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On the Relationship between Boltzmann-Langevin and Boltzmann-Uehling-Uhlenbeck Models †

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Abstract

A statistical averaging of the Boltzmann-Langevin equation is performed. It is shown that at the averaged level the fluctuations induce an additional collision term with a medium-modified transition rate, which can give rise to a critical scattering phenomena in the vicinity of unstable regions.

Transport models with self-consistent mean fields, like the Boltzmann-Uehling-Uhlenbeck (BUU) model, are widely applied to the description of heavy-ion collisions [1]. These mean-field transport models are very successful in describing the average properties of the one-body observables associated with nuclear collisions, such as nucleon spectra, collective flows and particle production [2, 3]. However, these approaches do not provide an adequate description, when an instability occurs during the dynamical evolution of the system, e.g., such as those in thermal fission or multifragmentation processes. The reason is that the mean-field transport models bring about a deterministic description for the average evolution and do not allow for any branching of dynamical trajectories in the instability region.

The stochastic transport models offer a more appropriate framework for the description of the unstable dynamic evolution. In these stochastic approaches, the transport theory is extended beyond the mean-field level by incorporating the correlations within a statistical approximation [4, 5, 6]. The correlations give rise to a stochastic collision term in the equation of motion, which acts as a source of continuous branching of the dynamical trajectories. In the semi-classical limit, this extended transport model is referred to as the Boltzmann-Langevin (BL) model for the phase-space density. Here, as a continuation of a previous work [7], we investigate the relation between the BL and BUU models. We demonstrate that at the averaged level the BL model contains a new (as compared with

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the BUU model) collision term arising from correlations induced by long-range density fluctuations.

For simplicity, we restrict our treatment to the semi-classical evolution of a spin-isospin averaged phase-space density and consider only elastic binary collisions. According to the BL model, the fluctuating phase-space density $\hat{f}(t, \boldsymbol{r}, \boldsymbol{p})$ obeys the equation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla_r - \nabla_r U[\hat{n}]\cdot\nabla_p\right)\hat{f}(t,\boldsymbol{r},\boldsymbol{p}) = K(t,\boldsymbol{r},\boldsymbol{p}) + \delta K(t,\boldsymbol{r},\boldsymbol{p}), \tag{1}$$

where $U[\hat{n}]$ is the fluctuating self-consistent mean field, which is assumed to be local and, hence, determined by the fluctuating local density,

$$\hat{n}(t,\boldsymbol{r}) = \frac{4}{(2\pi)^3}\int d^3p\,\hat{f}(t,\boldsymbol{r},\boldsymbol{p}), \tag{2}$$

and $K(t, \boldsymbol{r}, \boldsymbol{p})$ denotes the collision term of the BUU form. The additional term $\delta K(t, \boldsymbol{r}, \boldsymbol{p})$ represents a stochastic part of the collision term. In analogy with the treatment of the Brownian motion, it is regarded that eq. (1) describes a stochastic process, in which the entire phase-space density $\hat{f}(t, \boldsymbol{r}, \boldsymbol{p})$ is a stochastic variable and δK acts as a random force. The stochastic collision term vanishes on the average

$$\langle\delta K(t,\boldsymbol{r},\boldsymbol{p})\rangle = 0, \tag{3}$$

and is characterized by a correlation function

$$\langle\delta K(t,\boldsymbol{r},\boldsymbol{p})\,\delta K(t',\boldsymbol{r}',\boldsymbol{p}')\rangle = \delta_c(t-t')\,\delta_c(\boldsymbol{r}-\boldsymbol{r}')\,C(T,\boldsymbol{R},\boldsymbol{p},\boldsymbol{p}'), \tag{4}$$

where $T = \frac{1}{2}(t + t')$ and $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r} + \boldsymbol{r}')$. In the Markovian treatment, the quantities $\delta_c(t - t')$ and $\delta_c(\boldsymbol{r} - \boldsymbol{r}')$ are assumed to be sharp δ-functions. Here, we take them as "broad δ-functions"

$$\delta_c(t-t') = \frac{1}{\sqrt{2\pi}\tau_c}\exp\left[-\frac{1}{2}\left(\frac{t-t'}{\tau_c}\right)^2\right], \tag{5}$$

$$\delta_c(\boldsymbol{r}-\boldsymbol{r}') = \left(\frac{1}{\sqrt{2\pi}r_c}\right)^3\exp\left[-\frac{1}{2}\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{r_c}\right)^2\right]. \tag{6}$$

This is a more realistic parametrization of the correlation function. Here, the correlation length, $r_c \sim 1$ - 2 fm, is of the order of the two-body interaction range, and the correlation time is $\tau_c \simeq r_c/v$, with v being an average relative velocity of nucleons. The correlation function $C(t, \boldsymbol{r}, \boldsymbol{p}, \boldsymbol{p}')$ can be expressed as follows

$$\begin{aligned} C(t,\boldsymbol{r},\boldsymbol{p}_1,\boldsymbol{p}_1') = &\int d^3p_3\,d^3p_4\,W(1,1'\mid 3,4)\,[(1-\hat{f}_1)(1-\hat{f}_{1'})\hat{f}_3\hat{f}_4 + \hat{f}_1\hat{f}_{1'}(1-\hat{f}_3)(1-\hat{f}_4)] \\ &-2\int d^3p_2\,d^3p_4\,W(1,2\mid 1',4)\,[(1-\hat{f}_1)(1-\hat{f}_2)\hat{f}_{1'}\hat{f}_4 + \hat{f}_1\hat{f}_2(1-\hat{f}_{1'})(1-\hat{f}_4)] \\ +\delta(\boldsymbol{p}_1-\boldsymbol{p}'_1)&\int d^3p_2\,d^3p_3\,d^3p_4\,W(1,2\mid 3,4)\,[(1-\hat{f}_1)(1-\hat{f}_2)\hat{f}_3\hat{f}_4 + \hat{f}_1\hat{f}_2(1-\hat{f}_3)(1-\hat{f}_4)]. \end{aligned} \tag{7}$$

The correlation function is closely related to the collision term and is entirely determined by the one-body characteristics. This relation can be regarded as a consequence of the fluctuation-dissipation theorem associated with the stochastic evolution of the phase-space density. The BL equation (1) offers a stochastic description of the collision process, in contrast to the deterministic one of the BUU model. For a given initial condition, the BL equation (1) results in an ensemble of solutions. If the system evolves through an instability region, these solutions can largely diverge from each other, giving rise to large density fluctuations.

In order to carry out the ensemble averaging of the BL equation (1), we decompose the phase-space density and the mean-field as

$$\hat{f} = f + \delta f, \quad \hat{U} = U + \delta U, \tag{8}$$

where $f = \langle \hat{f} \rangle$ and $U = \langle \hat{U} \rangle$ are the averaged parts, while δf and δU denote the fluctuating parts of the phase-space density and the mean field, respectively. Note that $\langle \delta f \rangle = \langle \delta U \rangle = 0$ by definition. By performing the ensemble averaging, we readily obtain the transport equation for the averaged phase-space density

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_r - \nabla_r U \cdot \nabla_p \right) f = \langle K \rangle + K_{BL}, \tag{9}$$

where $\langle K \rangle$ is the average BUU collision term, and

$$K_{BL} = \langle \nabla_r \delta U \cdot \nabla_p \delta f \rangle \tag{10}$$

is an additional term indicating that the kinetic equation for the averaged phase-space density, emerging from the BL model, is not identical to the BUU one.

The calculation of the additional collision term K_{BL} in terms of the averaged characteristics is, in general, a highly complicated problem. Therefore, we consider a particular situation under certain approximations, which simplify the K_{BL} calculation and, at the same time, clarify the dissipation mechanism associated with this collision term. In this particular case

(i) We assume that the magnitude of fluctuations is small as compared with that of averaged quantities. As a result, the fluctuations can be treated in the linearized approximation.

(ii) We treat the deviation of the collision term from its averaged value in the relaxation time approximation

$$K(\hat{f}) - K(f) = -\Delta \, \delta f, \tag{11}$$

where $\tau_R = 1/\Delta$ is the relaxation time of the phase-space density. In accordance with this, we approximate the diagonal part of the correlation function of eq. (7) as

$$C_{diag.}(\boldsymbol{p}_1, \boldsymbol{p}_1') = \delta(\boldsymbol{p} - \boldsymbol{p}') \, 2 \, \Delta \, f(1 - f), \tag{12}$$

and neglect its off-diagonal parts.

(iii) We consider fluctuations of the space-time scale to be much shorter than that of averaged quantities. Hence, in the calculation of fluctuations, we neglect the space-time dependence of the averaged quantities: $f = f(\boldsymbol{p})$ etc.

As a result of the assumptions (i)–(iii), the fluctuations are determined by the linearized BL equation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla_r\right) \delta f - \nabla_p f \cdot \nabla_r \delta U = -\Delta\, \delta f + \delta K. \tag{13}$$

To solve this equation, it is natural to use the Fourier transformation

$$\delta f(\omega, \boldsymbol{k}, \boldsymbol{p}) = \int dt\; d^3r\; \exp(i\omega t - i\boldsymbol{k}\cdot\boldsymbol{r})\; \delta f(t, \boldsymbol{r}, \boldsymbol{p}), \tag{14}$$

and, similarly, for other quantities. In the Fourier representation, the solution of this equation reads

$$\delta f(\omega, \boldsymbol{k}, \boldsymbol{p}) = \frac{\boldsymbol{k}\cdot\nabla_p f}{\boldsymbol{k}\cdot\boldsymbol{v} - \omega - i\Delta} \delta U(\omega, \boldsymbol{k}) - i \frac{\delta K(\omega, \boldsymbol{k}, \boldsymbol{p})}{\boldsymbol{k}\cdot\boldsymbol{v} - \omega - i\Delta}, \tag{15}$$

where

$$\delta U(\omega, \boldsymbol{k}) = -i \frac{V(\boldsymbol{k})}{\varepsilon(\omega, \boldsymbol{k})} \int d^3p\; \frac{\delta K(\omega, \boldsymbol{k}, \boldsymbol{p})}{\boldsymbol{k}\cdot\boldsymbol{v} - \omega - i\Delta}, \tag{16}$$

$$\varepsilon(\omega, \boldsymbol{k}) = 1 - V(\boldsymbol{k}) \int d^3p\; \frac{\boldsymbol{k}\cdot\nabla_p f}{\boldsymbol{k}\cdot\boldsymbol{v} - \omega - i\Delta}. \tag{17}$$

By using these together with the $\langle \delta K\; \delta K\rangle$ correlator in the Fourier representation

$$\langle \delta K(\omega, \boldsymbol{k}, \boldsymbol{p})\; \delta K(\omega', \boldsymbol{k}', \boldsymbol{p}')\rangle = (2\pi)^4\, G_c(\omega, \boldsymbol{k})\; \delta(\omega + \omega')\; \delta(\boldsymbol{k} + \boldsymbol{k}')\; \delta(\boldsymbol{p} - \boldsymbol{p}')\; 2\; \Delta\; f(1 - f), \tag{18}$$

one can calculate all the required quantities. Here, the quantity

$$G_c(\omega, \boldsymbol{k}) = \exp\left[-\frac{1}{2}\left(\frac{\omega}{\omega_c}\right)^2 - \frac{1}{2}\left(\frac{\boldsymbol{k}}{k_c}\right)^2\right], \tag{19}$$

with $\omega_c = 1/\tau_c$ and $k_c = 1/r_c$, represents the cut-off determined by finite correlation lengths in time and space of the correlation function (4) of the stochastic collision term.

First, let us consider the correlation function of the phase-space density. A simple but somewhat lengthy calculation gives

$$\begin{aligned}\langle \delta f(\omega, \boldsymbol{k}, \boldsymbol{p})\;\; \delta f(\omega', \boldsymbol{k}', \boldsymbol{p}')\rangle &= (2\pi)^4\, G_c(\omega, \boldsymbol{k})\; \delta(\omega + \omega')\; \delta(\boldsymbol{k} + \boldsymbol{k}') \\ &\times\;\; [2\pi\delta(\boldsymbol{p} - \boldsymbol{p}')\; \delta(\boldsymbol{k}\cdot\boldsymbol{v} - \omega)\; f(1 - f) + \phi(\omega, \boldsymbol{k}, \boldsymbol{p}, \boldsymbol{p}')],\end{aligned} \tag{20}$$

where $\phi(\omega, \boldsymbol{k}, \boldsymbol{p}, \boldsymbol{p}')$ is an analytic (nonsingular, except for simple poles) function of all the variables. The inverse Fourier transformation into the coordinate representation results in

$$\begin{aligned}\langle \delta f(t, \boldsymbol{r}, \boldsymbol{p})\; \delta f(t', \boldsymbol{r}', \boldsymbol{p}')\rangle &= \delta_c^{(v)}(\boldsymbol{r} - \boldsymbol{r}' - \boldsymbol{v}(t - t'))\; \delta(\boldsymbol{p} - \boldsymbol{p}')\; f(1 - f) \\ &+\;\; \phi(t - t', \boldsymbol{r} - \boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p}'),\end{aligned} \tag{21}$$

where $\phi(t-t',\boldsymbol{r}-\boldsymbol{r}',\boldsymbol{p},\boldsymbol{p}')$ is a nonsingular function proportional to Δ. The quantity ϕ goes to zero in the limit $|\boldsymbol{r}-\boldsymbol{r}'|\to\infty$ and/or $|t-t'|\to\infty$. Here, the quantity $\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)$ is again a "broad δ-function" of the special kind

$$\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)=\frac{1}{(2\pi)^{3/2}(\boldsymbol{v}^2\tau_c^2+r_c^2)^{1/4}r_c}\exp\left\{-\frac{1}{2r_c^2}\left[(\boldsymbol{r}-\boldsymbol{v}t)^2-\frac{\tau_c^2}{\boldsymbol{v}^2\tau_c^2+r_c^2}(\boldsymbol{v}\cdot(\boldsymbol{r}-\boldsymbol{v}t))^2\right]\right\}. \tag{22}$$

In particular, the $\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)$ function indicates that the nonlocality of the correlation function (21) is larger in the longitudinal (with respect to $\boldsymbol{v}$) direction than that in the transverse one. When τ_c and r_c go to zero, $\delta_c^{(v)}(\boldsymbol{r}-\boldsymbol{v}t)$ transforms into conventional δ-function. The result of eq. (21) is a natural form of the correlation function, and it serves us as a test for consistency of our calculations.

The additional collision term K_{BL} can be evaluated by making use of the expressions for $\delta f(\omega,\boldsymbol{k},\boldsymbol{p})$ and $\delta U(\omega,\boldsymbol{k})$ given by eqs. (15) and (16). We find that it takes the form of the Balescu-Lenard type [8] collision term

$$\begin{aligned}K_{BL} &= 2\,\nabla_p\int\frac{d\omega\,d^3k}{(4\pi)^2}\,G_c(\omega,\boldsymbol{k})\,\boldsymbol{k}\left|\frac{V(\boldsymbol{k})}{\varepsilon(\omega,\boldsymbol{k})}\right|^2\\ &\times\int d^3p'\,\delta(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)\,\delta(\boldsymbol{k}\cdot\boldsymbol{v}'-\omega)\left[\boldsymbol{k}\cdot\nabla_p f\,f'(1-f')-\boldsymbol{k}\cdot\nabla'_p f'\,f(1-f)\right]\end{aligned} \tag{23}$$

with a medium-modified transition rate determined by the permittivity. Here, the G_c factor introduces a natural cut-off in momentum transfers, which is determined by inverse correlation lengths [cf. eq. (19)]. The collision term K_{BL} involves a contribution of collective excitations which correspond to zeros of the permittivity: $\varepsilon(\omega_{coll.}(\boldsymbol{k}),\boldsymbol{k})=0$. Hence, the collective modes appear as poles of the integrand in eq. (23). For weakly damping collective modes, their contribution into the K_{BL} collision term can be separated to give

$$K_{BL}^{(coll.)}=\nabla_p\int\frac{d^3k}{(2\pi)^3}\,G_c(\omega,\boldsymbol{k})\,\boldsymbol{k}\,|V(\boldsymbol{k})|^2\,\frac{\Gamma_+}{(Re\varepsilon(\omega,\boldsymbol{k}))^2+(V(\boldsymbol{k})\Gamma_-)^2}\,(\boldsymbol{k}\cdot\nabla_p)f, \tag{24}$$

where

$$\Gamma_+=2\Delta\int d^3p'\,\frac{f'(1-f')}{(\boldsymbol{k}\cdot\boldsymbol{v}'-\omega)^2}, \tag{25}$$

$$\Gamma_-=\Delta\int d^3p'\frac{\boldsymbol{k}\cdot\nabla_{p'}f'}{(\boldsymbol{k}\cdot\boldsymbol{v}'-\omega)^2}, \tag{26}$$

and $\omega=\boldsymbol{k}\cdot\boldsymbol{v}$. This is the semi-classical limit of the collision term, derived in ref. [7] in the quantal representation and with due account for the full correlation function, as well as the memory effect associated with finite duration of the binary collisions. This collision term arises from the correlations associated with the long-wavelength collective density fluctuations. It describes the dissipation mechanism resulting from the coupling between the single-particle motion and the collective vibrations. In the vicinity of the spinoidal instability, the magnitude of this dissipation mechanism increases due to the large density fluctuations. Therefore, it can slow down the expansion of nuclear matter, as well as induce the critical scattering phenomenon similar to the critical opalescence in liquids near the phase transition [9]. In addition, the collective term K_{BL} of eq. (23) contains non-collective contributions corresponding to the small-angle binary scattering with momentum transfer $k \leq k_c$. Hence, it gives rise to corrections to the BUU collision term in the range of small momentum transfers.

In conclusion, we have considered the evolution of the averaged phase-space density in the BL model. We have demonstrated that, besides the usual collision term of the BUU form, the equation of motion involves an additional collision term arising from correlations associated with long-wavelength density fluctuations. This additional term can strongly affect the averaged evolution of the system in the vicinity of the spinoidal region. In the limit of small fluctuations around a quasistatic state, we have derived the explicit expression for this collision term.

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