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Simple formulae for light-yield considerations in the design of scintillator-Fe and scintillator-Pb sampling calorimeters

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Abstract

Recent designs of sampling hadron and electromagnetic calorimeters for hadron collider experiments incorporate scintillating tile-fiber technology in which scintillation light is transported to phototubes through optical fibers. A major technical challenge has been to develop designs in which energy resolution is not limited by low photostatistics. Minimum light yield requirements in scintillator-Fe and scintillator-Pb sampling calorimeters are discussed, and several simple formulae are derived for estimating the contribution of photostatistics to energy resolution without resorting to detailed Monte Carlo calculations.

1. Introduction

Most major high energy physics experiments use sampling electromagnetic and hadron calorimeters to detect and measure the energy of particle interactions. In many experiments, including both fixed-target [1,2] and collider [3] configurations such calorimeters use plastic scintillator as the sampling medium.

The recent evolution in the design of sampling calorimeters has seen the development of scintillating tile detectors. In tile-fiber calorimeters, blue scintillation light produced in the tiles is wavelength-shifted (to green) by optical fibers and then transported through clear optical fibers to phototubes. The use of tile-fiber light collection has greatly simplified the design of large calorimeters with tower geometry. Recently, this technique has been implemented on a large large scale in the upgrade of the plug detector at the Collider Detector at Fermilab (CDF) [4,5]. Scintillating tile-fiber calorimeters are also being planned for two major detectors (CMS and ATLAS) at the Large Hadron Collider (LHC) at CERN and were previously under development for the SDC experiment at the Superconducting Super-Collider.

Significant light losses result from the wavelength-shifting and transport of light through optical fibers. Therefore, considerable development effort has aimed at producing uniform, high light-yield scintillating tiles. One way to increase the light yield from tiles is to use thicker scintillator. In terms of both cost and space considerations, however, this option has severe drawbacks. In seeking alternatives, it is important that we understand the light-yield requirements of sampling calorimeters using scintillator as the sampling medium.

Researchers designing calorimeters for hadron colliders at the LHC and elsewhere intend to use them to detect electrons, hadrons, and muons through the deposition of energy of these particles in the sampling medium. Critical to these measurements is to minimize the error (energy resolution) introduced by low photostatistics due to light losses. In this communication, we derive simple formulae for estimating the effect of photostatistics on the error in the measurement of the energy deposition of electrons, hadrons, and muons in calorimeters designed for their detection. An overview of the derivation is given in this section, and details are provided in subsequent sections.

The light yield of a single scintillating tile may be described in terms of n, the number of photoelectrons per minimum ionizing particle (mip). For this purpose, an equivalent particle (EP) [6] is defined to approximate the mean light yield of a single minimum ionizing particle traversing a single tile of a single sampling plane.

Section 2 treats hadronic energy detection, in which the calorimeter is composed of iron (Fe) planes of thickness d (in cm) interspersed with planes of plastic scintillator. In this case, the resolution σ of the hadronic energy E_{had} (in GeV) is related to n, the number of photoelectrons per mip per tile, as follows:

$$\frac{\sigma}{E_{had}} = \left(\frac{\sigma}{E_{had}}\right)_{intrinsic} \left[1 + 0.2/n\right]^{1/2}, \quad (1)$$

where

$$\left(\frac{\sigma}{E_{had}}\right)_{intrinsic} \approx \frac{1.0}{\sqrt{E_{had}}}\sqrt{\frac{d}{10\text{ cm}}} \quad (2)$$

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is the intrinsic hadronic energy resolution resulting from the finite intersampling distance d in iron.

Section 3 treats electromagnetic energy detection, in which the sampling calorimeter is composed of lead (Pb) sheets of thickness t (in cm) interspersed with planes of plastic scintillator. In this case, the resolution σ of the electron energy E_{em} (in GeV) is related to n as follows:

$$\frac{\sigma}{E_{em}} = \left(\frac{\sigma}{E_{em}}\right)_{\text{intrinsic}} \left[1 + 0.6/n\right]^{1/2}, \qquad (3)$$

where

$$\left(\frac{\sigma}{E_{em}}\right)_{\text{intrinsic}} \approx \frac{0.145}{\sqrt{E_{em}}}\sqrt{\frac{t}{X_t}} \qquad (4)$$

is the intrinsic electromagnetic resolution resulting from the finite intersampling distance in lead, and $X_t = 0.56$ cm is the radiation length of lead.

Eqs. (1) and (3) place minimum requirements on n, if photostatistics are to degrade the calorimeter's resolution by less than an additional 10% of the intrinsic resolution. For a scintillator–iron hadron calorimeter, $n > 1$, and for a scintillator-lead electromagnetic calorimeter, $n > 3$. Of particular note is that these formulae and conclusions are independent of the iron or lead intersampling distance. In Section 4, we show that requiring the hadron calorimeter to serve as a muon identifier leads to similar conclusions.

2. Sampling hadronic calorimeters

We first derive an estimate of the intrinsic energy resolution for a general scintillator-Fe sampling hadronic calorimeter. To do this, we examine the resolution of two such calorimeters representing typical designs. As a first example, the fixed-target Chicago–Columbia–Fermilab–Rochester (CCFR) calorimeter [1] consists of 3 m × 3 m steel plates of thickness 10.3 cm. Each plate is followed by a plastic scintillation counter of thickness 2.5 cm. The light yield of the scintillator is about 10 photoelectrons per minimum ionizing muon per counter. The total length of the CCFR calorimeter is 8 m. The electron-to-hadron response ratio is ≈ 1.1, making the calorimeter close to compensating, and the design therefore yields a resolution close to the best possible for a given intersampling distance. The resolution of the CCFR calorimeter is $0.87/\sqrt{E_{had}}$, with a constant term close to zero. Extrapolating from this example to finer (but finite) sampling, one expects the resolution of a hadron calorimeter to improve in proportion to the square root of the intersampling distance. Therefore, for a close-to-compensating scintillator-Fe hadron calorimeter, one expects the following:

$$\left(\frac{\sigma}{E_{had}}\right)_{\text{CCFR}} \approx \frac{0.87}{\sqrt{E_{had}}}\sqrt{\frac{d}{10\ \text{cm}}}. \qquad (5)$$

For a second, quite different example from a collider experiment, the CDF central hadronic calorimeter [3] is constructed of 5 cm thick steel plates arranged in towers. The energy sampling occurs in thin scintillator, and the tower size is relatively small, 0.3 m × 0.3 m. The calorimeter is only about a meter in depth and is non-compensating, with the ratio of electron to hadron response ≈ 1.3. The resolution of the CDF hadron calorimeter is $0.83/\sqrt{E_{had}}$ to which a constant term of 3.5% must be added in quadrature. For energies below 50 GeV, the resolution of a hadron calorimeter with the CDF geometry is therefore given by:

$$\left(\frac{\sigma}{E_{had}}\right)_{\text{CDF}} \approx \frac{1.16}{\sqrt{E_{had}}}\sqrt{\frac{d}{10\ \text{cm}}}. \qquad (6)$$

To formulate the instrinsic energy resolution (i.e., due to sampling alone) of a general scintillator-Fe hadron calorimeter, we take the average of the resolutions of the CCFR and CDF type hadron calorimeters (noting that the estimate is good to within ±15%) as follows:

$$\left(\frac{\sigma}{E_{had}}\right)_{\text{intrinsic}} \approx \frac{1.0}{\sqrt{E_{had}}}\sqrt{\frac{d}{10\ \text{cm}}}. \qquad (7)$$

In order to derive the contribution of photostatistics to the hadronic energy resolution, one needs to express the calibration and resolution of the calorimeter in terms of mip's. The first step is to define an equivalent particle (EP), the response of the calorimeter to a well-defined particle standard. In the CCFR experiment, 1 EP is defined as the truncated mean of the pulse height distribution of a 77-GeV muon traversing one scintillation counter [6]. The truncated mean of the energy deposition of a 77-GeV muon is about 5% higher than that of a 3-GeV muon (a 3-GeV muon may be assumed to be a minimum ionizing particle).

For the CCFR calorimeter [6], a 21-GeV hadronic energy deposition yields a phototube pulse which is the same as 100 EP, giving a calibration constant of 0.21 GeV/EP or ≈ 0.20 GeV/mip. In addition, if the sampling frequency in the CCFR hadron calorimeter were 5 cm instead of 10 cm, the calibration constant would increase by a factor of 2. Combining this information, the calibration of a hadron calorimeter in terms of mip's is

$$C_{had} \approx \frac{0.2d}{10\ \text{cm}}\left[\text{GeV/mip}\right]. \qquad (8)$$

Note that the dE/dx energy loss of a minimum ionizing particle in 10 cm of iron is about 0.12 GeV/mip. Therefore, the calibration of a a scintillating-Fe sampling calorimeter is a factor of 1.7 (that is, 0.2/0.12 = 1/0.6) higher than that calculated under the naive assumption that the calibration per mip is equal to the dE/dx energy loss of a minimum ionizing particle in a single iron plate. Because the CCFR calorimeter's reponse to hadrons and electrons is nearly equal (electron/hadron ≈ 1.1), the factor of 1/0.6 reflects the additional energy losses in the iron plates that are not fully accounted in the scintillator, for both hadrons and electrons.

Using the above calibration, we may express the total pulse height of any hadron shower as N_{mip}, the number of mip's, as follows:

$$N_{mip}(E_{had}) \approx \frac{E_{had}}{0.2}\frac{10\ \text{cm}}{d}, \quad (9)$$

where E_{had} is the hadron energy in GeV. Therefore, the total number of photoelectrons produced by an incident hadron of energy E_{had} is $N_{pe}(E_{had}) = nN_{mip}(E_{had})$, or

$$N_{pe}(E_{had}) \approx \frac{nE_{had}}{0.2}\frac{10\ \text{cm}}{d}. \quad (10)$$

The contribution of photostatistics to the hadron energy resolution is therefore $1/\sqrt{N_{pe}(E_{had})}$, or

$$\left(\frac{\sigma}{E_{had}}\right)_{photostatistics} = \sqrt{\frac{0.2}{n}}\frac{1.0}{\sqrt{E_{had}}}\sqrt{\frac{d}{10\ \text{cm}}}. \quad (11)$$

Adding this (Eq. (11)) in quadrature to the intrinsic hadronic energy resolution (Eq. (7)) yields

$$\frac{\sigma}{E_{had}} = \left(\frac{\sigma}{E_{had}}\right)_{intrinsic}\left[1 + 0.2/n\right]^{1/2}. \quad (12)$$

Eq. (12) implies that for $n = 3, 2, 1$, and 0.5 photoelectrons per mip per scintillator sampling, the energy resolution degrades by factors of 1.03, 1.05, 1.10, and 1.18, respectively. Therefore, $n > 1$ is required if photostatistics are to degrade the instrinsic resolution of the hadron calorimeter by less than 10%.

3. Sampling electromagnetic calorimeters

Del-Peso and Ross [7] have studied the resolution of sampling electromagnetic calorimeters by comparing various experimental data to EGS Monte Carlo simulations. Their approximate formula for the resolution of a typical scintillator-Pb sampling calorimeter, with 5 mm thick scintillator, is as follows:

$$\left(\frac{\sigma}{E_{em}}\right)_{intrinsic} \approx \frac{0.145}{\sqrt{E_{em}}}\sqrt{\frac{t}{X_t}}, \quad (13)$$

where t is the thickness of the lead plates in cm and $X_t = 0.56$ cm is the radiation length of lead. The dE/dx of a minimum ionizing particle in lead is 12.8 MeV/cm. This means that the dE/dx energy loss of a minimum ionizing particle in one radiation length of lead is 7.2 MeV. We may assume that the calibration (in mip's) of an electromagnetic calorimeter is related to the dE/dx energy loss of a minimum ionizing particle in a single lead plate by the same 1/0.6 factor as in the case for the CCFR calorimeter. In this case, the calibration of an electromagnetic calorimeter in terms of mip's is

$$C_{em} \approx \frac{7.2}{0.6}\frac{t}{X_t}\left[\text{MeV/mip}\right] = 0.012\frac{t}{X_t}\left[\text{GeV/mip}\right]. \quad (14)$$

to within 20%. Following the steps in the hadronic case, the calibration may be expressed in mip's as

$$N_{mip}(E_{em}) \approx \frac{E_{em}}{0.012}\frac{X_t}{t}. \quad (15)$$

The corresponding total number of photoelectrons for an electromagnetic shower of energy E_{em} in GeV is given by $N_{pe}(E_{em}) = nN_{mip}(E_{em})$, or

$$N_{pe}(E_{em}) \approx \frac{nE_{em}}{0.012}\frac{X_t}{t}. \quad (16)$$

Therefore, the contribution of photostatistics to the hadron energy resolution is $1/\sqrt{N_{pe}(E_{em})}$ or

$$\left(\frac{\sigma}{E_{em}}\right)_{photostatistics} = \sqrt{\frac{0.012}{n}}\frac{1.0}{\sqrt{E_{em}}}\sqrt{\frac{t}{X_t}}. \quad (17)$$

Adding this contribution (Eq. (17)) in quadrature to the intrinsic electromagnetic shower energy resolution (Eq. (13)) yields

$$\frac{\sigma}{E_{em}} = \left(\frac{\sigma}{E_{em}}\right)_{intrinsic}\left[1 + 0.57/n\right]^{1/2}. \quad (18)$$

Eq. (18) implies that for n of 3, 2, 1 and 0.5 photoelectrons per mip per scintillator sample, the overall electromagnetic shower energy resolution degrades by factors of 1.09, 1.13, 1.25, and 1.46, respectively. Therefore, $n > 3$ is required if photostatistics are to degrade the instrinsic resolution of the hadron calorimeter by less than 10%.

4. Muon detection

Experimenters typically identify a muon as a non-interacting particle, in the electromagnetic calorimeter and in a sequence of longitudinal segments of the hadronic calorimeter. A possible definition of a muon in terms of energy is a particle for which the energy deposition is within three standard deviations ($\pm 3\sigma$) of the energy deposition of a minimum ionizing particle. For such a criteria to be applied, the mean energy deposition of a muon should be greater than four standard deviations away from zero.

The total number of photoelectrons produced by a muon which traverses a longitudinal segment of a hadron calorimeter (for example, a back segment had2) is the product of the number of photoelectrons per mip per sampling (or per tile), times the number of longitudinal samplings (or tiles) in that segment l_{had2},

$$N_{pe}^{muon} = nl_{had2}. \quad (19)$$

Therefore, the contribution of photostatistics to the resolution of the muon energy deposited in the had2 segment of the calorimeter is

$$\left(\frac{\sigma}{E_{deposited}}\right)_{muon} = \frac{1}{\sqrt{nl_{had2}}}. \quad (20)$$

The requirement that the muon energy deposition to be more that four standard deviations away from zero implies $n > 16/l_{had2}$. For a longitudinal segment consisting of 16 counters, the muon identification requirement implies $n > 1$ photoelectron per mip per tile.

5. Conclusions

We have derived several formulae for estimating, with precision $\approx \pm 20\%$, the contribution of photostatistics to the energy resolution of sampling hadronic and electromagnetic calorimeters. As an intermediate step, we have expressed the energy calibration of each type of calorimeter in terms of the calorimeter's response to minimum ionizing particles. This in turn has allowed us to express the energy resolution in terms of n, the number of photons per mip per scintillator tile. The resulting expressions for the the fractional degradation of the energy resolution of such calorimeters due to low photostatistics are independent of the intersampling distance (thickness of iron or lead plates). These formulae may be used to estimate the improvement in resolution obtainable using thicker scintillator or more frequent samplings, relative to cost and space considerations, without resorting to Monte Carlo calculations.

Specifying that the degradation of the energy resolution be less than 10% in each case provides minimum requirements on the photostatistical capacities of sampling scintillators and related apparatus (for example, optical fibers). To satisfy this specification, our formulae require $n > 1$ for hadron calorimeters and $n > 3$ for electromagnetic calorimeters. Further specifying that the hadron calorimeter function as a muon identifier yields compatible results. All these specifications have recently been satisfied in a realistic mass production environment by the CDF plug-upgrade calorimeter group [4,5]. This group has developed high light-yield scintillating tile-fiber assemblies with $n > 2$, for the CDF plug-upgrade hadron calorimeter, and smaller tiles with $n > 4$, for the CDF plug-upgrade electromagnetic calorimeter.

References

[1] W.K. Sakumoto et al. (CCFR Collaboration), Nucl. Inst. and Meth. A 234(1990) 179.
[2] H. Abramowicz et al. (CDHS Collaboration), Nucl. Inst. and Meth. 180 (1981) 429.
[3] F. Abe et al. (CDF Collaboration), Nucl. Inst. and Meth. A 271(1988) 387.
[4] P. de Barbaro et al. Nucl. Inst. and Meth. A 315(1992) 317.
[5] P. de Barbaro et al., UR-1360 (1994).
[6] P. Auchincloss et al. (CCFR Collaboration), Nucl. Inst. and Meth. A 343(1994) 463.
[7] J. Del-Peso and E. Ross, Nucl. Inst. and Meth. A 276(1989) 459.