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THE HBT- INTERFEROMETRY OF EXPANDING INHOMOGENEOUS SOURCES

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The Bose-Einstein correlations of pions and kaons emitted from expanding inhomogeneous systems that eventually formed in ultra-relativistic A+A collisions are considered. It is shown that if the particle transversal masses are large enough in comparison with the freeze-out temperature, the *long-*, *side-* and *out-* interferometry radii are defined by the corresponding lengths of homogeneity in the radiating systems. The analysis of the momentum behavior of the interferometry radii has been done for the sources with a large temperature gradients and also for intensive relativistic transversal and longitudinal flows. The general structure of the correlation function for a such kind of the sources is found on the base of model-independent analysis. The simple analytical behavior of the *long-*, *side-* and *out-* interferometry radii as depending on the transversal momenta is found for the typical classes of the transversal flows and also for temperature inhomogeneous systems. The model-independent relations between the momentum slopes of the different interferometry radii for 3-dimensionally expanding systems is found.

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1. INTRODUCTION

The smallness of the effective emitting region is the basic condition for the interferometry effect to be revealed experimentally. The systems that form in ultrarelativistic A+A collisions are quasi-macroscopic ones producing $10^3 \div 10^5$ hadrons and are eventually thermalized. They are dense and involved in collective expansion due to contiguity with vacuum. So, the systems can have very small lengths of homogeneity for density, temperature and collective velocities. The spectra and correlations in small thermal relativistic quantum-field systems are not trivial and can be understood on the base of the space-time scales [1, 2]. This includes the total geometrical lengths that the thermalized system occupies, \bar{R}_i , the local lengths of homogeneity (hydrodynamic lengths), $\bar{\lambda}_i$, and the wave-length of the quanta, $\lambda_p \propto 1/p^0$. First it has been shown in Ref. [3] that in contrast to small homogeneous systems, $\bar{\mathbf{R}} = \bar{\boldsymbol{\lambda}}$, when the interferometry radii R coincide with the geometrical radii of the system, $\bar{\mathbf{R}} = \mathbf{R}$, for longitudinally expanding system where $\bar{R}_L \gg \bar{\lambda}_L$, the longitudinal interferometry radius is defined by the hydrodynamic length, $R_L \propto \bar{\lambda}_L$. In Ref. [2] the analogous statement has been proved for 3-dimensionally expanding systems, and the transversal *side-* and *out-* radii as well as the longitudinal one have been expressed through the correspondent lengths of homogeneity.

The attempts to study analytically the Bose-Einstein correlations for 3-dimensionally expanding hadron sources have been recently demonstrated also in the papers [4],[5]. The main shortcoming of the results that was obtained is the absence of the analysis of the region of applicability for the final analytical approximations. As will show in this paper they are suitable for “non-relativistic processes” only, i.e., for low p_T -pions and/or for slow transversal expansion, $\bar{\lambda}_T \gg \bar{R}_T$. Therefore these approximations agree with the general results of [2] in this limit only.

The main aim of this paper is to consider the general case of the relativistic transversal expansion and to find typical analytical approximations of the interferometry radii for the different classes of the transversal flows.

2. THE PHYSICAL ASSUMPTIONS AND FORMALISM

The theory of bosonic spectra and correlation functions for inhomogeneous thermalized systems has been proposed in Ref. [1]. It has been shown that if the wavelength of quanta, $\lambda_p \propto 1/p^0$, in a weakly interacting bosonic gas, is much smaller than the system's length of homogeneity, $\bar{\lambda}$, the Wigner function coincides approximately with the locally equilibrium Bose-Einstein distribution function ,

$$f\{x, p\} = (2\pi)^{-3} [\exp(\beta p \cdot u(x) - \beta\mu) - 1]^{-1} , \quad (1)$$

where $u^\mu(x)$ is the 4-velocity of a hadron gas, $\beta(x)$ is the inverse of the temperature, $\mu(x)$ is the chemical potential. In this paper we limit ourselves by the framework of this approximation. Then the single- and double- particle inclusive spectra are expressed through the product of the thermal averages of the creation and annihilation operators:

$$p^0 \frac{dN}{d\mathbf{p}} = \langle a_p^+ a_p \rangle , \quad p_1^0 p_2^0 \frac{dN}{d\mathbf{p}_1 d\mathbf{p}_2} = \langle a_{p_1}^+ a_{p_1} \rangle \langle a_{p_2}^+ a_{p_2} \rangle + \langle a_{p_1}^+ a_{p_2} \rangle \langle a_{p_2}^+ a_{p_1} \rangle \quad (2)$$

where the averages of the operators are expressed through the Wigner function (with the momentum argument $p = (p_1 + p_2)/2$) by the integration of it over freeze-out hypersurface σ :

$$\langle a^+(p_1) a(p_2) \rangle_\sigma = \int d\sigma_\mu p^\mu e^{i\Delta p \cdot x} f(x, p) \quad (3)$$

The expression (3) describes the operator's average also when the hypersurface σ is an arbitrary hypersurface that is situated within of the light cone of the future as to the decoupling 4-volume [1]. The latter means the collection of space-time points that correspond to the points of the last scattering of particles. In the general case the Wigner function $f_W(x, p)$, of course, has much more complicated form than the Bose-Einstein distribution on the freeze-out hypersurfaces. So, the calculations simplify greatly if the decoupling volume is narrow enough and one can consider it as the freeze-out hypersurface. If it is not the case, the different prescriptions are used, such as the introduction of a particle emission function instead of the Wigner function and integration of it over the all decoupling 4-volume instead of over the hypersurface according to Eq. (3). Partially, at least for single-particle spectra, this phenomenological prescription can be derived by means of the averaging of the Eq.(3) over different freeze-out hypersurfaces that can change from event to event. But it is the problem to prove

this prescription in the framework of the thermal QFT for an essentially 4-dimension decoupling volume in every collision event.

We will base on the general equation (3) to estimate in the main approximation the Bose-Einstein correlation function of the thermalized boson sources with 3-dim relativistic flows. We suppose that the decoupling 4-volume is narrow enough. This means for cylindrically symmetric sources expanding longitudinally in the boost-invariant manner, that at the initial stage when the system is very dense inside the hydrodynamic tube, the hadron emission occurs mostly near the surface of the tube. At the final stage the decay of the rest of the hydrodynamic tube can occur even simultaneously. So, the freeze-out hypersurface $\tau(r)$ is expected to be time-like one near the initial system's radius, $r \approx r_i$ (or at proper time, $\tau = \sqrt{t^2 - x_L^2} \approx \tau_i$) and space-like one at the final stage, $r \approx 0$ ($\tau \approx \tau_f$). We do not average over different hypersurfaces σ that may change from event to event and will use the average one. Our approach corresponds to the main approximation based on the saddle point method. Within the approximation we can ignore also the problem with negative particle density for some momenta p that may happen for time-like parts of the freeze-out hypersurface. (As to the latter problem, see details in Ref. [6]). At the same time to simplify our consideration we will use the gaussian distribution

$$\rho(r) \propto \exp(-r^2/2\bar{R}_T^2) \quad (4)$$

for the transversal size of the system instead of the fixing the constant radius as the average radius \bar{R}_T in the all initial expressions. It does not change essentially the results. If we suppose the average freeze-out hypersurface to be $\bar{\tau}(r) = \tau_f = const$ we can also to average round the mean value $\bar{\tau}_f$ with the weight $\propto \exp[-((\tau - \bar{\tau}_f)^2 / 2\Delta\tau^2)]$ (see, e.g., [4],[5]). It does not change essentially the results in the main approximation if $\Delta\tau / \bar{\tau}_f \ll 1$. However, in such the approach we lose the correlation between a radius and a time of decay. The correlations appear when the radiation process continuous during the matter evolution. There is the dependence between the radius and decoupling time, $\tau(r)$, because at each time τ most of the particles leave the system near its surface, at least at the initial stage. It is important to consider this realistic scenario that includes, of course, constant proper time freeze-out, $\bar{\tau}(r) = const$, as the specific case. We will not specify the concrete form of freeze-out hypersurface $\tau(r)$ and will

consider the problem in the main approximation: $\tau_f(r) = \bar{\tau}(r)$ (without averaging over different surface-trajectory $\tau(r)$).

In the cylindrically symmetric model with longitudinal boost-invariance we use the following notations:

$\tau(r) = \sqrt{t^2 - x_L^2}$ is the freeze-out proper time, $y_L = \text{arctanh } v_L = \frac{1}{2} \ln \frac{t+x_L}{t-x_L}$ is the longitudinal rapidity; $y_T \equiv y_T(r) = \text{arctanh } v_T$ is the transversal rapidity; \mathbf{r} is 2-dim transversal radius-vector, $\frac{d\tau}{dr} = \frac{d\tau}{dr} \frac{\mathbf{r}}{r}$; $x^\mu = (\tau \cosh y_L, \mathbf{r}, \tau \sinh y_L)$; the 4-velocity of the decaying fluid is $u^\mu(x) = \left(\cosh y_L \cosh y_T, \frac{\mathbf{r}}{r} \sinh y_T, \sinh y_L \cosh y_T \right)$. The standard representation of the particle 4-momenta through longitudinal rapidity is used: $p^\mu = (m_T \cosh \theta, \mathbf{p}_T, m_T \sinh \theta)$.

We start from the Eq. (3) with corresponding integral measure

$$d\sigma_\mu = \tau(r) dy_L d^2\mathbf{r} \left(\cosh y_L, \frac{d\tau}{dr}, -\sinh y_L \right) \quad (5)$$

The function $f(x, p)$ in Eq.(3) taking into account the finite average transversal radius of the system has the form $f(x, p) = f_W(x, p)\rho(r)$. To simplify the analytical analysis we introduce the weekly modified argument in gaussian form (4) for $\rho(r)$. In relativistic covariant form it looks like

$$\tilde{\rho}(r) \propto \exp \left[-\frac{\alpha}{2} (u(r, y_L) - u(0, y_L))^2 \right] = \exp \left[-\alpha (\cosh y_T(r) - 1) \right] \quad (6)$$

Because of the cylindrical symmetry $v_T(0) = y_T(0) = 0$ and putting $\frac{dv_T(0)}{dr} \equiv \frac{1}{R_v}$ we have at small $r \ll R_v$,

$$y_T(r) \approx v_T \approx v_T(0)r = \frac{r}{R_v} \quad \text{and} \quad \cosh y_T(r) \approx 1 + \frac{1}{2} y_T^2(r) \approx 1 + \frac{1}{2} \frac{r^2}{R_v^2}$$

So, in nonrelativistic approach for transversal expansion, $\bar{R}_T < R_v$, the distributions (4) and (6) are coincided inside the hydrodynamic tube $r < \bar{R}_T$, $\tilde{\rho}(r) \cong \rho(r)$. The physical meaning of the parameter α is then:

$$\alpha = R_v^2 / \bar{R}_T^2 \quad (7)$$

The numerical analysis with the prescriptions (4) and (6) for different types of the transversal flows demonstrates their closeness in the typical momentum region for the current interferometry analysis even for essentially relativistic flows. The main advantage of the distribution (6) is that it allows to give the analytical solution of the problem. So, the second factor in Eq. (3) we are fixing now is the function $f(x, p)$. At the $\beta p^0 \gg 1$ it looks like:

$$f(x, p) = f_W(x, p) \tilde{\rho}(r) \propto \frac{1}{(2\pi)^3} \exp[-\beta(r)(p_0 \cosh y_L \cosh y_T(r) - \mathbf{p}_T \frac{\mathbf{r}}{r} \sinh y_T(r) - p_L \sinh y_L \cosh y_T(r) - \mu(r))] \exp\left[-\frac{R_v^2}{R_f^2} (\cosh y_T(r) - 1)\right] \quad (8)$$

The third factor in the basic formula (3) is the exponent. To rewrite it in the convenient form let us choose one of the transversal axis, \mathbf{x}_o along the vector \mathbf{p}_T . It calls the *outward* - direction. The second transversal axis, \mathbf{x}_s , is chosen to be orthogonal to the first one. It calls the *sideward*-direction. The longitudinal axis, \mathbf{x}_L , is directed along the collision axis. In this coordinate system the exponent in Eq.(3) looks like:

$$\exp[i\Delta p \cdot x] = \exp\left[-i(q_{out} g_{out} + q_{side} g_{side} + q_{long} g_{long})\right] \quad (9)$$

where q_{out} , q_{side} , q_{long} are the momentum difference in the corresponding directions and

$$g_{out} = \left(x_o - t \frac{p_T}{p_0}\right) \quad g_{side} = x_s \quad g_{long} = \left(x_L - t \frac{p_L}{p_0}\right) \quad (10)$$

3. MODEL-INDEPENDENT STRUCTURE OF THE CORRELATION FUNCTIONS

The form of the correlation function follows from Eqs. (2)

$$C(p_1, p_2) = 1 + \left\langle a_{p_1}^+ a_{p_2} \right\rangle \left\langle a_{p_2}^+ a_{p_1} \right\rangle / \left\langle a_{p_1}^+ a_{p_1} \right\rangle \left\langle a_{p_2}^+ a_{p_2} \right\rangle \quad (11)$$

The average of operators is defined by the Eqs. (3),(5),(8),(9). To calculate Eq.(11) we use the saddle-point method at large parameter $\beta p_0 \gg 1$ in the Wigner function (1). Then (see Appendix) the correlation function is:

$$C(p, q) = 1 + \exp \left[- \sum_i \left(\left(-\beta(0) p_0 \frac{\partial^2 S(x)}{\partial x_i^2} \right)^{-1/2} \frac{\partial}{\partial x_i} \sum_j q_j g_j(x) \right) \Big|_{x=\bar{x}} \right]^2 \quad (12)$$

where

$$S(x) \beta(0) p_0 = \ln \left[(2\pi)^3 f(x, p) \right], \quad (13)$$

x_i are x_o, x_s or x_L ; \bar{x} is the saddle point that is defined from the extreme conditions and the conditions for the sign of the second derivations:

$$\frac{\partial S(x)}{\partial x_o} = \frac{\partial S(x)}{\partial x_s} = \frac{\partial S(x)}{\partial y_l} = 0. \quad (14)$$

It is immediately follows from Eqs. (8),(13),(14) that:

$$x_s = 0 \Rightarrow \bar{r} = \bar{x}_o; \quad \tanh \bar{y}_L = p_L/p_0 \Rightarrow \bar{y}_L = \theta = (\theta_1 + \theta_2). \quad (15)$$

As to the saddle point \bar{r} , it depends on the concrete form of the function $\beta(r)$ and $y_T(r)$. Now we will find the model-independent expressions for $C(p, q)$ using the conception of the length of homogeneity as it was firstly proposed in Refs. [1, 2].

The peculiarities of the spectra and correlations as we will show are strongly dependent on the ratio between different space-time lengths inherent in the system. The local length of homogeneity $\bar{\lambda}(x_0)$ is defined by the behavior of the Wigner function. It means the length within which the deviation of the Wigner function is relatively small and is about the function value

$$\frac{|f(p, x_0 + \bar{\lambda}) - f(p, x_0)|}{f(p, x_0)} = 1 \quad (16)$$

So the lengths of homogeneity at the point $\bar{x}(p)$ where the distribution function is maximal, i.e., Eqs. (14) are satisfied, are

$$(\bar{\lambda}_i(\bar{x}))^{-2} = \left| \frac{\partial^2 f(p, x)}{\partial^2 x_i} \right| \Big|_{\bar{x}(p)} / 2f(p, x) = -\frac{1}{2} \frac{\partial^2 S(\bar{x})}{\partial x_i^2} \beta(0) p_0 \quad (17)$$

where $i = \text{long, side, out}$. Using Eqs. (10), (12) and (15), (17) we have for the correlation function:

$$C(p, q) = 1 + \exp \left[-\frac{1}{2} R_{out}^2 q_{out}^2 - \frac{1}{2} R_{side}^2 q_{side}^2 - \frac{1}{2} R_{long}^2 q_{long}^2 + R_{out, long}^2 q_{out} q_{long} \right] \quad (18)$$

where

$$R_{out}^2 = \bar{\lambda}_{out}^2 \left(1 + \frac{p_T}{m_T} \left| \frac{d\tau}{dr}(\bar{r}) \right| \right)^2 + \bar{\lambda}_{long}^2 \left(\frac{p_T \tanh \theta}{m_T \cosh \theta} \right)^2, \quad R_{side}^2 = \bar{\lambda}_{side}^2 \quad (19)$$

$$R_{long}^2 = \bar{\lambda}_{long}^2 \cosh^{-4} \theta, \quad \bar{R}_{out, long}^2 = \bar{\lambda}_{long}^2 \left(\frac{p_T}{m_T} \right) \sinh \theta \cosh^{-4} \theta$$

The existence of the crossing *out-long* term in the correlation function (18) was recently discussed in Ref.[5].

The model-independent expressions (18), (19) for the correlation function $C(p, q)$ were derived in the saddle-point method approximation. In the framework of this approach the freeze-out trajectory $\tau(r)$ in (19) is treatment as the most probable one reflecting the dynamics of the boson emission process.

Let us consider some general consequences of the results represented by the Eqs. (18), (19). In the general case we have $R_{side} \neq R_{out}$ even in LCMS where $p_L = 0$ ($\theta \approx 0$). It is follow from Eq. (19)

$$R_{out}^2 = R_{*,out}^2 + \bar{R}_{long}^2 \left(\frac{p_T}{m_T} \right)^2 \sinh^2 \theta \quad (20)$$

where $R_{*,out} = R_{out}(p_L = 0) = \bar{\lambda}_{out}(\bar{x}) + \frac{p_T}{m_T} \Delta\tau$ and $\Delta\tau = \bar{\lambda}_{out}(\bar{x}) \left| d\tau(r)/dr \right|$ is the duration of the radiation in the period of its maximal intensity. It characterizes the time interval when the main number of the particles emitted and does not coincide with the total life-time of the system: $\Delta\tau < \tau_f - \tau_i$.

The form of the crossing *out-long* term is obvious from Eq.(19):

$$R_{out, long}^2 = R_{long}^2 \frac{p_T}{m_T} \sinh \theta \quad (21)$$

In boost-invariant approach it is equal to zero if $p_L = 0$ or $p_T = 0$. In other situations it is present in the correlation function. If one is doing the interferometry analysis in the LCMS

($\theta_{*,i} = 0$) for each bin in Lab system: ($\theta_i - \Delta\theta, \theta_i + \Delta\theta$), ($p_{T,i} - \Delta p_T, p_{T,i} + \Delta p_T$), it is possible to neglect of this crossing term if $\frac{p_{T,i} \sinh \Delta\theta}{m_{T,i}} \ll 1$.

4. THE LENGTHS OF HOMOGENEITY FOR TYPICAL PHYSICAL SCENARIO

The model-independent expressions (18), (19) define the correlation functions by means of the lengths of homogeneity in the decaying system. Now we analyze them in physically interesting situations.

1. The systems with a large temperature inhomogeneity.

Let us begin from the scenario, when the transversal lengths of homogeneity are formed mostly by the large temperature gradient. Then the saddle point $\bar{r} = 0$ and when the temperature decreases as gaussian, $\beta(r) = \beta(0) \exp[r^2/2R_\beta^2]$, when the transversal radius increases we have:

$$\bar{\lambda}_{out}^2 = \bar{\lambda}_{side}^2 = 2 \left(\frac{\beta(0)m_T}{R_\beta^2} + \frac{1}{\bar{R}_T^2} \right)^{-1}, \quad \bar{\lambda}_{long}^2 = \frac{2\bar{\tau}^2 \cosh^2 \theta}{\beta(0)m_T} \quad (22)$$

$$p_0 \frac{d^3 N}{d^3 p} \propto e^{-\beta(0)m_T}$$

If the ‘‘temperature’’ length of homogeneity is much less than the average transversal radius, $R_\beta \ll \bar{R}_T$, the *out*-, *side*- and *long*- interferometry radii are decreasing as inverse of $\sqrt{m_T}$ when p_T increases.

2. The system with large velocity inhomogeneity.

The most realistic one is scenario when freeze-out hypersurface is characterized by the almost constant temperature. If such a system possesses the essential transversal flows, it is easy to obtain the lengths of homogeneity using Eqs. (14), (17):

$$\lambda_{out}^2 = \frac{2}{\beta m} \left[y_T^2(\bar{r})(\sinh(\bar{y}_T - \eta_T) + \frac{\alpha}{\beta m} \sinh \bar{y}_T) + (y_T^2(\bar{r})(\cosh(\bar{y}_T - \eta_T) + \frac{\alpha}{\beta m} \cosh \bar{y}_T) \right]^1$$

$$\bar{\lambda}_{side}^2 = \frac{2\bar{r}^2}{\beta p_T \sinh \bar{y}_T} = \frac{2\bar{r}^2}{\beta p_T \bar{v}_T} (1 - \bar{v}_T^2)^{\frac{1}{2}} \quad (23)$$

$$\bar{\lambda}_{long}^2 = \frac{2\bar{\tau}^2 \cosh^2 \theta}{\beta m_T \cosh \bar{y}_T} = \frac{2\bar{\tau}^2 \cosh^2 \theta}{\beta m_T} (1 - \bar{v}_T^2)^{\frac{1}{2}}$$

and the spectra

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp \left[-(\beta m_T + \alpha)(1 - \bar{v}_T^2)^{\frac{1}{2}} \right] \quad (24)$$

where α is defined by the Eq.(7), $\bar{y}_T \equiv y_T(\bar{r})$, η_T is the average rapidity of the pair, $\tanh \eta_T = p_T/m_T$. It is important to note, that the transversal velocity at the saddle point as well as the longitudinal length of homogeneity, $\bar{\lambda}_{long}(\bar{x})$, and exponential factor in the single-particle spectra does not depend on a concrete model of transversal flows (i.e., on the form of the function $v_T(r)$):

$$\bar{v}_T \equiv \tanh \bar{y}_T = \frac{\beta p_T}{\beta m_T + \alpha} \quad (25)$$

The *out*- and *side*- lengths of homogeneity depend from concrete model of the transversal expansion. But in the two limited situation we can do some conclusion concerning of its behavior.

- Nonrelativistic transversal flows.

The corresponding condition is:

$$\bar{v}_T = \frac{\beta p_T}{\beta m_T + \alpha} \ll 1 \Rightarrow \bar{v}_T \approx y_T(\bar{r}) \approx \frac{\bar{r}}{R_v} \ll 1 \quad (26)$$

The parameter $\alpha = R_v^2/\bar{R}_T^2$ according to (7). Using this conditions we can get from Eqs. (23),(24) :

$$\bar{\lambda}_{out}^2 = \bar{\lambda}_{side}^2 \cong 2 \left(\frac{\beta m_T}{R_v^2} + \frac{1}{R_T^2} \right)^{-1}, \quad \bar{\lambda}_{long} \cong \frac{2\bar{\tau}^2 \cosh^2 \theta}{\beta m_T}, \quad p_0 \frac{d^3 N}{d^3 p} \propto e^{-\beta m_T} \quad (27)$$

It is important to emphasize, that according to Eq. (26) the applicability of the expressions (27) in the relativistic momentum region, $p_T \approx m_T$, is limited by the conditions $\beta m_T/R_v^2 \ll 1/\bar{R}_T^2$. If this conditions is violated, the behavior or the hydrodynamic lengths, as we will show, changes dramatically as compare with the results (27). In the region $\beta m_T \bar{R}_T^2 > R_v^2$ the predictions in the papers [4,5] that based actually on the formulas (27) without conditions of its applicability are incorrect. In particularly, the predictions of Ref.[4] as to the inverse of square root m_T -decreasing of the *out*-, *side*- and *long*-radii in relativistic momentum region are incorrect. They need the condition $\beta m_T \bar{R}_T^2 \gg R_v^2$ which strongly violates the applicability of the results (27). (In our signification the parameters in the Ref. [4] are

$R_v \equiv \tau_0$, $\bar{R}_T \equiv R_G$, $\alpha = \tau_0^2/R_G^2$). In Ref.[5] (where $\alpha \equiv 1/v^2$) the obvious violation of the condition (26) is happens also when the single-particle spectra is considered.

- Ultrarelativistic transversal flows

In the framework of the ultrarelativistic approach we suppose $\bar{v}_T \approx 1$ that means according to Eqs. (25), (7) that $\beta m_T/R_v^2 \gg 1/\bar{R}_T^2$ and $\bar{v}_T \approx p_T/m_T = \tanh \eta_T \approx 1$. Then we have

$$\bar{\lambda}_{out}^2 \cong \frac{2}{\beta m \bar{y}_T^2}, \quad \bar{\lambda}_{side}^2 \cong \frac{2\bar{r}^2 m}{\beta p_T^2}, \quad \bar{\lambda}_{long}^2 \cong \frac{2m\bar{\tau}^2 \cosh^2 \theta}{\beta m_T^2}, \quad p_0 \frac{d^3 N}{d^3 p} \propto e^{-\beta m \sqrt{1+2\alpha m_T/m}} \quad (28)$$

As distinct from $\bar{\lambda}_{long}(\bar{x})$ that does not depend on the transversal velocity distribution (see Eq. (23), (25)) and demonstrates the inverse of m_T decreasing for strongly relativistic flows, the momentum behavior of the transversal *out*- and *side*- lengths of homogeneity depends on a concrete model of transversal flows, $v_T(r)$ (or $y_T(r)$). However, it is possible to show that for monotonously growing functions $y_T(r)$ without singularity the ratio *out*- to *side*- lengths of homogeneity in the ultrarelativistic limit has the form (with accuracy to the powers of $\ln(p_T/m)$)

$$\frac{\bar{\lambda}_{out}(\bar{x})}{\bar{\lambda}_{side}(\bar{x})} \propto \frac{p_T}{m} \quad (29)$$

The detail behavior of the transversal lengths of homogeneity and the radii depends on a concrete model of the transversal expansion. We will demonstrate in the two typical scenarios.

5. THE MOMENTUM DEPENDENCE OF THE INTERFEROMETRY RADII FOR 3-DIMENSIONAL EXPANSION

I. First we consider the scenario with “hard” transversal flows:

$$y_T(r) = \frac{r}{R_v} \quad (30)$$

Then we have from Eqs.(23), (25) the solution for the saddle point:

$$\bar{r} = \frac{R_v}{2} \ln \left(\frac{1+\bar{v}_T}{1-\bar{v}_T} \right) \quad (31)$$

and for the lengths of homogeneity:

$$\bar{\lambda}_{out}^2 = \frac{2R_v^2}{\beta m_T + \alpha} (1-\bar{v}_T^2)^{-\frac{1}{2}}, \quad \bar{\lambda}_{side}^2 = \frac{R_v^2}{2\beta p_T \bar{v}_T} (1-\bar{v}_T^2)^{\frac{1}{2}} \ln^2 \left(\frac{1+\bar{v}_T}{1-\bar{v}_T} \right) \quad (32)$$

The transversal velocity is expressed through the momentum variables by the Eq.(25).

II. Let us consider model, realizing scenario of “soft” transversal flows:

$$y_T(r) = \ln \frac{\sqrt{R_v^2 + r^2} + R_v}{R_v} \Rightarrow v_T(r) = \frac{r}{\sqrt{r^2 + R_v^2}} \quad (33)$$

The model of the such type has been used in Ref. (4) with $R_v \equiv \tau$. Then we have from Eqs.(23), (25) the solution for the saddle point

$$\bar{r} = R_v \frac{\bar{v}_T}{(1 - \bar{v}_T^2)^{1/2}} \quad (34)$$

and homogeneity lengths

$$\bar{\lambda}_{out}^2 = \frac{2R_v^2}{\beta m_T + \alpha} (1 - \bar{v}_T^2)^{-\frac{3}{2}}, \quad \bar{\lambda}_{side}^2 = \frac{2R_v^2}{\beta m_T + \alpha} (1 - \bar{v}_T^2)^{-\frac{1}{2}} \quad (35)$$

Note that $\bar{\lambda}_{out}(\bar{x})$ for “hard” flows coincides formally with $\bar{\lambda}_{side}(\bar{x})$ for “soft” flows. The slope k of the transversal spectra $\propto \exp(-k m_T)$ in the region $m_T \approx 1$ GeV is $k \approx 5$ GeV⁻¹ from 200A GeV S+S collisions [7]. According to the Eq. (24) the slope $k = T^{-1} \sqrt{1 - \bar{v}_T^2}$, \bar{v}_T is defined by the Eq. (25). Then the experimental value of the slope k corresponds to $\alpha \approx 3$ if $T \equiv T_{f.o.} \approx m_\pi$. The corresponding *out-*, *long-* and *side-* lengths of homogeneity in LCMS are demonstrated in Figs.1-3 for the both types of the flows: “hard” and “soft”. The applicability region of the result presented is limited by the validity of the saddle point method and is approximately $p_T > m_\pi$. We show also the results for $\alpha = 15$ that correspond to more developed flows which one can expect for collisions with heavier nucleus. We choose there $\bar{R}_T = 3.9$ fm, $\bar{\tau} = 4.35$ fm / c . If $\frac{d\tau}{dr}(\bar{r}) \ll 1$, these lengths of homogeneity correspond to the interferometry radii: R_{out} , R_{side} and R_{long} .

6. CONCLUSIONS

We show that the experimentally observed particle spectra and correlation are very sensitive to the space-time structure of the emitting matter. The ratios between local lengths of homogeneity of the system and its geometrical sizes define the behavior of the spectra and correlations. This is reflected in different behavior of the interferometry radii as depending on momentum regions. In particular, the strong transversal expansion leads to more quick than in-

verse of $\sqrt{m_T}$ decreasing of the longitudinal interferometry radius. At the same time the side-radius decreases more quickly than the out-radius when the transversal momentum increases. The simple analytical behavior of the *long*-, *side*- and *out*- interferometry radii as depending on the transversal momenta has been found for typical classes of the transversal flows moreover for temperature inhomogeneous systems. The analytical approximations for spectra and correlations allow to clear up experimentally the character of the transversal flows in the systems formed in ultra-relativistic nucleus-nucleus collisions and the details of its evolution.

7. APPENDIX

To calculate the integral in the expression (3) for the correlation function (11) with Eqs. (5),(8),(9) we use the saddle point method at the large parameter $\lambda = p^0 \beta \gg 1$ in the distribution function (8). Then the structure of the integral we are interested in is (see, e.g., [8]):

$$F_\lambda(\Delta p) = \int \varphi(x) \exp[\lambda S(x)] e^{i\Delta p g(x)} dx \xrightarrow{\lambda \rightarrow \infty} \exp[\lambda S(\bar{x})] \sum_{k=0}^{\infty} c_k \lambda^{-k-\frac{1}{2}} \quad (\text{A1})$$

where in our case $\lambda S(x) = \ln((2\pi)^3 f(x))$, saddle point \bar{x} is defined from the equations $S'(x_0) = 0$, $S''(\bar{x}) < 0$ and

$$c_k = \frac{\Gamma\left(k + \frac{1}{2}\right)}{(2k)!} \left(\frac{d}{dx}\right)^{2k} \left[\varphi(x) \left(\frac{S(\bar{x}) - S(x)}{(x - \bar{x})^2}\right)^{-k-\frac{1}{2}} e^{i\Delta p g(x)} \right]_{x = \bar{x}}$$

Leaving the maximal coefficient at each power of Δp in the expansion of $|F_\lambda(\Delta p)|^2$, one can sum series (A1) up and find

$$|F_\lambda(\Delta p)|^2 = \left| \frac{2\pi}{\lambda S''(\bar{x})} \right| \varphi^2(\bar{x}) \exp[2\lambda S(\bar{x})] \exp\left[\frac{\Delta p^2 (g'(\bar{x}))^2}{\lambda S''(\bar{x})} \right] \quad (\text{A2})$$

In our case we have 3-dimensional integration in Eq. (3). However, if the mixed derivations are equal to zero or small in comparison with other ones, the saddle point method is reduced to the consequent using of this approach for each variable of the integration. After that we have for the correlation function the expression (12).

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FIGURE CAPTIONS

Figure 1: The p_T - behavior of the transversal *out-* and *side-*interferometry radii for “hard” (I) and “soft” (II) transversal expansion. The *out-*radius for the I-st class of the flows coincides with the *side-*radius for the II-nd one. The ratio of the average transversal hydrodynamic length to the average transversal radius of a system is $R_v/\bar{R}_T = \sqrt{\alpha} = \sqrt{3}$. The freeze-out temperature $T = m_\pi$, $\bar{R}_T = 3.9$ fm.

Figure 2: The p_T - behavior of the transversal *out-* and *side-*interferometry radii for “hard” (I) and “soft” (II) transversal expansion. The *out-*radius for the I-st class of the flows coincides with the *side-*radius for the II-nd one. The ratio of the average transversal hydrodynamic length to the average transversal radius of a system is $R_v/\bar{R}_T = \sqrt{\alpha} = \sqrt{15}$. The freeze-out temperature $T = m_\pi$, $\bar{R}_T = 3.9$ fm.

Figure 3: The p_T - behavior of the longitudinal *long-* radius that are universal for both classes of the transversal flows. The results are presented for $\alpha = 3$ and $\alpha = 15$. The comparison with the *long-*radius without transversal flows ($\alpha = \infty$) is presented. The proper time of the longitudinally expanding source when the emission is maximal is $\bar{\tau} = 4.35$ fm/c.

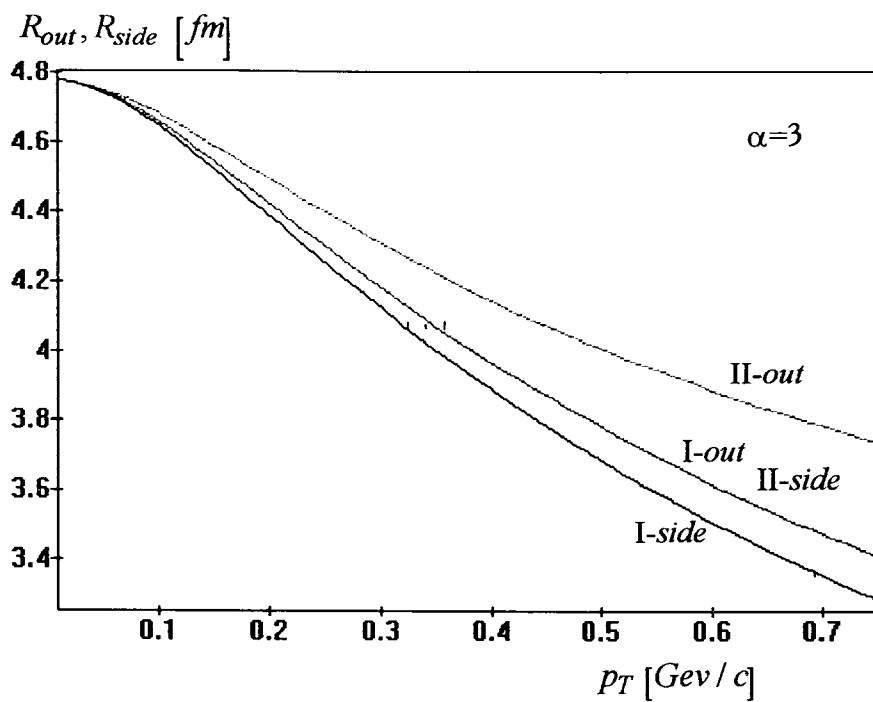


Fig. 1

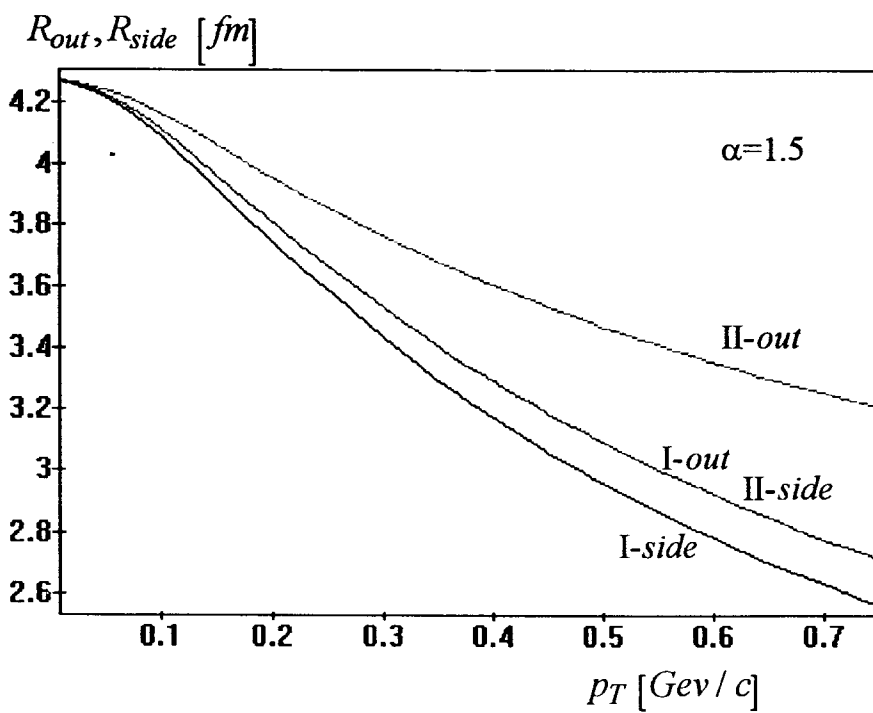


Fig. 2

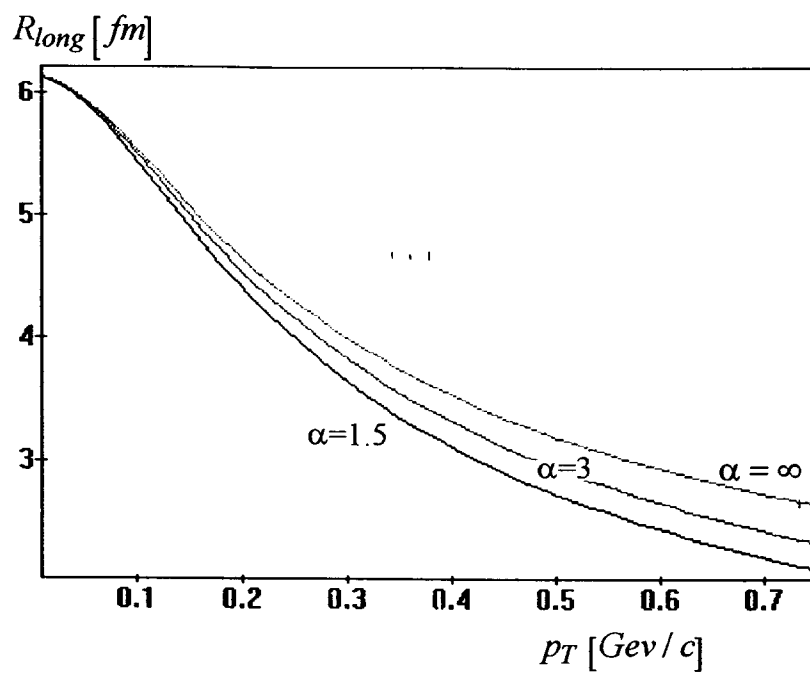


Fig. 3