

# Use and reuse of SMEFT

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**ABSTRACT:** In this work we address three questions: can we successfully describe (observed) deviations from the standard model in the SMEFT language? Can we learn something about the underlying, beyond the standard model, physics using the SMEFT language? If no deviation is observed, how to proceed? Given the myriad of viable BSM options with extended scalar sectors, we suggest a widespread use of SMEFT not just as a global fitting tool (that could miss out on deviations from extended scalar sectors) but also as a bookkeeping framework in which the results from SMEFT fits to individual observables are provided, reported, and archived in a consistent way. The compatibility of such individual results can then be assessed in the light of BSM models with extended scalar sectors.

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# 1 Introduction

The SMEFT [1–3] is a framework that consistently extends the standard model (SM) and allows to capture the effects of beyond-standard-model (BSM) physics in a reasonably general fashion.

In order to define the SM effective-field-theory (SMEFT) we start by considering a broader scenario: there is a “standard” theory,  $X$ , described by a Lagrangian based on a symmetry group  $G$ . The definition of the EFT extension of  $X$  (say, XEFT) requires a circumstantial description for which we need to consider  $X'$ , the ultraviolet (UV) completion of  $X$  or the next theory in a tower of EFTs.

The parameters of the “standard”  $X$  theory are always measured to within some error. Having uncertainties in the parameters leads to hypothesizing a higher structure where the SM Higgs boson mixes with additional scalars. Given the most recent results [4–8] we have to admit that this amount of mixing is observed to be rather constrained, especially because data continue to push the Higgs couplings towards the SM-like limits.

There are two main, non-exclusive, paths in going from  $X$  to  $X'$ :

1.  $X'$  is based on the group  $G$  and contains heavy degrees of freedom belonging to some representation of  $G$ .
2.  $X'$  is based on a larger group  $F$ , where  $G \subset F$  and  $X'$  must reduce to  $X$  at low energies.

An additional assumption is that there are no “undiscovered” degrees of freedom in  $X'$  that are both light and weakly-coupled. We can say that there are four players in the game: the standard theory  $X$ , the corresponding EFT extension XEFT, the beyond-standard theory  $X'$  and its low-energy limit LEX'.

Most of this work (moving from the results presented in ref. [3], in section 18 of ref. [9], and in ref. [10]) will be devoted to discuss the connection between XEFT and LEX'.

When  $X$  is the standard model,  $G = SU(3) \otimes SU(2) \otimes U(1)$ , the simplest examples of extensions are the SM singlet extension [11] (SESM), the THDMs containing two scalar doublets [12], or a non-supersymmetric  $SO(10)$  [13] which breaks down to the SM through a chain of different intermediate groups. Another example is the so-called 331 model [14]. For the SM, the EFTs are further distinguished by the presence (or absence) of a Higgs doublet in the construction. In the SMEFT, the EFT is constructed with an explicit Higgs doublet. This is in contrast with the HEFT [15, 16] (an electroweak chiral Lagrangian with a dominantly 0+ scalar) that does not include such a doublet, i.e. no special relationship is assumed between the Higgs scalar and the Goldstone fields (see section 2 of ref. [17]).

Additional selection criteria can be introduced for the SMEFT [18], in particular that a basis should be chosen from among Potentially-Tree-Generated (PTG) operators (as compared to LG, Loop-Generated operators).

After having discussed the definition of the SMEFT we will consider several aspects of its implementation, i.e. the observational and mathematical consistency of the SMEFT will be critically examined in the light of known (but often overlooked) theoretical results.

An interesting question is: what is so special about the SM? There seems to be no good answer so far: spontaneous breaking of the EW symmetry in the minimal way does not necessarily mean that  $SU(2) \otimes U(1)$  describes the most fundamental theory. We could imagine a scenario where, for some reason, people would have chosen a different “standard” theory, say  $SU(3) \otimes U(1)$  gauge theory of the electroweak interactions [14, 19]; what would be the present situation? Something similar to what we had during the LEP-LHC interregnum: one scalar boson discovered and few heavy states to be fitted, therefore an incomplete “standard” theory and no EFT.

## 1.1 Implementation of the SMEFT

An important aspect of the SMEFT is the so-called SMEFT representation (linear or quadratic).

### 1.1.1 SMEFT representations

Given any amplitude  $A$  its EFT expansion can be written as

$$A = A^{(4)} + \frac{1}{\Lambda^2} A^{(6)} + \dots \quad (1.1)$$

where  $\Lambda$  is the heavy scale. Therefore, when squaring the amplitude, “linear” means including the interference between  $A^{(4)}$  and  $A^{(6)}$ ; “quadratic” currently means including the square of  $A^{(6)}$  instead of the inclusion of all terms of  $\mathcal{O}(1/\Lambda^4)$ , on top of the  $\dim = 8$  terms in the expansion, i.e.  $A^{(8)}$ . The obvious criticism to this procedure is that one should not construct S-matrix elements at  $\mathcal{O}(1/\Lambda^4)$  using a canonically-transformed Lagrangian truncated at  $\mathcal{O}(1/\Lambda^2)$ . When including  $\mathcal{O}(1/\Lambda^4)$  terms the canonical normalization procedure induces changes to the shape of differential distributions, not only to their integral, as is the case for the aforementioned truncated Lagrangian.

The new results of ref. [20] put a new perspective on the “linear vs. quadratic” option, discussing (for the first time) a comparison between partial  $\mathcal{O}(1/\Lambda^4)$  and full  $\mathcal{O}(1/\Lambda^4)$ .

### 1.1.2 Canonical normalization

By canonical normalization we mean the problem induced when, given an effective Lagrangian, we find that the kinetic terms have a non-canonical normalization. This fact does not represent a real problem, as long as we remember the correct treatment of sources in going from amputated Green’s functions to S-matrix elements. To give an example we consider the SMEFT Lagrangian (Warsaw basis [21]) written in the mass eigenbasis; there will be terms like

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left( 1 + \frac{v^2}{\Lambda^2} \delta Z_h^{(6)} + \frac{v^4}{\Lambda^4} \delta Z_h^{(8)} \right) \partial_\mu h \partial_\mu h \\ & + \frac{1}{\Lambda^2} \left( a^{(6)} M_W^3 h Z_\mu Z_\mu + \dots \right) + \frac{1}{\Lambda^4} \sum_i a_i^8 \mathcal{O}_i^{(8)}, \end{aligned} \quad (1.2)$$

where  $v$  is the Higgs VEV and the  $a^{6,8}$  are Wilson coefficients; furthermore the  $\delta Z$  factors depend on Wilson coefficients. Note that the  $\dim = 8$  terms, including  $\delta Z_h^{(8)}$ , are not yet available. Canonical normalization means redefining the  $h$  field

$$h = \left[ 1 - \frac{1}{2} \frac{v^2}{\Lambda^2} \delta Z^{(6)} + \mathcal{O}(v^4/\Lambda^4) \right] \hat{h}. \quad (1.3)$$

As a consequence the Lagrangian becomes

$$\hat{\mathcal{L}} = -\frac{1}{2} \partial_\mu \hat{h} \partial_\mu \hat{h} + a^{(6)} \frac{M_W^3}{\Lambda^2} \left[ 1 - \frac{1}{2} \frac{v^2}{\Lambda^2} \delta Z_h^{(6)} \right] \hat{h} Z_\mu Z_\mu + \dots \quad (1.4)$$

showing terms of  $\mathcal{O}(1/\Lambda^4)$  which are neglected in the “quadratic” representation.

Our approach to canonical normalization is completed with a rescaling of the SM parameters so that the part of the Lagrangian quadratic in the fields is the SM Lagrangian; for instance we use

$$M_W \rightarrow M_W \left( 1 - \frac{g_6}{\sqrt{2}} a_{\phi W} \right). \quad (1.5)$$

### 1.1.3 Equivalent operators

Actually, there could be more  $\mathcal{O}(1/\Lambda^4)$  “missing” terms. First of all we need to define equivalent operators [22]: from the point of view of the S-matrix two operators are equivalent if (for simplicity we will consider the case of scalar fields)

$$\mathcal{O}_i - \mathcal{O}_j = F(\phi) \frac{\delta \mathcal{L}}{\delta \phi}, \quad (1.6)$$

and we have to decide which one is to be eliminated (the redundant one) in order to construct a basis. Redundant operators are eliminated by a field redefinition; the corresponding shift in the Lagrangian will eliminate redundant operators leaving a, neglected, higher order compensation which becomes relevant when we want to compare the SMEFT in two different bases and in the “quadratic” representation.

Building any EFT means promoting a theory with a finite number of terms into an effective field theory with an infinite number of terms and in doing so it is important to establish its consistency order-by-order.

### 1.1.4 A provisional summary of the SMEFT

In this paper, SMEFT will be understood as the SM extension containing dimension 6 terms in the so-called Warsaw basis [21]. However, in our approach, we have rescaled the Wilson coefficients: in front of an operator  $\mathcal{O}_i^{(k)}$  of dimension  $k$  and containing  $n$  fields we write

$$g^{n-2} \frac{a_i^{(k)}}{\Lambda^{k-4}} \quad (1.7)$$

where  $g$  is the  $SU(2)$  coupling constant. This rescaling is useful when discussing SMEFT at the one-loop level, as explained in ref. [23].

In conclusion: the SMEFT framework is useful because one can set limits on the effective coefficients in a model-independent way. This is why the SMEFT in the bottom-up approach, going beyond a global fit, is so useful: we do not know what the tower of UV-complete theories is (or if it exists at all) but we can formulate the SMEFT and perform calculations with it without needing to know what happens at arbitrarily high scales. On the other hand, interpreting such limits as bounds on UV models (BSM models) does require some assumptions on the UV dynamics.

Having defined SMEFT, we further note that most BSM scenarios have extended scalar sectors. The lack of direct discovery of BSM states suggests that the SM is “isolated,” including a small mixing between light and heavy scalars [24]. In other words, no “light” BSM scalars have been found and the light Higgs couplings seem to be SM-like. The small mixing scenario raises the following question: if there are more scalars then we have to conclude that there is a small mixing with any other scalar. This is no longer accidental but a systematic effect, a feature of nature as we presently understand it for which there is no clear theoretical motivation for.

Mixing is not a peculiarity of extended scalar sectors; for instance we could consider extensions of the SM with general new vector bosons. It is worth noting that there are classes of BSM models where mass mixing terms of SM and new vectors are explicitly forbidden; however, the general case includes interactions with the Higgs doublet that give rise to mass mixing of the Z and W bosons with the new vectors when the electroweak symmetry is broken.

### 1.1.5 The SMEFT beyond LO

An additional comment is about LO SMEFT vs. the inclusion of SMEFT loops, sometimes called NLO SMEFT. Here, by NLO SMEFT we mean the following:

- SMEFT vertices inserted in tree-level SM diagrams,

- tree-level (SMEFT-induced) diagrams with a non-SM topology,
- SMEFT vertices inserted in one-loop SM diagrams, and
- one-loop (SMEFT-induced) non-SM diagrams.

“NLO” SMEFT provides the general framework for consistent calculations of higher orders and allows for global fits, superseding any ad-hoc variation of the SM parameters. Ongoing and near-future experiments can achieve an estimated per mille accuracy on precision Higgs and EW observables, thus providing a window to indirectly explore the theory space of BSM physics. That is why “NLO SMEFT” is needed. To summarize: NLO results have already had an important impact on the SMEFT physics program. LEP constraints should not be interpreted to mean that effective SMEFT parameters should be set to zero in LHC analyses. It is important to preserve the original data, not just the interpretation results, as the estimate of the missing higher order terms can change over time, modifying the lessons drawn from the data and projected into the SMEFT. Considering projections for the precision to be reached, LO results for interpretations of the data in the SMEFT are challenged by consistency concerns and are not sufficient, if the cut off scale is in the few TeV range. The assignment of a theoretical error for SMEFT analyses (missing higher order uncertainty or MHOU) is always important.

There is a hierarchy in the MHOU, ranging from 1% to 100%; for instance, LO PDFs and NLO PDFs in LO SMEFT give results differing by a large factor.

We can say that there aspects of the problem which should be solved “today” but not at the price of forgetting aspects which will require our attention “tomorrow”. For instance, while QCD corrections are dominant it would be inaccurate to say that EW corrections are negligible (i.e. well below 10%), see refs. [25, 26] for an example. Furthermore, there are QCD corrections in the SMEFT which are unrelated to the SM ones and can be sizeable [27].

### 1.1.6 The SMEFT and renormalization

Further comments are: “renormalization” of any EFT should make UV finite all off-shell Green functions, i.e. not only those relevant for a single process. In order to make all, on-shell, S-matrix elements finite, we have to introduce renormalization for fields( $\Phi$ ) and parameters ( $p$ ), i.e.

$$\Phi = Z_\Phi \Phi_R, \quad p = Z_p p_R, \quad (1.8)$$

where the Z factors must be expanded order-by-order in  $1/\Lambda^2$ . The full renormalization program [23] includes a) construction of the self-energies and Dyson resummation of the propagators; b) construction of 3 (and higher) point functions, check of their  $\text{dim} = 4$  finiteness and complete removal of the residual  $\text{dim} = 6$  UV divergences by mixing Wilson coefficients.

There is a deep connection between UV poles and symmetry of the Lagrangian [28]; when including  $\text{dim} = 8$  operators we should realize that the SMEFT is computationally more complex than Quantum Gravity. To give an example we consider a Lagrangian containing scalar field and use the background-field-method [29] where we split the fields into a classical and a quantum part. All one-loop diagrams are generated by the part of the Lagrangian which is quadratic in the quantum fluctuations,

$$\mathcal{L}_2 = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \phi N^\mu \partial_\mu \phi + \frac{1}{2} \phi M \phi. \quad (1.9)$$

In principle the counter-Lagrangian contains 8 terms but if we define  $X = M - N^\mu N_\mu$  we can see that  $\mathcal{L}_2$  is invariant under the 't Hooft transformation (H)

$$\phi' = \phi + \lambda \phi, \quad N'_\mu = N_\mu - \partial_\mu \lambda + [\lambda, N_\nu], \quad X' = X + [\lambda, X], \quad (1.10)$$

where  $\lambda$  is an antisymmetric matrix. Therefore,  $\Delta\mathcal{L}$  will also be invariant, reducing the number of independent counterterms to 2. Any EFT containing  $\dim = 6$  and  $\dim = 8$  operators will have terms like

$$\frac{1}{2} \partial_\mu \phi_i g_{ij}^{\mu\nu}(\phi_c) \partial_\nu \phi_j, \quad (1.11)$$

with a matrix-valued metric tensor. To  $g$  there will correspond matrix-valued Riemann tensors, i.e. many more invariants for  $\Delta\mathcal{L}$  [30]. Note that if  $\mathcal{L}$  is invariant under a group  $G$  then the relation between  $G$ -invariance and  $H$ -invariance is crucial in proving closure under renormalization (not the same as strict renormalizability).

Once again today's priority goes to QCD NLO SMEFT, including those corrections that are unrelated to the SM. All relevant  $\dim = 6$  operators must be included and not only a subset, because subsets are, in general, not closed under renormalization; Wilson coefficients mix, e.g. there is a mixing between  $a_{q_{\text{BW}}}$  and  $a_{q_{\text{G}}}$  in  $Z \rightarrow \bar{q}q$ .

### 1.1.7 The SMEFT and scheme dependence

Scheme dependence is present in the SM and in the SMEFT predictions; it comes from "finite renormalization", i.e. the choice of experimental input quantities. A given observable  $\mathcal{O}$  will be written as

$$\mathcal{O} = \mathcal{O}_{\text{LO}} + \mathcal{O}_{\text{NLO}}, \quad \mathcal{O}_i = \mathcal{O}_i^{(4)} + g_6 \mathcal{O}_i^{(6)} + \dots \quad (1.12)$$

where

$$\mathcal{O}_i^{(4)} \equiv \mathcal{O}_i^{(4)}(g_{\text{R}}, \{\text{M}_{\text{R}}\}), \quad \mathcal{O}_i^{(6)} \equiv \mathcal{O}_i^{(6)}(g_{\text{R}}, \{\text{M}_{\text{R}}\}, \{\text{a}_{\text{R}}\}), \quad (1.13)$$

where counterterms and mixing of Wilson coefficients have been introduced and UV poles removed. By LO we mean the lowest order in perturbation theory where the SM(SMEFT) observable is computed, e.g.  $\mathcal{O}(g)$  for  $h \rightarrow \bar{b}b$  and  $\mathcal{O}(g^3)$  for  $h \rightarrow \gamma\gamma$  in the SM. The definition of NLO requires more attention; for instance, the QCD corrections to the gluon fusion process,  $gg \rightarrow h$ , require more than the two-loop calculation and must include

$$gg \rightarrow hg, \quad gq \rightarrow hq, \quad \bar{q}q \rightarrow hg. \quad (1.14)$$

On-shell finite renormalization requires  $M_{i,OS}^2$  to be a zero of the real part of the inverse propagator for particle  $i$ . Then

$$M_{i,\text{R}} = M_{i,OS} + \frac{g_{\text{R}}^2}{16\pi^2} \left[ dZ_{\text{M}_i}^{(4)} + g_6 dZ_{\text{M}_i}^{(6)} \right]. \quad (1.15)$$

in the  $G_{\text{F}}$ -scheme we require

$$g_{\text{R}} = g_{\text{exp}} + \frac{g_{\text{exp}}^2}{16\pi^2} \left[ dZ_g^{(4)} + g_6 dZ_g^{(6)} \right], \quad (1.16)$$

where  $g_{\text{exp}}$  will be expressed in terms of the Fermi coupling constant  $G_{\text{F}}$ . The expressions for the renormalized quantities are then replaced into  $\mathcal{O}$ , truncating in  $g_{\text{exp}}$  and in  $g_6$ . A complete  $\mathcal{O}(1/\Lambda^4)$  calculation requires to perform finite renormalization at  $\mathcal{O}(g_6^2)$  in order to be consistent. Well-known arguments on the running of  $\alpha_{em}$  and of  $G_{\text{F}}$  indicate that the preferred scheme is based on selecting  $\{G_{\text{F}}, M_{\text{W}}, M_{\text{Z}}\}$ . However, a consistency check is based on the choice  $\{\alpha_{em}, G_{\text{F}}, M_{\text{Z}}\}$  where  $M_{\text{W}}$  can be predicted, giving

$$M_{\text{W}} = M_{\text{W}}|_{\text{SM}} + \frac{\alpha_{em}}{\pi} g_6 \Delta_{\text{W}}, \quad (1.17)$$

where the SMEFT corrections contain 9 PTG and 9 LG Wilson coefficients.

Input parameter sets values are based on extraction of the the parameters from different experimental results using the SM and not the SMEFT. Therefore, there is a problem, not only for  $\alpha_s$  (although dominant) but for  $\alpha_s, M_W, M_Z$  etc. Finally, SMEFT is not (yet) included in the PDF parametrization and all EWPD (e.g. LEP) are SM-based (LEPEWWG fits) and QED/QCD deconvoluted.

### 1.1.8 The SMEFT vs. BSM

Once again, we are considering the following scenario [3]:

- the SM, valid for  $E \ll \Lambda$ ,
- the corresponding EFT extension (say SMEFT), and
- the next SM (NSM), some UV completion of the SM (or the next theory in a tower of effective theories).

We are interested in the low- $E$  limit of the NSM beyond the tree-level approximation.

### 1.1.9 The SMEFT, BSM models and heavy-light contributions

At the one-loop level we obtain local and non-local terms [3, 31, 32] corresponding to long distance propagation and hence to reliable, perturbative, predictions at low energy, as well as local effects which, by contrast, summarize the unknown effects from high energies. Having both local and non-local terms corresponds to a full implementation of the (one-loop) EFT program, including the logarithmic dependence upon the characteristic momentum transfer in the problem, see ref. [33]. To summarize: loop diagrams with light external legs and heavy internal ones admit a local low-energy limit; diagrams with light external legs and mixed internal legs may show normal-threshold singularities in the low-energy region and yield inherently non-local parts.

Any EFT Lagrangian and the corresponding EFT amplitudes have a different interpretation: the Lagrangian is local (as it should), the amplitudes generate long-distance kinematic logarithms.

As an example we consider a scalar 3-point function

$$i \pi^2 C_0(m, M, m) = \int d^d q \left[ (q^2 + m^2) ((q + p_1)^2 + M^2) ((q + p_1 + p_2)^2 + m^2) \right]^{-1}, \quad (1.18)$$

in the limit  $M \rightarrow \infty$ . The result is

$$C_0(m, M, m) \sim \frac{1}{M^2} \left[ 1 + \ln \frac{M^2}{m^2} - \beta \ln \frac{\beta + 1}{\beta - 1} \right] + \mathcal{O}(1/M^4), \quad (1.19)$$

where, using the Feynman prescription,  $\beta^2 = 1 + 4 m^2 / (P^2 - i 0)$  and  $P = p_1 + p_2$ . The result shows the normal threshold at  $P^2 = -4 m^2$ . This example should be compared with

$$i \pi^2 C_0(M, M, M) = \int d^d q \left[ (q^2 + M^2) ((q + p_1)^2 + M^2) ((q + p_1 + p_2)^2 + M^2) \right]^{-1}, \quad (1.20)$$

$$C_0 = \frac{1}{M^2} + \mathcal{O}(1/M^4).$$

To summarize: special attention is due for configurations where there is a hierarchy involving the heavy scale, the Mandelstam invariants describing the process, and the light masses,

$$\Lambda^2 \gg s_{ij \dots k} = -(p_i + p_j + \dots + p_k)^2 > (m_1 + m_2 + \dots + m_n)^2. \quad (1.21)$$

with the presence of normal-threshold effects and, eventually of anomalous-thresholds effects [34].



## 2 Using SMEFT

There is a large variety of directions in the SMEFT that allows for it to be a proxy for BSM scenarios. The question is: can any BSM model in nature be caught by using the SMEFT? Well, “a large variety” means that we expect to have enough directions in the SMEFT to fit nearly everything. But the underlying assumptions such as one single scalar doublet and one single heavy scale could affect the interpretation. Therefore, if the question is “can any BSM be caught by using an EFT with many assumptions?” the answer will be: observable by observable, yes. But will that lead us to what nature has in store? No, not necessarily. But it will help because of the sensitivity in individual observables.

It is possible that for the BSM model realized in nature the effects in the full set of observables used is such that the SMEFT result *seems* to be null. This can come about via an averaging effect, with different observables pulling a Wilson coefficient in opposite directions.

**Proposition 1** *Experiments cannot generate processes and reconstruct simulated event samples in every single BSM framework. SMEFT d.o.f. (the Wilson coefficients  $a_i$ ) are being tested by experiments, are being expanded and improved upon, and are rather comprehensive as to the types of BSM deformations they can encode. SMEFT d.o.f. can be used as a bookkeeping tool in exploring the likelihood function,  $L$ , for (sub)sets of observables [10].*

An example of the procedure is given in App. A of ref. [35] where pulls are introduced which can be interpreted in terms of fit robustness, bias and coverage. In our case we can make separate fits of data samples characterized by, precision EW data, Higgs boson production (LHC Run 1 and Run 2), VV production at LHC etc. Each fit yields estimates for the Wilson coefficients; estimates on one fit can be used to constrain the remaining fits. Note however that some of the data samples come with caveats, e.g. the correct interpretation of the  $h \rightarrow Z\gamma$  signal strength.

Furthermore, if present low-energy measurements are not sensitive to a subset of SMEFT operators, there would be a null result which could be interpreted as the impossibility of uncover the corresponding heavy sector while a new set of measurements could very well do it. It is important to be able to quantify the impact of a new measurement in the SMEFT parameter space without having to redo the full fit. Bayesian inference has been suggested in ref. [36].

An additional warning is that at high scales there are  $\dim = 8$  parameters with a greater impact than  $\dim = 6$  parameters. Inference about  $\dim = 6$  parameters will be different if  $\dim = 8$  is neglected; at the very least one should treat them as nuisance parameters and profile or marginalise them so as to obtain a (truncation) uncertainty [37].

### 2.1 SMEFT and kappa parameters

An alternative way of recording SMEFT fits has been introduced in ref. [23] where a connection between Wilson coefficients and kappa-parameters [38] was suggested.

In the original kappa-framework we replace  $\mathcal{L}_{\text{SM}}(\{m\}, \{g\})$  with  $\mathcal{L}_{\text{SM}}(\{m\}, \{\kappa_g g\})$ , where  $\{m\}$  denotes the SM masses,  $\{g\}$  the SM couplings and  $\kappa_g$  are the scaling parameters. This is the framework used during Run 1 of LHC.

In the SMEFT approach we define amplitudes: at LO

$$A_{\text{SMEFT}}^{\text{LO}} = \sum_{i=1,n} A_{\text{SM}}^{(i)} + i g_6 A_c, \quad g_6 = 1/(\sqrt{2} G_{\text{F}} \Lambda^2), \quad (2.1)$$

where the  $A_{\text{SM}}^{(i)}$  are the SM (gauge-parameter independent) sub-amplitudes and  $A_c$  is the SMEFT “contact” amplitude. For instance, in  $h \rightarrow \gamma\gamma$ , the SM sub-amplitudes are the ones due to top,

bottom and bosonic loops (the latter including W,  $\phi$  and FP ghosts). When dim = 6 operators are inserted (once) in loops we obtain

$$A_{\text{SMEFT}}^{\text{NLO}} = \sum_{i=1,n} \kappa_i A_{\text{SM}}^{(i)} + i g_6 A_c + g_6 \sum_{i=1,N} a_i A_{nf}^{(i)}, \quad (2.2)$$

where the  $a_i$  are Wilson coefficients and the  $\kappa_i$  are linear combinations of the Wilson coefficients. Furthermore, the amplitudes  $A_{nf}^{(i)}$  collect all loop contributions which do not factorize into the SM sub-amplitudes. The simplest example is  $h \rightarrow \gamma_\mu(p_1) + \gamma_\nu(p_2)$ . The amplitude becomes

$$A_{h\gamma\gamma}^{\mu\nu} = i \mathcal{A}_{h\gamma\gamma} (p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu\nu}). \quad (2.3)$$

We introduce  $g_F^2 = 4\sqrt{2} G_F M_W^2$  and obtain

$$\mathcal{A}_c = g_F \frac{M_h^2}{M_W} a_{AA}, \quad (2.4)$$

where  $a_{AA} = c_W^2 a_{\phi B} + s_W^2 a_{\phi W} + c_W s_W a_{\phi WB}$  and  $s_W$  is the sine of the weak-mixing angle. The kappa coefficients in the factorizable part of the amplitude can be written as

$$\kappa_i = 1 + g_6 \Delta\kappa_i = \frac{g_F^3 s_W^2}{8\pi^2} \rho_i, \quad \rho_i = 1 + g_6 \Delta\rho_i, \quad (2.5)$$

where the index  $i$  runs over W loops (i.e. the bosonic part), top quark loops, and b quark loops. The non-factorizable part of the amplitude depends on the following Wilson coefficients,

$$a_{qWB}, a_{AA}, a_{AZ}, a_{ZZ}, \quad (2.6)$$

where we have defined

$$\begin{aligned} a_{ZZ} &= s_W^2 a_{\phi B} + c_W^2 a_{\phi W} - s_W c_W a_{\phi WB}, \\ a_{AZ} &= \left(2c_W^2 - 1\right) a_{\phi WB} + 2s_W c_W \left(a_{\phi W} - a_{\phi B}\right). \end{aligned} \quad (2.7)$$

In the factorizable part of the amplitude, adopting the PTG scenario, we only keep  $a_{q\phi}$  and  $a_{\phi D}$ ,  $a_{\phi\Box}$ . These results tell us that the kappa-factors can be introduced also at the loop level; they are combinations of Wilson coefficients but we have to extend the scheme with the inclusion of process dependent kappa-factors and non-factorizable contributions.

The kappa-parameters form hyperplanes in the space of Wilson coefficients; each kappa-plane describes (tangent)flat-directions while normal directions are blind and there are correlations among different processes.

The generalized kappa parameters have two labels referring to the (gauge-parameter independent) SM sub-amplitude and to the process. The SMEFT requires relations among the  $\Delta\kappa$ , e.g.

$$\begin{aligned} \Delta\kappa_b^{h\gamma Z} - \Delta\kappa_t^{h\gamma Z} &= \Delta\kappa_b^{h\gamma\gamma} - \Delta\kappa_t^{h\gamma\gamma} \\ \left(\frac{3}{2} + 2c_W^2\right) (\Delta\kappa_t^{h\gamma\gamma} - \Delta\kappa_t^{h\gamma Z}) &= c_W^2 \Delta\kappa_W^{h\gamma Z} + \left(\frac{1}{2} + 3c_W^2\right) \Delta\kappa_W^{h\gamma\gamma}, \end{aligned} \quad (2.8)$$

where the labels  $t, b$ , and  $W$  refer to the top quark loop, etc. Another interesting relation concerns the  $h\bar{t}t$  vertex where we have

$$V_{h\bar{t}t}^{\text{SMEFT}} = V_{h\bar{t}t}^{\text{SM}} \left\{ 1 + g_6 \left[ \Delta\kappa_t^{\gamma\gamma} + \frac{1}{2} c_W^2 \Delta\kappa_W^{\gamma Z} - \frac{1}{2} (2 - s_W^2) \Delta\kappa_W^{\gamma\gamma} \right] \right\}. \quad (2.9)$$

In LO SMEFT the contact amplitude is non-zero while  $\kappa_t$ , etc. are set to one. If a deviation is measured it will reflect into some value for the Wilson coefficient controlling the LO SMEFT. However, at NLO SMEFT  $\kappa_i \neq 1$  and we get a degeneracy; i.e. the interpretation in terms of LO SMEFT and NLO SMEFT could be rather different.

Another way of describing the generalized kappa framework and its connection with the SMEFT is as follows: we can write down all amplitudes respecting the required symmetries and these amplitudes are in one-to-one correspondence with the operators of SMEFT.

Of course, as soon as we start discussing the  $Z\gamma$  decay, a question will arise: we need a more general classification of the Higgs decays according to kinematics. Otherwise we will end up in a situation where it is not clear whether some event is actually  $h \rightarrow 4$  fermions or rather  $h \rightarrow 2$  fermions + radiation [39]. For recent results on the Higgs decay into  $Z\gamma$  see ref. [40] where, however the most relevant theoretical results are (apparently) not used, see refs. [39, 41–44].

### 3 How to reuse SMEFT

The increasing interest in the SMEFT has led to the development of a wide spectrum of public codes which implement automatically different aspects of the SMEFT for phenomenological applications [9].

The question is: what happens if we only do global SMEFT fits to data from a nature with more scalars? We will present few examples of BSM models and discuss their low-energy limits (for a similar discussion see ref. [17]) and then we will introduce the concept that the SMEFT should not be understood (only) as a global fitting tool but (also) as a bookkeeping framework in which the results from SMEFT fits to individual observables are provided, reported, and archived. At the end of this section we will discuss statistical aspects of the procedure.

To summarize the New Physics (NP) scenario: given a BSM model, we

- compute all relevant observables in terms of the Lagrangian parameters,
- take into account loop effects and the renormalization procedure. We should keep in mind that there are subtle points in on-shell vs.  $\overline{\text{MS}}$  renormalization vs. gauge invariance: tadpoles matter, i.e. they only cancel in on-shell renormalization. When masses of heavy states and mixings are  $\overline{\text{MS}}$ -renormalized there could be problems: i.e. the  $\overline{\text{MS}}$ -renormalization of the mixing angles combined with the popular on-shell renormalization schemes gives rise to gauge-dependent results already at the one-loop level [45].
- Compare to experimental results, i.e. Observables  $\rightarrow$  Likelihood.

Admittedly this is time consuming to do for each BSM model.

- The SMEFT is a powerful tool to connect model-building to phenomenology without needing to fit hundreds of observables to data in each model.

Before discussing specific examples of BSM models we look for theoretical guidance. For instance, assuming an unbroken custodial invariance as suggested by precision electroweak measurements [46], implies  $\rho_{\text{LO}} = M_W^2 / (c_W^2 M_Z^2) = 1$  [47–49]. Note that Higgs doublets generally respect the custodial symmetry, except for certain combinations of the doublets associated with complex parameters. Models with Higgs triplets can violate the symmetry. Custodial symmetry can also be violated by terms of higher dimension arising from physics at some higher scale.

There is a comment to be made: the key point is not  $\rho_{\text{LO}} = 1$  but the UV finiteness of  $\rho$ . Therefore, renormalization is required in those BSM models where  $\rho$  is UV-divergent; there are more Lagrangian parameters and, as a consequence, more counterterms which can be used to

cancel the UV pole in  $\rho$ . After removing the divergences we are left with “finite” renormalization, i.e. a scheme connecting the renormalized parameters to an experimental input containing  $\rho_{exp}$ . We could claim that it is not a “natural” solution but it remains a solution.

### 3.1 BSM models

The archetype of BSM models is the so-called singlet extension of the SM (SESM). Here we summarize the approach followed in [50]. The only modification w.r.t. the SM is contained in the scalar potential

$$-\mu_2^2 \Phi^\dagger \Phi - \mu_1^2 \chi^2 - \frac{1}{2} \lambda_2 (\Phi^\dagger \Phi)^2 - \frac{1}{2} \lambda_1 \chi^4 - \lambda_{12} \chi^2 \Phi^\dagger \Phi. \quad (3.1)$$

$\Phi$  is a doublet containing  $h_2$ , the custodial singlet in  $2_L \otimes 2_R$ , while

$$\chi = \frac{1}{\sqrt{2}} (h_1 + v_s). \quad (3.2)$$

The mixing angle is defined by

$$h = \cos \alpha h_2 - \sin \alpha h_1, \quad H = \sin \alpha h_2 + \cos \alpha h_1, \quad (3.3)$$

where  $h$  and  $H$  are the mass eigenstates. We go to the mass eigenbasis, select

$$\Lambda = M_s = \frac{1}{2} g v_s, \quad (3.4)$$

take the limit  $\Lambda \rightarrow \infty$  and eliminate  $\lambda_2$ . At the same time,  $\lambda_1$  and  $\lambda_{12}$  (the “extra SM” parameters) remain free parameters,

$$\lambda_1 = t_1 g^2, \quad \lambda_{12} = t_3 g^2. \quad (3.5)$$

In this way  $\lambda_2$  is modified (w.r.t. the SM) as follows,

$$\lambda_2 = \frac{1}{4} g^2 \frac{M_h^2}{M_w^2} + g^2 \frac{t_3^2}{t_1} + \mathcal{O}(M_s^{-2}), \quad (3.6)$$

where  $M_h$  is the (bare) mass of the light Higgs boson and  $M_w$  is the (bare) mass of the W boson. It is worth noting that we did not “integrate out” the heavy degree of freedom in the weak eigenbasis (unphysical fields), where we can construct a manifestly  $SU(2) \otimes U(1)$  invariant low energy Lagrangian by integrating out the  $h_1$  field in the limit  $\mu_1 \rightarrow \infty$  (see ref. [51] for a discussion of this construction). In our approach the integration is performed in the (physical) mass eigenbasis; this is analogous to what is done in [52] when deriving the low-energy Effective Field Theory below the electroweak scale (LEFT).

Relevant in this context is the argument of ref. [53] on extended scalar sectors and mixing (see also ref. [54]): integration in the weak eigenbasis reproduces the effect of scalar mixing on interactions involving one Higgs scalar  $h$ , but fails to do so for the case of two scalars  $hh$ . Indeed, when integrating out the field  $h_1$  we obtain an effective Lagrangian where only the wave function of the Higgs field is modified w.r.t. the SM. After integrating out the heavy field  $H$  (in the mass eigenbasis) we obtain a Higgs-gauge interaction leading to a mismatch between the  $hVV$  and the  $hhVV$  couplings (w.r.t. their SM values). Furthermore, one of the operators in the Warsaw basis is  $\mathcal{O}_\phi = (\Phi^\dagger \Phi)^3$ , where  $\Phi$  is the  $SU(2)$  doublet. Since

$$\Phi^\dagger \Phi = \frac{1}{2} \left[ (h_2 + \sqrt{2} v)^2 + \phi^0 \phi^0 + 2 \phi^+ \phi^- \right], \quad (3.7)$$

there will be one Wilson coefficient,  $a_\phi$  for the couplings  $h_2^6$ ,  $h_2(\phi^0)^4$ , etc. When integrating out the  $h_1$  field in the weak eigenbasis (at LO) we obtain  $a_\phi = 0$ . But when integrating out the  $H$  field in the mass eigenbasis we obtain different coefficients in front of polynomials of scalar fields, a fact which becomes relevant when comparing SMEFT and SESM at the NLO level.

The key difference is that the SMEFT utilizes the complete Higgs doublet as a building block. On the other hand, the low-energy limit of some BSM model can be such that the physical Higgs boson excitation and the Goldstone bosons are independent objects. Another way of describing this fact is to look at the tree-generated scalar potential in the SESM (in the unitary gauge), where we obtain

$$U(h) = -\frac{1}{2} M_h^2 \left( h^2 + \frac{1}{2} \frac{g}{M_W} h^3 + \frac{1}{16} \frac{g^2}{M_W^2} h^4 \right) + \frac{1}{8} g \frac{t_3^2}{t_1^2} \frac{M_h^2}{\Lambda^2} \left( 2 M_W h^3 + \frac{7}{4} g h^4 + \frac{1}{4} \frac{g^2}{M_W} h^5 \right), \quad (3.8)$$

showing the effect of the portal interaction  $\lambda_{12}$  ( $t_{1,3}$  are given in Eq.(3.5)). The loop-generated (local) part of the potential will not be reproduced here and can be found in ref. [50]. The potential in Eq.(3.8) follows after canonical normalization. Any Lagrangian for low-energy theories requires canonical normalization; in the SESM only the  $h$  field requires a shift,

$$h \rightarrow Z_h h' \quad Z_h = 1 - \frac{g^2}{96 \pi^2} \frac{M_W^2}{\Lambda^2} \frac{t_3^2}{t_1^2} (t_1 - t_3)^2. \quad (3.9)$$

Actually, we can introduce shifts also for  $M_W$  and  $M_h$  so that the bare mass terms for physical states are SM-like. The shift in  $M_h$  gives the typical ‘‘fine-tuning’’ which is often present when we derive the small of a low state from some UV completion. For comparison, in the SMEFT the Higgs potential is

$$U_{\text{SMEFT}} = U_{\text{SM}} + \frac{g_6}{\sqrt{2}} \left[ a_\phi U_1^{(6)} + (4 a_{\phi_W} - a_{\phi_D} + 4 a_{\phi_\square}) U_2^{(6)} + (a_{\phi_D} - 4 a_{\phi_\square}) U_3^{(6)} \right], \quad (3.10)$$

where we have included the effect of canonical normalization. The four components are given by

$$\begin{aligned} U_{\text{SM}} &= -2 \frac{M_W}{g} \beta_h h - \frac{1}{2} \beta_h h^2 - \frac{1}{2} M_h^2 h^2 \\ &\quad - \frac{1}{4} g \frac{M_h^2}{M_W} h^3 - \frac{1}{32} g^2 \frac{M_h^2}{M_W^2} h^4, \\ U_1^{(6)} &= 2 g M_W h^3 + \frac{3}{2} G^2 h^4 + \frac{3}{8} \frac{g^3}{M_W} h^5 + \frac{1}{32} \frac{g^4}{M_W^2} h^6, \\ U_2^{(6)} &= -\frac{1}{8} \beta_h h^2 - \frac{1}{16} g \frac{M_h^2}{M_W} h^3 - \frac{1}{64} g^2 \frac{M_h^2}{M_W^2} h^4, \\ U_3^{(6)} &= -\frac{1}{4} \frac{g}{M_W} h \partial_\mu h \partial_\mu h - \frac{1}{16} \frac{g^2}{M_W^2} h^2 \partial_\mu h \partial_\mu h, \end{aligned} \quad (3.11)$$

where  $\beta_h$  is designed to cancel tadpoles, order-by-order in perturbation theory.

The behavior of the mixing angle  $\alpha$  is not selected a priori but follows from the hierarchy of VEVs,

$$\sin \alpha = \frac{t_3}{t_1} \frac{M_W}{M_s} \left[ 1 + \left( \frac{t_2}{t_1} - \frac{3}{2} \frac{t_3^2}{t_1^2} \frac{M_W^2}{M_s^2} \right) \right] + \mathcal{O}(M_s^{-5}). \quad (3.12)$$

where we have introduced

$$t_2 = \frac{1}{4} \frac{M_h^2}{M_w^2} + \frac{t_3^2}{t_1}, \quad (3.13)$$

and  $t_{1,3}$  are defined in Eq.(3.5). The SESM is the simplest BSM model where we can discuss the general strategy. Once again, the SMEFT limit of the SESM can be obtained by integrating out the heavy field  $\chi$ , while retaining the doublet  $\Phi$  even though  $\chi$  and  $\Phi$  are not the physical fields. Is the mixing subleading and diagonalization not needed? To answer this question we underline that the key parameter is the portal interaction between the doublet and the singlet fields [54], i.e.  $\lambda_{12}$ . Our result is  $\sin \alpha \propto M_w/M_s$ , see also ref. [53]; additional suppression of the heavy mode can be obtained by requiring that  $\lambda_{12} \propto g^2 M_w/M_s$  [55] (the so-called decoupling limit). Only in this case are mixing effects moved to higher-dimensional operators (higher than  $\dim = 6$ ). It is worth noting that the SM decoupling limit cannot be obtained by making assumptions about only one parameter, indeed the relevant expansion parameters are  $\lambda_{12}/\lambda_1$  and  $M_w/M_s$ . We have adopted the more conservative approach, considering the non-decoupling limit where we keep  $\lambda_{12}$  and  $\lambda_1$  as free parameters of the effective theory. In other words, the only assumption that we make is that the ratio of couplings is of the order of a perturbative coupling, i.e.  $\lambda_{12}/\lambda_1^2 < 1/2$ .

### Summary of the SESM

in order to achieve  $M_H \rightarrow \infty$  and  $M_h$  finite with a mixing angle  $\alpha \rightarrow 0$ , one can consider the limit  $\mu_1 \rightarrow \infty$  ( $\mu_1$  being the only new mass scale in the singlet sector). However, keeping  $M_w$  finite requires yet another modification. An obvious way out is provided by forcing decoupling of the singlet sector by taking  $\lambda_{12} \rightarrow 0$  as well <sup>1</sup>. It is important to realize that the decoupling theorem (the Appelquist Carrazzone theorem [56]) tell us that the effects of heavy particles go into local terms in a field theory, either renormalizable couplings or in non-renormalizable effective interactions suppressed by powers of the heavy mass. In the SESM (as in many other models) we should replace the last statement with “suppressed by powers of the heavy mass and/or by powers of the portal interaction”.

The non-decoupling assumption does not exclude low values of  $\lambda_{12}$  for which the effect of mixing becomes less relevant. For unitarity constraints on the Higgs portal see refs. [55, 57, 58]. One of the first examples of adding a scalar singlet in a gauge invariant way to the Higgs system can be found in ref. [59], where renormalization is also discussed.

In the singlet extension of the SM we will have

$$\Phi_{SESM} = [\Phi_{SM(h_2 \rightarrow h)}] + \frac{1}{\sqrt{2}} [(\cos \alpha - 1) h + \sin \alpha H] e, \quad (3.14)$$

where  $e^\dagger = (1, 0)$ . It is worth noting that  $\Lambda \neq M_H$  and that the limit  $\Lambda \rightarrow \infty$  should be computed in the mass eigenbasis, not in the weak eigenbasis. As a result,  $\alpha$  is a function of  $\Lambda$  and must also be expanded in powers of  $\Lambda$ . This means that parts of the linear multiplets are integrated out, while other states are retained. A large number of  $1/\Lambda^2$  terms come from the expansion of the mixing angle, not from the integration of the heavy fields, unless  $\lambda_{12}$  is further suppressed by an additional “external” choice or assumption. Therefore, we assume that the ratio  $t_3/t_1$  of quartic couplings is  $\mathcal{O}(1)$ , without excluding strongly coupled scenarios [54].

As an example of mixing, we consider the operator  $\mathcal{O}_{weak} = \widehat{\mathcal{O}} (D_\mu \Phi)^\dagger D^\mu \Phi$  where  $\widehat{\mathcal{O}}$  does not contain Higgs fields. We derive

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<sup>1</sup>We acknowledge an important discussion with S. Dittmaier

$$\begin{aligned} \mathcal{O}_{weak}^{(d+4)} = \widehat{\mathcal{O}}^{(d)} & \left[ (D_\mu \Phi_h)^\dagger D^\mu \Phi_h - \sqrt{2} \sin^2 \frac{\alpha}{2} g_{h_2 V V} h V V \right. \\ & \left. + \frac{1}{\sqrt{2}} \sin \alpha g_{h_2 V V} H V V \right], \end{aligned} \quad (3.15)$$

where  $g_{h_2 V V}$  is the SM Higgs-VV coupling and  $V = W, Z$ . Furthermore,  $\Phi_h$  is the SM scalar doublet where  $h_2$  has been replaced by the light Higgs field in SESM. We have also made explicit the dimension of the operators.

### THDM models

Another class of BSM models include additional doublets: the so-called THDM models. There are four THDM models that differ in the fermion sector: they are type I, II, X and Y, see ref. [12]. The THDM models contain five physical states, two of which are neutral and even under CP transformations, one is neutral and CP-odd, and the remaining two carry the electric charge  $\pm 1$  and are degenerate in mass. It is assumed that the resonance measured at the LHC is the lighter CP-even Higgs  $h$ , while the other particles are labelled  $H$ ,  $A$ , and  $H^\pm$ , respectively. The above-mentioned THDM types contain eight independent parameters in the Higgs potential.

Once again, the first problem in deriving the low-energy behavior of any BSM model is represented by the individuation of the cutoff scale; the SMEFT requires a unique scale, which implies a degeneracy of masses in the BSM model. Of course, multiple scales are relevant only if their effect is of the same order of  $\dim = 8$  operators. Two options have been discussed in the literature both in the unbroken phase [54] and in the mass eigenstates [50, 60], the latter based on the fact that custodial symmetry requires almost degenerate heavy states.

A more general situation would be the following: light masses ( $m_i$ ) and two heavy masses with  $m_i \ll M_j$  but  $|M_1^2 - M_2^2| \ll M_1^2 + M_2^2$ . Given  $M_-^2 = (M_1^2 - M_2^2)/2$ , the most general result is given by a triple expansion, in  $M_{1,2}^{-2}$  and  $M_-^{-2}$ , see refs. [3, 61] for details.

### SMEFT-predicted observables

Having discussed few examples of BSM models and their low-energy limits we are ready to formulate

**Proposition 2** *Can we successfully describe an observed SM-deviation in the SMEFT language? Can we learn something about the underlying BSM physics using the SMEFT framework? We can perform a fit of the SMEFT coefficients,  $\{\mathbf{a}\}$ , to a set of observables,  $\{\mathbf{O}\}$ . Take the best-fit results from the SMEFT fit to the data,  $\{\hat{\mathbf{a}}\}$ , and compute SMEFT-predicted observables  $\{\hat{\mathbf{O}}\}_{\text{SMEFT}} = \{\mathbf{O}\}_{\text{SMEFT}}(\{\hat{\mathbf{a}}\})$ .*

- *Within framework  $X$ , with parameters  $\vec{p}$ , we compute  $\{\mathbf{O}\}_X(\vec{p})$  and compare with  $\{\hat{\mathbf{O}}\}_{\text{SMEFT}}$ .*

*There are two possible scenarios: we can perform the calculation directly using the Lagrangian of framework  $X$  (for SESM see ref. [62, 63]) or we can compute the low-energy limit of  $X$  and use the corresponding effective Lagrangian (for SESM see ref. [50]).*

As discussed in section 2.1 there are “flat directions” in the space of Wilson coefficients; by this we mean that observables depend (at  $\dim = 6$ ) on linear combinations of Wilson coefficients. This problem will show up whenever a limited set of experimental data is considered, e.g.  $N_{\text{data}} < N_{\text{w}}$ ; of course, it is not only a question of how many points but also what sensitivity the observables have to which operator.

A problem will remain if some of the linear combinations of the  $\mathbf{a}^6$  is poorly constrained by itself, a different aspect of “flat directions” or “sloppiness” of the model. For instance, consider vector-like fermions with opposite hypercharge: the operator  $\mathcal{O}_{\text{WB}}^{(6)}$  is not generated while the operator  $\mathcal{O}_{\text{W}}$  is

LG, i.e. of  $\mathcal{O}(g^3/16\pi^2)$ . A way out could be to measure processes with an even number of B-legs. Another example [64], it is well known that the Higgs cross-sections alone cannot distinguish between Higgs couplings to gluons and top quarks. Furthermore, consider the process  $\bar{u}u \rightarrow Z h$  it depends on the following combinations of Wilson coefficients

$$\begin{aligned} a_{uV} &= a_{\phi u} + a_{\phi q}^{(1)} + a_{\phi q}^{(3)}, & a_{uA} &= a_{\phi u} - a_{\phi q}^{(1)} - a_{\phi q}^{(3)}, \\ a_n^{(1)} &= a_{\phi D} - 4 a_{\phi W}, & a_n^{(2)} &= a_{\phi D} + 4 a_{\phi W} + 4 a_{\phi \square}, \\ a_{ZZ} &= s_W^2 a_{\phi B} + c_W^2 a_{\phi W} - s_W c_W a_{\phi WB}. \end{aligned} \quad (3.16)$$

For instance the  $\bar{u}uZh$  contact vertex is

$$i \frac{gg_6}{2\sqrt{2}M_W c_W} \gamma^\mu \left( a_{uV} - a_{uA} \gamma^5 \right). \quad (3.17)$$

Alternative approaches have been proposed for dealing with the problem: diagonalization of the Fisher information matrix [65, 66] (FIM) or singular-value decomposition [67]. Sloppiness of a model is observed in many branches of physics. In those cases the Fisher information is ill-conditioned: one possibility is that the eigenvalues of the FIM are dependent on how the model has been parametrized, i.e. we should choose a more natural parametrization from a phenomenological point of view. In any case it is convenient to associate models with geometrical manifolds [68], with Wilson coefficients (or generalized kappas) as coordinates.

To summarize: there are combinations of Wilson coefficients to which measurements are not sensitive. Technically speaking the likelihood is flat (no curvature) when moving away from maximal likelihood. The results should be obtained from a fit after re-parametrizing observables into a “measurement” recombination of Wilson coefficients. Different operators cannot be disentangled by the measurement; therefore, only combinations are constrainable and it makes sense to use them as the new “coordinates”.

All in all we insist on the fact that the main emphasis should be given to SMEFT-reconstructed observables and not to a list of Wilson coefficients, e.g. SMEFT observables can be reconstructed by fitting linear combinations of Wilson coefficients. The problem becomes more complex when we include  $\mathcal{O}(1/\Lambda^4)$  terms. There are two reasons for that, there are 993 dim = 8 operators for one generation (44807 for 3 generations and each observable depends on linear combinations of dim = 6, 8 coefficients and on quadratic combinations of dim = 6 coefficients. The latter originate from the square of dim = 6 (one insertion) and from the interference between two dim = 6 insertions and dim = 4. In conclusion it would be interesting to present tables similar to the ones produced at LEP with observables, measurements, SMEFT fits and pulls <sup>2</sup>.

Starting from any BSM Lagrangian we can compute observables, including one-loop diagrams and renormalization. From any BSM model we can compute the low-energy limit, obtaining the corresponding effective Lagrangian, the LEBSM Lagrangian, which should be used consistently. No additional problem will arise if we restrict the LEBSM to tree-generated operators. Special care must be adopted when loop-generated operators are included, as discussed in ref. [50]. Consider, for example, the hVV vertex: the tree-level generated vertex can be used in any LO/NLO calculation, i.e. it can be consistently inserted in one-loop diagrams containing light particles. On the other hand, the loop-generated vertex can only be used, at tree level, in one loop calculations. I.e. it should not be inserted into loops of light particles.

One of the chief LHC statistical challenges is to devise techniques to test efficiently whether the data support the solid observation of an unexpected physics phenomenon or not. For practical reasons, we may need a choice of the observables to consider. This effectively means testing only

<sup>2</sup>see for instance <http://lepewwg.web.cern.ch/LEPEWWG/plots/winter2012/>



a limited class of BSM extensions for which such choice shows enhanced sensitivity to processes in the kinematic range of the LHC.

Furthermore, we will have to check the statistical consistency between the simulated distribution of the BSM signal and the SMEFT signal [69].

Significance tests tell us how (statistically) confident we can be that there is truly a difference between a BSM model and the SMEFT. For example: the null hypothesis, there is no “real” difference and the alternative hypothesis, there is a difference. A significance test should measure how much evidence there is in favor of rejecting the null hypothesis.

### 3.2 LHC, SM, BSM, EFT, distances and information

When we work with a family of differential distributions (SM, SMEFT or BSM), there would seem to be an obvious way to proceed: calculate the distance between distributions. We ask how close we can come to guessing a BSM model, based on an observation.

We assume that a global SMEFT fit has been performed, returning the best value of the Wilson coefficients,  $\{\hat{a}\}$  and their covariance matrix. Consider a differential kinematic distribution,  $D(x)$ ; it could be  $x = p_\perp$  and  $D = d\sigma/dp_\perp$  for a specific process; furthermore,  $x \in X$ . Given two distributions,  $f(x)$  and  $g(x)$  where

$$\int_X dx f(x) = \int_X dx g(x) = 1, \quad (3.18)$$

we need to define their “distance”,  $\text{dist}_B(f, g)$ ; one choice, used in the literature [69], would be the  $L^2$ -norm of  $f - g$ . Here we prefer to use the so-called Bhattacharyya distance [70], based on the following definitions:

$$\rho(f, g) = \int_X dx \sqrt{f g}, \quad \text{dist}_B = -\ln[\rho(f, g)]. \quad (3.19)$$

Note that  $X$  can be the full phase-space; but instead of integrating over the entire phase space, it could be relevant to study how the information is distributed in phase space.

This distance satisfies  $0 \leq \rho(f, g) \leq 1$  and  $\rho(f, g) = 1$  iff  $f = g$  while  $\rho(f, g) = 0$  iff  $f$  and  $g$  are orthogonal. If  $X$  is split into a chosen number of bins then

$$\rho(f, g) = \sum_{i=1}^n \sqrt{f_i g_i} \quad (3.20)$$

where  $n$  is the number of bins and  $f_i, g_i$  are the numbers of members of samples  $f$  and  $g$  in the  $i$ -th bin. The Bhattacharyya coefficient  $\rho$  will be zero if there is a multiplication by zero in every bin.

The Bhattacharyya distance is related to the Hellinger distance (which obeys the triangular inequality) by  $H^2 = 1 - \rho$ .  $H$  is a probabilistic analog of the Euclidean distance and can also be used to quantify the distance between measures from the same distribution indexed by different parameters,  $f(x; \theta_1, \dots, \theta_k)$  [71]. This is particularly relevant when we want to compare a given differential distribution  $D$  in some BSM model with the corresponding one in the SM. Here, for example,  $x = p_\perp$  and the  $\theta$  parameters are the BSM ones, i.e.  $\theta_i = 0, \forall i$  is the SM.

The concept of Hellinger information [71] is related to the Hellinger distance. Under certain regularity conditions it is closely related to Fisher information which has been shown to encode the maximum sensitivity of observables to model parameters for a given experiment [65, 72]. Hellinger information can be also used to describe information properties of the parametric set in situations where the Fisher information does not exist. By means of the Hellinger distance we can obtain the robust estimators of multivariate location and covariance, as proposed in ref. [73]. For additional usage of the Hellinger distance see ref. [74, 75].

The Fisher information metric [72] measures the amount of information a random variable  $X$  contains in reference to an unknown parameter  $\theta$ . The Fisher information distance is a consistent metric, enabling the approximation of the information distance when the specific parameterization of the manifold is unknown, and there have been many metrics developed for this approximation. The Hellinger distance is closely related to the information distance [76]. Furthermore, Hellinger distance analogs of likelihood ratio tests have been proposed for parametric inference in ref. [77]. Finally, quoting ref. [78], we can say that the classical Neyman-Pearson Lemma says that the “standard” test for distinguishing two distributions is the log-likelihood test but another classical test says that the optimal sample complexity is characterized by the square of the Hellinger distance.

Using a geometric framework we will discuss the interplay between SM, SMEFT and BSM models. First of all we analyze the impact of the BSM signal: for that we maximize the distance  $\text{dist}_B(D_{\text{BSM}}, D_{\text{SM}})$ , varying the BSM parameters; this should be done under the condition that the BSM model remains a weakly coupled theory, e.g. the running coupling constants do not exceed some critical value and the conditions of vacuum stability are satisfied for each value of the high scale  $\Lambda$ . Therefore, conditions are necessary to single out parameter regions in the BSM model which cannot be treated perturbatively [79]. If the maximal distance is less than some, preselected, value then  $D(x)$  or  $X$  are not a good choice.

Next we want to discuss BSM-SMEFT compatibility: for that we minimize the distance  $\text{dist}_B(D_{\text{BSM}}, D_{\text{SMEFT}})$  by varying both the BSM parameters and the Wilson coefficients under the following condition: we define a radius in the space of Wilson coefficients,  $r^2 = \sum_i a_i^2$  and require that  $r \geq \hat{r}$  where  $\hat{r}^2 = \sum_i \hat{a}_i^2$ . If the distance is greater than some, preselected, value then there will be a tension between the BSM model and the SMEFT.

At this point we can compare  $D_{\text{BSM}}$  and  $D_{\text{SMEFT}}$  at the minimum of their distance with the band corresponding to  $D_{\text{SMEFT}}$  reconstructed and derive informations on the goodness of the BSM model.

Finally, varying one SMEFT operator at a time is unlikely to be a useful description of UV-complete BSM physics. For instance, without flavour assumptions, one needs to deal with a large number of independent operators corresponding to three fermion generations. Because of this challenge, the complexity of the SMEFT analyses has, thus far, been restricted to a subset of higher-dimensional operators. More recently, a novel approach has been developed [36].

Multi-operator analysis and the combination of different observables is needed. The existence of additional operators in the HEFT-limit of BSM models may help.

### 3.3 BSM contiguity

The concept of SM isolation has been defined and discussed in ref. [80]. Here we propose an alternative approach. Within the  $\text{dim} = 6$  SMEFT approach we can only fit Wilson coefficients/ $\Lambda^2$ . For any BSM model the heavy scale is one of the parameters. Let  $\Lambda_{max}$  be the highest scale which can be tested at LHC. For a given differential distribution  $D(x)$  we define  $\hat{D}_{\text{SMEFT}}$  as the D-distribution as the D-distribution obtained by fitting the SMEFT to data. Given

$$\text{dist}_{\text{BSM}} = \text{dist}_B(D_{\text{BSM}}, \hat{D}_{\text{SMEFT}}), \quad (3.21)$$

we minimize w.r.t. the BSM parameters (including the heavy scale  $\Lambda_{\text{BSM}}$ , e.g.  $M_s$  in the SESM) Given a reference value for the distance,  $d_{ex}$  we have the following situations:

1.  $\min \text{dist}_B > d_{ex}$ , the BSM model is excluded,
2.  $\min \text{dist}_B < d_{ex}$  and  $\Lambda_{\text{BSM}} > \Lambda_{max}$ . The BSM model and the SM are not contiguous (i.e. isolation of the SM).
3.  $\min \text{dist}_B < d_{ex}$  and  $\Lambda_{\text{BSM}} < \Lambda_{max}$ : the BSM model and the SM are contiguous.

For a MC approach to the same problem see ref. [81].

### 3.4 How good is the truncation error?

Ref. [60] presents a discussion on the possible failure of  $\dim = 6$  operators, in the low-energy limit of a BSM model (LEBSM), in describing LHC kinematics. This problem is equivalent to discussing the “truncation error” introduced in expanding observables in powers of  $1/\Lambda$ .

Given two distributions  $f(x)$  and  $g(x)$ , with  $x \in X$ , their Hellinger distance can be written as

$$H^2(f, g) = \frac{1}{2} \int_X dx (\sqrt{f} - \sqrt{g})^2. \quad (3.22)$$

For a given process and a given distribution we want to compute the “distance” between  $f = D_{\text{BSM}}$ , i.e. distribution  $D$  computed in the full BSM model, and its truncated, low-energy, expansion, i.e.

$$g = D_{\text{SM}} \left( 1 + \Delta_v \frac{v^2}{\Lambda^2} + \Delta_e \frac{E^2}{\Lambda^2} \right), \quad (3.23)$$

where  $\Delta_v$  and  $\Delta_e$  are, process dependent, kinematic factors and we have separated the “scale”-growing contribution, i.e.  $E$  can be any scale describing the process while  $v$  is the Higgs VEV. We obtain that

$$H^2(D_{\text{BSM}}, D_{\text{LEBSM}}) = H^2(D_{\text{BSM}}, D_{\text{SM}}) - \int_X dx (\sqrt{D_{\text{BSM}}} - \sqrt{D_{\text{SM}}}) \sqrt{D_{\text{SM}}} \left( \Delta_v \frac{v^2}{\Lambda^2} + \Delta_e \frac{E^2}{\Lambda^2} \right), \quad (3.24)$$

is as a quantity which can “measure” uncertainties associated to the truncation at  $\mathcal{O}(1/\Lambda^2)$ .

### 3.5 SMEFT validity from unitarity bounds

We should keep in mind that unitarity constraints must always be understood as “perturbative unitarity” constraints. Given a strictly renormalizable model depending on a parameter  $p$ , the statement  $p < p_{\text{max}}$  means that for  $p > p_{\text{max}}$  the model becomes strongly interacting. For the SMEFT we should distinguish between one-at-a-time bounds and couple-channel bounds. For the sake of simplicity we consider the case of a single Wilson coefficient for which we have derived an upper bound

$$| Q^2 \frac{a_i^{(6)}}{\Lambda^2} | \leq A_i, \quad (3.25)$$

where  $Q$  is the scale where the SMEFT is being tested. Suppose that, as a result of a fit, we have derived  $| a_i^{(6)}/\Lambda^2 < B_i |$ . When the bound is saturated we conclude that

$$| Q^2 | < \frac{A_i}{B_i}, \quad (3.26)$$

or the SMEFT stops to be valid as a perturbative expansion. These are not statements that unitarity is violated in the SMEFT. Unitarity “would be violated”, if we could trust the perturbative expansion, which we cannot; there are perturbative unitarity bounds, but the bounds also imply that loops and higher dimensional operators must be important. It is interesting to observe that there is complementarity between (h) “pole” vs. “tail” measurements: derivative operators influence tail observables and pole observables in a different way. Tails are interesting, the accessible  $\Lambda$  can be higher but, unfortunately, predictions will break in “tails” (or new physics will be seen before the breaking); projecting data into the SMEFT will have a large intrinsic uncertainty, i.e. we do

not know what exactly is going on because the SMEFT interpretation becomes a series where the expansion parameter is close to 1 and/or the perturbative unitarity bound is saturated.

UV complete models show then phenomenon of delayed unitarity [82] which is best seen in VV-scattering. If we have a light Higgs boson (h) and an heavy one (H), then the scattering could get strong for a range of energies, until the high-energy UV physics starts unitarizing. The energy growing behavior is tamed only above  $M_H$  and it is expected if there is space enough between  $M_h$  and  $M_H$ .

#### 4 SMEFT vs. BSM models: a critical summary

There are several steps to be considered when comparing the SMEFT with a BSM model. First of all we recall the classification of ref. [18] where  $\text{dim} = 6$  operators can be PTG or LG; in any given BSM model, some operators may arise from tree diagrams, while others may only arise from loop corrections. The equivalence theorem [83] relates some operators arising from loops to operators arising from trees; for the apparent puzzle of equivalence of  $\mathcal{O}_{PTG}$ ,  $\mathcal{O}_{LG}$  see section 4 of ref. [18]. Imagine a situation where the SMEFT is defined by choosing as basis vectors PTG operators while the BSM model generates LG operators; this is exactly the scenario under discussion: the equivalence of two operators is a property of SMEFT while it is possible for the BSM theory to generate one but not the other. From this point of view we must be careful not to omit any basis operators, since their contributions to Green's functions can be very different.

For the sake of simplicity we consider the decay  $h \rightarrow 4$  leptons. We have the following situations:

- 1) in the SMEFT we take the best available prediction for the  $\text{dim} = 4$  part [84] and add tree diagrams containing one  $\text{dim} = 6$  operator, see ref. [85]
- 2) In the BSM model we include tree diagrams and take the large  $\Lambda$  limit (once  $\Lambda$  has been identified).

It follows that a comparison between 1) and 2) is not adequate since LG (local) operators have been included in the SMEFT. Therefore,

- 3) we consider the BSM model at one-loop and take the large  $\Lambda$  limit. However, in most cases, the result includes mixed heavy-light contributions which are not present in 1). As a consequence.
- 4) we include loops with one  $\text{dim} = 6$  operator insertion in the SMEFT predictions. Always following the PTG/LG classification, LG insertions call for a two-loop calculation in the BSM model.

There is an interesting connection between ultraviolet and infrared [86, 87]: the Froissart unitarity bound and dispersion relations imply a connection between unitarity in the UV (BSM models) and positivity in the IR (SMEFT). The implication is that part of the observable parameter (Wilson coefficients) space is inconsistent with causality and analyticity.

#### 5 SMEFT and LEP pseudo-observables

The main question is “how to use (LEP 1) EWPD (POs) in the SMEFT analysis?”. In order to understand LEP 1 Pseudo-Observables (hadronic peak cross-section etc.) we have to clarify the strategy which was used [88].

What the experimenters did <sup>3</sup> was just collapsing (and/or transforming) some “primordial quantities” (say number of observed events in some predefined set-up) into some “secondary quantities”

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<sup>3</sup>Partially based on an old discussions with Manel Martinez.

or realistic-observables (RO) which are closer to the theoretical description of the phenomena. In this step, if the number of quantities is reduced, this implies that some assumptions have been made on the behaviour of the primordial quantities. The validity of these assumptions is judged on statistical grounds. Within these assumptions (QED deconvolution, resonance approach, etc.) the secondary quantities are as “observable” as the first ones. At this point, let us clarify that even the “primordial quantities”, are obtained through many assumptions (event classification, detector response, etc.) which, as in the previous case, can be judged just on statistical grounds.

The practical attitude of the experiments was to stay with a Model-Independent fit, i.e. from ROs  $\rightarrow$  POs (plus a SM remnant) for each experiment, and these sets of POs were averaged. The result of this procedure are best values for POs. The extraction of Lagrangian parameters was based on the LEP-averaged POs.

At LEP all QED initial state corrections and QED+QCD final state corrections were deconvoluted. The rationale for the de-convolution was based on the fact that all experiments used different kinematic cuts and selection criteria, while an objective requirement was put forward by the scientific community for having universal results anchored to the Z-peak. Assuming a structure function representation for the initial fermions, in turn, allowed us to de-convolute the measurements and to access the hard scattering at the nominal peak. Therefore, the transition from ROs to POs involves certain assumptions that reflect our understanding of SM effects. In particular it was used the fact that in the SM there are several effects, such as the imaginary parts or Z- $\gamma$  interference or the pure QED background, having a (usually) negligible influence on the line shape. Therefore, POs are determined by fitting ROs but we will have some ingredients which are still taken from the SM, making the model-independent results (slightly) dependent upon the SM. In other words, the information needed should contain a complete definition of lineshape and asymmetry POs, together with the residual SM dependence in model-independent fits; this includes a description on what is actually taken from the SM.

Another approach was also used at LEP, i.e. extraction of Lagrangian parameters directly from the ROs, which are not raw data but rather educated manipulations of raw data, e.g. distributions defined for some simplified setup.

The main question to be answered is: are POs valid/usable even in the case where the SM is replaced by the SMEFT? It is hard to believe that we will repeat the extraction of the SMEFT Lagrangian parameters (this time including Wilson coefficients) directly from the ROs. Therefore, the adopted strategy is to perform a SMEFT fit to the LEP POs; guidance for estimating the corresponding uncertainty can be based on the following fact: it has been tested by each LEP experiment how the results on the SM parameters differ between a SM fit to its own measured ROs, and a SM fit to the POs which themselves are derived in a fit to the same ROs. For each experiment, the largest difference in central values, relative to the fitted errors, was observed for  $M_Z$ , up to 30% of the fit error. For the other four SM parameters, the observed differences in fitted central values and errors were usually below 10%–15% of the fit error on the parameter.

Another effect has to do with scheme dependence. We should keep in mind that one of the key ingredients in computing the LEP POs has been  $\alpha_{em}$  at the mass of the Z. Define the running of  $\alpha_{em}$  as

$$\alpha_{em}(M_Z) = \alpha_{em}(0) \left[ 1 - \Delta\alpha^{(5)}(M_Z) - \Delta\alpha_t(M_Z) - \Delta\alpha_t^{\alpha_s}(M_Z) \right]^{-1}, \quad (5.1)$$

showing the top contribution, the mixed weak-QCD effects, with  $\Delta\alpha^{(5)}(M_Z) = \Delta\alpha_l(M_Z) + \Delta\alpha_{had}^{(5)}(M_Z)$ , showing the leptonic part and the hadronic one. The SMEFT effect (neglecting LG operators in loops) is equivalent to replace

$$\Delta\alpha^{(5)}(M_Z) \rightarrow (1 - \kappa_\alpha) \Delta\alpha_z^{(5)}, \quad (5.2)$$

with  $\kappa_\alpha = 0.188 a_{\phi D}$  at  $\Lambda = 3 \text{ TeV}$ , to give an example. As a result we obtain

$$|\kappa_\alpha \Delta\alpha_t| > |\Delta\alpha_l|, \quad |\kappa_\alpha \Delta\alpha_t| \approx |\Delta\alpha_t^{\alpha\alpha_s}|, \quad (5.3)$$

i.e. there are SMEFT effects of the same order of magnitude than the SM  $\mathcal{O}(\alpha_{em} \alpha_s)$  ones.

## 6 Moving towards dimension 8

There has been recent progress in building a  $\text{dim} = 8$  basis [89–91]; imagine we use such a basis, say a generalized Warsaw basis, and that global fits have been performed giving the best values for the Wilson coefficients,  $\{\hat{a}^{6,8}\}$ . We recall a well-known result: removing a redundant operator  $\mathcal{O}_R^{(6)}$  with coefficient  $a_R^6$  will propagate  $a_R^6$  into the Wilson coefficients of  $\text{dim} = 8$  operators [3, 92].

In the bottom-up approach the shift due to the field redefinition which eliminates  $\mathcal{O}_R^{(6)}$  can be absorbed into the coefficients of operators which are already present in the theory. Therefore, in this approach, we only “measure” combinations of Wilson coefficients, linear in  $\{a^8\}$  and quadratic in  $\{a^6\}$ . To give an example we consider the operator

$$\mathcal{O}_R^{(6)} = \Phi^\dagger \Phi (D_\mu \Phi)^\dagger D_\mu \Phi. \quad (6.1)$$

The term containing

$$g^2 \frac{a_R^6}{\Lambda^2} \mathcal{O}_R^{(6)} \quad (6.2)$$

can be eliminated by the transformation [93]

$$\Phi \rightarrow \Phi - g^2 \frac{a_R^6}{\Lambda^2} (\Phi^\dagger \Phi) \Phi, \quad (6.3)$$

which induces higher order compensations, e.g.

$$\mathcal{O}_{\phi B}^{(6)} \rightarrow \mathcal{O}_{\phi B}^{(6)} - g^2 \frac{a_R^6}{\Lambda^2} \mathcal{O}_{\text{hB}}^{(8)}, \quad (6.4)$$

where the  $\text{dim} = 8$  operator is one of the operators of Tab. 4 in ref. [37]. However, consider some BSM model and compute its low-energy limit, obtaining some operator  $\mathcal{O}_a^{(8)}$  with a coefficient  $d_a$  which depends on the BSM parameters; in this limit we may also obtain some operator  $\mathcal{O}_R^{(6)}$  which is redundant in the basis where the fits have been performed and whose  $\text{dim} = 8$  compensation contains  $\mathcal{O}_a^{(8)}$ . As a consequence, when deriving relations between the set of Wilson coefficients  $\{a^8\}$  and the BSM parameters, we should keep in mind that  $d_a$  should not be compared with  $\hat{a}_a^8$ . Wrong relations should not be confused with “systematic”.

## 7 Precision calls

Precision calls <sup>4</sup>: the heavy particles in any BSM model are unstable and their description requires the introduction of the corresponding complex poles [94]. The Dyson-resummed propagator for particle  $\phi$  is

$$\Delta_\phi(s) = \left[ s - M_\phi^2 + \Sigma_\phi(s) \right]^{-1}, \quad (7.1)$$

where  $M_\phi$  is the renormalized mass and  $\Sigma_\phi$  is the renormalized  $\phi$  self-energy (to all orders but with one-particle-irreducible diagrams). The complex pole is defined as the complex solution of

$$s_\phi - M_\phi^2 + \Sigma_\phi(s_\phi) = 0. \quad (7.2)$$

<sup>4</sup>Inspired by [https://en.wikipedia.org/wiki/Night\\_Calls\\_\(album\)](https://en.wikipedia.org/wiki/Night_Calls_(album)).

To lowest order accuracy we can use

$$\Delta_\phi^{-1} = s - s_\phi, \quad (7.3)$$

where the complex pole is conventionally parametrized as

$$s_\phi = \mu_\phi^2 - i \gamma_\phi \mu_\phi. \quad (7.4)$$

Let us define the following quantities:

$$\overline{M}_\phi^2 = \mu_\phi^2 + \gamma_\phi^2, \quad \mu_\phi \overline{\Gamma}_\phi = \overline{M}_\phi \gamma_\phi. \quad (7.5)$$

It follows that

$$\frac{1}{s - s_\phi} = \left(1 + i \frac{\overline{\Gamma}_\phi}{\overline{M}_\phi}\right) \left(s - \overline{M}_\phi^2 + i \frac{\overline{\Gamma}_\phi}{\overline{M}_\phi} s\right)^{-1}, \quad (7.6)$$

which is equivalent to say that we have introduced a running width with parameters which are not the on-shell ones. Therefore, the low-energy limit of the propagator is controlled by barred parameters and not by the on-shell mass. Let  $M_{\phi\text{OS}}$  and  $\Gamma_{\phi\text{OS}}$  be the on-shell mass and width of the  $\phi$ , if  $\Gamma_{\phi\text{OS}}/M_{\phi\text{OS}} \ll 1$  we can write a perturbative solution of Eq.(7.2),

$$\mu_\phi^2 = M_{\phi\text{OS}}^2 - \Gamma_{\phi\text{OS}}^2 + \text{h.o.}, \quad \gamma_\phi = \Gamma_{\phi\text{OS}} \left[1 - \frac{1}{2} \left(\frac{\Gamma_{\phi\text{OS}}}{M_{\phi\text{OS}}}\right)^2\right] + \text{h.o.}, \quad (7.7)$$

and the difference between barred and on-shell quantities is  $\mathcal{O}(\Gamma_{\phi\text{OS}}^4/M_{\phi\text{OS}}^4)$ . Outside this region we have to solve Eq.(7.2) numerically.

To give an example we consider the SESM and introduce  $\bar{t}^2 = t_3^2/t_1$ , so that

$$\sin \alpha \sim 2 \bar{t} \frac{M_W}{M_H}. \quad (7.8)$$

Let  $\Gamma_\phi(s)$  be the total width of particle  $\phi$  at virtuality  $s$ ; we obtain

$$\Gamma_H(M_H^2) = \sin^2 \alpha \Gamma_h^{\text{SM}}(M_H^2) + \Gamma(H \rightarrow hh). \quad (7.9)$$

The first component behaves like

$$4 \left(\bar{t} \frac{M_W}{M_H}\right)^2 \Gamma_h^{\text{SM}}(M_H^2), \quad (7.10)$$

where, for  $M_H = 1 \text{ TeV}$  the best calculation gives  $\Gamma_h^{\text{SM}} = 647 \text{ GeV}$ . The second component [95], computed at LO, gives

$$\Gamma(H \rightarrow hh) = \frac{K^2}{32 \pi M_H} \left(1 - 4 \frac{M_h^2}{M_H^2}\right)^{1/2}, \quad (7.11)$$

$$K = g \sin \alpha \cos \alpha \left(M_h^2 + \frac{1}{2} M_H^2\right) \left(\frac{\cos \alpha}{M_W} + 2 \sqrt{t_1} \frac{\sin \alpha}{M_H}\right).$$

Therefore, in the large  $M_H$  limit we obtain

$$K \sim g \bar{t} M_H, \quad \Gamma(H \rightarrow hh) \sim \frac{g^2 \bar{t}_1^2}{32 \pi} M_H. \quad (7.12)$$

For ‘‘perturbative’’ values of  $\bar{t}$  the ratio  $\Gamma_H/M_H$  remains small, the difference between barred and on-shell parameters is negligible, i.e.  $s/M_H^2$  and  $\Gamma_H/M_H$  can be taken as the correct expansion parameters.

The advantage of Eq.(7.9) is that we can use the best available SM calculation for  $\Gamma_h^{\text{SM}}$ . However, Eq.(7.9) is valid only at LO and, when loops are included, should be modified into

$$\Gamma_H(M_H^2) = \sin^2 \alpha \Gamma_h^{\text{SM}}(M_H^2) + \Gamma(H \rightarrow hh) + \Delta \Gamma_H. \quad (7.13)$$

The reason is that in  $\Gamma_h^{\text{SM}}$  we are using  $M_H$  for Higgs couplings and Higgs propagators (in loops). Let us define

- $\Gamma_{\text{H}}^s$  as the part of  $\Gamma_{\text{H}}$  containing loops with internal h and/or H lines,
- $\Gamma_{\text{h}}^{\text{SM},s}$  as the part of  $\Gamma_{\text{h}}^{\text{SM}}$  containing loops with internal h lines of mass  $M_{\text{H}}$ .

Then the additional contribution can be written as

$$\Delta\Gamma_{\text{H}} = \Gamma_{\text{H}}^s - \sin^2 \alpha \Gamma_{\text{h}}^{\text{SM},s}. \quad (7.14)$$

It is worth noting that there are many different ingredients entering the calculation of  $\Delta\Gamma_{\text{H}}$ , i.e. loop diagrams, wave function factors, tadpoles and UV(finite) renormalization.

The presence of a resonance should be taken into account when considering the range of validity of the low-energy expansion of any BSM model, i.e. we should not enter the region where the shape of the resonance is already visible. Always using the SESM as an example we can ask for which values of  $M_{\text{H}}$  the ratio  $\Gamma_{\text{H}}/M_{\text{H}}$  is sizeable. For instance (at LO) we obtain

$$\Gamma(\text{H} \rightarrow \text{ZZ}) \sim \sin^2 \alpha \frac{G_{\text{F}} M_{\text{H}}^3}{16 \sqrt{2} \pi}, \quad (7.15)$$

and consider the ratio

$$x_{\text{z}} = \frac{\Gamma(\text{H} \rightarrow \text{ZZ})}{M_{\text{H}}}. \quad (7.16)$$

To obtain  $x_{\text{z}} = 1/2$  we need  $\sin \alpha M_{\text{H}} = 1.74 \text{ TeV}$ , which for  $\sin \alpha = 0.2$  puts the H resonance at 8.7 TeV. If we require  $\sin \alpha = 0.2$  and  $M_{\text{H}} = 3 \text{ TeV}$  then  $\Gamma(\text{H} \rightarrow \text{ZZ}) \approx 180 \text{ GeV}$  and  $\Gamma(\text{H} \rightarrow \text{WW}) \approx 360 \text{ GeV}$  which gives a rough estimate of the scale where LESESM gives reliable predictions.

## 8 Examples

In this section we will discuss significant differences between SMEFT results and BSM models. In order to discuss the low-energy limit of a BSM model, i.e. how the “expansion” is performed we consider, once again, the following integral

$$\text{I} = \mu_{\text{r}}^{\varepsilon} \int d^d q \left[ (q^2 + M_{\text{h}}^2) ((q + p_1)^2 + M_{\text{s}}^2) ((q + p_1 + p_2)^2 + M_{\text{h}}^2) \right]^{-1}, \quad (8.1)$$

where  $M_{\text{s}}$  is a “large” scale and  $\varepsilon = 4 - d$ ; this integral will appear whenever there is a real, heavy, field S added to the SM Lagrangian, e.g. with an interaction  $\Phi^\dagger \Phi S$ . To understand the details of the procedure we can say that there are 3 ways of “expanding” the integral (see also ref. [96]):

1. the heavy propagator is expanded as follows,

$$\frac{1}{(q + p_1)^2 + M_{\text{s}}^2} = \frac{1}{M_{\text{s}}^2} \left( 1 - \frac{(q + p_1)^2}{M_{\text{s}}^2} + \dots \right) \quad (8.2)$$

giving

$$\text{I} \sim \frac{i\pi^2}{M_{\text{s}}^2} \left[ \frac{1}{\varepsilon} - \ln \frac{M_{\text{h}}^2}{\mu_{\text{r}}^2} + 2 - \beta \ln \frac{\beta + 1}{\beta - 1} + \dots \right], \quad (8.3)$$

which corresponds to  $M_{\text{s}}^2 \gg |q^2| \sim |p_i^2|$ . This expression must be combined with EFT counterterms and it must be stressed that the soft part (the last two terms in the square bracket) cancels out in the matching procedure (although not a throwaway).



2. The heavy propagator is expanded while respecting the UV structure of the integral [97] (at one loop)

$$\frac{1}{(q+p_1)^2 + M_s^2} = \frac{1}{q^2 + M_s^2} \left( 1 - \frac{p_1^2 + 2p_1 \cdot q}{q^2 + M_s^2} + \dots \right) \quad (8.4)$$

giving the result of Eq.(1.19),

$$I \sim \frac{i\pi^2}{M_s^2} \left[ 1 + \ln \frac{M_s^2}{M_h^2} - \beta \ln \frac{\beta+1}{\beta-1} \right], \quad (8.5)$$

corresponding to  $M_s^2 \sim |q^2| \gg |p_i^2|$ . This result includes both soft and hard terms.

3. The expansion can be performed à la Mellin-Barnes (instead of a Taylor expansion of the “heavy” propagators), see ref.[3] for details. Our findings are that the Mellin-Barnes expansion should always be the preferred method.

It is worth noting that the expansion introduces kinetic logarithms. When we consider the addition of a complex, scalar, field to the SM and compute processes like  $gg \rightarrow \bar{t}t$  the non-local terms will contain also di-logarithms. Obviously, heavy-light diagrams (above their normal threshold) develop an imaginary part leading to  $\pi^2$ -enhanced terms in the corresponding cross-section (another reason to include loops).

Most of our examples are dealing with the (real) singlet extension of the SM. The SESM Lagrangian contains several parameters. When taking the  $M_s = \Lambda \rightarrow \infty$  limit we have

$$\sin \alpha \sim f_2 \frac{M_W}{\Lambda} \quad \text{and} \quad M_H \sim f_1 \left( 1 + f_2^2 \frac{M_W^2}{\Lambda^2} \right) \Lambda, \quad (8.6)$$

where  $M_W$  is the bare W mass and  $f_{1,2}$  are functions of the SESM parameters.

### 8.1 hhZZ

Let us consider the process  $Z_\mu(p_3) + Z_\nu(p_4) \rightarrow h(p_1) + h(p_2)$ . The LO SESM amplitude is the sum of 7 diagrams and in the limit  $\Lambda \rightarrow \infty$  we obtain

#### SMEFT

$$A_{\mu\nu} = \left[ 1 + \frac{1}{3\sqrt{2}G_F\Lambda^2} \Delta \right] A_{\mu\nu}^{\text{SM}} + \frac{1}{\sqrt{2}G_F\Lambda^2} \frac{g^2}{c_w^2} [F_1 \delta_{\mu\nu} + F_2 T_{\mu\nu}^{\text{sh}} + F_3 T_{\nu\mu}^{\text{sh}} + F_4 T_{\mu\nu}^{\text{sz}} + F_5 T_{\mu\nu}^t + F_6 T_{\mu\nu}^u] \quad (8.7)$$

#### SESM

$$A_{\mu\nu} = \left( 1 - 2\lambda_2^2 \frac{M_W^2}{\Lambda^2} \right) A_{\mu\nu}^{\text{SM}} + \frac{g^2}{c_w^2} \lambda_2^2 \frac{M_W^2}{\Lambda^2} \left( \frac{T_{\mu\nu}^{\text{sh}}}{t - M_Z^2} + \frac{T_{\nu\mu}^{\text{sh}}}{u - M_Z^2} \right) \quad (8.8)$$

where we have introduced the following quantities:

$$\begin{aligned} T_{\mu\nu}^{\text{sh}} &= M_Z^2 \delta_{\mu\nu} + p_{1\mu} p_{2\nu}, \\ T_{\mu\nu}^{\text{sz}} &= p_{3\mu} p_{4\nu} + \left( \frac{1}{2} s - M_Z^2 \right) \delta_{\mu\nu}, \\ T_{\mu\nu}^t &= (M_h^2 - M_Z^2 - t) \delta_{\mu\nu} - p_{1\mu} p_{4\nu} - p_{3\mu} p_{2\nu}, \\ T_{\mu\nu}^u &= (M_h^2 - M_Z^2 - u) \delta_{\mu\nu} - p_{3\mu} p_{1\nu} - p_{2\mu} p_{4\nu}, \end{aligned} \quad (8.9)$$

$$\begin{aligned}
\Delta &= 6 a_{\phi_W} - a_{\phi_D} + 10 a_{\phi_\square}, \\
F_1 &= 12 \frac{M_W^2}{s - M_h^2} a_\phi + \frac{1}{4} \frac{s}{s - M_h^2} (a_{\phi_D} - 4 a_{\phi_\square}) - \frac{1}{6} (7 a_{\phi_D} - 4 a_{\phi_\square}), \\
F_2 &= \frac{1}{6} \frac{1}{t - M_Z^2} (5 a_{\phi_D} - 8 a_{\phi_\square}), \\
F_3 &= \frac{1}{6} \frac{1}{u - M_Z^2} (5 a_{\phi_D} - 8 a_{\phi_\square}), \\
F_4 &= \frac{1}{M_Z^2} \left( 3 \frac{M_h^2}{s - M_h^2} + 1 \right) a_{zz}, \\
F_5 &= \frac{2}{t - M_Z^2} a_{zz}, \\
F_6 &= \frac{2}{u - M_Z^2} a_{zz}.
\end{aligned} \tag{8.10}$$

Furthermore,  $s$ ,  $t$ , and  $u$  are Mandelstam invariants,  $c_w$  is the cosine of the weak-mixing angle, and  $G_F$  is the Fermi coupling constant.

It is worth noting that in the SMEFT result we have made no distinction between PTG and LG operators. We observe that the largest number of  $1/\Lambda^2$  terms in SESM are due to the expansion of the mixing angle. Furthermore, there are terms in SMEFT which are not reproduced by the SESM expansion, not even at higher orders in  $1/\Lambda$ . We can twist SMEFT for this process but this would require (among other things) to set  $a_{zz}$  to zero and this Wilson coefficient also multiplies the transverse part of the  $hZZ$  LO vertex, contributing to SM-deviations in the decay  $h \rightarrow 4$ -fermions.

The comparison has been performed using the LO SESM prediction. There will be many more terms when SESM prediction is computed at the NLO level. The full SESM Lagrangian in the low-energy limit (the LESESM Lagrangian) has been presented in ref. [50]. It is convenient to write a generic term in the LESESM Lagrangian as follows:

$$\mathcal{O} = \frac{M_W^l}{\Lambda^n} \bar{\psi}^a \psi^b \partial^c (\Phi^\dagger)^d \Phi^e A^f, \tag{8.11}$$

where Lorentz, flavor and group indices have been suppressed,  $\psi$  stands for a generic fermion fields,  $\Phi$  for a generic scalar and  $A$  for a generic gauge field. All light masses are scaled in units of the (bare)  $W$  mass. We define dimensions according to

$$\text{codim } \mathcal{O} = \frac{3}{2} (a + b) + c + d + e + f, \quad \dim \mathcal{O} = \text{codim } \mathcal{O} + l. \tag{8.12}$$

Terms in the Lagrangian can be classified according to their dimension and their codimension. For instance, we obtain

$$\begin{aligned}
\mathcal{L}_{6,6} &= -\frac{1}{8} g^2 \frac{t_3^2}{t_1^2} \partial_\mu \Phi_h^2 \partial_\mu \Phi_h^2 - \frac{1}{384} \frac{g^4}{\pi^2} \frac{t_3^2}{t_1} \partial_\mu \Phi_h^2 \partial_\mu \Phi_h^2 \\
&\quad - \frac{1}{4096} \frac{g^6}{\pi^2} \frac{t_3^3}{t_1^3} \beta_1 \Phi_h^6 - \frac{1}{3072} \frac{g^6}{\pi^2} \frac{t_3^3}{t_1} (5 + 9 A_0) \Phi_h^6 \\
&\quad + \frac{3}{1024} \frac{g^6}{\pi^2} \frac{t_3^3}{t_1} B_{00} \Phi_h^2,
\end{aligned} \tag{8.13}$$

where  $\Phi_h^2 = h^2 + \phi^0 \phi^0 + 2 \phi^+ \phi^-$ . Furthermore, we have introduced

$$\begin{aligned}
A_0 &= \frac{2}{d-4} + \gamma + \ln \pi - 1 + \ln \frac{M_H^2}{\mu_R^2}, \\
B_{00} &= -A_0 - 1,
\end{aligned}
\tag{8.14}$$

where  $d$  is the space-time dimension,  $\gamma$  is the Euler-Mascheroni constant and  $\mu_R$  is the 't Hooft scale. The coefficient  $\beta_1$  is due to tadpoles and is given by

$$\beta_1 = -6 t_1^2 A_0. \tag{8.15}$$

It is worth noting that the presence of a  $\pi^2$  signals a loop-generated term. The remaining components of the LESESM Lagrangian contain many more terms, for instance  $\mathcal{L}_{6,5}$  contains

$$-\frac{1}{512} \frac{g^5}{\pi^2} \frac{t_3}{t_1} \left[ 6 + \left( 9 + \frac{t_3^2}{t_1^2} \right) A_0 \right] \Phi_h^2 h \left( 2 M_W^2 W_\mu^+ W_\mu^- + M_Z^2 Z_\mu Z_\mu \right), \tag{8.16}$$

which is a loop-generated term. Beyond the tree-generated terms the SESESM Lagrangian is not as trivial as it may seem (simple  $\sin(\cos)\alpha$ -rescaling of the SM results); for instance, consider the  $h^2H$  vertex

$$-\frac{g}{2} (2 M_h^2 + M_H^2) \sin \alpha \cos \alpha \left( \frac{\cos \alpha}{M_W} + \frac{\sin \alpha}{\Lambda} \right). \tag{8.17}$$

In the limit  $\Lambda \rightarrow \infty$  this vertex is not suppressed and that is why many loop diagrams containing heavy Higgs (internal) lines appear in the LESESM Lagrangian at  $\mathcal{O}(1/\Lambda^2)$ .

## 8.2 hVV

For instance, we can write

$$h \rightarrow V^\mu(p_1) + V^\nu(p_2) = F_D^{\text{VV}} \delta^{\mu\nu} + F_T^{\text{VV}} T^{\mu\nu}, \tag{8.18}$$

with  $T^{\mu\nu} = p_1 \cdot p_2 \delta^{\mu\nu} - p_1^\nu p_2^\mu$ . Introduce  $a_{zz} = s_W^2 a_{\phi_B} + c_W^2 a_{\phi_W} - s_W c_W a_{\phi_{WB}}$  and  $\rho = M_Z^2 / (c_W^2 M_Z^2)$  and define

$$\kappa^{\text{ww}} = -g M_W, \quad \kappa^{\text{zz}} = -g \frac{M_Z^2}{M_W} \rho \tag{8.19}$$

and also

$$\delta \kappa^{\text{ww}(zz)} = a_{\phi_W} + a_{\phi_\square} \mp \frac{1}{4} a_{\phi_D} \tag{8.20}$$

to derive

$$\begin{aligned}
h \rightarrow W_\mu^-(p_1) + W_\nu^+(p_2) &= \kappa^{\text{ww}} \left( 1 + \frac{g_6}{\sqrt{2}} \delta \kappa^{\text{ww}} \right) \delta_{\mu\nu} - \sqrt{2} \frac{g g_6}{M_W} a_{\phi_W} T_{\mu\nu}, \\
h \rightarrow Z_\mu(p_1) + Z_\nu(p_2) &= \kappa^{\text{zz}} \left( 1 + \frac{g_6}{\sqrt{2}} \delta \kappa^{\text{zz}} \right) \delta_{\mu\nu} - \sqrt{2} \frac{g g_6}{M_W} a_{zz} T_{\mu\nu}
\end{aligned}
\tag{8.21}$$

where  $\sqrt{2} g_6 = 1/(G_F \Lambda^2)$ . As a consequence, SMEFT predicts a change in the normalization of the SM-like term and the appearance of the transverse term. Note that  $a_{\phi_D}$  induces a breaking of custodial symmetry.

This result should be compared with

$$\left(1 + \frac{c_1^{\text{VV}}}{\Lambda^2} h + \frac{c_2^{\text{VV}}}{\Lambda^2} h^2 + \dots\right) V_\mu V^\mu + \dots \in \mathcal{L}_{\text{HEFT}}, \quad (8.22)$$

or, more generally, with

$$\left(1 + \frac{d_1^{\text{VV}}}{\Lambda^2} h + \frac{d_2^{\text{VV}}}{\Lambda^2} h^2 + \dots\right) F_{\mu\nu}^a F^{a\mu\nu} + \dots \in \mathcal{L}_{\text{HEFT}}, \quad (8.23)$$

The  $c_1^{\text{VV}}$  and  $c_2^{\text{VV}}$  give information on the doublet structure of the scalar field and can be computed in any UV completion of the SM. For the hVV vertex in the SESM we will have SM-like terms of  $\mathcal{O}(g)$ , tree-generated terms of  $\mathcal{O}(g/\Lambda^2)$  and loop-generated terms of  $\mathcal{O}(g^3/\pi^2)$  containing both  $\mathcal{O}(1)$  and  $\mathcal{O}(1/\Lambda^2)$  components.

The SMEFT prediction is

$$R_w = \frac{c_2^{\text{ww}}}{c_1^{\text{ww}}} = \frac{1}{2} \frac{g}{M_w} \left(1 - \frac{g_6}{\sqrt{2}} \delta\kappa^{\text{ww}}\right), \quad R_z = \frac{c_2^{\text{zz}}}{c_1^{\text{zz}}} = \frac{1}{2} \frac{g}{M_w} \left[1 - \frac{g_6}{\sqrt{2}} (\delta\kappa^{\text{zz}} - a_{\phi_D})\right] \quad (8.24)$$

where  $\delta\kappa^{\text{ww}} = \delta\kappa^{\text{zz}}$  if  $a_{\phi_D} = 0$  (custodial symmetry). If the fit to data yields  $R_w \neq R_z$ , the SMEFT interpretation will be  $a_{\phi_D} \neq 0$ , though several new terms will appear if  $\text{dim} = 8$  operators are included. It is worth noting that in the SESM expansion,  $F_T^{\text{VV}}$  starts at  $\mathcal{O}(1/\Lambda^4)$  and arises only from loops with both light and heavy internal propagators. The general conclusion is that it is important to compare processes with one h leg and two h legs, e.g. the h decay into 4 leptons vs. double Higgs production (in vector boson scattering).

There are subtle points in comparing the SMEFT with BSM models; we can have SMEFT at LO, including both PTG and LG operators (vertices) and the SMEFT with loops where one  $\text{dim} = 6$  operator is inserted (eventually PTG only). At the same time we can have the full BSM model, its low-energy limit at the LO level, the same limit including loops. It is interesting to observe that, in the SESM, heavy-light contributions are  $\mathcal{O}(1/\Lambda^4)$  for the hVV vertex but they are  $\mathcal{O}(1/\Lambda^2)$  for the hhvV vertex.

### 8.3 hZ production

As a final example we consider the  $p_\perp$  distribution of the Z boson in the process  $\bar{q}q \rightarrow Zh$ . In SESM, at LO there is a simple rescaling of the SM predictions. In SMEFT (at LO) we have an expression containing 9 Wilson coefficients, 3 of them loop generated (the full SMEFT amplitude has been presented in section 6.3 of ref. [3]).

There are two possible scenarios: scenario a) it is possible to obtain an almost constant SMEFT/SM ratio with special values of the Wilson coefficients, e.g.  $a_{AZ} = -1/(16\pi^2)$  and other coefficients set to zero.

Scenario b) shows a completely different behavior. For instance, with  $a_{\phi_D V} = 1$  and other coefficients set to  $-1$  we obtain that the SMEFT(linear)/SM ratio becomes negative at around 350 GeV, while the SMEFT(quadratic)/SM ratio remains positive, but exploding for increasing values of  $p_\perp$ . For  $p_\perp \approx 500$  GeV the ratios are  $-2$  for the linear representation and  $+4$  for the quadratic one, an evident sign of the breakdown of the SMEFT.

The SESM option (and many others) is excluded if scenario b) is the result of the SMEFT fit.

### 8.4 $\bar{t}t h$

Here we consider  $gg \rightarrow \bar{t}t h$ . In the SMEFT we will have to assemble several vertices; as a result we will have factorizable (fct) and non-factorizable (nfct) contributions, the latter changing the shape of distributions. For instance, the  $g\bar{t}t$  vertex will become

$$\begin{aligned}
V_{ij}^{\mu, a} |_{fct} &= \frac{i}{2} g_s \gamma^\mu \phi \lambda_{ij}^a \left( 1 + \frac{g_6}{\sqrt{2}} a_{\phi G} \right), \\
V_{ij}^{\mu, a} |_{nfc} &= -\frac{i}{4} g_s g_6 a_{tG} \lambda_{ij}^a \sigma^{\mu\nu} p_\nu,
\end{aligned} \tag{8.25}$$

where  $p$  is the (ingoing) gluon momentum. Next we will have the (factorizable)  $h\bar{t}t$  vertex,

$$V = -\frac{1}{2} \frac{m_t}{M_W} \left[ 1 + \frac{g_6}{\sqrt{2}} (a_{\phi W} + a_{t\phi} - \frac{1}{4} a_{\phi D} + a_{\phi\Box}) \right], \tag{8.26}$$

the (factorizable) 3-gluon vertex which receives a correction  $g_6/\sqrt{2} a_{\phi G}$ . Here  $g_s$  is the strong coupling constant. Furthermore,

$$\text{Tr}(\lambda^a \lambda^b) = 2 \delta^{ab}, \quad [\lambda^a, \lambda^b] = 2i f^{abc} \lambda_c. \tag{8.27}$$

Finally, there are vertices with no counterpart in the SM,  $gg\bar{t}t$  and  $gghh$ :

$$V_{ij}^{\mu\nu, ab} = \frac{1}{8} \frac{g_s^2 g_6}{M_W} f^{cab} \lambda_{ij}^c \sigma^{\mu\nu} a_{tG}, \tag{8.28}$$

$$V^{\mu\nu, ab} = \frac{g^2 g_6}{\sqrt{2} M_W^2} \delta^{ab} (p_3^\nu p_4^\mu - p_3 \cdot p_4 \delta^{\mu\nu}) a_{\phi G}. \tag{8.29}$$

In the SESM the only change w.r.t. the SM is a rescaling of the  $\bar{t}t$  coupling by  $\cos \alpha$ . For the THDM the rescaling is given by

$$\sin(\alpha - \beta) - \cot \beta \cos(\alpha - \beta) \tag{8.30}$$

## 8.5 $h \rightarrow \bar{b}b$

Some interesting feature appears when we consider the process  $h \rightarrow \bar{b}b$  in the SESM. Beyond LO we will have SM-like diagrams rescaled by  $\cos \alpha$ , producing  $\mathcal{O}(1/\Lambda^2)$  contributions with normal-threshold logarithms for an off-shell decaying light Higgs. But there is more: loop diagrams with internal H-lines. Here the expansion has two relevant parameters,  $M_W/M_s$  and  $\lambda_{12}$  so that we will have mixed heavy-light corrections which are not  $M_s$ -suppressed but  $\lambda_{12}^2$ -suppressed, i.e. the amplitude for these diagrams starts with

$$i \frac{g^3}{8 \pi^2} \frac{M_b^3}{M_W M_h^2} \frac{t_3^2}{t_1}, \tag{8.31}$$

where the  $t_i$  are defined in Eq.(3.5).

## 8.6 THDM examples

An example of complete calculations done in a BSM model, their low-energy limit and the corresponding, generalized, kappa-framework is as follows: consider  $h \rightarrow \gamma\gamma$  in THDM type I. There are two doublets containing fields  $h_1, h_2$  which mix with an angle  $\beta$ ; diagonalization in the neutral sector requires an angle  $\alpha$ . The amplitude becomes

$$\begin{aligned}
A_{h\gamma\gamma}(s) &= i \frac{g^2 s_W^2}{8\pi^2} (p_1 \cdot p_2 \delta^{\mu\nu} - p_2^\mu p_1^\nu) \\
&\times \left\{ \frac{\cos\alpha}{\sin\beta} \sum_f A_f^{\text{SM}} - \sin(\alpha - \beta) A_W^{\text{SM}} \right. \\
&\left. + \left[ (s + M_{sb}^2) \cos(\alpha - \beta) \cos(2\beta) - (s + 2M_{sb}^2 + 2M_{H^+}^2) \sin(\alpha - \beta) \sin(2\beta) \right] A_{H^+} \right\},
\end{aligned} \tag{8.32}$$

where  $M_{sb}$  is the  $Z_2$  soft-breaking scale [12, 50] and the h-virtuality is  $s$ . The coefficients in front of the SM sub-amplitudes are kappas and  $A_{H^+}$  is the resolved  $H^+$ -loop which will become the contact term in the low-energy expansion,  $M_{H^+} \rightarrow \infty$ .

As far as the low-energy limit is concerned, one of the possible scenarios is as follows: there are two doublets and the scalar potential will contain a term

$$\mu_3^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1). \tag{8.33}$$

The scale  $\Lambda$  is defined by  $\mu_3^2 = \sin\beta \cos\beta \Lambda^2$ , so that all heavy masses are proportional to  $\Lambda$  with subleading corrections. If we denote the VEVs by  $v_i$  and introduce  $v^2 = v_1^2 + v_2^2$  the heavy  $\Lambda$  limit will give

$$\alpha = -\frac{1}{2} \frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right), \quad \beta = \frac{1}{2} \left[ \pi - \frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right) \right]. \tag{8.34}$$

## 9 Conclusions

The price one has to pay for using an EFT in going beyond the SM is that EFTs are only valid in a limited domain. This prompts the important question as to whether there is a last fundamental theory in this tower of EFTs, each one superseding the previous one with rising energies. Should one ultimately expect from physics theories to only be valid as approximations and in a limited domain <sup>5</sup>?

Mathematics suffers from some of the same inherent difficulties as theoretical physics <sup>6</sup>: great successes during the 20th century and increasing difficulties to do better, as the easier problems get solved. The conventional vision is: some very different physics occurs at Planck scale, SM is just an effective field theory. What about the next SM? Are we expecting a new weakly-coupled renormalizable model or a tower of EFTs? There is an alternative vision, the SM could be close to a fundamental theory; indeed, the lesson from experiments since 1973 is that it is extremely difficult to find a flaw in the SM. Perhaps the SM includes elements of a truly fundamental theory.

Returning to the conventional vision [98, 99] we can say that a key ingredient of top-down EFT studies is matching a given UV theory onto its low-energy EFT. After the Higgs boson discovery, we have a paradigm shift, i.e. we use the SMEFT in fits to the data. The “fitted” Wilson coefficients (in the Warsaw basis) can be used to derive SMEFT-predicted observables which become the pseudo-data and we may take any specific BSM model, compute the corresponding low-energy limit and confront the BSM parameters with the pseudo-measurements.

Of course, there is another scenario: depending on the results for the fits, the corresponding interpretation could tell us that the required Wilson coefficients are too large to allow for a meaningful interpretation in terms of a weakly coupled UV completion. It is also possible that part of the observable parameter (Wilson coefficients) space will be inconsistent with causality and analyticity.

<sup>5</sup>Kuhlmann, Meinard, “Quantum Field Theory”, The Stanford Encyclopedia of Philosophy (Winter 2018 Edition), Edward N. Zalta (ed.).

<sup>6</sup>See Peter Woit. “The state of high-energy particle physics: a view from a neighboring field”, US Naval Observatory Colloquium December 6, 2018.

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