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Study of Jet Production Rates in the Four Flavor Continuum and a Test of QCD

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A Study of Jet Production Rates in the Four Flavor Continuum and a Test of QCD

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Abstract

We present an analysis of jet production rates within the framework of QCD. These rates are measured in e^+e^- annihilation into hadrons in the energy region below the $\Upsilon(4S)$ resonance where only four flavors are produced. All previous jet studies have been performed in the five flavor continuum. However, in the four flavor continuum, QCD predictions are substantially different. The measured jet rate distributions are compared to the predictions of $\mathcal{O}(\alpha_s^2)$ QCD computed for the four flavor continuum. From fits of the theory to the data we obtain $\alpha_s(10.53\text{ GeV})=0.164\pm0.004\pm0.014$, where the first error is due to the experimental uncertainties and the second due to the theoretical uncertainties. This value of α_s extrapolated to M_Z in the five flavor continuum yields $\alpha_s(M_Z)=0.113\pm0.002\pm0.006$.

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I. INTRODUCTION

Within the framework of quantum chromodynamics (QCD), multihadron production in e^+e^- collisions is associated with the production of partons which subsequently materialize into collimated clusters of colorless hadrons called jets. The first observation of jets in e^+e^- collisions was reported in 1975 from SPEAR [1] at a center of mass energy (E_{cm}) of 6-8 GeV. Since then, jets have emerged as powerful tools for performing tests of perturbative QCD [2-5]. The theoretical predictions of the relative production rates of multijet events have been found to be in good agreement with measurements made in the e^+e^- continuum and around the Z peak. The values of the strong coupling constant, α_s, determined from these measurements at different energies have been found to be consistent with each other [4,6]. All these studies have been performed in the regime of five flavors. In the four flavor regime, however, the QCD predictions of the next to leading order coefficients are substantially different due to the different contributions from vacuum polarisation diagrams [7]. QCD has never been tested with the same process on both sides of a flavor threshold. Thus, it is desirable to test the evolution of α_s as predicted by the renormalization group equation (RGE) by the determination of α_s from essentially the same observable but in the region of lower energy and different number of active flavors. Comparison of the results obtained from such a study with those in the five flavor continuum at higher energies would then constitute a unique test of QCD. Due to its large data sample and that it is performed at the highest possible energy in the four flavor continuum, the CLEO experiment at the Cornell Electron Storage Ring (CESR) is well suited for this test.

II. EVENT SELECTION

The data used in this analysis were collected with the CLEO II detector, described elsewhere [8], at CESR at an average energy of E_{cm} = 10.53 GeV. The detector components important for this analysis are the tracking system consisting of a six-layer straw tube chamber, a ten-layer vertex drift chamber, and a 51-layer main drift chamber, and an electromagnetic calorimeter consisting of 7800 CsI crystals, all operated inside a 1.5-T solenoidal magnet. Crystals in which energy is deposited are clustered to form photon candidates in the barrel ($|\cos\theta| < 0.80$, where θ is the angle of the cluster with respect to the positron beam axis) or in the endcap ($18^0 \leq \theta \leq 36^0$). No particle identification is used in this analysis. The charged tracks and electromagnetic clusters are taken to be pions and photons, respectively.

Hadronic events are selected by requiring at least five charged tracks in the central detector and a total energy in the calorimeter greater than 15% of the center-of-mass energy. This largely suppresses the background from electromagnetic and two-photon processes. Background from beam gas and beam wall interactions is further suppressed by vetoing events which had a large momentum imbalance. We require that the fitted event vertex be within ± 5 cm of the beam interaction point along the beam axis. Any event having a charged track with momentum greater than 6 GeV is rejected. Only electromagnetic clusters which are not matched with any charged track are used in the analysis. The clusters are required to have a minimum energy of 50 MeV in the endcaps or 30 MeV in the barrel. The

thrust axis of the selected events is required to point to inside the central tracking chamber. These event selection criteria have an efficiency of about 70%.

III. MEASUREMENT OF JET PRODUCTION RATES

The jet finding algorithms [9,10,12–14] used in this analysis are described in the appendix. In brief, resolvable jets of particles in an event are defined by requiring that the jet resolution parameter $y_{ij} = M_{ij}^2/E_{vis}^2$ for any jet pair (i,j) exceeds a threshold value y_{cut}. The visible energy, E_{vis}, is the total energy of all observed particles. Different definitions of the "mass" M_{ij} and different schemes to combine two unresolvable jets into one give rise to different algorithms.

Using the jet finding algorithms, JJ, JP and DP, we measure the jet production rates in the hadronic event sample in the four flavor continuum. To correct the data for the effects of limited detector resolution and acceptance, and for initial state photon radiation, we use the JETSET shower Monte Carlo (MC) [15] model to determine the correction factors. We use the standard values of the parameters which are known to describe the experimental data well in the range of E_{cm} = 30 - 91 GeV [17]. We have found in a separate study that with these parameter values this model also gives a good description of the global features of our multihadron data. To determine the correction factors, jet rate distributions are produced from two MC samples: (I) without detector simulation and initial state radiation, (II) with detector simulation and initial state radiation and subject to the same reconstruction procedure and event selection criteria as the experimental data. For a given value of y_{cut}, the correction factor C_i is defined as

$$C_i(y_{cut}) = \frac{(U_i^{MC}/N_U^{MC})}{(D_i^{MC}/N_D^{MC})}; \tag{1}$$

where U_i^{MC} is the number of i-jet events in sample (I) which contains a total of N_U^{MC} events, and D_i^{MC} is the number of i-jet events in sample (II) which contains N_D^{MC} events that survived the reconstruction and selection process. The corrected experimental distribution for the i-jet rate, R_i, is given by

$$R_i = C_i\,\hat{R}_i; \tag{2}$$

where $\hat{R}_i$ is the uncorrected experimental distribution.

The experimental measurements of 2-jet rates, thus corrected for detector effects and initial state radiation, are presented in Table I, for the JJ, JP and DP algorithms. The jet rates are given for different values of y_{cut} ranging from 0.06 to 0.25 (corresponding to minimum jet pair masses of about 2.58 GeV/c^2 to 5.26 GeV/c^2) for the JJ and JP algorithms and from 0.03 to 0.25 (corresponding to minimum relative transverse momentum of about 1.82 GeV/c to 5.26 GeV/c) for the DP algorithm. We have listed the jet rates only in the y_{cut} region where the detector corrections are less than 30%.

Because QCD calculations are performed at the parton level, the experimental data must be corrected for hadronization before comparison with the QCD predictions. Figures 1a, 2a, and 3a present the jet rates at the parton and the hadron level as predicted by JETSET. The experimental data, corrected for detector effects and initial state radiation only, are compared to the predictions at hadron level MC in Figures 1b, 2b and 3b for the JJ, JP and DP algorithms, respectively. We have checked that the data and the MC also agree well at the detector level. The detector correction factors are presented in Figures 1c, 2c and 3c.

IV. DETERMINATION OF α_s

To $\mathcal{O}(\alpha_s^2)$, the strong coupling constant may be written [18]:

$$\alpha_s(\mu) = \frac{12\pi}{(33-2n_f)\ln(\mu^2/\Lambda^2_{\overline{MS}})}\left[1 - 6\,\frac{(153-19n_f)}{(33-2n_f)^2}\frac{\ln(\ln(\mu^2/\Lambda^2_{\overline{MS}}))}{\ln(\mu^2/\Lambda^2_{\overline{MS}})}\right] \quad (3)$$

where n_f is the number of active quark flavors (taken to be four in our continuum data), $\Lambda_{\overline{MS}}$ is the QCD parameter referring to the $\overline{MS}$ renormalization scheme, and μ is the renormalization scale which may be related to the e^+e^- c.m. energy by a factor f_μ through the relation

$$\mu = f_\mu\, E_{cm}. \quad (4)$$

The relative rates of 3- and 4-jet events, R_3 and R_4, have been calculated in $\mathcal{O}(\alpha_s^2)$ QCD [19] perturbation theory and parametrized [10] as below:

$$R_3 \equiv \frac{\sigma_3}{\sigma_t} = \frac{1}{\sigma_c}\left[\frac{\alpha_s(\mu)}{2\pi}A_3(y_{cut}) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left(A_3(y_{cut})2\pi b_0\ln(\frac{\mu^2}{E^2_{cm}}) + B_3(y_{cut})\right)\right]; \quad (5)$$

$$R_4 \equiv \frac{\sigma_4}{\sigma_t} = \frac{1}{\sigma_c}\left[B_4(y_{cut})\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\right]; \quad (6)$$

where

$$\sigma_c = 1 + \frac{\alpha_s}{\pi} + 1.52\left(\frac{\alpha_s}{\pi}\right)^2; \quad (7)$$

$$b_0 = \frac{33-2n_f}{12\pi}; \quad (8)$$

and σ_t is the total cross-section for e^+e^- annihilation into hadrons. The theoretical value of the 2-jet rate, R_2, is computed from the condition $R_2 + R_3 + R_4 = 1$. Values for the coefficients A and B are derived [10] by integrating the $\mathcal{O}(\alpha_s^2)$ matrix elements [19] using a program supplied by P. Nason [20].

We test these QCD predictions in the following fashion. The value of $\Lambda_{\overline{MS}}$ is determined by comparing the above equations with the measurements corresponding to a y_{cut} value of 0.06. There the hadronization and detector corrections are small while the statistics is still large. Taking this value of $\Lambda_{\overline{MS}}$ as input into the above equations, the agreement (or disagreement) between the theoretical predictions and the measurements at other y_{cut} points is taken as a test of the theory. Figures 4a, 5a and 6a show the comparison of the theoretical

predictions with the data for the JJ, JP and DP algorithms, respectively. The theory agrees well with the data except at small values of y_{cut} where 4-jets dominate. This discrepancy is not surprising as the 4-jet rates are calculated to leading order only.

All the points in the R_2 distribution are correlated as each point subsumes the previous point. Thus we do not use the R_2 distribution to determine the value of α_s. Instead, we define the differential 2-jet distribution, $D_2(y_{cut})$, as

$$D_2(y_{cut} - \Delta y_{cut}/2) = \frac{R_2(y_{cut}) - R_2(y_{cut} - \Delta y_{cut})}{\Delta y_{cut}}, \quad (9)$$

where Δy_{cut} is the difference between the two y_{cut} values corresponding to the adjacent R_2 points. This distribution may be thought of as the y_{cut} distribution for which the jet multiplicity of events changes from 3 to 2. Now, all the points are statistically independent of each other. The QCD parameter $\Lambda_{\overline{MS}}$ is determined for each algorithm by fitting the predicted D_2 distribution to the measured distribution for two ways of treating the renormalization scale: first setting f_μ=1, and second taking f_μ as a free parameter in the fit. The y_{cut} regions in which the fits are made are chosen by the following requirements: 1) the measured 5-jet rates should be less than 1% as theoretical calculations for 5-jet rates are not available, and 2)the measured 4-jet rates should be less than 1% as theoretical calculations for 4-jet rates are only available to leading order. The second condition is removed in the fits where f_μ is taken as a free parameter and is expected to optimize the 4-jet prediction, but we avoid the low y_{cut} region where detector corrections are large (*i.e.* greater than 30%). The $\Lambda_{\overline{MS}}$ values thus obtained may then be translated to α_s by using equation 3. The comparison of theoretical predictions with the data emerging from these fits is presented in figures 4b, 5b and 6b. The results of the fits are listed in Tables II and III. The methods to estimate the errors in Table III are described in the next section.

Although the JJ, JP and DP algorithms predict different jet rates, when applied consistently at theoretical and experimental levels they yield values of the physical parameter α_s which are consistent. This compatability of QCD calculations with the data has also been observed by LEP experiments [25]. We use the JJ algorithm only as a consistency check, as it is more sensitive to hadronization. The results from the JP and DP algorithms are very similar, and we quote the α_s value from the DP algorithm as our final result which is

$$\alpha_s(10.53\text{GeV}) = 0.164 \pm 0.004 \pm 0.014 \quad (10)$$

where the first error is the experimental error and the second error has been obtained by adding in quadrature all the other errors in Table III. This value extrapolated to M_Z in the five flavor regime, using the formulae of Ref. [26], yields

$$\alpha_s(M_Z) = 0.113 \pm 0.002 \pm 0.006. \quad (11)$$

Note that the fractional error in α_s decreases considerably in extrapolating from 10.53 to M_Z. This is due to the energy dependence of α_s in equation 3 used for extrapolation. This is one of the reasons for a growing interest among high energy physicists for determining α_s from low energy processes.

V. ESTIMATE OF EXPERIMENTAL AND THEORETICAL UNCERTAINTIES

The procedures we use to estimate the uncertainties on the determination of α_s from several sources are described below:

1. **Experimental uncertainties:** The experimental uncertainties on the final results come mainly from the detector correction factors which may depend upon the MC and upon the detector simulation procedure. We have used the JETSET MC model with parameters which are known to describe the e^+e^- data well from a center of mass energy of 30 to 91 GeV and we have checked that it also describes well the global features of our data. We estimate the errors due to the possible remaining problems in our simulation procedure by performing the analysis in three different ways: 1) using both the charged tracks and clusters from the calorimeters, 2) using charged tracks only, and 3) using clusters only. The α_s values determined from analysis 1) are the quoted central values. The larger of the deviations of α_s values from procedures 2) and 3) from that from 1) is taken as a symmetric error which, added in quadrature with the statistical error from the data and the correction factors, provides the quoted experimental error.
2. **Hadronization uncertainties:** The data, which are collected at the hadron level, must be corrected for hadronization before comparison with the QCD calculations which are performed at the parton level. Our quoted results are extracted from the data corrected to the parton level using the QCD parton shower MC model in JETSET [15] with the string hadronization scheme. The analysis was repeated with the data corrected to the parton level by using the HERWIG MC [21] model, tuned to match our data at the hadron level, with the cluster hadronization scheme. The models agree better at the hadron level than at the parton level as they differ in their treatments of the parton shower. The difference in final results obtained by using corrections computed from these two models is quoted as a symmetric error due to hadronization.
3. **Uncertainty in parton virtuality:**
   The QCD shower models contain a parameter, Q_0, which defines the cutoff value on the virtuality of the parton where the QCD shower process is halted and the hadronization process begun. Changing the value of Q_0 changes the final parton multiplicity in the shower before hadronization starts, thus the hadronization corrections change as well. We obtain our central values, using Q_0=1. We repeated our analysis for several values of Q_0 in the range of 0.7 to 2, and take the resulting deviation from the central value of α_s as the error due to parton virtuality.
4. **Uncertainty due to renormalization scale:**
   One may notice from equation 5 that the jet-rate distributions do depend upon the renormalization scale μ. Had QCD been solved to all orders, the values of α_s determined from the data would have been independent of the choice of the scale. The results from $\mathcal{O}(\alpha_s^2)$ QCD are expected to be dependent on scale. One choice would be $\mu = E_{cm}$, i.e. f_μ=1. Although QCD itself would not suggest at which value of the scale the fits should be made, a number of QCD inspired theoretical procedures have been suggested to make an optimal choice of the scale [22]. Some experimental procedures have also been presented to make an optimal choice of the scale [4,23,24]. It has been shown [4,23] that various experimental procedures to optimize the scale give similar results to those obtained by treating f_μ as a free parameter in the fit and that the values of f_μ so obtained agree with the values predicted by various theoretical procedures. As there is no agreement on this issue, we regard the different choices of scale as a source of systematic error. In order to accommodate all these approaches we perform fits of the theory to the data in two different ways: 1) by fixing f_μ=1, and 2) by treating f_μ as a free parameter in the fit. The average of the two α_s values thus obtained is quoted as the central result and the difference between the central value and either extreme is taken as a symmetric error due to the renormalization scale. This method has also been applied in determining the reported LEP results with which we shall compare our results. This is the dominant error in both cases.
5. **Uncertainty from the fit range:** We have already explained our criteria for choosing the y_{cut} range in which the fits are made. We use different ranges to check the consistency of our results. We find that the variations are not larger than the statistical errors. However, we quote this variation as an error due to the choice of the y_{cut} range.

As no renormalization scale dependence is expected if QCD is solved to all orders, it may be argued that the errors due to the renormalization scale are a measure of the uncertainty introduced by the missing higher order terms in the theory [4,23,24]. Changing the value of Q_0 changes the final parton multiplicity in the shower and the hadronization corrections; thus the error due to Q_0 is correlated with the errors due to hadronization and the renormalization scale. However, taking a conservative approach, we estimate and quote these errors separately. The final values of α_s along with all the errors are listed in Table III.

VI. COMPARISON WITH OTHER RESULTS

In the following, we compare the value of α_s(10.53 GeV)=0.164±0.004±0.014 obtained from this analysis with results from other analyses.

1. Our result agrees well with $\alpha_s(M_\Upsilon) = 0.167^{+0.015}_{-0.011}$ from Υ decay [6] at about the same energy. As these are two different physical processes, the agreement between the α_s values obtained from them provides a convincing consistency check of the underlying theory.
2. The measured α_s value at 10.53 GeV in the four flavor continuum extrapolated to 29 GeV in the five flavor continuum yields α_s(29 GeV)=0.135 ± 0.003 which is in good agreement with α_s(29 GeV)=0.133 ± 0.004 extracted from Mark II jet data [27] using the same procedure, where only the experimental errors are shown.

3. Our α_s value extrapolated to $\alpha_s(M_Z) = 0.113 \pm 0.002 \pm 0.006$ is also consistent with $\alpha_s(M_Z) = 0.120 \pm 0.006$ extracted from the LEP and SLC jet data [5] using the same procedure and calculations, where all errors are included.

The data collected at CESR present a unique situation for this analysis. At about the same energy we have the data in the four flavor continuum (10.53 GeV) and on the $\Upsilon(4S)$ resonance (10.58 GeV). One can ask this question: can the QCD calculations, performed for the continuum, make a distinction between the continuum and the resonance data? We repeated our analysis on a sample of $\Upsilon(4S) \to B\bar{B}$ events where at least one of the B mesons was fully reconstructed, from which we do not expect results consistent with those from our analysis of the continuum data. We obtained α_s(10.58 GeV)=0.316±0.021 where the errors are experimental only. This value is about seven standard deviations away from that extracted from the continuum data.

VII. SUMMARY AND DISCUSSION

In summary, we have tested the predictions of $\mathcal{O}(\alpha_s^2)$ QCD for jet rates in the four flavor e^+e^- continuum by comparing them to data taken at $E_{cm} = 10.53$ GeV, where only four flavors are produced. As the QCD predictions in the four flavor continuum are different from those in the five flavor continuum due to the different contributions from the vacuum polarization diagrams and have not been tested before, the agreement between the theory and the data constitutes a new test of the validity of QCD.

The value of α_s we obtain agrees well with $\alpha_s(M_\Upsilon) = 0.167^{+0.015}_{-0.011}$ from Υ decay [6]. As these are two different physical processes at about the same energy, the agreement between the α_s values obtained from them provides a convincing consistency check of the underlying theory. The measured α_s value at 10.53 GeV in the four flavor continuum extrapolated to 29 GeV in the five flavor continuum is in good agreement with α_s(29 GeV)=0.133 ± 0.004 extracted from Mark II jet data [27] using the same procedure, where the shown errors are experimental errors only. Our α_s value extrapolated to M_Z is also consistent with $\alpha_s(M_Z) = 0.120 \pm 0.006$ extracted from the LEP and SLC jet data [5] using the same procedure and calculations, where all errors are included. We repeated our analysis on a sample of $\Upsilon(4S)$ data and find that the QCD calculations do make a distinction between continuum and resonance data.

The hadronization process is not understood from first principles at any energy. We have computed the hadronization corrections and the uncertainties on them using the method applied by LEP experiments. Our four flavor continuum data at $E_{cm} = 10.53$ GeV is about as far above the charm production threshold ($2m_c$) as the LEP data at $E_{cm} = 91$ GeV is above the bottom production threshold ($2m_b$) on the logarithmic scale ($\log(10.53)/\log(2m_c) \approx \log(91)/\log(2m_b)$) which is the scale of QCD. Furthermore, the value of α_s determined from this analysis is in excellent agreement with that from the cross section ratios of Υ(1S) decays, where hadronization is only a small effect. This gives us some confidence to believe in the hadronization corrections applied in this analysis. Moreover, the corrections are small (< 10 %).

One should note that we have used QCD calculations of the same process from which α_s has been determined at 29 and 91 GeV by comparing the calculations to the data at those

energies. The α_s value extracted from our data, following the same procedure as at higher energies, and extrapolated to those energies, agrees with the higher energy results. This is a unique test of the validity of the renormalization group equation of QCD used for the extrapolation and which governs the running of α_s. Previously, all such comparisons have been made for one process on one side of the flavor threshold to a different process on the other side of the threshold.

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y_{cut}	R_2 for Algorithm		
	JJ	JP	DP
0.03	—	—	54.64±0.75
0.04	—	—	69.60±0.87
0.05	—	—	78.92±0.71
0.06	34.93±1.28	54.22±1.20	85.21±0.58
0.07	43.22±1.44	64.08±1.56	89.37±0.56
0.08	50.82±1.69	72.32 ±1.25	92.11±0.38
0.09	57.44±1.46	78.67 ±0.88	94.22±0.23
0.10	63.39±1.43	83.98 ±0.84	95.51±0.24
0.12	73.47±1.06	91.11 ±0.48	97.38±0.17
0.14	81.14±1.02	95.31 ±0.39	98.51±0.08
0.17	89.04±0.84	98.48 ±0.25	99.39±0.08
0.20	94.02±0.57	99.59 ±0.16	99.79±0.04
0.25	98.16±0.25	99.96 ±0.03	99.99±0.02

Table I: Measured 2-jet event production rates, R_2, in % of the total hadronic cross section, computed with the JJ, JP, and DP algorithms. The data are corrected for detector effects and initial state radiation. The errors include statistical and experimental systematic uncertainties.

Algorithm	y_{cut} fit range	f_μ	$\Lambda_{\overline{MS}}$	α_s(10.53GeV)	χ^2
JJ	0.08-0.20	1	372± 36	0.178±0.004	1.62/5
	0.07-0.20	0.05±0.02	96 ± 7	0.132±0.002	1.64/5
JP	0.06-0.20	1	391± 25	0.180±0.003	1.60/7
	0.06-0.20	0.109±0.013	187± 11	0.151±0.002	2.59/9
DP	0.05-0.20	1	351± 31	0.175±0.004	6.55/8
	0.04-0.20	0.118±0.030	192± 15	0.152±0.003	2.27/8

Table II: Values of α_s(10.53GeV) determined from fitting $\mathcal{O}(\alpha_s^2)$ QCD to the data, with the experimental errors only.

Algorithm	α_s	Δ_{exp}	Δ_{had}	Δ_{Q_0}	Δ_μ	Δ_y
JJ	0.156	±0.004	±0.014	±0.004	±0.023	±0.002
JP	0.165	±0.004	±0.005	±0.007	±0.015	±0.002
DP	0.164	±0.004	±0.003	±0.007	±0.012	±0.002

Table III: Final values of α_s(10.53GeV) determined from fitting $\mathcal{O}(\alpha_s^2)$ QCD to the data, with all errors.

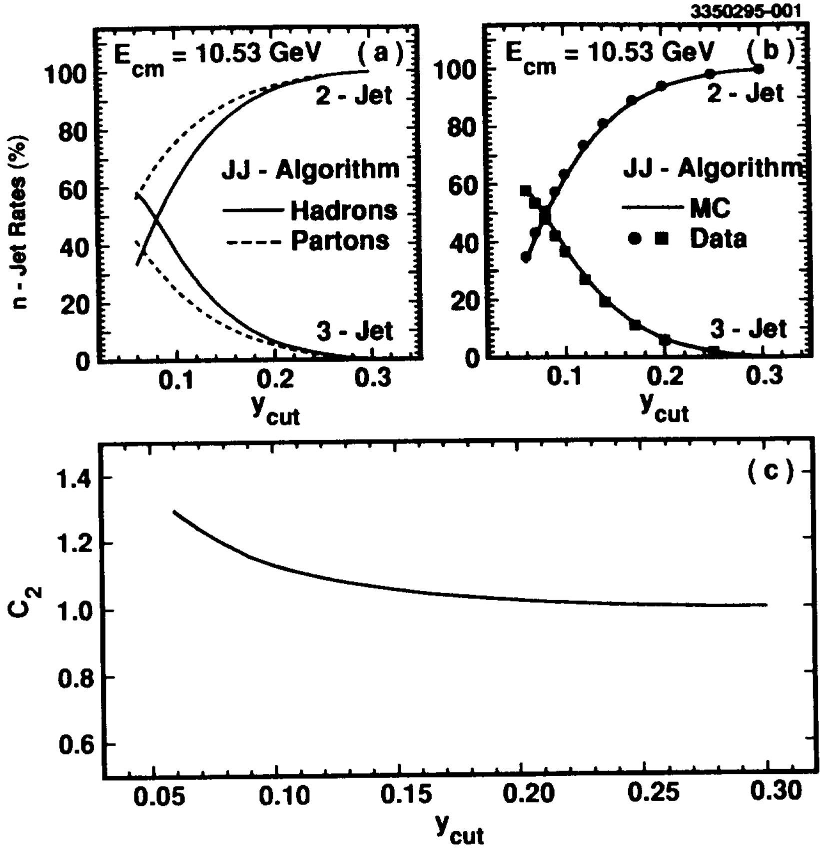

Figure 1: a) Jet rates at parton and hadron levels computed from JETSET MC using the JJ algorithm. b) Comparison of measured jet rates, corrected for detector effects and initial state radiations, to the predictions of the JETSET MC. c) Detector correction factor, C_2, (*i.e.* the ratio of jet rates measured at generator level MC and after smearing the MC with the detector effects) as a function of y_{cut}, for the JJ algorithm.

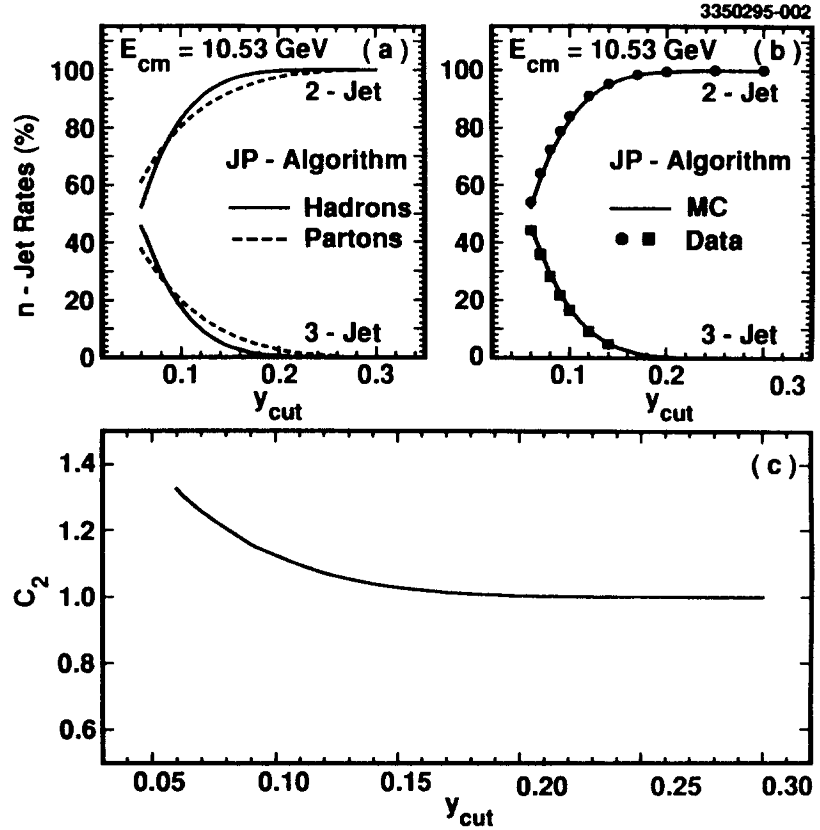

Figure 2: a) Jet rates at parton and hadron levels computed from JETSET MC using the JP algorithm. b) Comparison of measured jet rates, corrected for detector effects and initial state radiations, to the predictions of the JETSET MC. c) Detector correction factors, C_2, (*i.e.* the ratio of jet rates measured at generator level MC and after smearing the MC with the detector effects) as a function of y_{cut}, for the JP algorithm.

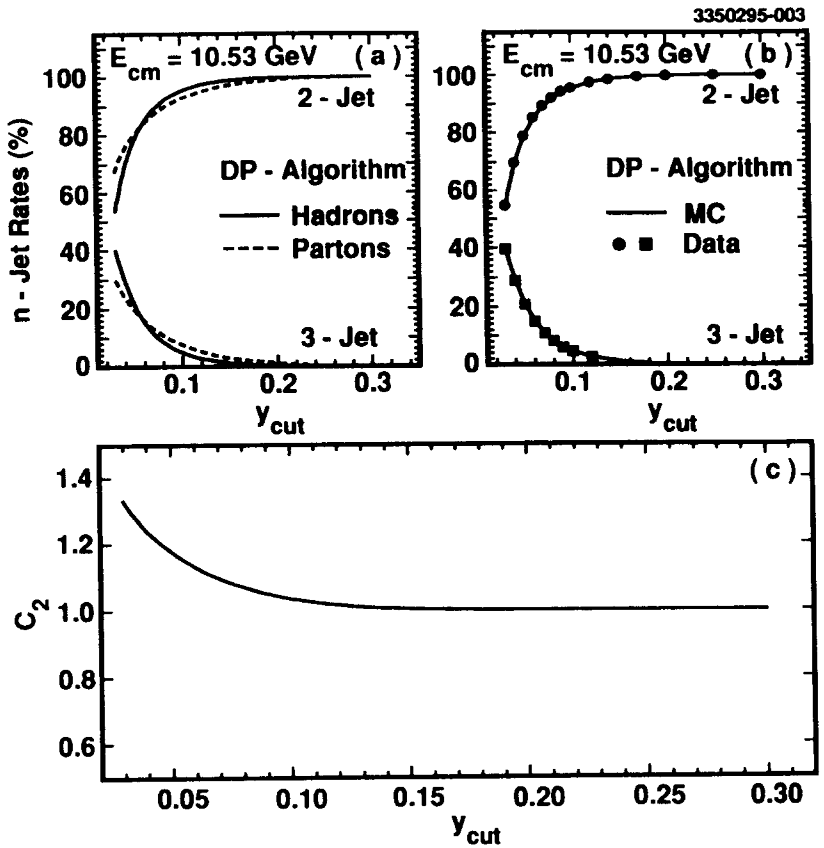

Figure 3: a) Jet rates at parton and hadron levels computed from JETSET MC using the DP algorithm. b) Comparison of measured jet rates, corrected for detector effects and initial state radiations, to the predictions of the JETSET MC. c) Detector correction factors, C_2, (i.e. the ratio of jet rates measured at generator level MC and after smearing the MC with the detector effects) as a function of y_{cut}, for the DP algorithm.

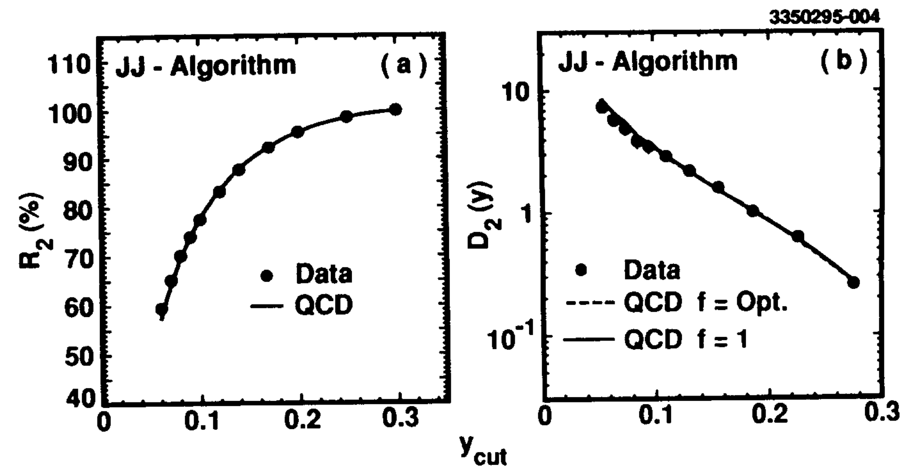

Figure 4: a) Measured R_2 distribution using the JJ algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions computed by using the value of α_s which was determined by comparing data and theory at one point, y_{cut}=0.06, only. b) Measured D_2 distribution using the JJ algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions. The QCD parameters used in the predictions were determined from the fits. The fit ranges are shown in Table II.

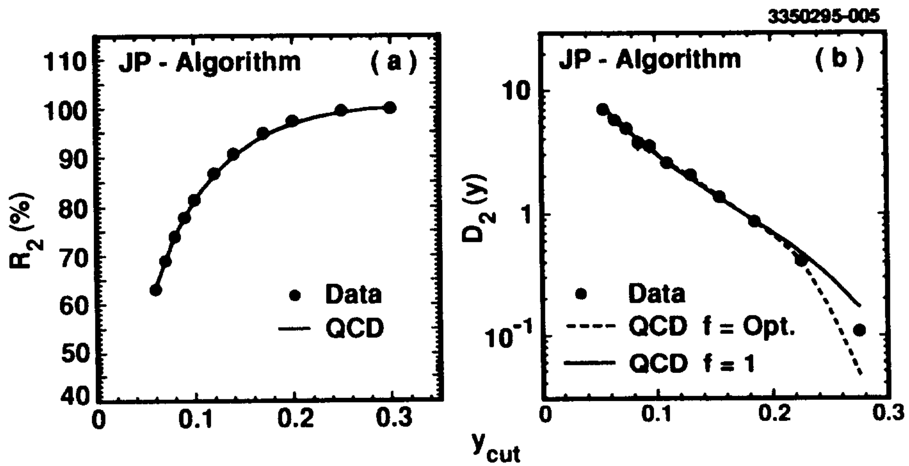

Figure 5: a) Measured R_2 distribution using the JP algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions computed by using the value of α_s which was determined by comparing data and theory at one point, y_{cut}=0.06, only. b) Measured D_2 distribution using the JP algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions. The QCD parameters used in the predictions were determined from the fits and the fit ranges are shown in Table II.

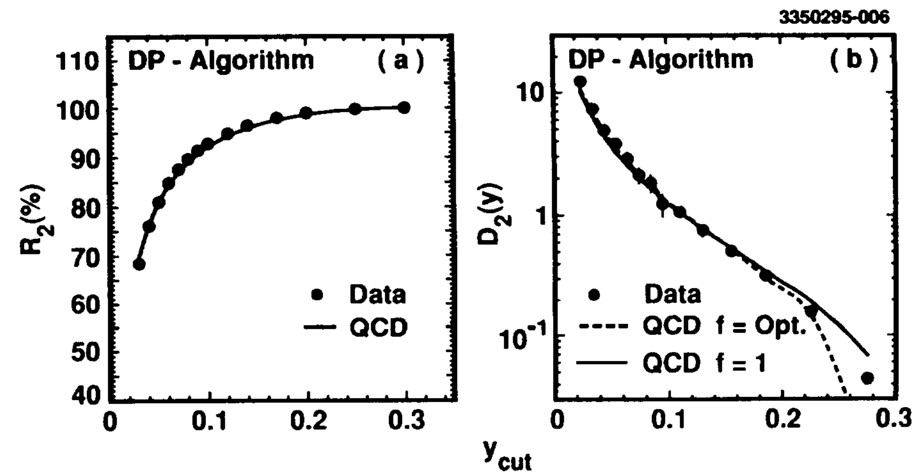

Figure 6: a) Measured R_2 distribution using the DP algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions computed by using the value of α_s which was determined by comparing data and theory at one point, y_{cut}=0.06, only. b) Measured D_2 distribution using the DP algorithm, compared to $\mathcal{O}(\alpha_s^2)$ QCD predictions. The QCD parameters used in the predictions were determined from the fits and the fit ranges are shown in Table II.

Appendix: Definition of Jets in Theory and Experiment

The criteria of a good jet definition [9] are that

1. it be defined at any order of perturbation theory;
2. it yield finite cross sections at any order of perturbation theory;
3. it be simple to implement in both theoretical calculations and experimental analyses; and that
4. it be insensitive to hadronization.

In the earliest studies, jet finding algorithms were based on event shape parameters [2]. Soon, it was realized that a jet definition of a more quantitative nature was required in order to make a direct comparison between experiment and theory. This gave rise to the so called cluster jet algorithms. In the cluster algorithm, more compatible with QCD calculations, one begins by considering all particles in an event as clusters. A jet resolution variable y_{ij}, defined in general

$$y_{ij} = \frac{M_{ij}^2}{E_{vis}^2}, \tag{12}$$

is computed for all possible pairs (i,j); where M_{ij} is the combined "mass" of clusters i and j, and E_{vis} is the visible energy of the event. The pair (i,j) with the smallest value of y_{ij} is combined into a single cluster with 4-momentum p_k, determined by some recombination scheme, for example,

$$\begin{aligned} \vec{p_k} &= \vec{p_i} + \vec{p_j}, \\ E_k &= P_k, \end{aligned} \tag{13}$$

called the P-scheme [4,10], or

$$\begin{aligned} \vec{P_k} &= \frac{E_k}{|\vec{p_i} + \vec{p_j}|}(\vec{p_i} + \vec{p_j}), \\ E_k &= E_i + E_j, \end{aligned} \tag{14}$$

called the E_0-scheme [4,10]. This process is repeated until the value of y_{ij} for all the possible pairs of the remaining clusters exceeds a prescribed threshold value, y_{cut}. The remaining number of clusters at this stage is taken as the number of jets in the event corresponding to the given value of y_{cut}. We use the following two definitions of M_{ij}^2:

$$M_{ij}^2 = (p_i + p_j)^2 \tag{15}$$

first presented by Z. Kunszt in the context of higher order QCD calculations [11]. This was implemented by the JADE collaboration using E_0 scheme in their analysis giving rise to the popular JADE algorithm [12] which we shall call the JJ algorithm; and

$$M_{ij}^2 = 2E_{min}(1 - cos\theta_{ij}) \tag{16}$$

first suggested by Dokshitzer [13], where E_{min} is the minimum of E_i^2 and E_j^2. It is trivial to verify that $2E_{min}(1 - cos\theta_{ij}) = (P_{min}^t)^2$, where P_{min}^t is the minimum relative transverse momentum of i and j for small angles θ_{ij}. These two definitions of M_{ij}^2, combined with the "P" recombination scheme constitute the JADE algorithm and Durham algorithm in the P-scheme, respectively which we shall refer to as the JP and DP algorithms. There are other recombination schemes which can be used to vary the JADE and Durham algorithms [4,10]. However, we use only the P-scheme as, based on a Monte Carlo (MC) study [14] in the energy range from 10 - 91 GeV, we have found that the JADE and the Durham algorithms in the P-scheme are relatively less sensitive to hadronization, especially at the lower energy end. However, we also use JJ algorithm just for a consistency check. These jet algorithms were implemented consistently in the theory and data.

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