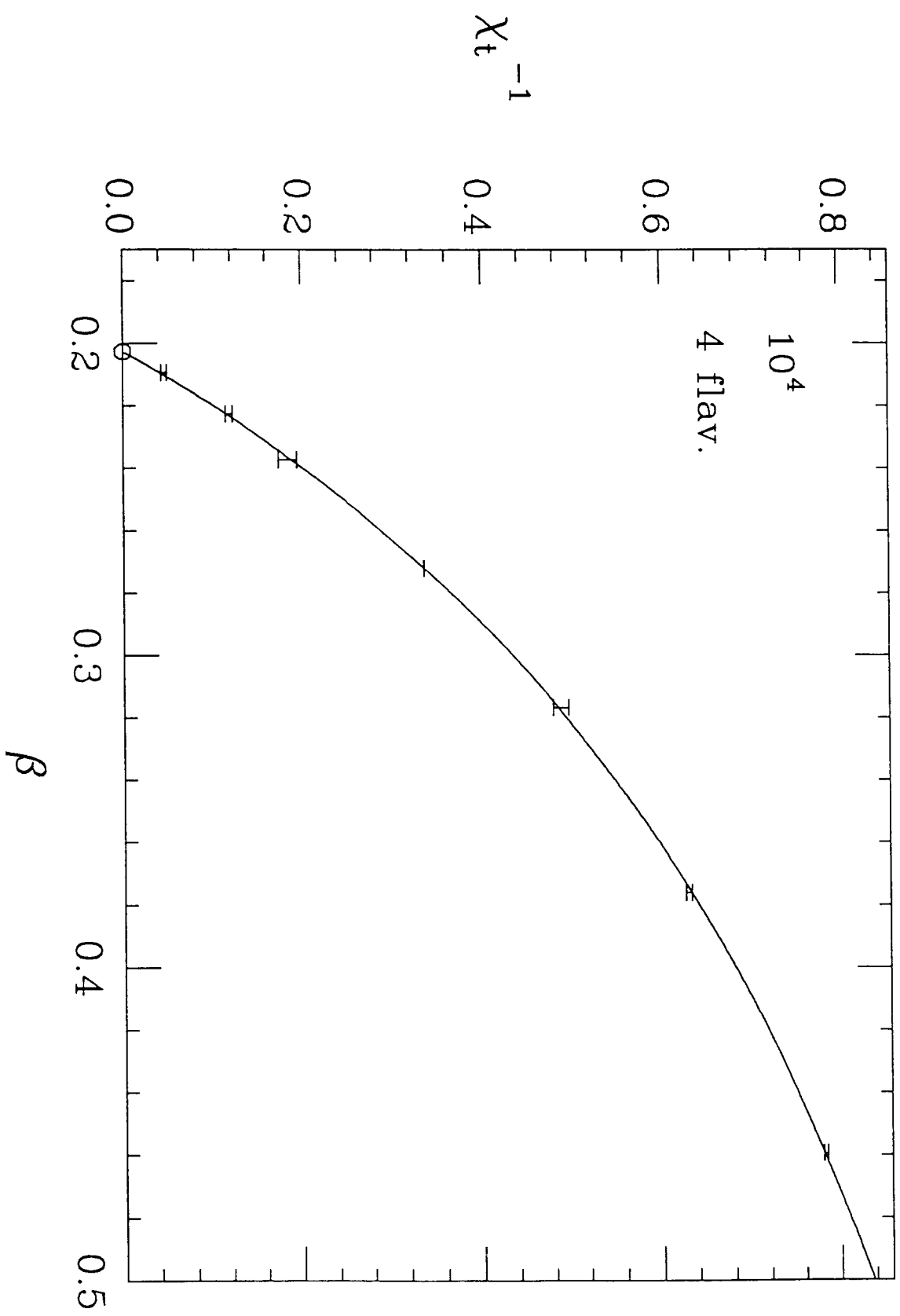


Fig. 1

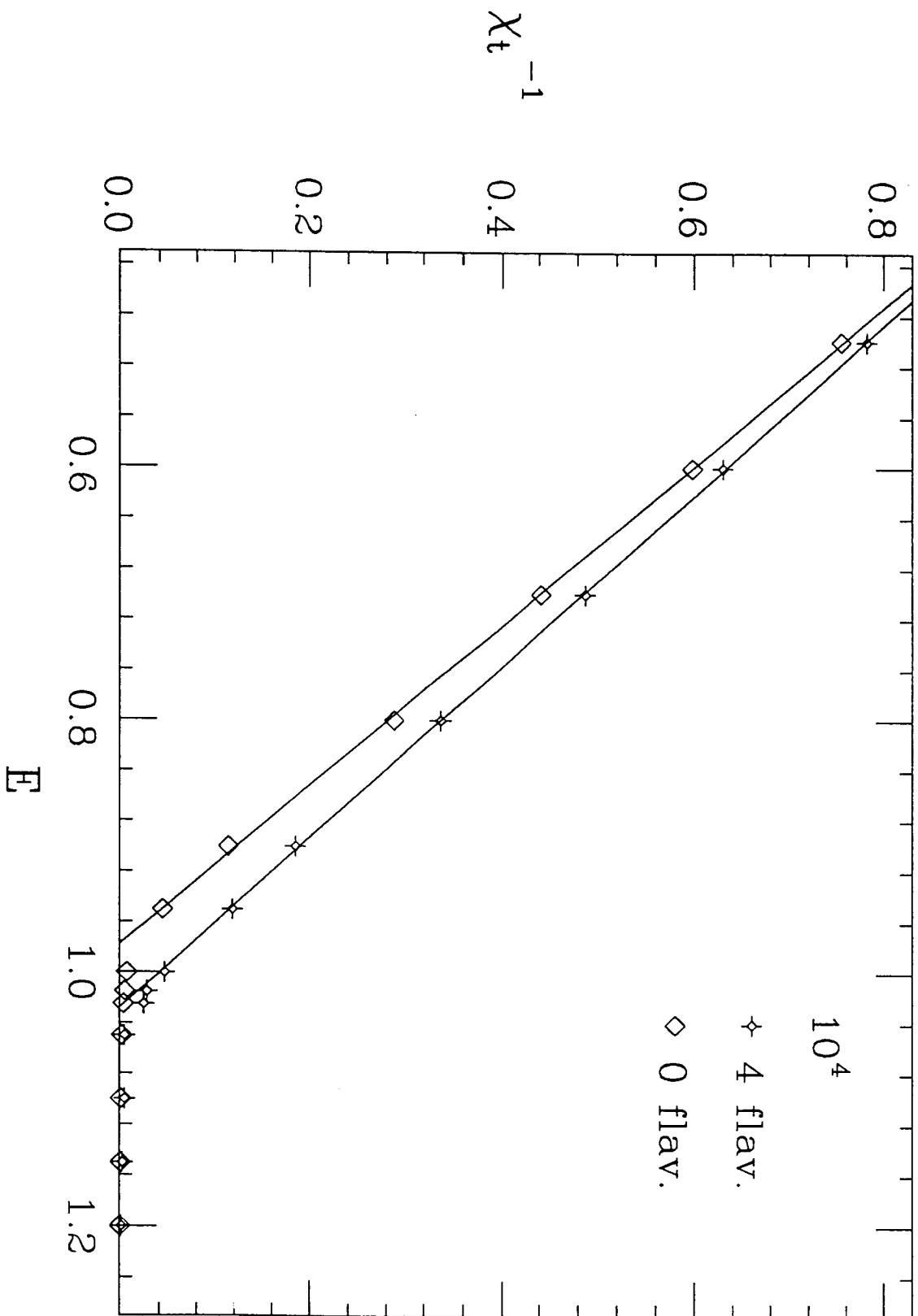


Fig. 2

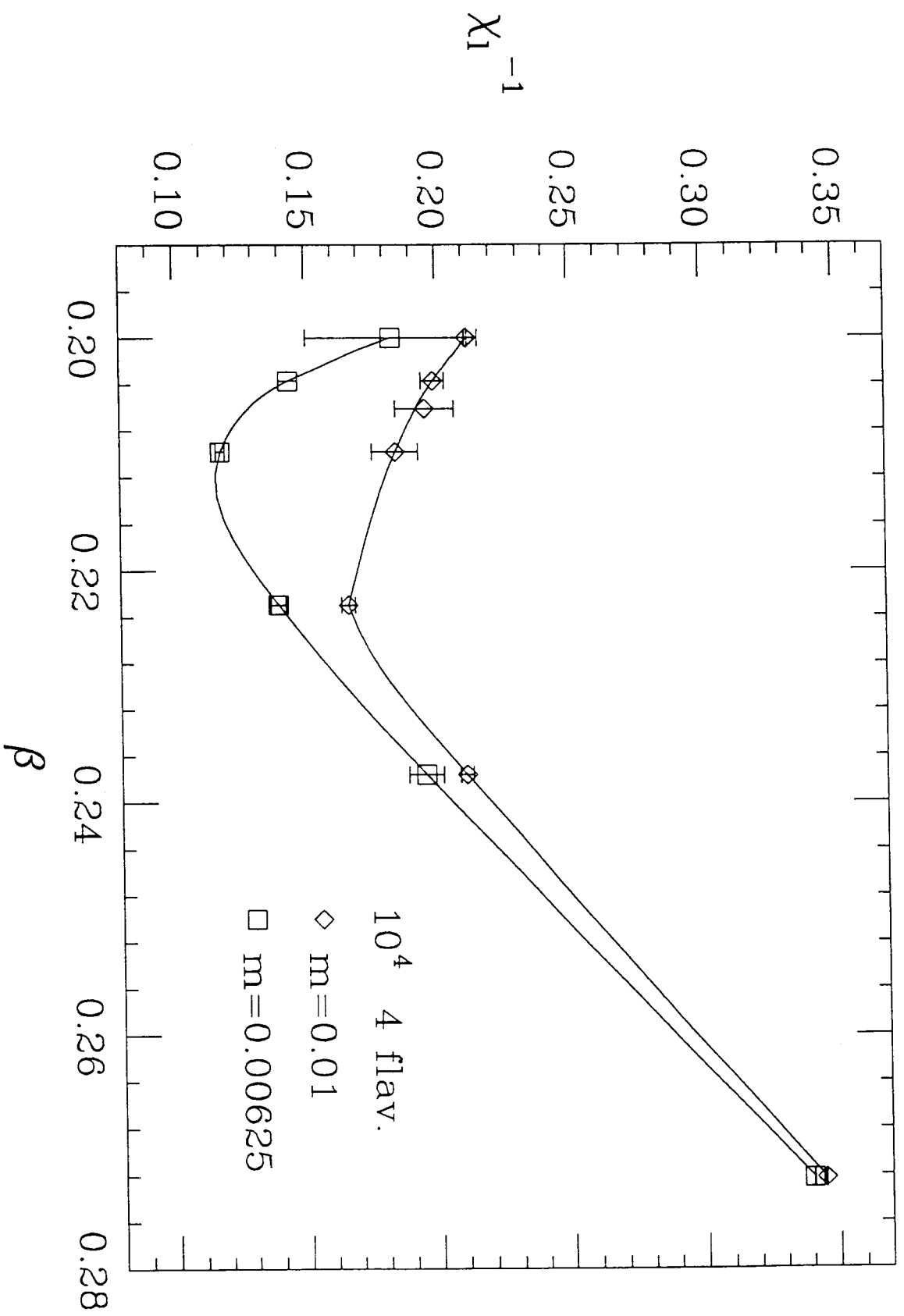


Fig. 3-a

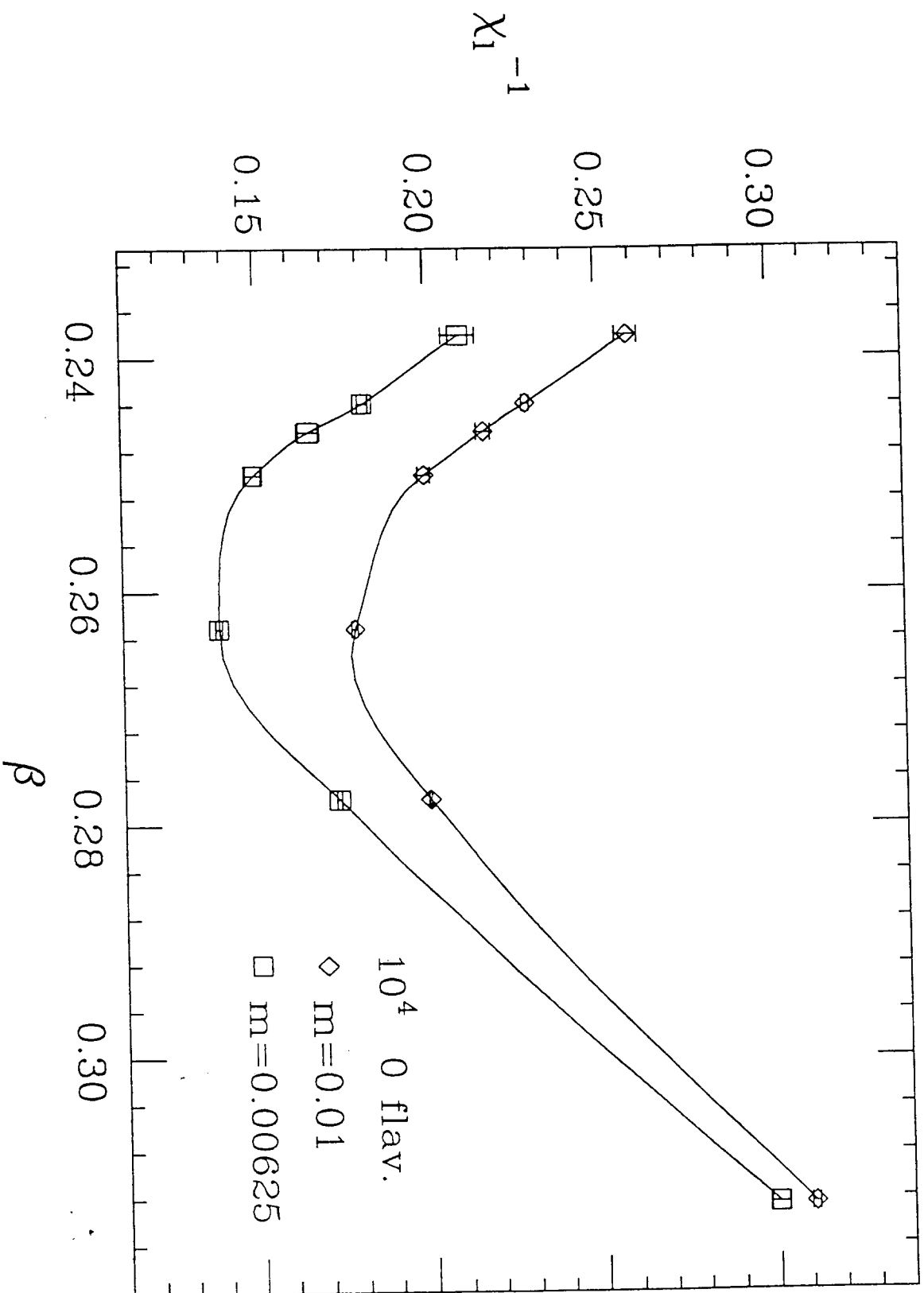


Fig. 3-b

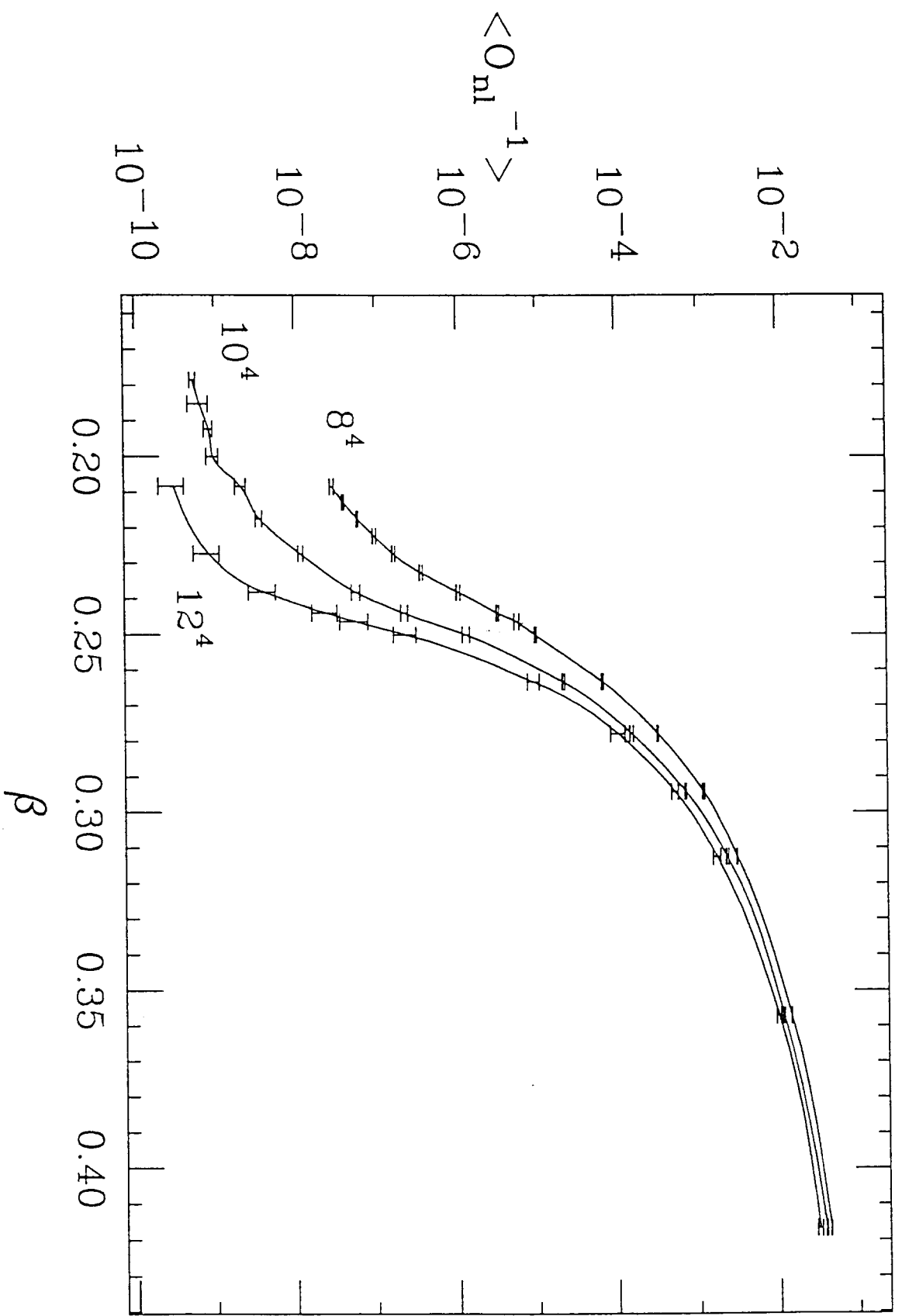


Fig. 4