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INSTITUTE OF THEORETICAL PHYSICS ACADEMIA SINICA

AS-ITP-94-44
September 1994

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PACS number(s): 11.10.Kk 11.25.Hf 04.60.Kz

Abstract

A direct relation between the superconformal formalism for 2d-quantum gravity and the super W-constrained SKP hierarchy is found, without the need to invoke intermediate matrix model technology. The super Kontsevich-Miwa transform of the SKP hierarchy is used to establish an identification between super W constraints on the SKP tau function and decoupling equations corresponding to super Virasoro null vectors. The super Kontsevich-Miwa transformation maps the super W-constrained SKP hierarchy to the (p, p') minimal model.

1 Introduction

Much progress has been made in the non-perturbative description of two-dimensional quantum gravity and gravity-coupled matter, since matrix models, which appeared as a discretized approach to them, resulted at the double scaling limit in integrable hierarchies subjected to Virasoro and possibly high W constraints[1-9]. Recently, using the Kontsevich-Miwa transform, Semikhatov et al. found[10-11] a relation between the Virasoro constraints imposed on the tau functions of the KP hierarchy and the decoupling equation corresponding to the null vector in minimal conformal field theories extended by a scalar current, which plays a role similar to that of the Liouville field. The importance of the method based on the Kontsevich-Miwa transform[12-14] is therefore to relate directly the Virasoro-constrained KP hierarchy and two dimensional quantum gravity, bypassing the matrix model technology. This connection is more meaningful for the non-perturbative description of two-dimensional quantum super gravity and super gravity-coupled matter, because there is no actual generalized matrix model (for example, supermatrix model) for the discrete super counterpart of two dimensional quantum gravity[15].

Similar to the non-super case, the relation between the Super Virasoro-constrained KP hierarchy and two dimensional super quantum gravity is achieved through so-called the Super Kontsevich-Miwa transform proposed by C.S. Huang and D.H. Zhang[16]. In that paper, we start with the level-$\frac{3}{2}$ null vector decoupling equation in the (p,p') superconformal minimal models extended by a super current of weight-$\frac{1}{2}$ and lead to the super Virasoro constraints imposed on the super tau function of the Rabin SKP hierarchy[17] by using the super Kontsevich-Miwa transform and vice versa, provided that the Miwa parameter n_0 satisfies

$$n_0^2 = \frac{p}{p'}. \tag{1.1}$$

Because the derivation is tedious, the question arises as to whether the higher super W constraints on the super KP hierarchy are amenable to the above scheme. It is needed to confirm by concrete example our general conjecture about the existence of a bridge between matter interacting with the continuum quantum super gravity and the W-constrained Rabin SKP hierarchy suggested in our last paper. A positive answer to this question is given in this paper, based on the results obtained for Super $W^{(3)}$ constraints in addition to the previous results for the super $W^{(2)}$ (Super Virasoro)case. We verify that a single level-$\frac{l}{2}$ decoupling equation corresponds to the complete set of Super $W^{(l')}$ constraints (including the Super Virasoro ones) $G^{(l'-1/2)}_{p+1/2}\tau = 0, L^{(l')}_p\tau = 0, p \geq -l'+1$ and $2 \leq l' \leq \frac{(l+1)}{2}$, while these do not lead back to lower-level decoupling equations.

2 '*Dressing*' the decoupling equation.

What we need is the N=1 superconformal (p,p') minimal models extended by a super current J(z) of weight $\frac{1}{2}$. Expanding the energy-momentum tensor and the super current in modes:

$$T(z) = \sum_m z^{-m-3/2}\frac{1}{2}G_m + \theta\sum_m z^{-m-2}L_m, \tag{2.1}$$

$$J(z) = \sum_m z^{-m-1/2}\psi_m + \theta\sum_m z^{-m-1}j_m, \tag{2.2}$$

the standard (anti)commutation relations between their modes are

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{1}{12}(c+1/2)(m^3-m)\delta_{m+n,0}, \quad (2.3)$$

$$[L_m, G_n] = (\frac{1}{2}m - n)G_{m+n}, \quad (2.4)$$

$$\{G_m, G_n\} = 2L_{m+n} + \frac{1}{3}(c+1/2)(m^2 - \frac{1}{4})\delta_{m+n,0}, \quad (2.5)$$

$$[j_m, j_n] = m\delta_{m+n,0}, \quad (2.6)$$

$$[j_m, \psi_n] = 0, \quad (2.7)$$

$$\{\psi_m, \psi_n\} = \delta_{m+n,0}, \quad (2.8)$$

$$[L_m, j_n] = -nj_{m+n}, \quad (2.9)$$

$$[L_m, \psi_n] = (-\frac{1}{2}m - n)\psi_{m+n}, \quad (2.10)$$

$$[G_m, j_n] = -n\psi_{m+n}, \quad (2.11)$$

$$\{G_m, \psi_n\} = j_{m+n}, \quad (2.12)$$

where the contribution to the central charge of the super current J has been written explicitly.

A primary super field $\Phi_{h,q}$ with conformal weight h and U(1) charge q is defined by

$$L_n\Phi_{h,q} = G_n\Phi_{h,q} = j_n\Phi_{h,q} = \psi_n\Phi_{h,q}, \quad for\ n > 0$$

$$L_0\Phi_{h,q} = h\Phi_{h,q}, \quad (2.14)$$

$$j_0\Phi_{h,q} = q\Phi_{h,q}. \quad (2.15)$$

¿From the Eqs.(2.3-15), we can obtain the null vector of level $\frac{5}{2}$

$$\chi_{5/2} = aG_{-5/2} + b_1G_{-3/2}L_{-1} + b_2j_{-1}G_{-3/2} + b_3\psi_{-1/2}G_{-3/2}G_{-1/2} + c_1G_{-1/2}(L_{-1})^2$$
$$+c_2(j_{-1})^2G_{-1/2} + c_3L_{-2}G_{-1/2} + c_4j_{-2}G_{-1/2} + c_5j_{-1}G_{-1/2}L_{-1} + d\psi_{-5/2} + e_1\psi_{-3/2}L_{-1}$$
$$+e_2j_{-1}\psi_{-3/2} + e_3\psi_{-1/2}\psi_{-3/2}G_{-1/2} + f_1\psi_{-1/2}(L_{-1})^2 + f_2(j_{-1})^2\psi_{-1/2} + f_3\psi_{-1/2}L_{-2}$$
$$+f_4\psi_{-1/2}j_{-2} + f_5j_{-1}\psi_{-1/2}L_{-1}, \quad (2.16)$$

where

$$a = \frac{1}{6}(4h^2 - 4hq^2 + 4h + q^4 - 2q^2), \quad (2.17)$$

$$b_1 = -\frac{1}{2}(2h - q^2 + 2), \quad (2.18)$$

$$b_2 = \frac{1}{2}q(2h - q^2 + 2), \quad (2.19)$$

$$b_3 = 0, \quad (2.20)$$

$$c_1 = 1, \quad (2.21)$$

$$c_2 = \frac{1}{6}(2h + 5q^2 + 2), \quad (2.22)$$

$$c_3 = -\frac{1}{3}(2h - q^2 + 2), \quad (2.23)$$

$$c_4 = \frac{q}{3}(2h - q^2 - 1), \quad (2.24)$$

$$c_5 = -2q, \quad (2.25)$$

$$d = -\frac{q}{6}(4h^2 - 4hq^2 - 8h + q^4 + 4q^2), \quad (2.26)$$

$$e_1 = \frac{q}{2}(2h - q^2 - 2), \quad (2.27)$$

$$e_2 = -\frac{1}{3}(2h^2 + hq^2 + 2h - q^4 - 4q^2), \quad (2.28)$$

$$e_3 = -\frac{1}{6}(2h - q^2 + 2), \quad (2.29)$$

$$f_1 = -q, \tag{2.30}$$

$$f_2 = -\frac{q}{3}(4h + q^2 + 4), \tag{2.31}$$

$$f_3 = \frac{q}{3}(2h - q^2 + 2), \tag{2.32}$$

$$f_4 = -\frac{1}{6}(4h^2 + 4h - q^4 - 4q^2), \tag{2.33}$$

$$f_5 = \frac{1}{2}(2h + 3q^2 + 2), \tag{2.34}$$

and

$$c = (4h^2 - 4hq^2 - 26h + q^4 + 13q^2 + 6)/(2q^2 - 4h - 4). \tag{2.35}$$

After inserting eq.(2.16) into correlation functions, one gets the following decoupling equation:

$$\mathcal{O} < \cdots \Phi_{h_i,q_i}(z_i) \cdots \Phi_{h_1,q_1}(z_1)\Phi_{h,q}(z_0) >= 0, \tag{2.36}$$

$$\begin{aligned}\mathcal{O} = & -D_0\partial_0\partial_0 + \sum_{i\geq 1}\{\tfrac{1}{Z_{i0}}\{b_1(D_i - 2\theta_{i0}\partial_i)\partial_0 + c_3\partial_i D_0 + c_5 q_i D_0\partial_0 \\ & -f_1 q_i\theta_{i0}\partial_0\partial_0\} + \tfrac{1}{Z_{i0}^2}\{a(D_i - 2\theta_{i0}\partial_i) - c_3(\tfrac{1}{2}\theta_{i0}\partial_{\theta_i} + \tfrac{1}{2}\theta_i\theta_0\partial_i + h_i)D_0 \\ & +c_4 q_i D_0 - e_1 q_i\theta_{i0}\partial_0 + 2b_1 h_i\theta_{i0}\partial_0\} + \tfrac{1}{Z_{i0}^3}\theta_{i0}(4h_i a - dq_i)\} \\ & + \sum_{i\geq 1}\cdot\sum_{j\geq 1}\{\tfrac{1}{Z_{i0}Z_{j0}}\{-b_2 q_i(D_j - 2\theta_{j0}\partial_j) - c_2 q_i q_j D_0 + f_3 q_i\theta_{i0}\partial_j + f_5 q_i q_j\theta_{j0}\partial_0\} \\ & +\tfrac{1}{Z_{i0}Z_{j0}^2}\{(e_2 q_i q_j - 2b_2 q_i h_j)\theta_{j0} - e_3 q_i q_j\theta_{i0}\theta_{j0}D_0 + (f_4 q_i q_j - f_3 q_i h_j)\theta_{i0} \\ & -f_3 q_i\theta_{i0}(\tfrac{1}{2}\theta_{j0}\partial_{\theta_j} + \tfrac{1}{2}\theta_j\theta_0\partial_j)\} \\ & -f_2\sum_{i\geq 1}\cdot\sum_{j\geq 1}\cdot\sum_{k\geq 1}\tfrac{1}{Z_{i0}Z_{j0}Z_{k0}}q_i q_j q_k\theta_{k0}\end{aligned} \tag{2.37}$$

where $\theta_{i0} = \theta_i - \theta_0$, $Z_{ij} = z_i - z_j - \theta_i\theta_j$, $D_i = \partial_{\theta_i} + \theta_i\partial_i$ and $\partial_i = \partial_{z_i}$

We now need to use the super Kontsevich-Miwa transformation which has been given in Ref.[16]

$$t_r = \frac{1}{r}\cdot\sum_i n_i z_i^{-r}, r \geq 1 \tag{2.38}$$

$$\tau_{r-1/2} = -\sum_i n_i\theta_i z_i^{-r}, r \geq 1 \tag{2.39}$$

where t_r and $\tau_{r-1/2}$ are Grassman even and odd flow parameters of the SKP hierarchies, respectively, and $\{z_i, \theta_i\}$ is a set of infinitely many points on the super Rieman sphere in order to make the flow parameters t_r and $\tau_{r-1/2}$ independent.

For the purpose of converting the decoupling equation into Super W-constraints on the SKP hierarchy via the Super Koutsevich-Miwa transformation one should take:

$$q_i = in_i, i = 1, 2, \cdots, \tag{2.40}$$

$$q = in_0, \tag{2.41}$$

i.e. the Miwa parameter play the role of U(1) charge. Since the terms in Eq.(2.37) which are not in the image of the Super Kontsevich-Miwa transformation of the Super W-constraints should be excluded, the weights and $U(1)$ charges must satisfy the following relations

$$h = -\frac{q^2 + 4}{4} = \frac{n_0^2 - 4}{4} \tag{2.42}$$

and

$$h_i = \frac{h}{q}q_i = \frac{n_0^2 - 4}{4n_0}n_i, i = 1, 2, \cdots \tag{2.43}$$

We think that Eq.(2.43) is a general character of correlation functions in any level decoupling equation when we convert the decoupling equation into Super W-constraints on the SKP hierarchy via the Super Kontsevich-Miwa transformation. We will give the explanation in section 4.

Using Eqs.(2.40-43), the decoupling operator (2.37) reduces to

$$\begin{aligned}\mathcal{O} = &-D_0\partial_0\partial_0 + \sum_{i\geq 1}\{\tfrac{1}{Z_{i0}}\{-\tfrac{3}{4}n_0^2(D_i - 2\theta_{i0}\partial_i)\partial_0 - \tfrac{1}{2}n_0^2\partial_i D_0 \\ &+2n_0 n_i D_0\partial_0 - n_0 n_i \theta_{i0}\partial_0\partial_0\} + \tfrac{1}{Z_{i0}^2}\{\tfrac{n_0^2(3n_0^2-4)}{8}(D_i - 2\theta_{i0}\partial_i) \\ &+\tfrac{n_0^2}{4}(\theta_{i0}\partial_{\theta_i} + \theta_i\theta_0\partial_i)D_0 - \tfrac{n_0(3n_0^2-4)}{8}n_i D_0 + \tfrac{n_0(3n_0^2-4)}{8}\theta_{i0}n_i\partial_0\}\} \\ &+\sum_{i\geq 1}\cdot\sum_{j\geq 1}\{\tfrac{1}{Z_{i0}Z_{j0}}\{\tfrac{3}{4}n_0^3 n_i(D_j - 2\theta_{j0}\partial_j) - \tfrac{3}{4}n_0^2 n_i n_j D_0 - \tfrac{1}{2}n_0^3 n_i\theta_{i0}\partial_j \\ &+\tfrac{5}{4}n_0^2 n_i n_j\theta_{j0}\partial_0\} + \tfrac{1}{Z_{i0}Z_{j0}^2}\{-\tfrac{1}{4}n_0^2 n_i n_j\theta_{i0}\theta_{j0}D_0 + \tfrac{1}{4}n_0^3 n_i\theta_{i0}(\theta_{j0}\partial_{\theta_j} + \theta_j\theta_0\partial_j)\}\}.\end{aligned} \tag{2.44}$$

3 From the decoupling equation to the W_3 constraints

In the obtained decoupling operator, expressing D_i, ∂_i and ∂_{θ_i} in Eq.(2.44) in terms of ∂t_r and $\partial\tau_{r-1/2}$ by means of the Super Kontsevich-Miwa transformation Eqs.(2.38-39), a very tedious calculation leads to:

$$\begin{aligned}\tfrac{1}{2n_0^3}\mathcal{O} = &\sum_{p\geq -2} z_0^{-p-3}(\tfrac{1}{2}G^{(5/2)}_{p+1/2} + \theta_0 L_p^{(3)}) + \tfrac{3}{8}\cdot\sum_{i\geq 1}\cdot\sum_{p\geq -1}\tfrac{n_i}{z_i - z_0}z_i^{-p-2}G^{(3/2)}_{p+1/2} \\ &-\tfrac{1}{4}\cdot\sum_{i\geq 1}\cdot\sum_{p\geq -1}\tfrac{n_i\theta_i}{z_i - z_0}z_i^{-p-2}L_p^{(2)} + \theta_0\{\sum_{i\geq 1}\cdot\sum_{p\geq -1}\tfrac{n_i}{z_i - z_0}z_i^{-p-2}L_p^{(2)} \\ &-\sum_{i\geq 1}\cdot\sum_{p\geq -1}\tfrac{n_i\theta_i}{z_i - z_0}(p+2)z_i^{-p-3}\tfrac{1}{2}G^{(3/2)}_{p+1/2} - \tfrac{3}{8}\cdot\sum_{i\geq 1}\cdot\sum_{p\geq -1}\tfrac{n_i\theta_i}{(z_i - z_0)^2}z_i^{-p-2}G^{(3/2)}_{p+1/2}\}\end{aligned} \tag{3.1}$$

In the above formula the $G^{(5/2)}_{p+1/2}$ operator with weight 5/2 is given by

$$G^{(5/2)}_{p+1/2} = G^{(5/2)}_{p+1/2}(2) + G^{(5/2)}_{p+1/2}(1) + G^{(5/2)}_{p+1/2}(0) + G^{(5/2)}_{p+1/2}(-1) + G^{(5/2)}_{p+1/2}(-2) \tag{3.2}$$

where

$$G^{(5/2)}_{p+1/2}(2) = \sum_{s\geq 2}^{p}\sum_{r\geq 1}^{s-1}\frac{\partial}{\partial\tau_{p-s+1/2}}\frac{\partial}{\partial t_{s-r}}\frac{\partial}{\partial t_r}, \tag{3.3}$$

$$G^{(5/2)}_{p+1/2}(1) = \sum_{s\geq 1}^{p}\{\frac{1}{n_0}(p+s+2) - \frac{1}{4}n_0 s - \frac{5}{4}n_0 p - \frac{5}{2}n_0\}\frac{\partial}{\partial t_{p-s+1}}\frac{\partial}{\partial\tau_{s-1/2}}, \tag{3.4}$$

$$\begin{aligned}G^{(5/2)}_{p+1/2}(0) = &\sum_{r\geq 0}^{p}\sum_{k\geq 1}\tfrac{3}{4}kt_k\tfrac{\partial}{\partial\tau_{r+k-1/2}}\tfrac{\partial}{\partial t_{p-r+1}} + \sum_{r\geq -1}^{p-1}\cdot\sum_{k\geq 1, r+k\geq 1}\tfrac{3}{4}\tau_{k-1/2}\tfrac{\partial}{\partial t_{r+k}}\tfrac{\partial}{\partial t_{p-r}} \\ &+(\tfrac{9}{16}n_0^2 - \tfrac{7}{4} + \tfrac{1}{n_0^2})(p+1)(p+2)\tfrac{\partial}{\partial\tau_{p+1/2}},\end{aligned} \tag{3.5}$$

$$\begin{aligned}G^{(5/2)}_{p+1/2}(-1) = &\sum_{r\geq -1}^{p}\sum_{k\geq 1, r+k\geq 1}\tfrac{1}{2}kt_k\tfrac{\partial}{\partial t_{r+k}}\tfrac{\partial}{\partial\tau_{p-r+1/2}} \\ &+\sum_{r\geq -1}^{p}\sum_{k\geq 1, r+k\geq 1}\tfrac{1}{4}(r+2k-1)\tau_{k-1/2}\tfrac{\partial}{\partial\tau_{r+k-1/2}}\tfrac{\partial}{\partial\tau_{p-r+1/2}} \\ &-\tfrac{(3n_0^2-4)}{8n_0}\sum_{k\geq 1, p+k\geq 1}(p+2)\tau_{k-1/2}\tfrac{\partial}{\partial t_{p+k}},\end{aligned} \tag{3.6}$$

$$\begin{aligned}G^{(5/2)}_{p+1/2}(-2) = &-\sum_{k\geq 1, p+k\geq 0}\{\tfrac{1}{2}(3n_0 - \tfrac{4}{n_0})(p+2) + \tfrac{1}{8}(3n_0 - \tfrac{12}{n_0})(k-1)\}kt_k\tfrac{\partial}{\partial\tau_{p+k+1/2}} \\ &-\sum_{k\geq 1, p+k\geq 1}\{\tfrac{1}{8}(7n_0 - \tfrac{4}{n_0})(p+2) + \tfrac{1}{8}(n_0 - \tfrac{4}{n_0})(k-1)\}\tau_{k-1/2}\tfrac{\partial}{\partial t_{p+k}} \\ &-\sum_{k\geq 1}\cdot\sum_{r\geq 1, p+k\geq 1}^{p+k}\tfrac{1}{4}r\tau_{k-1/2}\tfrac{\partial}{\partial\tau_{r-1/2}}\tfrac{\partial}{\partial\tau_{p+k-r+1/2}} + \sum_{k\geq 1}\cdot\sum_{s\geq 1, p+k\geq 1}^{p+k}\tfrac{3}{4}kt_k\tfrac{\partial}{\partial t_{p+k-s+1}}\tfrac{\partial}{\partial\tau_{s-1/2}} \\ &+\sum_{k\geq 1}\cdot\sum_{s\geq 1, p+k\geq 2}^{p+k-1}\tfrac{1}{4}\tau_{k-1/2}\tfrac{\partial}{\partial t_s}\tfrac{\partial}{\partial t_{p+k-s}} + \sum_{k'\geq 1}\cdot\sum_{k\geq 1}\tfrac{3}{4}k't_{k'}kt_k\tfrac{\partial}{\partial\tau_{p+k+k'+1/2}} \\ &+\sum_{k'\geq 1}\cdot\sum_{k\geq 1, p+k'+k\geq 1}\tfrac{5}{4}\tau_{k'-1/2}kt_k\tfrac{\partial}{\partial t_{p+k'+k}} + \sum_{k'\geq 1}\cdot\sum_{k\geq 1, p+k'+k\geq 1}\tfrac{1}{4}k\tau_{k'-1/2}\tau_{k-1/2}\tfrac{\partial}{\partial\tau_{p+k+k'-1/2}}.\end{aligned} \tag{3.7}$$

The $L_p^{(3)}$ operator with weight 3 in eq.(3.1) is

$$L_p^{(3)} = L_p^{(3)}(3) + L_p^{(3)}(2) + L_p^{(3)}(1) + L_p^{(3)}(0) + L_p^{(3)}(-1) + L_p^{(3)}(-2) \tag{3.8}$$

where

$$L_p^{(3)}(3) = \sum_{s\geq 2}^{p-1}\cdot\sum_{r\geq 1}^{s-1}\frac{1}{2}\frac{\partial}{\partial t_{p-s}}\frac{\partial}{\partial t_{s-r}}\frac{\partial}{\partial t_r}, \tag{3.9}$$

$$\begin{aligned}L_p^{(3)}(2) = &\sum_{s\geq 1}^{p-1}\{(-\tfrac{5}{8}n_0 + \tfrac{1}{2n_0})(s+1) + (-\tfrac{3}{8}n_0 + \tfrac{1}{2n_0})(p+2)\}\tfrac{\partial}{\partial t_s}\tfrac{\partial}{\partial t_{p-s}} \\ &+\sum_{s\geq 2}^{p}\sum_{r\geq 1}^{s-1}r\tfrac{\partial}{\partial\tau_{p-s+1/2}}\tfrac{\partial}{\partial t_{s-r}}\tfrac{\partial}{\partial\tau_{r-1/2}},\end{aligned} \tag{3.10}$$

$$\begin{aligned}L_p^{(3)}(1) = &\sum_{r\geq 1}^{p}\{\tfrac{1}{2}n_0 r(\tfrac{r}{2} + \tfrac{3}{4}p + 2) - \tfrac{1}{2n_0}r(r+1)\}\tfrac{\partial}{\partial\tau_{r-1/2}}\tfrac{\partial}{\partial\tau_{p-r+1/2}} \\ &+\{\tfrac{1}{32}(9n_0^2 - 28) + \tfrac{1}{2n_0^2}\}(p+1)(p+2)\tfrac{\partial}{\partial t_p},\end{aligned} \tag{3.11}$$

$$L_p^{(3)}(0) = \sum_{r\geq -1}^{p-1} \sum_{k\geq 1, r+k\geq 1} \frac{1}{2}(r+2k-1)\tau_{k-1/2}\frac{\partial}{\partial\tau_{r+k-1/2}}\frac{\partial}{\partial t_{p-r}}$$
$$+\sum_{r\geq -1}^{p-1} \sum_{k\geq 1, r+k\geq 1} kt_k \frac{\partial}{\partial t_{r+k}}\frac{\partial}{\partial t_{p-r}} \tag{3.12}$$
$$+\sum_{r\geq -1}^{p-1} \sum_{k\geq 1, r+k\geq 1} \frac{1}{2}(\frac{3}{4}p - r - \frac{1}{2})\tau_{k-1/2}\frac{\partial}{\partial t_{r+k}}\frac{\partial}{\partial\tau_{p-r-1/2}}$$
$$+\sum_{r\geq 0}^{p}\sum_{k\geq 1} \frac{1}{2}(\frac{3}{4}p - r + \frac{1}{2})kt_k\frac{\partial}{\partial\tau_{r+k-1/2}}\frac{\partial}{\partial\tau_{p-r+1/2}},$$
$$L_p^{(3)}(-1) = 0, \tag{3.13}$$

$$L_p^{(3)}(-2) = \sum_{k\geq 1, p+k\geq 1}\{-\frac{1}{8}(11n_0 - \frac{12}{n_0})(p+2) + (\frac{1}{n_0} - \frac{n_0}{4})(k-1)\}kt_k\frac{\partial}{\partial t_{p+k}}$$
$$-\sum_{k\geq 1, p+k\geq 1}\frac{1}{16}\{(10n_0 - \frac{8}{n_0})p^2 + (23n_0 - \frac{28}{n_0})pk + (7n_0 + \frac{4}{n_0})p + (4n_0 - \frac{16}{n_0})k^2$$
$$+(34n_0 - \frac{8}{n_0})k - 18n_0 + \frac{8}{n_0}\}\tau_{k-1/2}\frac{\partial}{\partial\tau_{p+k-1/2}}$$
$$+\sum_{s\geq 1}^{p+k-1}\sum_{k\geq 1, p+k\geq 2}\frac{1}{2}kt_k\frac{\partial}{\partial t_s}\frac{\partial}{\partial t_{p+k-s}} \tag{3.14}$$
$$+\sum_{r\geq 1}^{p+k-1}\sum_{k\geq 1, p+k\geq 2}\frac{1}{2}(\frac{1}{4}p + k - \frac{1}{2})\tau_{k-1/2}\frac{\partial}{\partial t_{p+k-r}}\frac{\partial}{\partial\tau_{r-1/2}}$$
$$+\sum_{r\geq 1}^{p+k}\sum_{k\geq 1, p+k\geq 1}\frac{1}{2}rkt_k\frac{\partial}{\partial\tau_{p+k-r+1/2}}\frac{\partial}{\partial\tau_{r-1/2}} + \sum_{k'\geq 1}\sum_{k\geq 1, p+k'+k\geq 1}k't_{k'}kt_k\frac{\partial}{\partial t_{p+k'+k}}$$
$$+\sum_{k'\geq 1}\sum_{k\geq 1, p+k'+k\geq 1}\frac{1}{2}(\frac{5}{4}p + 3k' + k - \frac{3}{2})\tau_{k'-1/2}kt_k\frac{\partial}{\partial\tau_{p+k'+k-1/2}}$$
$$+\sum_{k'\geq 1}\sum_{k\geq 1, p+k'\geq 0, p+k'+k\geq 2}\frac{1}{2}k'\tau_{k'-1/2}\tau_{k-1/2}\frac{\partial}{\partial t_{p+k'+k-1}}.$$

The $G_{p+1/2}^{(3/2)}$ operator is

$$G_{p+1/2}^{(3/2)} = G_{p+1/2}^{(3/2)}(1) + G_{p+1/2}^{(3/2)}(0) + G_{p+1/2}^{(3/2)}(-1) \tag{3.15}$$

where

$$G_{p+1/2}^{(3/2)}(1) = \sum_{r\geq 1}^{p}\frac{\partial}{\partial t_{p-r+1}}\frac{\partial}{\partial\tau_{r-1/2}}, \tag{3.16}$$
$$G_{p+1/2}^{(3/2)}(0) = \frac{1}{2}(\frac{4}{n_0} - n_0)(p+1)\frac{\partial}{\partial\tau_{p+1/2}}, \tag{3.17}$$
$$G_{p+1/2}^{(3/2)}(-1) = \sum_{k\geq 1}kt_k\frac{\partial}{\partial\tau_{p+k+1/2}} + \sum_{k\geq 1, p+k\geq 1}\tau_{k-1/2}\frac{\partial}{\partial t_{p+k}}, \tag{3.18}$$

and the $L_p^{(2)}$ operator is

$$L_p^{(2)} = L_p^{(2)}(2) + L_p^{(2)}(1) + L_p^{(2)}(0) + L_p^{(2)}(-1) \tag{3.19}$$

where

$$L_p^{(2)}(2) = \sum_{r\geq 1}^{p-1}\frac{1}{2}\frac{\partial}{\partial t_r}\frac{\partial}{\partial t_{p-r}} \tag{3.20}$$
$$L_p^{(2)}(1) = \frac{1}{4}(\frac{4}{n_0} - n_0)(p+1)\frac{\partial}{\partial t_p} + \sum_{r\geq 1}^{p}\frac{1}{2}(p-r+1)\frac{\partial}{\partial\tau_{r-1/2}}\frac{\partial}{\partial\tau_{p-r+1/2}}, \tag{3.21}$$
$$L_p^{(2)}(0) = 0, \tag{3.22}$$
$$L_p^{(1)}(-1) = \sum_{k\geq 1, p+k\geq 1}kt_k\frac{\partial}{\partial t_{p+k}} + \sum_{k\geq 1, p+k\geq 1}(\frac{p-1}{2} + k)\tau_{k-1/2}\frac{\partial}{\partial\tau_{p+k-1/2}}. \tag{3.23}$$

In all of the above expressions we have defined

$$G_{p+1/2}^{(5/2)}(i) = 0, -2 \leq p \leq i-1,$$
$$L_p^{(3)}(i) = 0, -2 \leq p \leq i-1, \tag{3.24}$$
$$G_{p+1/2}^{(3/2)}(i) = 0, -1 \leq p \leq i-1, \tag{3.26}$$
$$L_p^{(2)}(i) = 0, -1 \leq p \leq i-1. \tag{3.27}$$

For the super tau function of the super W_3 constrained Rabin SKP hierarchy in the super Kontsevich parameterization we assume the ansatz:

$$\tau\{z_i\} = lim_{j\to\infty} < \Phi(z_j)\cdots\Phi(z_1)\Phi(z_0) > . \tag{3.28}$$

Then, from Eqs.(2.36) and (3.1), one has

$$\mathcal{O}\tau(z_i) = 0 \tag{3.29}$$

Therefore, it follows that:

$$G^{(5/2)}_{p+1/2}\tau = 0, p \geq -2, \tag{3.30}$$

$$L^{(3)}_p\tau = 0, p \geq -2, \tag{3.31}$$

$$G^{(3/2)}_{p+1/2}\tau = 0, p \geq -1, \tag{3.32}$$

$$L^{(2)}_p\tau = 0, p \geq -1. \tag{3.33}$$

Obviously, $G^{(3/2)}_{p+1/2}$ and $L^{(2)}_p$ are essentially the same as the $G_{p+1/2}$ and L_p in Ref.[16].

Thus we have recovered a set of super W_3 constants on the SKP tau function. From Eqs.(3.1-23), one knows that the constraints depend on n_0 and then, via $n_0^2 = \frac{4p}{p'}$, on the corresponding superconformal minimal model.

It is clear from the above derivation that we can reverse the argument. That is, to begin with a set of super W_3 constraints, we construct the supersymmetric combination Eq.(3.29) and convert the derivatives w.r.t. the flow parameters into those w.r.t. z_i, then we arrive at the decoupling equation. In this way, we have proved the equivalence between a set of super W_3 constraints and the level-$\frac{5}{2}$ decoupling equation in the (p, p') superconformal model.

4 Conclusion

We have explicitly revealed the correspondence between the super W_3 constraints and the level-$\frac{5}{2}$ decoupling equation. One can directly generalize the approach to super higher W constrained SKP hierarchies. There are two series of null vectors in the NS sector of the N=1 superconformal model[18]:the level-$\frac{l}{2}$ $(l = 3, 5, \cdots)$ one based on (dressed) $(l, 1)$ (or, $(1, l)$) superprimary fields and the level-$2l$ $(l = 3, 5, \cdots)$ one based on the (dressed) $(2l, 2)$(or, $(2, 2l)$) field. The level-$\frac{l}{2}$ $(l = 3, 5, \cdots)$ decoupling equation corresponds to a set of super $W_{l'}$, $2 \leq l' \leq \frac{l+1}{2}$, constraints on the SKP hierarchy.

The dimension (in the matter sector) of the superprimary field $\Phi_{l,1}$ is (see Ref.[18])

$$\Delta = \frac{1}{2}\left(\frac{l^2 - 1}{4}\frac{p}{p'} - \frac{l-1}{2}\right). \tag{4.1}$$

The dressing of $\Phi_{l,1}$ is determined by its U(1) charge $q = in_0$ with n_0 being the Miwa parameter which is related to the model by

$$n_0^2 = \frac{1}{4}(l-1)^2\frac{p}{p'}. \tag{4.2}$$

If one lets

$$\frac{p}{p'} = 1 + \frac{Q^2 + Q\sqrt{Q^2 + 4}}{2} \tag{4.3}$$

then one has

$$\frac{l-1}{2n_0} - \frac{2n_0}{l-1} = Q \tag{4.4}$$

For $l = 5$, Eq.(4.4) gives

$$\frac{2}{n_0} - \frac{n_0}{2} = Q \tag{4.5}$$

which is the super symmetric counterpart of the relation in the bosonic case. If we consider the different Q given in Ref.[16] and in this paper and denote n_0 by Q in the operators $L^{(2)}_p$, $G^{(3/2)}_{p+1/2}$ according to eq.(4.4), we find that the $G^{(3/2)}_{p+1/2}$ and $L^{(2)}_p$ are completely the same as $G_{p+1/2}$ and L_p in Ref.[16].

Now, we can explain that the conformal weights and U(1) charges of the super primary fields $\Phi(z_i)$ in the ansatz $\tau(z) = lim_{j\to\infty} < \Phi(z_j)\cdots\Phi(z_1)\Phi(z_0) >$ should satisfy $\frac{h_i}{q_i} = \frac{h}{q}$. From the level-$\frac{l}{2}$ $(l = 3, 5, \cdots)$ decoupling equation, by means of super Kontsevich-Miwa transformation, we obtain a set of $W_{l'}$ constraints on the SKP hierarchy from $l' = 2$ to $l' = \frac{l+1}{2}$

which contain super W_2 constraints. We have shown [16] that the super W_2 constraints ($G^{(3/2)}_{p+1/2}\tau = 0$ and $L^{(2)}_p\tau = 0$) make the conformal weights and U(1) charges of the super primary fields $\Phi(z_i)$ in the $\tau(z) = lim_{j\to\infty} < \Phi(z_j)\cdots\Phi(z_1)\Phi(z_0) >$ satisfy $\frac{h_i}{q_i} = \frac{h}{q}$. Therefore, at any level the conformal weights and U(1) charges of the super primary fields $\Phi(z_i)$ in the ansatz $\tau(z) = lim_{j\to\infty} < \Phi(z_j)\cdots\Phi(z_1)\Phi(z_0) >$ should satisfy $\frac{h_i}{q_i} = \frac{h}{q}$

Acknowledgement

We would like to thank Y.B.Dai, H.Y.Guo, B.Y.Hou, and B.H.Zhao for discussions. This work was supported in part by the Foundation of National Education Committee of China and in part by the National Science Foundation of China.

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