ИФВЭ
IHEP

FEDERAL SCIENCE CENTRE

INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 95-9

V.V.Kiselev

f_B^{stat} and μ_π^2 in quasiclassical approximation of sum rules

Protvino 1995

UDK 539.1.01 M–24

Abstract

Kiselev V.V. f_B^{stat} and μ_π^2 in quasiclassical approximation of sum rules: IHEP Preprint 95-9. – Protvino, 1995. – p. 5, refs.: 19.

In the framework of sum rules with a use of quarkonium mass spectrum, evaluated in the quasiclassical approximation, estimates of leptonic constant $f_B^{stat} \simeq 260$ MeV in a static limit and for the average heavy quark momentum squared $\mu_\pi^2 \simeq 0.4$ GeV^2 are obtained.

Аннотация

Киселев В.В. f_B^{stat} и μ_π^2 в квазиклассическом приближении правил сумм: Препринт ИФВЭ 95-9. – Протвино, 1995. – 5 с., библиогр.: 19.

В рамках правил сумм с использованием спектра масс кваркония, рассчитанного в квазиклассическом приближении, сделаны оценки лептонной константы $f_B^{stat} \simeq 260$ МэВ в статическом пределе и среднего квадрата импульса тяжелого кварка $\mu_\pi^2 \simeq 0,4$ ГэВ^2.

1. In the Heavy Quark Effective Theory [1], used for the description of strong interaction dynamics of heavy quarks, there are some dimensionful parameters, which determine an accuracy of the leading approximation in infinitely heavy quark limit as well as values of power corrections over $\Lambda/m_Q \ll 1$, where Λ is a scale, determining the heavy quark virtuality inside hadrons. Among such parameters in physics of heavy mesons $(Q\bar{q})$ with a single heavy quark, the most important quantities are the difference between masses of meson and heavy quark $\bar{\Lambda} = M(Q\bar{q}) - m_Q$, the leptonic constant of heavy meson f_Q^{stat} in the static limit $m_Q \to \infty$, and the square of heavy quark momentum μ_π^2 inside the meson. Since those values are determined by QCD at large distances, for estimates one uses nonperturbative approaches, among which the most powerful tool is sum rules [2].

As for the $\bar{\Lambda}$ value, its estimates in the framework of QCD sum rules have been obtained in refs.[3,4], where $\bar{\Lambda} = 0.57 \pm 0.07$ GeV. Moreover, the 'optical' sum rule by Voloshin [5] allows one to get the inequality [6]

$$\bar{\Lambda} > 2\delta_1(\rho^2 - \frac{1}{4}) \simeq 0.59 \text{ GeV} , \tag{1}$$

where ρ^2 is the slope of universal Isgur-Wise function [7], and δ_1 is the difference between the masses of the lightest vector S-wave state and P-wave state for $(Q\bar{q})$ system at $m_Q \to \infty$.

Further, estimates of f_B^{stat} in the framework of QCD sum rules and in lattice computations are in agreement with each other and result in [1]

$$f_B^{stat} = 240 \pm 40 \text{ MeV} . \tag{2}$$

The sum rule estimation of average square of the heavy quark momentum inside the meson gives the value [1,8]

$$\mu_\pi^2 = 0.5 \pm 0.1 \text{ GeV}^2 , \tag{3}$$

and the inequality [6]

$$\mu_\pi^2 > 3\delta_1^2(\rho^2 - \frac{1}{4}) \simeq 0.45 \text{ GeV}^2 . \tag{4}$$

Note, however, that the values of the parameters δ_1 and ρ^2 are presently rather uncertain, so that bounds (1) and (4) are not the most conservative ones. A special discussion of the μ_π^2 value can be found, for instance, in ref.[1], where the role of a field theory analog for the virial theorem is considered.

In the present letter we consider the QCD sum rules with a use of S-wave level mass spectrum, calculated in the quasiclassical approximation, [9] and obtain estimates of the f_B^{stat} and μ_π^2 values, which agree with the results, given above.

2. In recent papers [9,10] the QCD sum rules for leptonic constants of S-wave levels in the $(Q_1\bar{Q}_2)$ quarkonium have been considered with the use of the state mass spectrum, calculated in the quasiclassical approximation. For the $1S$-level one has got the expression

$$f^2 \cdot M = \frac{16\alpha_s}{\pi}\, \mu_\pi^2\, \mu\ , \tag{5}$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of quarkonium, $\mu_\pi^2 = 2\mu\langle T\rangle$ is the average square of quark momentum inside the quarkonium with the mass $M \simeq m_1 + m_2$. Note, in the broad region of average distances between quarks: 0.1 fm < r < 1 fm, where the coulomb-like potential of heavy quark is transformed into the linearly rising confining potential, the average kinetic energy $\langle T\rangle$ is a constant value, independent of μ (i.e. flavours), [11,12]

$$\langle T\rangle = \text{const.} \tag{6}$$

This leads to that in the mentioned region of distances, the heavy quark potential is close to the logarithmic one [12], and the quantization by the Bohr–Sommerfeld procedure results in

$$\frac{dM_n}{dn} = \frac{2\langle T\rangle}{n}\ . \tag{7}$$

In accordance with eq.(7) and from spectroscopic data on the charmonium and bottomonium [13], one can get the estimate

$$\langle T\rangle = 0.43 \pm 0.01 \text{ GeV}. \tag{8}$$

However, the polynomial interpolation of masses for the excited states in heavy quarkonia and heavy mesons[1] leads to the value

$$\langle T\rangle = 0.38 \pm 0.01 \text{ GeV}, \tag{9}$$

that is closer to the corresponding parameter of the logarithmic potential [12]. Therefore, in the following estimates we use the value

$$\langle T\rangle = 0.40 \pm 0.03 \text{ GeV}. \tag{10}$$

In the case of a heavy quarkonium $(Q\bar{Q})$ with a hidden flavour one has $4\mu = M$ and

$$\frac{f^2}{M} = \frac{2\alpha_s}{\pi}\,\langle T\rangle \simeq \text{const.}, \tag{11}$$

where we neglect logarithmic corrections. Relation (11) is in a good agreement with experimental values of leptonic constants for ψ- and Υ-particles [9,10].

[1] $(Q\bar{q})$ masses are in agreement with estimates in potential models.

Since the threshold of the hadronic continuum in the system with two heavy quarks is determined by masses of heavy mesons $(Q_1\bar{q})$ and $(\bar{Q}_2 q)$, one finds [14]

$$\bar{\Lambda} = \langle T\rangle \ln n_{th} \simeq 0.6 \pm 0.1 \text{ GeV}, \tag{12}$$

where n_{th} is the number of S-levels of heavy quarkonium below the threshold of hadronic continuum ($n_{th}(b\bar{b}) = 4$), so that the estimation error is, in general, due to the variation of n_{th}, $\delta\bar{\Lambda} = \langle T\rangle \delta n_{th}/n_{th} \simeq 0.1$ GeV.

For a heavy meson $(Q\bar{q})$ a motion of the light current quark in a medium of quark-gluonic condensate plays an essential role. Therefore the most consistent consideration of sum rules requires the use of the operator product expansion for quark currents with the account of vacuum expectation values for operators of higher dimensions. However, one can make the reasonable approximation and consider the case, when the condensate influence generally results in the appearance of an effective mass for the light quark. Such constituent quark can be considered as the nonrelativistic object, moving in the potential of static heavy quark[2] So, the potential quark models are quite successful in the heavy meson spectroscopy (see, for example, ref.[15]). Further, introducing the constituent light quark, one considers the phenomenological effective theory, where the parameters have a meaning at the tree level with no loops. Hence, one can consider the phenomenological expressions, where one does not include condensates, since the latters are implicitly taken into account by means of the introduction of some phenomenological parameters such as the constituent mass.

Within the offered approach, the approximation means that

$$\mu \simeq M(Q\bar{q}) - m_Q = \bar{\Lambda} \ .$$

However, in the potential quark model, one usually has

$$m_q \simeq 0.35 \text{ GeV} \ .$$

So, in the following we suppose

$$\mu = 0.45 \pm 0.15 \text{ GeV} \ . \tag{13}$$

Then one has

$$\mu_\pi^2 \simeq 2\mu \ \langle T\rangle \simeq 0.4 \pm 0.1 \text{ GeV}^2 \ . \tag{14}$$

In ref.[16] one has shown that spectroscopic data on the ψ- and Υ-families give

$$\alpha_s(\psi, \Upsilon) = 0.20 \pm 0.01 \ , \tag{15}$$

that agrees with α_s estimates[3] from experimental values of the leptonic and radiative decay branching fractions for ψ and Υ [17] as well as with lattice computations for the $(b\bar{b})$ system spectroscopy [18], where the estimate, close to (15), takes place, too. However, for

[2] This approximation means that the "brown muck" is considered as a whole, i.e. with no internal structure.

[3] The recent result by M.Voloshin gives $\alpha_s^{MS}(m_b) = 0.185 \pm 0.003$ [19].

the heavy-light systems, one has to evaluate α_s at the scale μ_π [3,4]. Using the one-loop expression for the "running" coupling in QCD

$$\alpha_s(m) = \frac{2\pi}{(11 - 2n_f/3)\ln m/\Lambda} ,$$

one finds that $\alpha_s(m_Z) = 0.12$ at $\Lambda = 0.1$ GeV and $n_f = 5$. At the low virtualities ($n_f = 3$) one has

$$\alpha_s(\mu_\pi) = 0.39 \pm 0.04 . \tag{16}$$

Then in accordance with eq.(5) one has

$$f_B^{stat} = 260 \pm 60 \text{ MeV} . \tag{17}$$

3. Thus, in the framework of sum rules with the use of quarkonium spectroscopy, considered in the quasiclassical approximation, one finds the estimates of f_B^{stat} and μ_π^2, which agree with the values, obtained in QCD sum rules for the heavy meson currents.

As one can see, the obtained estimates of f_B^{stat} and μ_π^2 practically are near the bounds, derived in the sum rules [5,6].

This work is, in part, supported by ISF grant NJQ000 and the Program "Russian State Stipendia for Young Scientists". Author expresses special thanks to prof. V.Obraztsov for his hospitality at DELPHI, CERN, where this work has been done.

References

[1] M.Neubert, Phys.Rep. 245 (1994) 259.

[2] M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl.Phys. B147 (1979) 385, 448; L.J.Reinders, H.Rubinstein, T.Yazaki, Phys.Rep. 127 (1985) 1.

[3] E.Bagan, P.Ball, V.Braun and H.Dosch, Phys.Lett. B278 (1992) 457.

[4] M.Neubert, Phys.Rev. D46 (1992) 1076.

[5] M.Voloshin, Phys.Rev. D46 (1992) 3062.

[6] I.Bigi, A.G.Grozin, M.Shifman, N.G.Uraltsev, A.Vainshtein, Phys.Lett. B339 (1994) 160.

[7] N.Isgur and M.B.Wise, Phys.Lett. B232 (1989) 113, B237 (1990) 527.

[8] P.Ball and V.Braun, Phys.Rev. D49 (1994) 2472.

[9] V.V.Kiselev, Nucl.Phys. B406 (1993) 340.

[10] V.V.Kiselev, Preprint IHEP 94-63, Protvino, 1994, submitted to Zh.Exp.Teor.Fiz. [JETP].

[11] E.Eichten et al., Phys.Rev. D21 (1980) 203.
E.Eichten, Preprint FERMILAB-Conf-85/29 T, 1985.
A.Martin, Phys.Lett. 93B (1980) 338.

[12] C.Quigg and J.L.Rosner, Phys.Lett. B71 (1977) 153.

[13] L.Montanet et al. PDG, Phys.Rev. D50 (1994) 1173.

[14] V.V.Kiselev, Pisma v Zh.Exp.Teor.Fiz. 60 (1994) 498, [JETP Lett.].

[15] S.Godfrey, N.Isgur, Phys.Rev. D32 (1985) 189.

[16] V.V.Kiselev, Preprint IHEP 94-92, Protvino, 1994.

[17] T.Appelquist, H.D.Politzer, Phys.Rev.Lett. 34 (1975) 43;
A.De Rujula, S.L.Glashow, Phys.Rev.Lett. 34 (1975) 46;
V.A.Novikov et al., Phys.Rep. 41C (1978) 1.

[18] C.T.H.Davies et al., Florida State Uni. Preprint FSU-SCRI-94-79, 1994.

[19] M.Voloshin, Univ. of Minnesota Preprint UMN-TH-1326-95, 1995.

Received January 12, 1995

В.В.Киселев.
f_B^{stat} и μ_π^2 в квазиклассическом приближении правил сумм .

Оригинал-макет подготовлен с помощью системы LaTeX.
Ответственный за выпуск В.В.Киселев.

Подписано к печати 23.01.1995 г. Формат 60 × 84/8.
Офсетная печать. Печ.л. 0,38. Уч.-изд.л. 0,52. Тираж 250. Заказ 191.
Индекс 3649. ЛР №020498 06.04.1992.

Институт физики высоких энергий, 142284, Протвино Московской обл.

Индекс 3649

ПРЕПРИНТ 95-9, ИФВЭ, 1995