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thys.risselada@cern.ch

New Ideas for the MAD-X Aperture Calculation

Thys Risselada
CERN, CH-1211 Geneva, Switzerland

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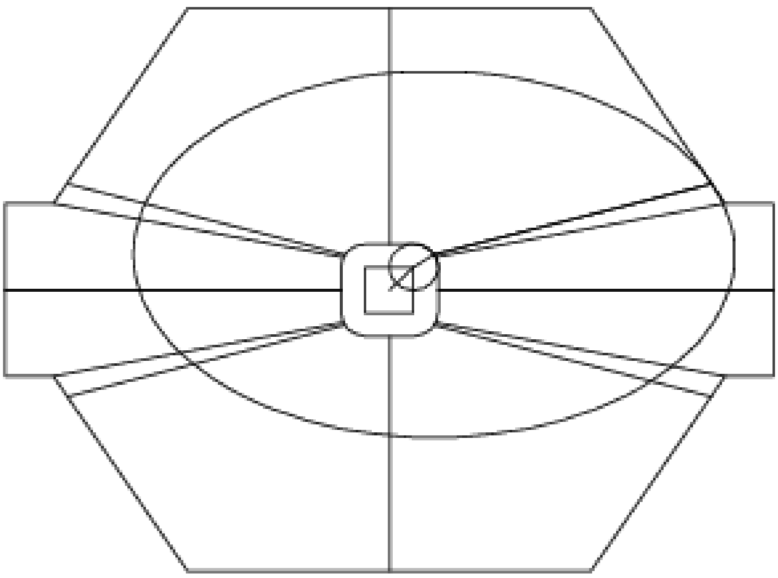
Summary

If a beam screen shape is made up of straight and circular segments the N1 values may be calculated exactly for each segment by a simple geometric construction.



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1 Introduction

The MAD-X [1] Aperture Module uses nested loops over the orientation angles of three vectors involved in the calculation of the N1 value of a given physical element. The number of angles requested by the user determines the accuracy of the aperture evaluation.

Doing so, it ignores the information concerning the intervals between the examined angles. The knowledge of how a beam screen shape is made up of straight and circular segments, together with their end point coordinates themselves, contains already sufficient information to find the relevant angle values directly, so loops over the angles are no longer required.

In the proposed calculation each beam screen segment of a given physical element is then analyzed separately, in the absence of the other segments. The smallest N1 value of all segments is the N1 value of the element. One of the advantages is that this method takes automatically care of exotic (e.g. concave, see below) screen shapes.

The LHC beam screen shape contains both straight and circular segments [2]. In the HE-LHC study the screen shape [3] is modeled by line segments only.

2 The tolerance envelope with linear segments

The aperture module evaluates the room left for the betatron motion of the beam between the tolerance envelope and the mechanical aperture limit.

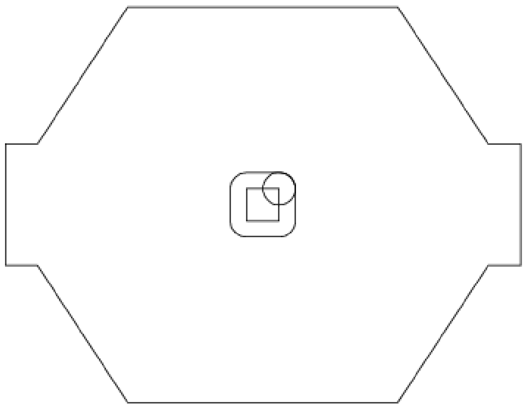

Figure 1: Combining circular and rectangular tolerances.

As in MAD-X, in the proposed method all H/V and radial tolerances are grouped together in the center:

- H/V mechanical tolerances, $D_x \times \Delta p/p$ and $D_y \times \Delta p/p$ in the rectangle
- radial mechanical tolerance and orbit distortion in the circle

The tolerance envelope is the convolution (Minkowski Addition) of the rectangle and the circle. It is the locus of points outside the rectangle all having a same distance (equal to the sum of the radial tolerances) to this rectangle. This envelope contains only straight and circular segments.

3 Choose the radial and rectangular tolerance vectors

It is possible to determine *a priori* the orientations of the radial and the rectangular tolerance vectors to the total tolerance vector, thus allowing to find the correct N1 value:

- choose the rectangle corner closest to the beam screen. If two corners are equally close the point halfway between them must be chosen
- the radial tolerance vector starts from this point, and must be chosen perpendicular (in X-Y space) to the beam screen

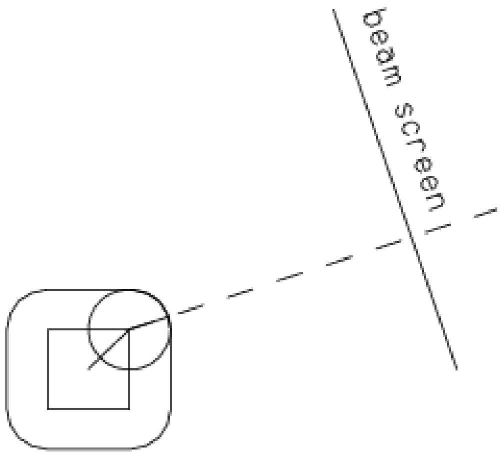

Figure 2: The chosen tolerance vectors in physical coordinates

The next step is to find the point where the betatron ellipse will touch the beam screen. This requires the use of *normalized* coordinates.

3.1 The betatron beam ellipse in normalized coordinates

The X and Y coordinates are divided by the X and Y betatron sigmas of the beam. The betatron ellipse is now a circle and the radius pointing to the touch-point is perpendicular to the beam screen.

This radius corresponds to the shortest distance in sigma space between the tolerance envelope and the beam screen. After the rectangular and the radial tolerance vectors, this N1 vector completes the picture. The result is exact.

3.2 If the touch-point lies outside the segment limits

This may happen to the shorter segments. The segment end point closest to the tolerance envelope then becomes the new touch-point.

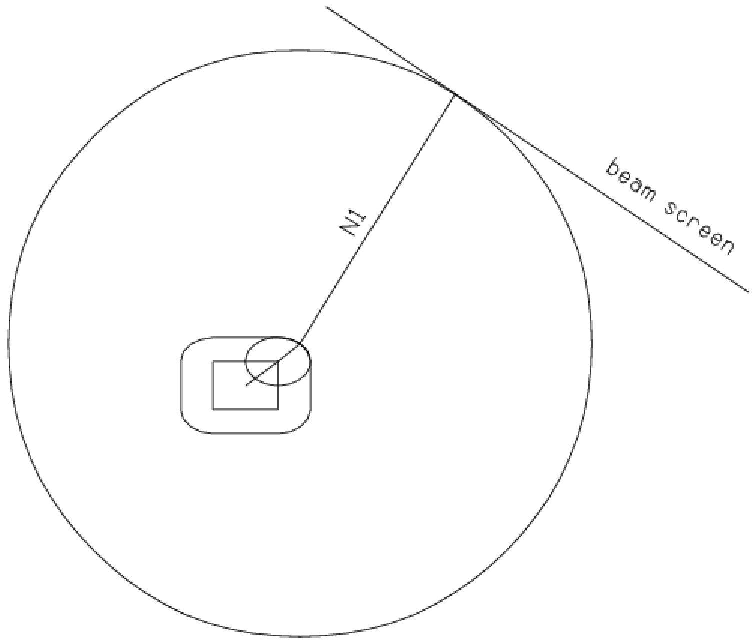

Figure 3: the two tolerance vectors and the N1 vector in normalized coordinates

In this case a second analysis is required to reestablish the smallest distance between this new touch-point and the tolerance envelope, as the starting point on the tolerance envelope will be displaced.

A circle is drawn around the new touch-point and its radius is adjusted until it just touches the tolerance envelope, by using an iterative method calculating the distance of a given point to a given ellipse (for the circular parts of the tolerance envelope) and to a given line (for the straight parts). This also determines the new orientation of the radial tolerance vector.

This procedure works for any length and position of the segment, so also concave screen shapes (see below) are handled correctly and do not require a special treatment.

A screen shape is called concave if it is indented towards the beam pipe center at one or more segment junctions. Such junction points (“CC” in Fig. 5) are likely to be touched by the betatron envelope.

On the other hand, a junction indented outwards (”CV”) will never be touched itself by the beam, and the beam envelope can only touch one of the two adjacent segments. The N1 value of the *element* will thus anyway be smaller than the N1 of this junction, so here it is not necessary to apply the above mentioned iterative method relocating the starting point of the N1 vector on the tolerance envelope.

3.3 The tolerance envelope with circular segments

The beam screen shape of the LHC is limited by both a rectangle and a circle, with dimensions producing only two straight and two circular segments. The N1 of the circular

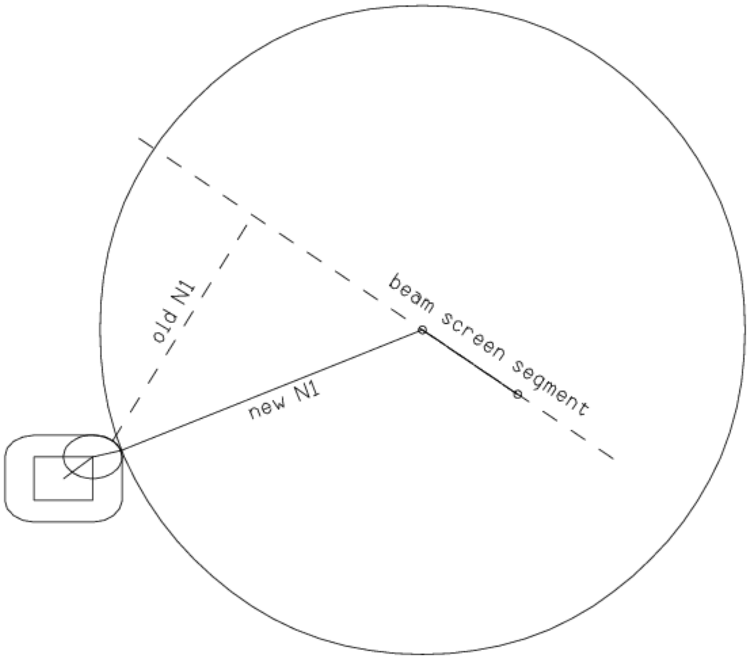

Figure 4: Case of a touch-point outside the segment limits

segments may be determined by the above mentioned iterative method using the normalized coordinates.

The geometric construction may be further simplified by removing the radial tolerances from the central tolerance envelope and subtracting them directly from the beam screen radius (see small circle in Fig. 6), as both shapes are perfectly circular.

With a circular screen segment it is mandatory to locate the starting point of the N1 vector on the tolerance envelope at one of its corners. Furthermore, in the iterative method the initial azimuth angle value must be chosen with great care, particularly with round beams as N1 is then almost independent of the azimuth angle. Without this precaution the N1 vector might end up in the wrong quadrant.

4 Truncated halo shapes: $halo_X < halo_R, halo_Y < halo_R$

Truncated halo shapes were useful when only H and V collimators were used. With the generalization of skew collimators the halos became more circular, and truncated halos are now less used in aperture studies. They are not used for the LHC. "Betatron" and "halo" there have the same meaning.

With truncated halos the azimuth angles of the halo envelope are divided into 3 zones (Fig. 7). In zones 1 and 3 the ellipses are truncated by vertical or horizontal lines. The N1 calculation is 1-dimensional. In zone 2 the halo envelope has the usual elliptical shape.

On long enough segments the beam will touch either inside zone 2 or at a transition between zones (1-2, or 2-3 as in Fig. 7), depending on the slope of the beam screen segment.

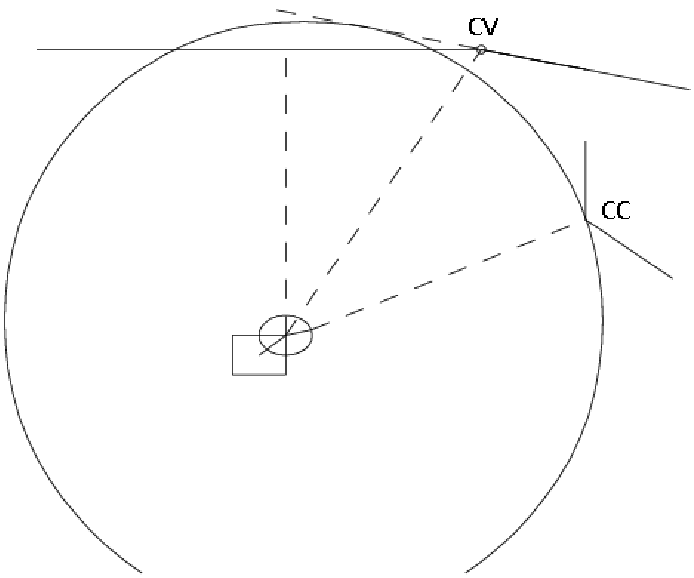

Figure 5: Examples of concave and convex segment junctions

In zone 2 the previously shown method for non-truncated ellipses may again be applied.

In zones 1 or 3 the touch-point on the beam screen will be found at the angle of the zone transition. As previously, for a too short segment the touch-point will be displaced to one of its endpoints.

In both these latter cases the touch-point is known, but not the starting point on the tolerance envelope. Its position must then be recalculated using the above mentioned iterative method.

As with elliptic halos, the concave screen shapes are handled correctly also in the case of truncated halos and do not require a special treatment.

5 Comparison with MAD-X version 5.05.02 (July 2019)

5.1 Elliptic halos

The agreement with MAD-X is perfect for purely horizontal and vertical beam screen segments. For circular segments the agreement is also rather good, as the N1 value then depends in a smooth way on the azimuth angle.

The agreement is less good for segments with other slopes, and particularly at segment end points, as shown in Fig. 8 for the HE-LHC screen. MAD-X (dashed line) does not always find the shortest vector. The MAD-X N1 value may then be too large by 0.25 sigma.

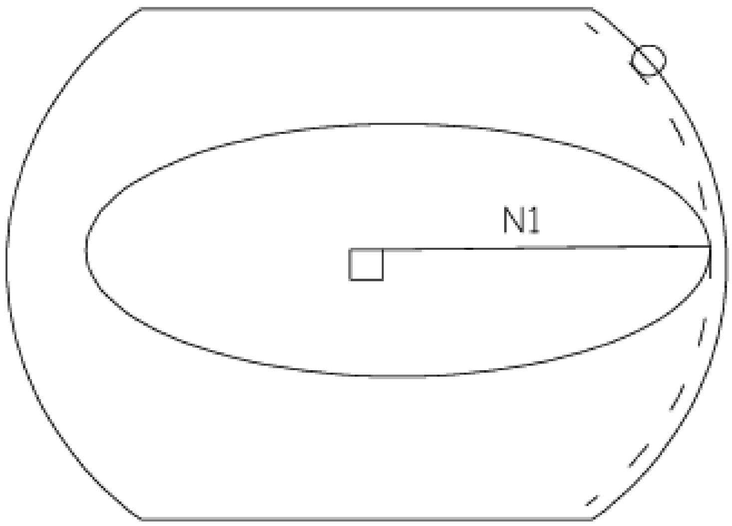

Figure 6: Circular segments

5.2 Truncated halos

Surprisingly, with truncated halos the agreement is about 10 times better than with full ellipses. Using `haloP=6, haloR=6, haloX=5, haloY=5` the N1 differences w.r.t. MAD-X stay below 0.03 sigma.

Inspection of the output reveals that MAD-X concentrates all candidate N1 vectors (dashed lines in Fig. 9) at the azimuth angle of the transition between the flat and the round parts of the beam shape to improve the sampling density. This may seem useful, but it might fail with concave screen shapes.

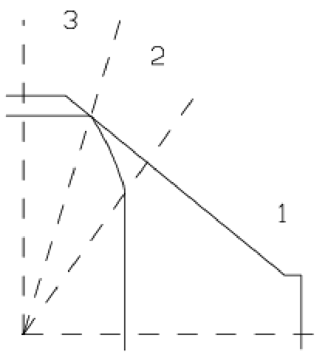

Figure 7: Truncated halos, with the three zones

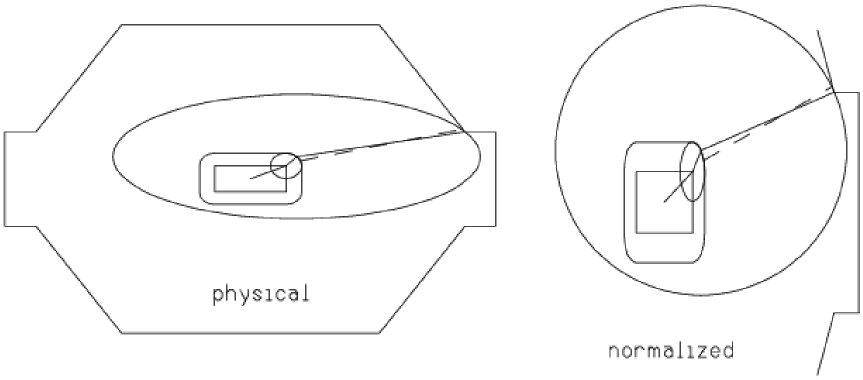

Figure 8: Elliptic halos, in physical (left) and normalized (right) coordinates

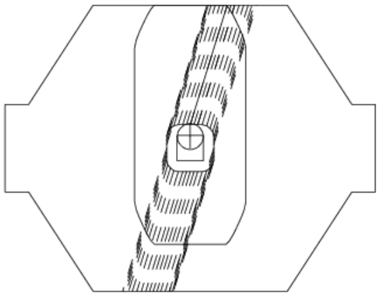

Figure 9: Truncated halos

5.3 Concave beam screen shapes

An attempt was made to run MAD-X with an extreme concave beam screen shape, but the analysis failed. The use of the *notsimple* option made no change. The MAD-X analysis (dashed line in Fig. 10) starts from the wrong point on the tolerance envelope. The N1 value may then be too large by a full sigma with a truncated halo, or by 0.2 sigma with an elliptic halo. The geometric analysis (solid line) analyzes this case correctly.

As shown above, the HE-LHC beam screen, which is only slightly concave, is analyzed by MAD-X with an N1 too large by 0.25 sigma for some elements.

6 Combined envelopes of tolerances and betatron ellipses

As mentioned above, the N1 value of a *single element* is found by identifying the "worst case" addition of tolerance and betatron vectors. On the other hand, once the minimum N1 value is known for a *range of elements* it may be useful to plot the combined envelopes in each element (separately or superimposed), all calculated using the minimum N1 value in the *range*. Adding betatron ellipses in the same plot would overload the figures. The method described below to create these plots was developed for *round* halos.

The coordinates of these envelopes may be found in a simple way if they are drawn in

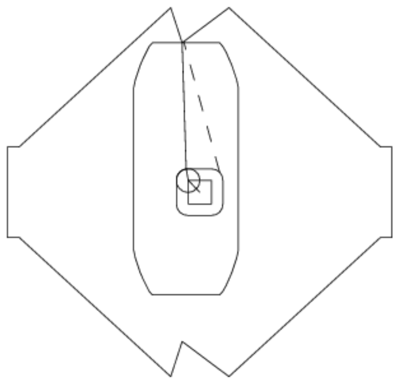

Figure 10: Concave beam screen shape

a normalized X,Y space where betatron ellipses are circles. The combined envelope is then the locus of points outside the tolerance envelope all having the same distance N1 to this envelope.

A circle with radius N1 drawn around any of these points will just touch the tolerance envelope (see Fig. 11). At the touch-point, the local slopes of the circle and the tolerance envelope will be equal, and the center-to-touch-point vector is perpendicular to these slopes. The tolerance envelope is made up of simple (straight or elliptic) segments, so its position and slope are known everywhere in the presence of radial tolerances or closed orbit distortion. This allows to find the coordinates of the center of the circle, and thus of the combined envelope.

Without radial tolerances or closed orbit the combined envelope contains only straight and circular segments in normalized X-Y space.

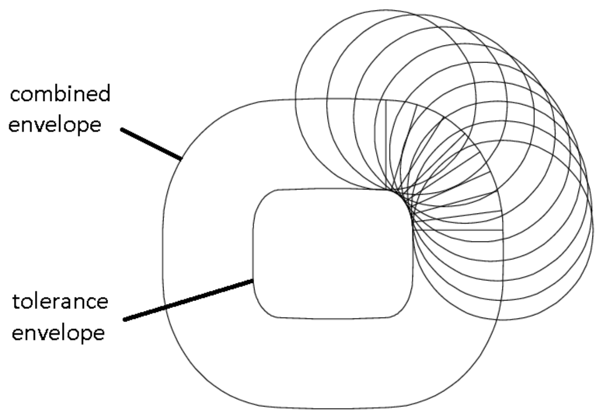

Figure 11: Combining tolerances and betatron ellipses in normalized coordinates

Displaying superimposed plots of combined envelopes obtained with the minimum N1 in a range of elements allows to show if the beam screen shape is well adapted to these envelopes.

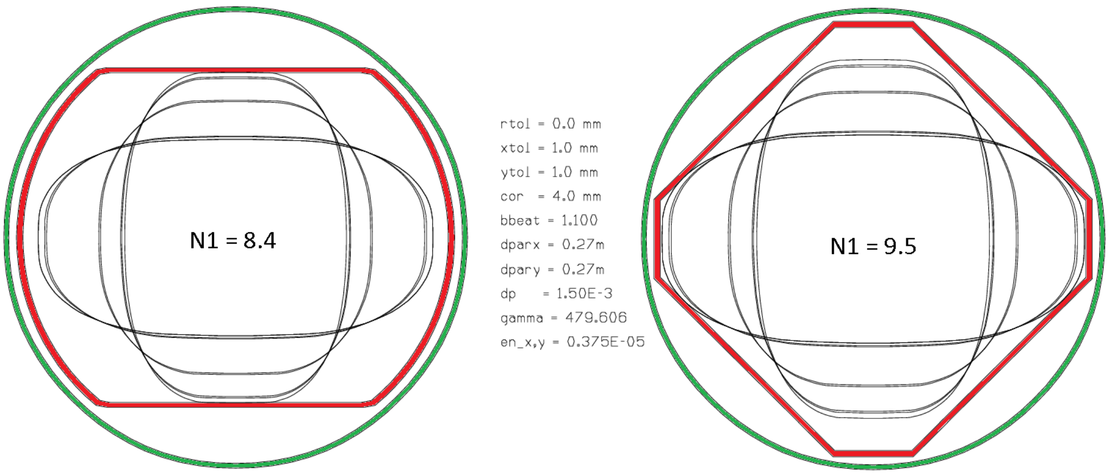

Figure 12: Two beam screens (red) in competition for the LHC in 1995. The identical green circles are added to guide the eye. The combined envelopes are shown for the main bends and quads in one arc cell. MAD-X defaults are taken for the aperture parameters (in use at the end of the LHC study, see list), but halos are assumed to be round.

For the LHC several beam screen cross sections were studied between 1990 and 1995. The present LHC beam screen (see Fig. 12, left) was finally in competition with a square screen rotated by 45 degrees (right). Unfortunately, plots showing both aperture and beam envelopes were not yet available. The comparisons proposed in the present report of the combined envelopes with the screen cross sections show that the required aperture at 45 degrees was overestimated in those days.

7 Conclusions

The geometric analysis is efficient with screen shapes containing geometric (straight or circular) segments. Elliptic segments were not included in this study, as they would require a more complex analysis, but truncated halo shapes are covered. Results are available segment by segment for each element.

This analysis allows an independent check of the MAD-X aperture module, as the methods used are completely different.

A few weak points were found in MAD-X 5.05.02:

- some numerical instabilities were still present in the aperture calculation
- an error exists in one of the combinations of tolerances (orbit combined with H/V mechanical tolerances)
- the analysis of concave screen shapes is not reliable

Lastly, transverse beam envelopes plotted together with the screen shape are absolutely essential for the design of beam screens, as the N1 alone does not show how the available aperture is filled by the beam.

8 Outlook

The above listed weak points were discussed with the MAD-X team. A private MAD-X 5.05.02 test version [4] allowed to confirm and correct the error in the tolerance combination. A recent public MAD-X version (5.06.00) includes this correction.

Acknowledgments

Discussions with the MAD-X team are warmly acknowledged.

References

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