Comment on "More Axions from Strings"

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Abstract

We comment on a claim that axion strings show a long-term logarithmic increase in the number of Hubble lengths per Hubble volume [1], thereby violating the standard "scaling" expectation of an O(1) constant. We demonstrate that the string density data presented in Ref. [1] are consistent with standard scaling, at a string density consistent with that obtained by us [2,3] and other groups. A transient slow growth in Hubble lengths per Hubble volume towards its constant scaling value is explained by standard network modelling [3].

The paper [1] reports on and interprets the results of a set of numerical simulations of axion string networks in a complex U(1) field model of the axion, aiming to pin down the axion number density in the post-inflationary PQ symmetry-breaking scenario, and thereby provide an accurate prediction of the axion mass for dark matter searches.

A central claim of the paper (first made in Ref. [4]) is that the long-established picture of scaling in string networks [5, 6] does not apply to global strings, and that

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the average number of Hubble lengths of string per Hubble volume (proportional to ξ in their notation) grows logarithmically with cosmic time in the long term, rather than tending to an O(1) constant. If confirmed, this would have important implications for axion dark matter in the axion string scenario: in particular, the axion mass estimate would be significantly changed. Resonant cavity axion detectors [7] benefit greatly from accurate mass estimates in order to reduce the search time.

Data from earlier simulations in the same field theory [8–15] analysed in the framework of the standard scaling scenario were consistent with $\xi \simeq 1$. Following the appearance of Ref. [4], other groups have also reported slow growth of ξ in simulations [16–19]. This is argued in Refs. [1, 4] to be the true long-term behaviour of an axion string network, replacing the standard scaling model.

In the standard scaling model, the string network evolves towards constant string density parameter, which is easily understood in terms of the reduced or increased rate of loop production in under- or over-dense networks. The loops evaporate into axions and massive scalar modes [20]. This model is given a mathematical expression as the "one-scale" model [21] and the "velocity-dependent one-scale" model (VOS) [22, 23], which adjusts the loop production rate according to the root mean square velocity of the strings. The VOS model in its simplest form has been checked against numerical simulations of gauge string networks [24, 25] and it has recently been shown to give a good description of the approach to scaling in global string networks [3].

On the other hand, in Ref. [1] it is not clear what the model is, beyond a hypothesis that the string density parameter grows logarithmically at late times. An argument is given for expecting logarithms based on the increase in the effective string tension and the decoupling of the axion field in an idealised model of axion strings [26], which is the subject of ongoing discussion [27], but no dynamical model is proposed. In particular, it is not made clear how the logarithmic decoupling of the axion field should affect the string density in this way. Others have tried adapting the VOS model with a time-dependent string mass per unit length [28–30], but this still results in a constant string density parameter at late times. The fit of the simplest VOS model to the global string network evolution close to the scaling fixed point is already very good [3], so the adaptation is not motivated by the simulations themselves.

The growth of ξ observed in the numerical simulations of Ref. [1] are presented as strong evidence for the hypothesis that the growth is logarithmic in the long term. However, as shown in Refs. [2,3], growth of ξ in simulations can be understood in terms of the slow approach of the dynamical variable ξ to a constant-ξ scaling solution with $\xi_\infty = 1.19 \pm 0.20$. As we demonstrate below, the data presented in Ref. [1], are consistent with $\xi \to O(1)$, and with the results of Refs. [2,3], when properly analysed in a framework allowing for transients. Criticisms of the claims of long-term logarithmic growth made in Ref. [2], which was published before Ref. [1] appeared, are not addressed in Ref. [1], and our paper is not cited.

The important quantities under study are the string density parameter ξ, defined in terms of the total string length ℓ as

$$\xi = \ell t^2/\mathcal{V}, \tag{1}$$

where t is cosmic time and $\mathcal{V}$ is the simulation volume, and the mean string separation

$$L = \sqrt{\mathcal{V}/\ell}. \tag{2}$$

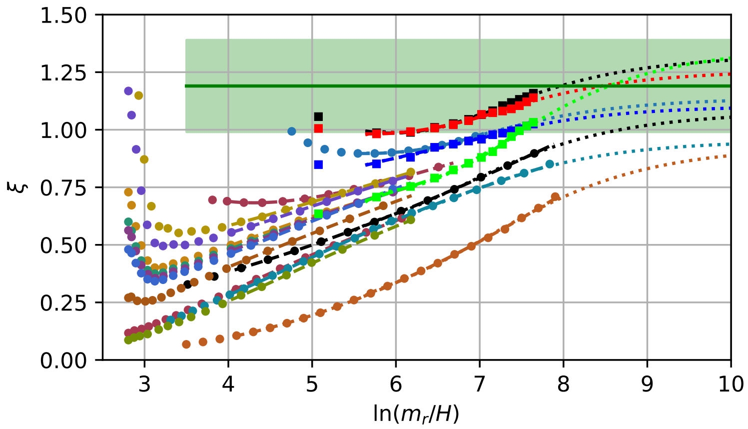

Figure 1: Data from Figure 1 of Ref. [1] (round markers) and [2] (square markers). The dashed lines give fits to (3) with all parameters free, over ranges $\log(m_r/H) > 4$ or $\log(m_r/H) > 5.5$. The dotted line shows the fit to a function (5) with long-term linear growth, as predicted by standard scaling, and an extrapolation. The estimate of the asymptotic value of ξ in [2] is shown as the green band. All data are broadly consistent with an asymptotic value $\xi(t \to \infty) \simeq 1$.

To compare the results of Ref. [1] with ours, we have digitised data from Figure 1 of Ref. [1] and present it together with data from [2] in Fig. 1. Data from Ref. [1] is presented as round dots and data from [2] as square markers. Fig. 1 also shows the estimate of the asymptotic value of ξ in [2] as a green band. Note that Ref. [2] was more conservative about the preparation of the string networks, and waited longer before starting to record data. Note also that the choice made in Refs. [1,4] to plot against the logarithm of cosmic time, rather than cosmic time, emphasises the early phase of the simulations, where the effect of initial conditions will be greater.

One can see that the simulations of [1] and [2] give a consistent picture of the evolution of $\xi(t)$, and are distinguished only by the generally lower string density in Ref. [1], resulting from the choice of initial conditions. It is reassuring that the two data sets obtained from different codes, initial conditions, data collection methods, and number of simulations at each initial string density are in broad agreement. The different conclusions are therefore a result of the analysis rather than the simulations.

Eq. (3) of Ref. [1] gives the authors' hypothesis for the behaviour of ξ, which we reproduce here

$$\xi = c_1 \log(m_r/H) + c_0 + \frac{c_{-1}}{\log(m_r/H)} + \frac{c_{-2}}{\log^2(m_r/H)}, \tag{3}$$

where m_r is the mass of the scalar field, $H = 1/2t$ is the radiation era Hubble rate, and c_n are fit parameters.

In Ref. [1] it is claimed that the coefficients of the first two terms (c_1 and c_0) are universal parameters, and the value $c_1 = 0.24(2)$ is presented, which in the long-term logarithmic growth hypothesis would describe the network at large times. It is not clear what the authors mean by the universality of c_1; and besides, the authors themselves cast doubt on the claim of the universality of c_1 when they state that fits with a $\log^2$ term also give good results.

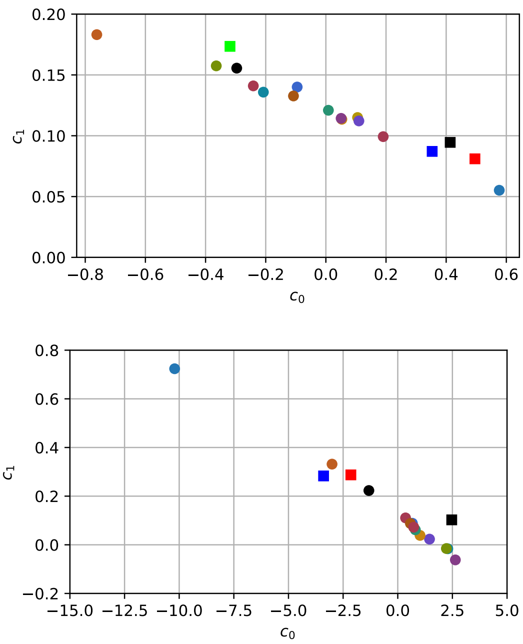

Figure 2: Values for the parameters c_0 and c_1 of Eq. 3 for the data plotted in Fig. 1, with the same colour and shape code. Top: data fitted with only those two parameters. Bottom: data fitted with all four parameters. Note that in the 4-parameter fit, the data set from Ref. [2] marked with the green square has $(c_0, c_1) = (-30, 1.8)$ and is not plotted.

We take universality to mean behaviour which is independent of the initial state. In order to check this universality, we have fitted the digitised data from [1], together with the data from [2], using just two parameters (c_0 and c_1) from Eq. 3, and also using all four parameters. For the 2-parameter fit, all fits were taken over the range $\log(m_r/H) > 5.5$. For the 4-parameter fit, the fit ranges for the data from Ref. [1] were the same as quoted in that work, (mostly $\log(m_r/H) > 4$, with one curve using $\log(m_r/H) > 5.5$) and $\log(m_r/H) > 5.5$ for the data from [2]. The fit ranges were chosen for comparison purposes. In practice, starting the fit too early risks biasing the estimates of the asymptotic behaviour with transients associated with the evolution away from the initial state.

The resulting parameters c_0 and c_1 are shown in Fig. 2. One immediately notices that there exists a correlation between c_0 and c_1, and that they vary widely with initial string density, suggesting that neither of them is universal. It can also can be seen in the figure that fitting for just two parameters the values for c_1 are between 0.05 and 0.2. However, when fitting for the four parameters, the spread is much bigger (bottom of Fig. 2), even disregarding the two outliers (the green square and blue dot). This casts another doubt over the universality of logarithmic growth.

For one set, marked with the black curve, we obtained $c_1 \simeq 0.22$ when fitting with four parameters. This is consistent with the value $c_1 = 0.24(2)$ given in Ref. [1] as the universal value in their model. This is not the mean and uncertainty obtained from all fits. Rather, the claim of universality seems to rest on the fact that one can fit all curves with values of c_1 and c_0 fixed from the "preferred" simulation set, with the other two parameters left free. The reason for privileging one set of initial conditions is unclear. It should also be noted that the value of c_1 of the preferred simulation is not as stable as the quoted uncertainty seems to suggest: for example, its value is $c_1 \simeq 0.16$ when fitting with two parameters over the range $\log(m_r/H) > 5.5$.

Let us now examine the consistency with standard scaling, which we recall is underpinned by the VOS model. The VOS model describes the network with two parameters, representing the efficiency with which loops are removed from long strings, and the efficiency with which mean curvature produces average acceleration. It predicts that the mean string separation $L = t/\sqrt{\xi}$ should grow linearly with cosmic time at late times, while the root mean square velocity tends to a constant.

In Ref. [2] estimates for the asymptotic linear growth rate were extracted from the data by fits to

$$L = x_* t + L_0. \tag{4}$$

In standard scaling x_* is predicted to be a universal parameter, while L_0 is a phenomenological fit parameter to reduce the effect on estimates of x_* of the initial conditions and evolution of the RMS velocity.

We plot L against t in Fig. 3, also with fits to (4) in the last half of the simulation, where the effect of initial conditions should be reduced. Again, data from Ref. [1] is plotted as dots, with data from [2] is plotted with square markers.

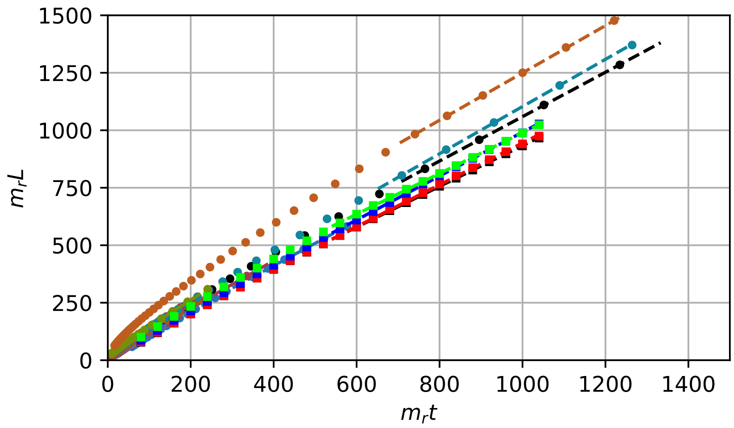

Figure 3: Data from Figure 1 of Ref. [1] (dots), and from [2] (square markers), showing the mean string separation L. Dashed lines give fits to a straight line (5) over the last half of the simulation from which the asymptotic coefficient linear growth is extracted.

Fig. 4 shows the best fit value of the parameters x_* and L_0, and it is clear that all simulations are consistent with $x_* \simeq 1$, as predicted by standard scaling. There is no obvious correlation between the non-universal parameter L_0 and the proposed universal one x_*, indicating that the effect of different initial string densities is mostly captured by L_0. One can improve the accuracy of the estimate of x_* and the description of the transients by including velocity data as well [3]. We have also looked for a slow increase in the value of x_* in our data, finding none [2].

As a further comparison with the four parameter model of Eq. (3), we also fitted the data to a 4 parameter model with linear growth,

$$L = x_* t + L_0 + L_1/t + L_2/t^2 \tag{5}$$

over the range $\log(m_r/H) > 5.5$, corresponding to $t \gtrsim 120$. The bottom panel of Fig. 4 shows the values of x_* and L_0 for this case. We also show the fits as dotted lines in Fig. 1, along with an extrapolation to $\ln(m_r H) = 10$. They are barely distinguishable from the logarithmic growth model as a fit to the data, and show how the apparent logarithmic growth turns over to a constant in the standard scaling picture.

It is also notable that the values of x_* do not vary significantly from the 2-parameter fit. Clearly L_0 changes, but this is to be expected, as it shares the information about the initial transients with L_{-1} and L_{-2}. The sensitivity of the parameter x_* to changes in the fit range were investigated in [2], and form part of the error budget in the result $x_* = 1.19 \pm 0.20$. Taking fits starting from earlier times biases the result to slightly lower values: we have not attempted a full analysis here.

The tendency of ξ to increase through most of the simulations is also cited in Ref. [1]

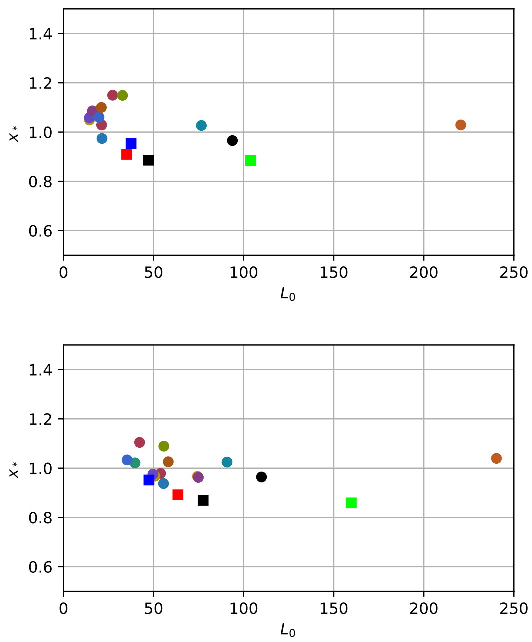

Figure 4: Fit paramerers to the data from Figure 1 of Ref. [1] (dots), and from [2] (square markers). Top: fit parameters x_* and L_0 from the 2-parameter fit (4). Bottom: fit parameters x_* and L_0 from the extended 4-parameter fit (5).

to support of the claim of long-term growth. This tendency is clear in our simulations too [2, 3], and can be understood as a transient as the network approaches scaling from low-density configurations (see for example Fig. 9 in Ref. [3]). It was also shown in [3] that the tendency to approach the fixed point from low densities could be understood in the framework of the VOS model as the result of an initial burst of loop production thinning out the network.

Therefore, both sets of simulations support the standard scaling model, with consistent values of the asymptotic string density parameter $\xi_* = 1/x_*^2 \simeq 1$, which is stable between different initial string densities and to the number of parameters in the fits used to extract it. The value can be understood as approximately one Hubble-sized loop being produced per Hubble time per Hubble volume, and subsequently decaying in about a Hubble time [20]. The VOS model gives us the framework to extrapolate the result.

On the other hand, the alternative long-term logarithmic growth model presented in Refs. [1, 4] lacks a dynamical framework which justifies the logarithmic fits, or their extrapolation. The method for extracting the coefficient of the logarithm is ad-hoc. Appealing to the excellence of the fits is not enough, as any smooth function over a finite interval can be arbitrarily well approximated by an expansion in a set of basis functions. The instability of the coefficient of the logarithm between different initial string densities and number of parameters in the fits is a sign that the model is not the correct description of the long-term behaviour.

It was also pointed out in Ref. [3] that there is a potentially decisive test between the two scenarios: whether simulations give asymptotic values of ξ significantly larger than O(1) or not. Simulations of the U(1) complex field models to date have final values of $\xi \simeq 1$, including the ones presented in Ref. [1] as we have established here. The slowly-growing ξ evolution shown in the simulations always has $\xi \lesssim 1$ at late times, as is consistent with a transient bringing the the system to the standard constant-ξ scaling solution with $\xi \simeq 1$.

In summary, the standard scaling scenario with $\xi \simeq 1$ is consistent with all simulations to date, and the proposal in Refs. [1,4] to replace the standard scaling scenario by the long-term logarithmic growth model is unjustified. Observational predictions based on long-term logarithmic growth [1,4,31] are therefore unsubstantiated.

Note added: Recently, another group [32] has used adaptive mesh refinement to simulate axion strings to $\log(m_r/H) \simeq 9$. The initial string density of their only simulation is close to that of the "preferred" simulation set of Ref. [1]. We have checked that the data for ξ in Ref. [32] are consistent with an approach to standard scaling with $\xi_* = 1.31 \pm 0.05$, in accord with our results. Therefore, although the data is presented within the framework of the logarithmic growth model, it supports standard scaling.

References

[1] M. Gorghetto, E. Hardy, and G. Villadoro, *More Axions from Strings*, *SciPost Phys.* **10** (2021) 050, [arXiv:2007.0499].

[2] M. Hindmarsh, J. Lizarraga, A. Lopez-Eiguren, and J. Urrestilla, *Scaling Density of Axion Strings*, *Phys. Rev. Lett.* **124** (2020), no. 2 021301, [arXiv:1908.0352].

[3] M. Hindmarsh, J. Lizarraga, A. Lopez-Eiguren, and J. Urrestilla, *Approach to scaling in axion string networks*, arXiv:2102.0772.

[4] M. Gorghetto, E. Hardy, and G. Villadoro, *Axions from Strings: the Attractive Solution*, *JHEP* **07** (2018) 151, [arXiv:1806.0467].

[5] M. Hindmarsh and T. Kibble, *Cosmic strings*, *Rept.Prog.Phys.* **58** (1995) 477–562, [hep-ph/9411342].

[6] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects.* Cambridge University Press, 2000.

[7] **ADMX** Collaboration, T. Braine et al., *Extended Search for the Invisible Axion with the Axion Dark Matter Experiment*, *Phys. Rev. Lett.* **124** (2020), no. 10 101303, [arXiv:1910.0863].

[8] M. Yamaguchi, M. Kawasaki, and J. Yokoyama, *Evolution of axionic strings and spectrum of axions radiated from them*, *Phys. Rev. Lett.* **82** (1999) 4578–4581, [hep-ph/9811311].

[9] M. Yamaguchi, *Scaling property of the global string in the radiation dominated universe*, *Phys. Rev. D* **60** (1999) 103511, [hep-ph/9907506].

[10] M. Yamaguchi, J. Yokoyama, and M. Kawasaki, *Evolution of a global string network in a matter dominated universe*, *Phys. Rev. D* **61** (2000) 061301, [hep-ph/9910352].

[11] M. Yamaguchi and J. Yokoyama, *Quantitative evolution of global strings from the Lagrangian view point*, *Phys. Rev.* **D67** (2003) 103514, [hep-ph/0210343].

[12] T. Hiramatsu, M. Kawasaki, T. Sekiguchi, M. Yamaguchi, and J. Yokoyama, *Improved estimation of radiated axions from cosmological axionic strings*, *Phys. Rev.* **D83** (2011) 123531, [arXiv:1012.5502].

[13] T. Hiramatsu, M. Kawasaki, K. Saikawa, and T. Sekiguchi, *Production of dark matter axions from collapse of string-wall systems*, *Phys. Rev.* **D85** (2012) 105020, [arXiv:1202.5851]. [Erratum: Phys. Rev.D86,089902(2012)].

[14] M. Kawasaki, K. Saikawa, and T. Sekiguchi, *Axion dark matter from topological defects*, *Phys. Rev.* **D91** (2015), no. 6 065014, [arXiv:1412.0789].

[15] A. Lopez-Eiguren, J. Lizarraga, M. Hindmarsh, and J. Urrestilla, *Cosmic Microwave Background constraints for global strings and global monopoles*, *JCAP* **1707** (2017) 026, [arXiv:1705.0415].

[16] M. Kawasaki, T. Sekiguchi, M. Yamaguchi, and J. Yokoyama, *Long-term dynamics of cosmological axion strings*, *PTEP* **2018** (2018), no. 9 091E01, [arXiv:1806.0556].

[17] A. Vaquero, J. Redondo, and J. Stadler, *Early seeds of axion miniclusters*, *JCAP* **1904** (2019) 012, [arXiv:1809.0924].

[18] M. Buschmann, J. W. Foster, and B. R. Safdi, *Early-Universe Simulations of the Cosmological Axion*, *Phys. Rev. Lett.* **124** (2020), no. 16 161103, [arXiv:1906.0096].

[19] V. B. Klaer and G. D. Moore, *Global cosmic string networks as a function of tension*, *JCAP* **06** (2020) 021, [arXiv:1912.0805].

[20] A. Saurabh, T. Vachaspati, and L. Pogosian, *Decay of Cosmic Global String Loops*, *Phys. Rev. D* **101** (2020), no. 8 083522, [arXiv:2001.0103].

[21] T. W. B. Kibble, *Evolution of a system of cosmic strings*, *Nucl. Phys.* **B252** (1985) 227. [Erratum: Nucl. Phys.B261,750(1985)].

[22] C. J. A. P. Martins and E. P. S. Shellard, *Quantitative string evolution*, *Phys. Rev.* **D54** (1996) 2535–2556, [hep-ph/9602271].

[23] C. Martins and E. Shellard, *Extending the velocity dependent one scale string evolution model*, *Phys.Rev.* **D65** (2002) 043514, [hep-ph/0003298].

[24] J. Moore, E. Shellard, and C. Martins, *On the evolution of Abelian-Higgs string networks*, *Phys.Rev.* **D65** (2002) 023503, [hep-ph/0107171].

[25] J. R. C. C. C. Correia and C. J. A. P. Martins, *Extending and Calibrating the Velocity dependent One-Scale model for Cosmic Strings with One Thousand Field Theory Simulations*, *Phys. Rev. D* **100** (2019), no. 10 103517, [arXiv:1911.0316].

[26] A. Dabholkar and J. M. Quashnock, *Pinning Down the Axion*, *Nucl. Phys. B* **333** (1990) 815–832.

[27] M. Dine, N. Fernandez, A. Ghalsasi, and H. H. Patel, *Comments on Axions, Domain Walls, and Cosmic Strings*, arXiv:2012.1306.

[28] C. J. A. P. Martins, *Scaling properties of cosmological axion strings*, *Phys. Lett.* **B788** (2019) 147–151, [arXiv:1811.1267].

[29] C.-F. Chang and Y. Cui, *Stochastic Gravitational Wave Background from Global Cosmic Strings*, *Phys. Dark Univ.* **29** (2020) 100604, [arXiv:1910.0478].

[30] C.-F. Chang and Y. Cui, *Gravitational Waves from Global Cosmic Strings and Cosmic Archaeology*, arXiv:2106.0974.

[31] M. Gorghetto, E. Hardy, and H. Nicolaescu, *Observing invisible axions with gravitational waves*, *JCAP* **06** (2021) 034, [arXiv:2101.1100].

[32] M. Buschmann, J. W. Foster, A. Hook, A. Peterson, D. E. Willcox, W. Zhang, and B. R. Safdi, *Dark Matter from Axion Strings with Adaptive Mesh Refinement*, arXiv:2108.0536.