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Medium-Modified Interaction Induced by Fluctuations †

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Abstract

Statistical averaging of the Boltzmann-Langevin equation is performed. It is shown that at the averaged level the fluctuations induce an additional collision term with a medium-modified transition rate, which can give rise to a critical scattering phenomena in the vicinity of unstable regions. In order to illustrate the effect of this additional collision term, the single-particle relaxation time in nuclear matter is estimated. It is shown that the fluctuation-induced contribution to the relaxation time is indeed comparable with that of the BUU collision term in the vicinity of spinoidal region.

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1 Introduction

Transport models with self-consistent mean fields, like the Boltzmann-Uehling-Uhlenbeck (BUU) model, are widely applied to the description of heavy-ion collisions [1]. These mean-field transport models are very successful in describing the average properties of the one-body observables associated with nuclear collisions, such as nucleon spectra, collective flows and particle production [2, 3]. However, these approaches do not provide an adequate description, when an instability occurs during the dynamical evolution of the system, e.g., such as those in thermal fission or multifragmentation processes. The reason is that the mean-field transport models bring about a deterministic description for the average evolution and do not allow for any branching of dynamical trajectories in the instability region.

The stochastic transport models offer a more appropriate framework for the description of the unstable dynamic evolution. In these stochastic approaches, the transport theory is extended beyond the mean-field level by incorporating the correlations within a statistical approximation [4, 5, 6]. The correlations give rise to a stochastic collision term in the equation of motion, which acts as a source of continuous branching of the dynamical trajectories. In the semi-classical limit, this extended transport model is referred to as the Boltzmann-Langevin (BL) model for the phase-space density. Here, as a continuation of a previous work [7], we investigate the relation between the BL and BUU models. We demonstrate that at the averaged level the BL model contains a new (as compared with the BUU model) collision term arising from correlations induced by long-range density fluctuations [8].

The paper is organized as follows. In section 2, a brief review of the BL model is presented. In section 3, we carry out a statistical averaging of the BL equation and derive the fluctuation-induced collision term. In section 4, we present an estimation of the relaxation time associated with the single-particle motion in nuclear matter. Finally in section 5, we give some conclusions.

2 Boltzmann-Langevin Model

The derivation of the BL model was performed in refs. [4]. Here we present a brief survey of the model. Binary collisions play a twofold part during the dynamical evolution of a system: (i) to produce dissipation by randomizing the momentum distribution of the constituent particles and, in addition, (ii) to induce fluctuations by propagating correlations in the phase space. These two effects of the binary collisions, i.e. the dissipation and fluctuations, can be incorporated into the equation of motion yielding a stochastic

transport equation for the single-particle density. For simplicity, we restrict our treatment to the semi-classical evolution of a spin-isospin averaged phase-space density and consider only elastic binary collisions. According to the BL model, the fluctuating phase-space density $\hat{f}(t, \boldsymbol{r}, \boldsymbol{p})$ is determined by the equation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_r - \nabla_r U[\hat{n}] \cdot \nabla_p\right) \hat{f}(t, \boldsymbol{r}, \boldsymbol{p}) = K(t, \boldsymbol{r}, \boldsymbol{p}) + \delta K(t, \boldsymbol{r}, \boldsymbol{p}). \tag{1}$$

Here $U[\hat{n}]$ is a fluctuating self-consistent mean field, which is assumed to be local and, hence, determined by the fluctuating local density,

$$\hat{n}(t, \boldsymbol{r}) = \frac{g}{(2\pi)^3} \int d^3p \; \hat{f}(t, \boldsymbol{r}, \boldsymbol{p}), \tag{2}$$

and

$$K(t, \boldsymbol{r}, \boldsymbol{p}_1) = \frac{g}{(2\pi)^3} \int d^3p_2 \; d^3p_3 \; d^3p_4 \; W(1,2 \mid 3,4) \; [(1-\hat{f}_1)(1-\hat{f}_2)\hat{f}_3\hat{f}_4 - \hat{f}_1\hat{f}_2(1-\hat{f}_3)(1-\hat{f}_4)] \tag{3}$$

is the collision term of the Boltzmann-Uehling-Uhlenbeck type. Here, g is the spin-isospin degeneracy factor of the nucleon ($g = 4$), $\hat{f}_i = \hat{f}(t, \boldsymbol{r}, \boldsymbol{p}_i)$ and

$$W(12 \mid 34) = \frac{1}{2m^2} \frac{d\sigma}{d\Omega_{cm}} \delta(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_3 - \boldsymbol{p}_4) \; \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \tag{4}$$

is the transition rate expressed in terms of the center-of-mass scattering cross section, $d\sigma/d\Omega_{cm}$.

An additional term, $\delta K(t, \boldsymbol{r}, \boldsymbol{p})$, in eq. (1) presents a fluctuating part of the collision term. In analogy with the treatment of the Brownian motion, it is regarded that eq. (1) describes a stochastic process, in which the entire phase-space density $\hat{f}(t, \boldsymbol{r}, \boldsymbol{p})$ is a stochastic variable and δK acts as a random force. The stochastic collision term vanishes on the average

$$\langle \delta K(t, \boldsymbol{r}, \boldsymbol{p}) \rangle = 0, \tag{5}$$

and is characterized by a correlation function

$$\langle \delta K(t, \boldsymbol{r}, \boldsymbol{p}) \; \delta K(t', \boldsymbol{r}', \boldsymbol{p}') \rangle = \delta_c(t - t') \; \delta_c(\boldsymbol{r} - \boldsymbol{r}') \; C(T, \boldsymbol{R}, \boldsymbol{p}, \boldsymbol{p}'), \tag{6}$$

where $T = \frac{1}{2}(t + t')$ and $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r} + \boldsymbol{r}')$. In the Markovian treatment, the quantities $\delta_c(t - t')$ and $\delta_c(\boldsymbol{r} - \boldsymbol{r}')$ are assumed to be sharp δ-functions. Here, we take them as "broad δ-functions"

$$\delta_c(t - t') = \frac{1}{\sqrt{2\pi}\tau_c} \exp\left[-\frac{1}{2}\left(\frac{t - t'}{\tau_c}\right)^2\right], \tag{7}$$

$$\delta_c(\boldsymbol{r}-\boldsymbol{r}') = \left(\frac{1}{\sqrt{2\pi}r_c}\right)^3 \exp\left[-\frac{1}{2}\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{r_c}\right)^2\right]. \tag{8}$$

This is a more realistic parametrization of the correlation function. Here, the correlation length, $r_c \sim 1$ - 2 fm, is of the order of the two-body interaction range, and the correlation time is $\tau_c \simeq r_c/v$, with v being an average relative velocity of nucleons. The correlation function $C(t, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{p}')$ can be expressed as follows

$$\begin{aligned} C(t,\boldsymbol{r},\boldsymbol{p}_1,\boldsymbol{p}'_1) = & \int d^3p_3\, d^3p_4\, W(1,1' \mid 3,4)\, [(1-\hat{f}_1)(1-\hat{f}_{1'})\hat{f}_3\hat{f}_4 + \hat{f}_1\hat{f}_{1'}(1-\hat{f}_3)(1-\hat{f}_4)] \\ & -2\int d^3p_2\, d^3p_4\, W(1,2 \mid 1',4)\, [(1-\hat{f}_1)(1-\hat{f}_2)\hat{f}_{1'}\hat{f}_4 + \hat{f}_1\hat{f}_2(1-\hat{f}_{1'})(1-\hat{f}_4)] \\ & +\delta(\boldsymbol{p}_1-\boldsymbol{p}'_1)\int d^3p_2\, d^3p_3\, d^3p_4\, W(1,2 \mid 3,4)\, [(1-\hat{f}_1)(1-\hat{f}_2)\hat{f}_3\hat{f}_4 + \hat{f}_1\hat{f}_2(1-\hat{f}_3)(1-\hat{f}_4)]. \end{aligned} \tag{9}$$

The correlation function is closely related to the collision term and is entirely determined by the one-body characteristics. This relation can be regarded as a consequence of the fluctuation-dissipation theorem associated with the stochastic evolution of the phase-space density. The BL equation (1) offers a stochastic description of the collision process, in contrast to the deterministic one of the BUU model. For a given initial condition, the BL equation (1) results in an ensemble of solutions. If the system evolves through an instability region, these solutions can largely diverge from each other, giving rise to large density fluctuations.

3 Averaging of the Boltzmann-Langevin Equation

In order to carry out the ensemble averaging of the BL equation (1), we decompose the phase-space density and the mean-field as

$$\hat{f} = f + \delta f, \quad \hat{U} = U + \delta U, \tag{10}$$

where $f = \langle \hat{f} \rangle$ and $U = \langle \hat{U} \rangle$ are the averaged parts, while δf and δU denote the fluctuating parts of the phase-space density and the mean field, respectively. Note that $\langle \delta f \rangle = \langle \delta U \rangle = 0$ by definition. By performing the ensemble averaging, we readily obtain the transport equation for the averaged phase-space density

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_r - \nabla_r U \cdot \nabla_p\right) f = \langle K \rangle + K_{BL}, \tag{11}$$

where $\langle K \rangle = \langle K(\{\hat{f}\}) \rangle$ is the averaged collision term. The additional term

$$K_{BL} = \langle \nabla_r \delta U \cdot \nabla_p \delta f \rangle \tag{12}$$

on the right-hand side indicates that the kinetic equation for the averaged distribution function, emerging from the BL model, in general, is not identical to the BUU equation.

The calculation of the additional collision term K_{BL} in terms of the averaged characteristics is, in general, a highly complicated problem. Therefore, we consider a particular situation under certain approximations, which simplify the calculation of K_{BL} and, at the same time, clarify the dissipation mechanism associated with this collision term. In this particular case

(i) We assume that the magnitude of fluctuations is small as compared with that of averaged quantities. As a result, the fluctuations can be treated in the linearized approximation.

(ii) We consider fluctuations of the space-time scale to be much shorter than that of the averaged quantities. Hence, in the calculation of fluctuations, we neglect the space-time dependence of the averaged quantities: $f = f(\boldsymbol{p})$ etc.

(iii) We assume the collisional damping in the system, determined by the collision term (K), to be weak. More precisely, we consider collisional relaxation time to be much longer than the characteristic inverse frequencies of the collective modes. In particular, this weak damping assumption supports the previous one on the space-time scale separation.

This particular case under consideration may roughly be associated with the average evolution of the premultifragmentation stage of nuclear collision, when the spatial gradients are small, the expansion is slow and the collisional relaxation of the system is less effective than the mean-field dynamics.

As a result of the assumptions (i)–(iii), the fluctuations are determined by the linearized BL equation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_r\right)\delta f - \nabla_p f\cdot\nabla_r \delta U = -\hat{I}\delta f + \delta K \tag{13}$$

where $\hat{I}$ is the linearized collision operator:

$$\begin{aligned}\hat{I}\delta f_1 &= \frac{g}{(2\pi)^3}\int d^3p_2\, d^3p_3\, d^3p_4\; W(1,2\mid 3,4)\\ &\times \Big[(\tilde{f}_2 f_3 f_4 + f_2\tilde{f}_3\tilde{f}_4)\delta f_1 + (\tilde{f}_1 f_3 f_4 + f_1\tilde{f}_3 f_4)\delta f_2\\ &- (\tilde{f}_4 f_1 f_2 + f_4\tilde{f}_1\tilde{f}_2)\delta f_3 - (\tilde{f}_3 f_1 f_2 + f_3\tilde{f}_1\tilde{f}_2)\delta f_4\Big],\end{aligned} \tag{14}$$

where $\tilde{f} = 1 - f$ is the Pauli-blocking factor. To solve this equation, it is natural to use the Fourier transformation

$$\delta f(\omega,\boldsymbol{k},\boldsymbol{p}) = \int dt\, d^3r\, \exp(i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r})\,\delta f(t,\boldsymbol{r},\boldsymbol{p}), \tag{15}$$

and, similarly, for other quantities. In the Fourier representation eq. (13) reads

$$i\,(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)\,\delta f(\omega,\boldsymbol{k},\boldsymbol{p})-i\boldsymbol{k}\cdot\nabla_p f\;\delta U(\omega,\boldsymbol{k})=-\hat{I}\delta f+\delta K(\omega,\boldsymbol{k},\boldsymbol{p}), \tag{16}$$

where

$$\delta U(\omega,\boldsymbol{k})=V(\boldsymbol{k})\,\delta n(\omega,\boldsymbol{k}), \tag{17}$$

$$\delta n(\omega,\boldsymbol{k})=\frac{g}{(2\pi)^3}\int d^3p\;\delta f(\omega,\boldsymbol{k},\boldsymbol{p}), \tag{18}$$

$$V(\boldsymbol{k})=\left(\frac{\delta U}{\delta n}\right)_{\boldsymbol{k}}. \tag{19}$$

While assumption (iii) does not necessary invoke the smallness of the transition rate W but can be associated with the Pauli blocking of final states, we shall formally perform the expansion in terms of W, keeping in mind that the Pauli blocking factors always accompany this W. It is worthwhile to note that the $\hat{I}$ operator is of the first order in the transition rate W [cf. eq. (14)], and the δK quantity is, roughly speaking, of the order of $W^{1/2}$ [cf. eqs. (6) and (9)].

Taking into account the assumption of the weak collisional damping [cf. (iii)], we should solve this equation in the lowest order in W. One can easily do that by analyzing the analytical structure of the solution. First, it is obvious that δf is of the order of δK, since δK is the only inhomogeneous term in the equation. Then, it could seem that the $\hat{I}\delta f$ term is of the third order in δK and, hence, can be neglected in the linear approximation in δK. However, it is not exactly so. As seen from eq. (16), the δf solution has a pole of the type of $(\boldsymbol{k}\cdot\boldsymbol{v}-\omega-i\Delta)^{-1}$, where Δ takes into account the collisional damping and is defined below [cf. eq. (20)]. Hence, the contribution of the $\hat{I}\delta f$ term into the solution is

$$(\boldsymbol{k}\cdot\boldsymbol{v}-\omega-i\Delta)^{-1}\;\hat{I}\;(\boldsymbol{k}\cdot\boldsymbol{v}-\omega-i\Delta)^{-1}$$

The diagonal part of $\hat{I}\delta f$

$$(\hat{I}\delta f)_{diag}=\Delta\,\delta f,$$

where

$$\Delta(\boldsymbol{p}_1)=\frac{g}{(2\pi)^3}\int d^3p_2\;d^3p_3\;d^3p_4\;W(1,2\mid 3,4)\;(\tilde{f}_2 f_3 f_4+f_2\tilde{f}_3\tilde{f}_4), \tag{20}$$

gives a double-pole contribution

$$\propto(\boldsymbol{k}\cdot\boldsymbol{v}-\omega-i\Delta)^{-2}$$

This double-pole contribution is of the first order in δK and, hence, should be kept. All the other terms of $\hat{I}\delta f$ are of the third order in δK and can be neglected.

Thus, in the weak damping approximation (i.e. in the linear approximation in δK) we get

$$\delta f(\omega, \boldsymbol{k}, \boldsymbol{p}) = \frac{\boldsymbol{k} \cdot \nabla_p f}{\boldsymbol{k} \cdot \boldsymbol{v} - \omega - i\Delta} \delta U(\omega, \boldsymbol{k}) - i \frac{\delta K(\omega, \boldsymbol{k}, \boldsymbol{p})}{\boldsymbol{k} \cdot \boldsymbol{v} - \omega - i\Delta}, \tag{21}$$

where

$$\delta U(\omega, \boldsymbol{k}) = -i \frac{V(\boldsymbol{k})}{\varepsilon(\omega, \boldsymbol{k})} \frac{g}{(2\pi)^3} \int d^3p \, \frac{\delta K(\omega, \boldsymbol{k}, \boldsymbol{p})}{\boldsymbol{k} \cdot \boldsymbol{v} - \omega - i\Delta}, \tag{22}$$

$$\varepsilon(\omega, \boldsymbol{k}) = 1 - V(\boldsymbol{k}) \frac{g}{(2\pi)^3} \int d^3p \, \frac{\boldsymbol{k} \cdot \nabla_p f}{\boldsymbol{k} \cdot \boldsymbol{v} - \omega - i\Delta}. \tag{23}$$

By using these together with the $\langle \delta K \, \delta K \rangle$ correlation function in the Fourier representation

$$\langle \delta K(\omega, \boldsymbol{k}, \boldsymbol{p}) \, \delta K(\omega', \boldsymbol{k}', \boldsymbol{p}') \rangle = (2\pi)^4 \, G_c(\omega, \boldsymbol{k}) \, \delta(\omega + \omega') \, \delta(\boldsymbol{k} + \boldsymbol{k}') \, C(\boldsymbol{p}, \boldsymbol{p}'), \tag{24}$$

one can calculate all the required quantities. Here, the quantity

$$G_c(\omega, \boldsymbol{k}) = \exp\left[-\frac{1}{2} \left(\frac{\omega}{\omega_c} \right)^2 - \frac{1}{2} \left(\frac{\boldsymbol{k}}{k_c} \right)^2 \right], \tag{25}$$

with $\omega_c = 1/\tau_c$ and $k_c = 1/r_c$, represents the cut-off determined by finite correlation lengths in time and space of the correlation function (6) of the stochastic collision term.

First, let us consider the correlation function of the phase-space density. A simple but somewhat lengthy calculation gives

$$\begin{aligned} \frac{g}{(2\pi)^3} \, \langle \delta f(\omega, \boldsymbol{k}, \boldsymbol{p}) \delta f(\omega', \boldsymbol{k}', \boldsymbol{p}') \rangle &= (2\pi)^4 \, G_c(\omega, \boldsymbol{k}) \, \delta(\omega + \omega') \, \delta(\boldsymbol{k} + \boldsymbol{k}') \\ &\times \left[\pi \delta(\boldsymbol{p} - \boldsymbol{p}') \, \delta(\boldsymbol{k} \cdot \boldsymbol{v} - \omega) \, \frac{\Delta_- f(\boldsymbol{p}) + \Delta_+ \tilde{f}(\boldsymbol{p})}{\Delta} + \phi(\omega, \boldsymbol{k}, \boldsymbol{p}, \boldsymbol{p}') \right], \end{aligned} \tag{26}$$

where

$$\Delta_{\pm}(\boldsymbol{p}_1) = \frac{g}{(2\pi)^3} \int d^3p_2 \, d^3p_3 \, d^3p_4 \, W(1, 2 \mid 3, 4) \begin{pmatrix} \tilde{f}_2 f_3 f_4 \\ f_2 \tilde{f}_3 \tilde{f}_4 \end{pmatrix}, \tag{27}$$

and $\phi(\omega, \boldsymbol{k}, \boldsymbol{p}, \boldsymbol{p}')$ is an analytic (nonsingular, except for simple poles) function of all the variables. The ϕ function, which is proportional to W, has a magnitude of the first order of smallness. The singular part of this correlation function originates from the diagonal part of the $C(\boldsymbol{p}, \boldsymbol{p}')$ correlation function (6)

$$C_{diag}(\boldsymbol{p}, \boldsymbol{p}') = \delta(\boldsymbol{p} - \boldsymbol{p}') \, \frac{(2\pi)^3}{g} \, [\Delta_- f(\boldsymbol{p}) + \Delta_+ \tilde{f}(\boldsymbol{p})]. \tag{28}$$

The inverse Fourier transformation into the coordinate representation results in

$$\begin{aligned} \frac{g}{(2\pi)^3} \langle \delta f(t, \boldsymbol{r}, \boldsymbol{p}) \, \delta f(t', \boldsymbol{r}', \boldsymbol{p}') \rangle &= \delta_c^{(v)}(\boldsymbol{r} - \boldsymbol{r}' - \boldsymbol{v}(t - t')) \, \delta(\boldsymbol{p} - \boldsymbol{p}') \, \frac{\Delta_- f + \Delta_+ \tilde{f}}{2\Delta} \\ &+ \phi(t - t', \boldsymbol{r} - r', \boldsymbol{p}, \boldsymbol{p}'), \end{aligned} \tag{29}$$

where $\phi(t-t', \boldsymbol{r}-\boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p}')$ is a nonsingular function proportional to W. The quantity ϕ goes to zero in the limit $\mid \boldsymbol{r}-\boldsymbol{r}' \mid \to \infty$ and/or $\mid t-t' \mid \to \infty$. Here, the quantity $\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)$ is again a "broad δ-function" of the special kind

$$\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t) = \frac{1}{(2\pi)^{3/2}(v^2\tau_c^2+r_c^2)^{1/4}r_c} \exp\left\{-\frac{1}{2r_c^2}\left[(\boldsymbol{r}-\boldsymbol{v}t)^2 - \frac{\tau_c^2}{v^2\tau_c^2+r_c^2}(\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{v}t))^2\right]\right\}. \tag{30}$$

In particular, the $\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)$ function indicates that the nonlocality of the correlation function (29) is larger in the longitudinal (with respect to $\boldsymbol{v}$) direction than that in the transverse one. When τ_c and r_c go to zero, $\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)$ transforms into conventional δ-function. In equilibrium we have

$$\frac{\Delta_- f + \Delta_+ \hat{f}}{2\Delta} = f\hat{f} \tag{31}$$

[cf. eqs. (20) and (27)]. Substituting this into eq. (29), we arrive at the natural form of the correlation function in equilibrium. The factor $g/(2\pi)^3$ in the left-hand side of eq. (29) has appeared due to the normalization condition (2) for the phase-space density. This result serves us as a test for consistency of our calculations.

Using the result of eq. (29), we can readily demonstrate that the averaged collision term, $\langle K\rangle = \langle K(\{\hat{f}\})\rangle$, coincides with the BUU one, $K_{BUU} = K(\{\langle\hat{f}\rangle\})$, in the first order in W, i.e. within the validity of our consideration [cf. assumption (iii)]. Indeed, the difference

$$\langle K\rangle - K_{BUU} = \frac{g}{(2\pi)^3}\int d^3p_2\, d^3p_3\, d^3p_4\, W(1,2\mid 3,4)\,(\langle\delta f_3\delta f_4\rangle - \langle\delta f_1\delta f_2\rangle) \tag{32}$$

is determined by the $\langle\delta f\delta f'\rangle$ correlation function. (For simplicity, we have omitted the Pauli factors here.) The contribution of the $\delta(\boldsymbol{p}-\boldsymbol{p}')$ part of this correlation function into the $\langle K\rangle - K_{BUU}$ difference is identically zero, as one can easily check. Physically, it means that the gain and loss terms cancel each other when there is no real scattering, i.e. when the momenta of particles are not changed in the collision. The ϕ function is proportional to W, hence, it makes the contribution of the second order in W, which is beyond the frame of our treatment. Thus, we obtain

$$\langle K\rangle = K_{BUU}. \tag{33}$$

The additional collision term K_{BL} can be evaluated by making use of the expressions for $\delta f(\omega, \boldsymbol{k}, \boldsymbol{p})$ and $\delta U(\omega, \boldsymbol{k})$ given by eqs. (21) and (22)

$$\begin{aligned} K_{BL} &= -Im\,\nabla_p \int \frac{d\omega\, d^3k}{(2\pi)^4}\, G_c(\omega,\boldsymbol{k})\,\boldsymbol{k}\,\left|\frac{V(\boldsymbol{k})}{\varepsilon(\omega,\boldsymbol{k})}\right|^2 \frac{1}{\boldsymbol{k}\cdot\boldsymbol{v}-\omega-i\Delta} \\ &\times \left(\frac{g}{(2\pi)^3}\right)^2 \int d^3p'\, d^3p''\, \frac{\boldsymbol{k}\cdot\nabla'_p f'\, C(\boldsymbol{p},\boldsymbol{p}'') - \boldsymbol{k}\cdot\nabla_p f\, C(\boldsymbol{p}',\boldsymbol{p}'')}{(\boldsymbol{k}\cdot\boldsymbol{v}'-\omega-i\Delta')(\boldsymbol{k}\cdot\boldsymbol{v}''-\omega+i\Delta'')}. \end{aligned} \tag{34}$$

This expression reminds the Balesku-Lenard collision term in Coulomb plasmas [9]. This similarity becomes more evident, when one makes use of the explicit form of the $C(\boldsymbol{p},\boldsymbol{p}')$ correlator. The collision term K_{BL} can then be expressed in the form of two terms

$$K_{BL} = K_{BL}^{(binary)} + K_{BL}^{(triple)}. \tag{35}$$

Here, the first term is a binary collision term of the Balescu-Lenard type

$$\begin{aligned} K_{BL}^{(binary)} &= \nabla_p \int \frac{d\omega\, d^3k}{(4\pi)^2}\, G_c(\omega,\boldsymbol{k})\, \boldsymbol{k}\, \left|\frac{V(\boldsymbol{k})}{\varepsilon(\omega,\boldsymbol{k})}\right|^2 \frac{g}{(2\pi)^3}\int d^3p'\, \delta(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)\, \delta(\boldsymbol{k}\cdot\boldsymbol{v}'-\omega) \\ &\times \left[\boldsymbol{k}\cdot\nabla_p f\, \frac{\Delta'_-\, f' + \Delta'_+\, \tilde{f}'}{\Delta'} - \boldsymbol{k}\cdot\nabla'_p f'\, \frac{\Delta_-\, f + \Delta_+\, \tilde{f}}{\Delta}\right], \end{aligned} \tag{36}$$

which originates from the diagonal part of the C correlation function (28) and involves the medium-modified V/ε interaction determined by the permittivity. The factor G_c introduces a natural cut-off in momentum transfers, which is determined by inverse correlation lengths [cf. eq. (25)]. In fact, $K_{BL}^{(binary)}$ presents a small scattering-angle expansion of a full Boltzmann-Uehling-Uhlenbeck collision term with the V/ε interaction. The $(\Delta_-\, f + \Delta_+\, \tilde{f})/\Delta$ construction simulates the Pauli blocking [cf. eq. (31)]. Assuming $(\Delta_-\, f + \Delta_+\, \tilde{f})/\Delta \approx 2f\tilde{f}$, we can reconstruct the full BUU form of $K_{BL}^{(binary)}$

$$K_{BL}^{(binary)} = \frac{g}{(2\pi)^3}\int d^3p_2\, d^3p_3\, d^3p_4\, W_{BL}^{(binary)}(1,2\mid 3,4)\, (\tilde{f}_1\tilde{f}_2 f_3 f_4 - f_1 f_2 \tilde{f}_3\tilde{f}_4) \tag{37}$$

where

$$W_{BL}^{(binary)}(12\mid 34) = \frac{G_c(\omega,\boldsymbol{k})}{(2\pi)^2}\left|\frac{V(\boldsymbol{k})}{\varepsilon(\omega,\boldsymbol{k})}\right|^2 \delta(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_3-\boldsymbol{p}_4)\, \delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4) \tag{38}$$

is the transition rate expressed in terms of Fourier-transformed $\delta U/\delta n$ [cf. eq. (19)] and permittivity (23), with $\boldsymbol{k} = \boldsymbol{p}_3 - \boldsymbol{p}_1$ and $\omega = (\varepsilon_3 - \varepsilon_1)/2$. The nondiagonal part $(C - C_{diag})$ yields a kind of triple collision term

$$\begin{aligned} K_{BL}^{(triple)} &= -Im\nabla_p \int \frac{d\omega\, d^3k}{(2\pi)^4}\, G_c(\omega,\boldsymbol{k})\, \boldsymbol{k}\, \left|\frac{V(\boldsymbol{k})}{\varepsilon(\omega,\boldsymbol{k})}\right|^2 \\ &\times \frac{g}{(2\pi)^3}\int d^3p'\, \frac{\boldsymbol{k}\cdot\nabla'_p f'\, Q(\omega,\boldsymbol{k},\boldsymbol{p}) - \boldsymbol{k}\cdot\nabla_p f\, Q(\omega,\boldsymbol{k},\boldsymbol{p}')}{(\boldsymbol{k}\cdot\boldsymbol{v}-\omega-i\Delta)(\boldsymbol{k}\cdot\boldsymbol{v}'-\omega-i\Delta')}, \end{aligned} \tag{39}$$

where

$$Q(\omega,\boldsymbol{k},\boldsymbol{p}_1) = \frac{g}{(2\pi)^3}\int d^3p_2\, d^3p_3\, d^3p_4\, W\, (\varphi_2-\varphi_3-\varphi_4)\, (\tilde{f}_1\tilde{f}_2 f_3 f_4 + f_1 f_2 \tilde{f}_3\tilde{f}_4), \tag{40}$$

$$\varphi_j = (\boldsymbol{k}\cdot\boldsymbol{v}_j - \omega + i\Delta_j)^{-1}. \tag{41}$$

The $K_{BL}^{(binary)}$ term is the leading order contribution to K_{BL}, since $K_{BL}^{(triple)}$ contains higher order smallness associated with W.

The collision term K_{BL} involves a contribution of collective excitations which correspond to zeros of the permittivity: $\varepsilon(\omega_{coll.}(\boldsymbol{k}), \boldsymbol{k}) = 0$. Hence, the collective modes appear as poles of the integrand in eq. (36). For weakly damping collective modes, their contribution into the K_{BL} collision term can be singled out to give

$$K_{BL}^{(coll.)} = \nabla_p \int \frac{d^3k}{(2\pi)^3} G_c(\omega, \boldsymbol{k})\, \boldsymbol{k}\, |V(\boldsymbol{k})|^2 \frac{\Gamma_+}{(Re\ \varepsilon(\omega, \boldsymbol{k}))^2 + (V(\boldsymbol{k})\Gamma_-)^2} (\boldsymbol{k} \cdot \nabla_p) f, \quad (42)$$

where

$$\Gamma_+ = \left(\frac{g}{(2\pi)^3}\right)^2 \int d^3p'\, d^3p''\, \frac{C(\boldsymbol{p}', \boldsymbol{p}'')}{(\boldsymbol{k} \cdot \boldsymbol{v}' - \omega)(\boldsymbol{k} \cdot \boldsymbol{v}'' - \omega)}, \quad (43)$$

$$\Gamma_- = \frac{g}{(2\pi)^3} \int d^3p' \frac{\boldsymbol{k} \cdot \nabla_{p'} f'}{(\boldsymbol{k} \cdot \boldsymbol{v}' - \omega)^2} \Delta(\boldsymbol{p}'), \quad (44)$$

and $\omega = \boldsymbol{k} \cdot \boldsymbol{v}$. This is the semi-classical limit of the collision term derived in ref. [7] in the quantum representation by taking into account the full correlation function as well as the memory effect associated with finite duration of the binary collisions. This collision term arises from the correlations associated with the long-wavelength collective density fluctuations. It describes the dissipation mechanism resulting from the coupling between the single-particle motion and the collective vibrations. In the vicinity of the spinoidal instability, the magnitude of this dissipation mechanism increases due to the large density fluctuations. Therefore, it can slow down the expansion of nuclear matter, as well as induce the critical scattering phenomenon similar to the critical opalescence in liquids near the phase transition [10]. In addition, the collective term K_{BL} of eq. (36) contains non-collective contributions corresponding to the small-angle binary scattering with momentum transfer $k \leq k_c$. Hence, it gives rise to corrections to the BUU collision term in the range of small momentum transfers.

4 Single-Particle Relaxation Time

In order to illustrate the effect of the collision term $K_{BL}^{(binary)}$ on the collision dynamics, we estimate a relaxation time associated with the single-particle motion. The single-particle relaxation time, τ, is defined as [11]

$$\frac{1}{\tau} = \frac{1}{\tau_{BUU}} + \frac{1}{\tau_{fluct.}}, \quad (45)$$

where

$$\begin{pmatrix} 1/\tau_{BUU} \\ 1/\tau_{fluct.} \end{pmatrix} = \frac{1}{(2\pi)^3} \int d^3p_2\, d^3p_3\, d^3p_4 \begin{pmatrix} W(1,2 \mid 3,4) \\ W_{BL}^{(binary)}(1,2 \mid 3,4) \end{pmatrix} [f_2 \tilde{f}_3 \tilde{f}_4 + \tilde{f}_2 f_3 f_4]. \quad (46)$$

Here τ_{BUU} [$= 1/\Delta$, cf. eq. (20)] is the relaxation time associated with the conventional BUU collision term, while $\tau_{fluct.}$ is a contribution of the additional collision term of eq. (37), arising from fluctuations. The first term in square brackets represents the rate at which nucleons of momentum $\boldsymbol{p}_1$ are scattered to new state, while the second term represents the blocking, due to the presence of a nucleon of momentum $\boldsymbol{p}_1$, of the processes that scatter nucleons into momentum $\boldsymbol{p}_1$. For simplicity, we consider the nuclear matter at low temperatures T: $T \ll \varepsilon_F$, where ε_F is the Fermi energy. In thermal equilibrium

$$[\tilde{f}_2\tilde{f}_3\tilde{f}_4 + \tilde{f}_2 f_3 f_4] = f_2\tilde{f}_3\tilde{f}_4/\tilde{f}_1. \tag{47}$$

Frequently, an alternative definition of the relaxation time is used [12, 13], in which only contribution of the loss term [i.e. the first term in square brackets of eq. (46)] is taken into account. As seen from (47), the relaxation time within such an alternative definition differs from that of eq. (46) by the factor $1/\tilde{f}_1$.

According to refs. [12, 13], at low temperatures and small deviations of the energy of the incident nucleon from the Fermi surface ($|\varepsilon - \varepsilon_F| \ll \varepsilon_F$) the general form of τ_{BUU} is

$$\frac{1}{\tau_{BUU}(T,\varepsilon)} = \beta_{BUU}\frac{(\pi T)^2 + (\varepsilon - \varepsilon_F)^2}{\varepsilon_F^2} v_F n, \tag{48}$$

where v_F is the Fermi velocity, n is the nucleon density, and β_{BUU} is some constant coefficient determined by cross sections. Collins and Griffin [14] performed extensive calculations of the nucleon mean free path, λ, within the above-mentioned alternative definition for τ_{BUU}. In terms of our definition of τ_{BUU} (46), it reads

$$\lambda = v\ \tau_{BUU}\{1 + \exp[-(\varepsilon - \varepsilon_F^{(0)})/T]\}, \tag{49}$$

where v is the velocity of the incident nucleon. For nuclear matter at the normal nuclear density and low temperatures, their results are well fitted by the formula

$$\lambda(T,\varepsilon) = \alpha\frac{1 + \exp[-(\varepsilon - \varepsilon_F^{(0)})/T]}{(\pi T)^2 + (\varepsilon - \varepsilon_F^{(0)})^2}, \tag{50}$$

where $\alpha \simeq = 1150$ fm MeV^2. For the parameters of the normal nuclear matter they used $n_0 = 0.17$ fm^{-3} and $\varepsilon_F^{(0)} = 38$ MeV. By comparing eqs. (48) and (50), we can deduce the value of the β_{BUU} coefficient

$$\beta_{BUU} = \frac{\varepsilon_F^{(0)\,2}}{\alpha n_0} \simeq 7.4\ fm^2. \tag{51}$$

For the relaxation time $\tau_{fluct.}$ we can use the corresponding result obtained for liquid He^3 [11, 15]

$$\frac{1}{\tau_{fluct.}(T,\varepsilon)} = \beta_{fluct.}\frac{(\pi T)^2 + (\varepsilon - \varepsilon_F)^2}{\varepsilon_F^2} v_F n + \left(\frac{1}{\tau_{fluct.}}\right)_{non-an}, \tag{52}$$

where

$$\beta_{fluct.} = \frac{\pi k_c}{32n} \left| \frac{F_0}{1+F_0} \right|^2 , \tag{53}$$

$$F_0 = \frac{gmp_F}{2\pi^2} V, \tag{54}$$

and p_F is the Fermi momentum. Here, F_0 is the conventional Landau parameter of the nuclear Fermi liquid [11, 12, 16], and $(1/\tau_{fluct.})_{non-an}$ is a non-analytic function of $(\varepsilon - \varepsilon_F)$ which is important in the very vicinity of the phase transition. The precise expression for this term can be found in refs. [11, 12]. Bellow, this term is not taken into account, since we are not allowed to go too close to the phase-transition point within our treatment due to our assumption of small fluctuations [cf. (i)]. We have deliberately represented the $\tau_{fluct.}$ time in the form similar to that of τ_{BUU}, which allows for a comparison of two relaxation times by simply comparing the values of $\beta_{fluct.}$ and β_{BUU}. In order to estimate $\beta_{fluct.}$, we utilize the parametrization of the nuclear effective potential [1, 17]

$$U[n] = A\frac{n}{n_0} + B\left(\frac{n}{n_0}\right)^{\sigma}, \tag{55}$$

where $n_0 = 0.145$ fm^{-3}, $A = -356$ MeV, $B = 303$ MeV, and $\sigma = 7/6$. This effective potential is extensively used in calculations of heavy-ion reactions and corresponds to a soft nuclear equation of state. According to eq. (19), we obtain

$$V(\boldsymbol{k}) = A\frac{1}{n_0} + B\frac{\sigma}{n_0}\left(\frac{n}{n_0}\right)^{\sigma-1} \tag{56}$$

and correspondingly

$$F_0 = \frac{3}{2}\left(\frac{n}{n_0}\right)^{1/3}\left[\frac{A}{\varepsilon_F^{(0)}} + \sigma\frac{B}{\varepsilon_F^{(0)}}\left(\frac{n}{n_0}\right)^{\sigma-1}\right], \tag{57}$$

where $\varepsilon_F^{(0)} = 34.4$ MeV [1, 17]. As seen, the effective interaction $V(\boldsymbol{k})$ does not depend on $\boldsymbol{k}$ within this parametrization.

Performing the actual calculation, we find that the effect of fluctuation on the relaxation time becomes very small at the normal nuclear density n_0, $\beta_{fluct.} \simeq 0$, indicating that the collision term K_{BL} is negligible at the normal nuclear density. We can determine the critical density n_c, specifying the onset of the spinoidal instability zone, from the condition that the Landau parameter becomes $F_0 = -1$. Using the parametrization of the effective potential, from eq. (57) we find $n_c = 0.625\ n_0$. When approaching this critical point, $n \to n_c$, the inverse of the relaxation time becomes very large, $1/\tau_{fluct.} \to \infty$, as it is seen from eq. (53). This corresponds to the critical-scattering phenomena at the liquid-gas phase transition. However, due to our assumption of small fluctuations [cf. (i)] we are not allowed to come close to the critical point within our treatment. We can only claim

that in the vicinity of the critical point the magnitude of $\beta_{fluct.}$ is comparable with that of β_{BUU}. For instance, taking the correlation length as $r_c = 2$ fm, we find $\beta_{fluct.} \simeq 6.1$ fm ^{2} at $n = 0.7\ n_0$. Thus, the effect of the collision term K_{BL}, indeed, becomes important when approaching the spinoidal region.

5 Conclusions

We have considered the evolution of the averaged phase-space density in the BL model. We have demonstrated that, besides the usual collision term of the BUU form, the equation of motion involves an additional collision term arising from correlations associated with long-wavelength density fluctuations. In the limit of small fluctuations around a quasistatic state, we have derived the explicit expression for this collision term.

In order to illustrate the effect of the collision term induced by fluctuations, we have estimated the relaxation time associated with the single-particle motion. It has been demonstrated that the relaxation time due to this dissipation mechanism is comparable with that determined by the BUU collision term already at densities $n \simeq 0.7 n_0$. Therefore, it appears that the fluctuation-induced collision term may strongly affect the average evolution of the system in the vicinity of the spinoidal region. In particular, the degree of equilibration in the expanding nuclear system just prior to the multifragmentation stage may be higher than that predicted by the BUU calculations.

The critical scattering in the vicinity of a phase transition is a fairly general phenomena [10]. Similar results for the collision term have been obtained in refs. [18, 19, 20] within the semiclassical limit of the time-dependent G-matrix theory [2]. Indeed, the V/ε interaction is the solution for the G-matrix in the semiclassical approximation. This fact is not surprising, since both the BL model and the time-dependent G-matrix theory present different truncation schemes of the BBGKY hierarchy within the theory of correlation dynamics and are aimed to take into account many-body correlations beyond the conventional single-particle approaches. The fact that these different approximate approaches bring about similar results gives confidence in the consistency of these models and in their ability to treat nuclear systems in the phase-transition domain.

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