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and SAURO SUCCI

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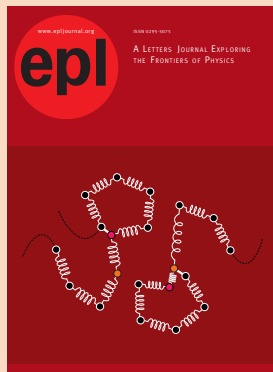
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# Estimating today's cosmological constant via the Zel'dovich-holographic connection

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**Abstract** – This letter proposes an approach to the vacuum energy and the cosmological constant (CC) paradox based on the Zel'dovich's ansatz, which states that the observable contribution to the vacuum energy density is given by the gravitational energy of virtual particle-antiparticle pairs, continually generated and annihilated in the vacuum state. The novelty of this work is the use of an ultraviolet cut-off length based on the holographic principle, which is shown to yield current values of the CC in semi-quantitative agreement with experimental observations.

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**Introduction.** – The Cosmological Constant (CC) problem or vacuum catastrophe stands for the stark mismatch between the currently observed values of the vacuum energy density (the small value of the CC) and theoretical large value of zero-point energy suggested by quantum field theory. It is also associated with a possible explanation for the dark energy driving the Universe accelerated expansion. Its theoretical value should therefore match observations. Unfortunately, due to about 120 orders of magnitude mismatch, it bears the reputation of “the worst prediction in the history of physics” [1], see also [2–6]. This note sets out to estimate the current experimental values of the CC by revisiting an original idea proposed by Zel'dovich and combining it with the holographic principle.

**The cosmological constant paradox.** – Despite being responsible for showing everyone that space-time is a dynamic entity, co-evolving with the matter that inhabits it, Einstein, for once, was not prepared for the idea that the entire Universe could be a dynamic entity as well. As a result, when faced with the irrefutable evidence that his equations did not admit a static universe as a solution, he resolved to add an *ad hoc* term, the CC, for obtaining one. Shortly later, however, it was for experimental data

to show that our Universe is actually expanding, at which point he famously termed the CC his “biggest blunder”. However, to say with Joyce, “errors are the portal of discovery”, and the CC has taken central stage in modern physics, mostly because of its potential connections with dark energy and the accelerated expansion of the Universe. Not without a huge riddle, though: the CC has dimensions of an inverse length squared and since its physical origin is generally attributed to vacuum fluctuations at the Planck scale, it is natural to assume that its value in Planck units should be of order 1, namely

$$\Lambda L_{\text{P}}^2 \sim 1. \quad (1)$$

By contrast, for the product  $\Lambda L_{\text{P}}^2$  cosmological observations deliver a value of  $\sim 10^{-122}$ , namely 122 orders of magnitude smaller, making of (1), as mentioned before, “the worst prediction ever in the history of physics” [2–6]. Despite intensive efforts, the puzzle is still standing. Here, we begin by observing that  $10^{-122}$  is surprisingly close to the square of the ratio of the Planck length and the Universe radius  $10^{2(-35-27)} = 10^{-124}$ , thereby providing a strong clue towards a theory where the “natural” equation (1) would be replaced by a much more accurate prediction:

$$\Lambda L_{\text{P}}^2 = \left( \frac{L_{\text{P}}}{L_{\text{IR}}} \right)^2, \quad (2)$$

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where  $L_{\text{IR}} \sim 10^{26}$  m is pretty close to the radius of the Universe  $L_{\text{U}} \sim 10^{27}$  m. One such theory was indeed proposed back in 2008, within the framework of modified gravity [7].

In the following, it is shown that the same relation can be obtained by a straightforward combination of a previous argument by Zel'dovich with the holographic principle.

**Revisiting Zel'dovich's ansatz.** – Zel'dovich argued that since the bare zero-point energy is unobservable, the observable contribution to the vacuum energy density,  $e_v$ , is given by the gravitational energy of virtual particle-antiparticle pairs, continually generated and annihilated in the vacuum state [8,9]. Therefore,

$$e_v(l) \sim \frac{Gm^2(l)}{l} \frac{1}{l^3}. \quad (3)$$

In the expression above, also according to Zel'dovich, the vacuum contains particles with an effective density  $m(l)/l^3$ . Additionally, by considering the Compton's expression for the wavelength, the effective mass of the particles at scale  $l$  is taken as  $m(l) \sim \hbar/(cl)$ . Substituting this in eq. (1), and defining a local CC as

$$\Lambda(l) = \frac{Ge_v(l)}{c^4}, \quad (4)$$

one readily obtains

$$\Lambda(l)L_{\text{P}}^2 \sim \left(\frac{L_{\text{P}}}{l}\right)^6, \quad (5)$$

where  $L_{\text{P}} = (\hbar G/c^3)^{1/2}$  is the Planck length. Next, we observe that the measured CC is likely to result from the average of the local CCs over the full spectrum of active scales [10], ranging from a UV cutoff to an IR one, which we shall be taken here as the *current* radius of the Universe. It is worth emphasizing that in the present approach, such scales are not intended as regulatory devices to tame infinities but bear a physical meaning instead. They fix the boundaries of the spectrum of dynamically active scales arising from the collective motion of the nonlinearly interacting effective degrees of freedom [11,12]. As a mere analog, in fluid turbulence the IR cutoff is the macroscopic scale  $L_{\text{IR}}$  of the problem, the molecular mean free path  $L_{\mu}$  is the underlying microscale, and the Kolmogorov dissipative length  $L_d = L_{\text{IR}}^{1/4} L_{\mu}^{3/4}$  represents the shortest dynamically active scale supporting coherent hydrodynamic motion. The effective UV cutoff of turbulence is therefore provided by  $L_d > L_{\mu}$  rather than  $L_{\mu}$ , consistently with the macroscopic (supramolecular), nature of fluid turbulence as a self-interacting classical vector field theory [11]. For a more detailed discussion on the analogy between IR-UV- coupling in fluid turbulence and the “naturalness” problem see ref. [13].

Under the plausible assumption that the average is dominated by the region around the UV cutoff, we obtain

$$\Lambda L_{\text{P}}^2 \sim \left(\frac{L_{\text{P}}}{L_{\text{UV}}}\right)^6, \quad (6)$$

where  $L_{\text{UV}}$  denotes the (yet unspecified) UV cutoff length and we have neglected the IR contribution since  $L_{\text{IR}} \gg L_{\text{UV}}$ .

To fix  $L_{\text{UV}}$ , we invoke the holographic principle, which states that the minimum observable length scale is not the Planck length but a much larger holographic scale, given by [14–16]:

$$L_{\text{H}} = L_{\text{IR}}^{1/3} L_{\text{P}}^{2/3}. \quad (7)$$

Hence, we stipulate

$$L_{\text{UV}} = L_{\text{H}}. \quad (8)$$

Inserting eq. (8) in eq. (5), and taking into account the expression (9), we finally obtain

$$\Lambda L_{\text{P}}^2 = \frac{L_{\text{P}}^6}{L_{\text{IR}}^2 L_{\text{P}}^4} = \left(\frac{L_{\text{P}}}{L_{\text{IR}}}\right)^2, \quad (9)$$

which is the sought relation (2). Based on the above expression, retrieving the sought 122 orders of magnitude implies  $L_{\text{IR}}/L_{\text{P}} \sim 10^{61}$ , very close to the radius of the Universe, *i.e.*,  $L_{\text{IR}} \sim 10^{27}$  meters. A few words of caution are in order. First, we assumed that the average is dominated by the UV cutoff, meaning that the probability of realizing a structure at scale  $l$  is peaked around  $l = L_{\text{UV}}$ , formally a Dirac delta centered about  $l = L_{\text{UV}}$ . In view of the steep (–6) decay in space of the local CC, this is a plausible assumption, yet one that does not follow, to the best of our knowledge, from any first principle. Second, it is worth appreciating that the value of the CC is extremely sensitive to changes in the exponents defining the UV cutoff length. For instance a local quantum field theory scaling  $L_{\text{UV}} = L_{\text{IR}}^{1/2} L_{\text{P}}^{1/2}$  yields  $\Lambda L_{\text{P}}^2 = \left(\frac{L_{\text{P}}}{L_{\text{IR}}}\right)^3$ , corresponding to  $L_{\text{IR}}/L_{\text{P}} \sim 10^{40} \sim 10^5$  meters, while a Casimir scaling  $L_{\text{UV}} = L_{\text{IR}}^{2/3} L_{\text{P}}^{1/3}$ , see [17,18], delivers  $\Lambda L_{\text{P}}^2 = \left(\frac{L_{\text{P}}}{L_{\text{IR}}}\right)^4$ , corresponding to  $L_{\text{IR}}/L_{\text{P}} \sim 10^{30} \sim 10^{-5}$  meters! Consequently, the above results should be taken essentially as a semi-quantitative estimate resulting from the simple idea of combining Zel'dovich's assumption with the holographic principle.

**Time dependence.** – All along this text, we have deliberately related the IR cut-off,  $L$ , to the *current* radius of our Universe, in order to emphasize that the present analysis does not encompass Universe's entire expansion chronology. In other words, our explanation does not cover the value of the CC across full time span since the Big Bang until now, but it only addresses the value of the CC at the current time. It does so, though, by proposing an alternative and possibly more economic explanation (in terms of assumptions) as compared to previous

ones [19,20]<sup>1</sup>. As is well known, our Universe expansion chronology is parametrised by a dimensionless quantity, known as the cosmic scale factor  $a(t)$ . Based on its time dependence, three characteristic eras can be distinguished: a radiation-dominated era encompassing the time scale from inflation until about 47000 years after the Big Bang, where  $a(t) \sim t^{1/2}$ ; a matter-dominated era, between about 47000 years and 9.8 billion years after the Big Bang, where  $a(t) \sim t^{2/3}$ ; and finally, the so-called dark energy dominated era in which  $a(t) \sim e^{H_0 t}$  ( $H_0$  being the actual value of the Hubble “constant”), and where our Universe is currently undergoing an accelerated expansion as suggested by observations [21–25]. In the early universe, the mass-energy density effect was larger than the cosmological constant one, so the universal expansion was slowing down (note that any power-law expansion implies a  $1/t$  decay of the Hubble parameter  $H = \dot{a}/a$ ). However, at around six billion years after the Big Bang, the mass-energy effect became so diluted that the cosmological constant one took over. As the universe evolved further, the mass-energy effect became less and less important as compared to the cosmological constant effect, as confirmed by experimental sources [22]. Finally, we note that the experimental evidence of a positive and small CC together with a potential eternal expansion of our Universe opens up the possibility that our Universe may be asymptotically approaching a de Sitter one [26]. Meaning a universe with no ordinary matter content but with a positive cosmological constant driving its expansion. In this context, our treatment might also offer a possible clue towards the explanation of the value of the CC in the mid-term and far-future regimes of our Universe.

**Conclusions.** – In this letter, we have proposed a straightforward approach to the vacuum energy and the CC paradox, based on the Zel'dovich's ansatz combined with the holographic principle. The result is in a quantitative agreement with the experimental value, which is rather remarkable considering the simple nature of the supporting assumptions. This result suggests that, as originally proposed by Zel'dovich, the observable vacuum energy density today is given by the gravitational energy of virtual particle-antiparticle pairs, continually produced and annihilated in the vacuum state. Nevertheless, this argument alone does not suffice, as it requires a merger with the holographic principle, in order to select the appropriate UV cut-off length.

It should also be pointed out that Zel'dovich had to consider the proton mass as the “typical” mass scale for producing a reasonably good order of magnitude result without a proper justification. Even then, his ansatz remained off the modern value by nine orders of magnitude. The approach suggested here does not necessitate any such

restriction and provides a much closer agreement with the experimental results.

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<sup>1</sup>A possible way to reinstate a time-independent picture is to identify the IR cutoff with the curvature radius of a slightly non-flat Universe. We owe this remark to David Spergel.

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