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Quantization of a Friedmann-Robertson-Walker model in N=1 Supergravity with Gauged Supermatter

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ABSTRACT

The theory of N = 1 supergravity with gauged supermatter is studied in the context of a k = + 1 Friedmann minisuperspace model. It is found by imposing the Lorentz and supersymmetry constraints that there are *no* physical states in the particular $SU(2)$ model studied. When the model is truncated by setting the spin-1, spin-$\frac{1}{2}$ multiplet to zero, a very restricted set of non-zero quantum states is found.

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The subjects of supersymmetric quantum gravity and cosmology have achieved a number of interesting results and conclusions during the last ten years or so. In finding a physical state, it is sufficient to solve the Lorentz and supersymmetry constraints of the theory because the algebra of constraints of the theory leads to anti-commutation relations implying that a physical wave functional Ψ will also obey the Hamiltonian constraints [1,2]. Using the triad Arnowitt-Deser-Misner canonical formulation, Bianchi models of class A have been studied in pure $N = 1$ supergravity with and without a cosmological constant [3-8] and have been found to have very simple allowed quantum states. [Supersymmetry (as well as other considerations) forbids mini-superspace models of class B.]. Other approaches can be found Ref. [9-15]

Clearly, a richer and more interesting class of minisuperspace models is given by coupling supermatter to $N = 1$ supergravity in 4 dimensions. In particular, from (1+3) dimensional N=1 supergravity a dimensional reduction allows one to obtain a (1+0)-dimensional theory with N=4 supersymmetry by making a suitable homogeneous Ansatz. In [16-18] such an Ansatz for the gravitational and gravitino fields was introduced in order to yield a Friedmann $k = +1$ geometry and a homogeneous gravitino field on the S^3 spatial sections. The Hamiltonian structure of the resulting theory was found, leading to the quantum constraint equations. The Hartle–Hawking wave-function [19] can be found as one solution of the quantum constraints. Following the model described in ref. [23], a Friedmann-Robertson-Walker (FRW) minisuperspace in N=1 supergravity coupled to locally supersymmetric supermatter (a massive complex scalar with spin-$\frac{1}{2}$ partner) was considered in [16-18]. In the massless case [17,18], the general solution of the quantum

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constraints can be found as an integral expression, admitting a ground quantum wormhole state [20].

One would like to extend this understanding to more general supergravity models involving spin-1 fields. We apply here the canonical formulation to the more general theory of $N = 1$ supergravity coupled to supermatter, and in particular its supersymmetry constraints, especially in the case with zero analytic potential $P(\Phi^I)$ [cf. ref. [21]. Such a study was performed in ref. [22]. Our objective here is to study a $k = 1$ supersymmetric FRW mini-superspace quantum cosmological model with a family of spin-0 as well as spin-1 gauge fields together with their odd (anti-commuting) spin-$\frac{1}{2}$ partners. Assuming an Ansatz for the the gravitational and gravitino fields as well as for the supermatter fields and their fermionic partners such as to to respect the homogeneity and isotropy of the FRW S^3 spatial sections, $N = 1$ supergravity plus supermatter in 4 dimensions may be reduced to a N=4 locally supersymmetric FRW quantum cosmological model in 1 dimension. Our supermatter model is chosen to correspond to a two-dimensional spherically symmetric Kähler geometry [21]. In the following the supersymmetry constraints will be derived from the reduced theory with supermatter. Subsequently, we solve for the components of the wave function the set of coupled partial differential equations which are obtained from the quantum constraints. We will then find that there are *no* solutions for the quantum states of the FRW universe analysed here. In addition, we also find that when the model is truncated by setting the spin-1, spin-$\frac{1}{2}$ multiplet to zero, a very restricted set of non-zero quantum states is obtained. A discussion and interpretation of our results, together with a summary of our research and indications of further possibilities completes this paper.

Let us begin by specifying our model in some detail. The more general gauged N=1 supergravity theory coupled to supermatter [21] depends on the tetrad $e^{AA'}{}_{\mu}$, where A, A' are two-component spinor indices using the conventions of [1] and μ is a space-time index, the odd (anti-commuting) gravitino field $\left(\psi^{A}{}_{\mu}, \tilde{\psi}^{A'}{}_{\mu}\right)$, a vector field $A^{(a)}_{\mu}$ labelled by an index (a), its odd spin-$\frac{1}{2}$ partners $\left(\lambda^{(a)}_{A}, \bar{\lambda}^{(a)}_{A'}\right)$, a family of scalars (Φ^I, Φ^{J^*}) and their odd spin-$\frac{1}{2}$ partners $\left(\chi^I_A, \bar{\chi}^{J^*}_{A'}\right)$. Its Lagrangian is given in Eq. (25.12) of ref. [21]: it is too long to write out here. The indices $I, \ldots, J^*, \ldots$ are Kähler indices, and there is a Kähler metric $g_{IJ^*} = K_{IJ^*}$ on the space of (Φ^I, Φ^{J^*}), where K_{IJ^*} is a shorthand for $\partial^2 K/\partial\Phi^I\partial\Phi^{J^*}$ with K the Kähler potential. Each index (a) corresponds to an independent (holomorphic) Killing vector field $X^{(a)}$ of the Kähler geometry. The gauge group is the isometry group of the Kähler manifold. Killing's equation implies that there exist real scalar functions $D^{(a)}(\Phi^I, \Phi^{I^*})$ known as Killing potentials, such that

$g_{IJ^*}X^{J^*(a)} = i\,\frac{\partial}{\partial\phi^I}\,D^{(a)}\ ,\ g_{IJ^*}X^{I(a)} = -i\,\frac{\partial}{\partial\phi^{J^*}}\,D^{(a)}$.

We choose the geometry to be that of a $k = +1$ Friedmann model with S^3 spatial sections, which are the spatial orbits of $G = SO(4)$ - the group of homogeneity and isotropy. The tetrad of the four-dimensional theory can be taken to be:

$$e_{a\mu} = \begin{pmatrix} N(\tau) & 0 \\ 0 & aE_{ai} \end{pmatrix}\ ,\quad e^{a\mu} = \begin{pmatrix} N(\tau)^{-1} & 0 \\ 0 & a(\tau)^{-1}E^{\hat{a}i} \end{pmatrix} \tag{1}$$

where $\hat{a}$ and i run from 1 to 3. E_{ai} is a basis of left-invariant 1-forms on the unit S^3 with volume $\sigma^2 = 2\pi^2$. The spatial tetrad $e^{AA'}{}_i$ satisfies the relation

$$\partial_i e^{AA'}{}_j - \partial_j e^{AA'}{}_i = 2a^2\epsilon_{ijk}e^{AA'k} \tag{2}$$

as a consequence of the group structure of SO(3), the isotropy (sub)group.

This Ansatz reduces the number of degrees of freedom provided by $e_{AA'\mu}$. If supersymmetry invariance is to be retained, then we need an Ansatz for $\psi^A{}_\mu$ and $\tilde{\psi}^{A'}{}_\mu$ which reduces the number of fermionic degrees of freedom, so that there is equality between the number of bosonic and fermionic degrees of freedom. One is naturally led to take $\psi^A{}_0$ and $\tilde{\psi}^{A'}{}_0$ to be functions of time only. In the four-dimensional Hamiltonian theory, $\psi^A{}_0$ and $\tilde{\psi}^{A'}{}_0$ are Lagrange multipliers which may be freely specified. We further take

$$\psi^A{}_i = e^{AA'}{}_i\tilde{\psi}_{A'}\ ,\ \tilde{\psi}^{A'}{}_i = e^{AA'}{}_i\psi_A\ , \tag{3}$$

where we introduce the new spinors ψ_A and $\tilde{\psi}_{A'}$ which are functions of time only. [It is possible to justify the Ansatz (3) by requiring that the form (1) of the tetrad be preserved under suitable homogeneous supersymmetry transformations [16,17].] Moreover, it turns out that the constraints obeyed by classical solutions of the 1-dimensional theory lead to a 4-dimensional energy-momentum tensor which is isotropic, consistent with the assumption of a Friedmann geometry.

In the Euclidean context, it is natural to regard ψ_A and $\tilde{\psi}_{A'}$ as independent quantities. The Ansatz for $\psi^A{}_i$ is preserved under a combination of a non-zero (spatially homogeneous) supersymmetry transformation and possible local Lorentz and coordinate transformations [16,17] if, in the case without supermatter, we impose the additional constraint $\psi^B\tilde{\psi}^{B'}e_{BB'i} = 0$. This implies that $\psi^B\tilde{\psi}^{B'} \propto n^{BB'}$, which can be written in the equivalent form :

$$J_{AB} = \psi_{(A}\tilde{\psi}^{B'}n_{B)B'} = 0\ , \tag{4}$$

together with its hermitian conjugate, thereby defining J_{AB}. The constraint $J_{AB} = 0$ has a natural interpretation as the reduced form of the Lorentz rotation constraint arising

in the full theory [1]. By requiring that the constraint $J_{AB} = 0$ be preserved under the same combination of transformations as used above, one finds equations which are satisfied provided the supersymmetry constraints $S_A = 0$, $S_{A'} = 0$ (see below) hold. By further requiring that the supersymmetry constraints be preserved, one finds additionally that the Hamiltonian constraint $\mathcal{H} = 0$ should hold. When matter fields are taken into account (see next paragraphs) the generalisation of the J_{AB} constraint is :

$$J_{AB} = \psi_{(A}\bar{\psi}_{B)} - \chi_{(A}\bar{\chi}_{B)} - \lambda^{(a)}_{(A}\bar{\lambda}^{(a)}_{B)} = 0\ . \tag{5}$$

Now, consider the supermatter fields. First, we choose for the gauge group of our model the *compact* group $\hat{G} = SU(2)$. In this case [21]

$$D^{(1)} = \frac{1}{2}\left(\frac{\varphi + \bar{\phi}}{1 + \phi\bar{\phi}}\right),\ D^{(2)} = -\frac{i}{2}\left(\frac{\bar{\phi} - \varphi}{1 + \phi\bar{\phi}}\right),\ D^{(3)} = -\frac{1}{2}\left(\frac{1 - \phi\bar{\phi}}{1 + \phi\bar{\phi}}\right), \tag{6}$$

with $K = \ln(1 + \phi\bar{\phi})$. Hence $g_{\varphi\bar{\varphi}} = \frac{1}{(1+\varphi\bar{\varphi})^2}$, $g^{\varphi\bar{\varphi}} = (1 + \phi\bar{\phi})^2$ and the Levi-Civita connection of the Kähler manifold is given by $\Gamma^{\varphi}_{\varphi\varphi} = g^{\varphi\bar{\varphi}}\frac{\partial g_{\varphi\bar{\varphi}}}{\partial\varphi} = -2\frac{\bar{\phi}}{(1+\phi\bar{\phi})}$ and its complex conjugate. The rest of the components are zero. The scalar super-multiplet, consisting of a complex massive scalar field ϕ and massive spin-$\frac{1}{2}$ field $\chi, \bar{\chi}$ are chosen to be spatially homogeneous, depending only on time. The odd spin-$\frac{1}{2}$ partner $(\lambda^{(a)}, \bar{\lambda}^{(a)})$, $a = 1,2,3$, is chosen to depend only on time as well. As far as the vector field $A^{(a)}_\mu$ is concerned we adopt here the Ansatz formulated in ref. [24-27] and choose

$$\mathbf{A}_\mu(t)\,\omega^\mu = \frac{f(t)}{2}T_c\omega^c\ . \tag{7}$$

where $\{\omega^\mu\}$ represents the moving coframe $\{\omega^\mu\} = \{dt, \omega^b\}$, $(b = 1,2,3)$, of one-forms, invariant under the left action of $SU(2)$ and T_a are the generators of the $SU(2)$ gauge group. Notice in the above form for the gauge field the A_0 component is taken to be identically zero. Thus, we will not have in our FRW case a gauge constraint $Q^{(a)} = 0$. However, in the case of larger gauge group some of the gauge symmetries will survive, giving rise, in the one-dimensional model, to local internal symmetries with a reduced gauge group. Therefore, a gauge constraint can be expected to play an important role in such a case and a study of such a model would be interesting.

Using the Ansätze previously described, the action of the full theory (Eq. (25.12) in ref. [21]) can be reduced to one with a finite number of degrees of freedom. Starting from the action so obtained, we study the Hamiltonian formulation of this model [22]. The procedure to find the expressions of S_A and $\bar{S}_{A'}$ is simple. First, we have to calculate the conjugate momenta of the dynamical variables and then evaluate the reduced Hamitonian. Afterwards, we read out the coefficients of $\psi_0{}^A$ and $\bar{\psi}_0{}^{A'}$ from this expression in order to get the S_A and $\bar{S}_{A'}$ constraints, respectively.

The contributions from the spin-0 field ϕ to the $\bar{S}_{A'}$ constraint are seen to be

$$\frac{1}{\sqrt{2}}\bar{\chi}_{A'}\left[\pi_\phi + \frac{i\sigma^2 a^3}{2\sqrt{2}}\frac{\phi}{(1+\phi\bar{\phi})}n_{BB'}\bar{\lambda}^{(a)B'}\lambda^{(a)B} - \frac{5i}{2\sqrt{2}}\frac{\sigma^2 a^3\phi}{(1+\phi\bar{\phi})^3}n_{BB'}\bar{\chi}^{B'}\chi^B\right.$$
$$\left.-\frac{3i}{2\sqrt{2}}\frac{\sigma^2 a^3\phi}{(1+\phi\bar{\phi})}n_{BB'}\psi^B\bar{\psi}^{B'} - \frac{3}{\sqrt{2}}\frac{\sigma^2 a^3}{(1+\phi\bar{\phi})^2}n_{BB'}\chi^B\bar{\psi}^{B'}\right] + \frac{\sigma^2 a^2 gf}{\sqrt{2}(1+\phi\bar{\phi})^2}\sigma^a{}_{AA'}n^{AB'}\bar{\chi}_{B'}X^a . \tag{8}$$

where $\sigma^2 = 2\pi^2$. The contributions to the $\bar{S}_{A'}$ constraint from the spin-1 field are

$$-i\frac{\sqrt{2}}{3}\pi_f\sigma^a{}_{BA'}\lambda^{(a)B} + \frac{\sigma^2 a^3}{6}\sigma^a{}_{BA'}\lambda^{(a)B}n_{CB'}(\sigma^{bCC'}\bar{\psi}_{C'}\bar{\lambda}^{(b)B'} + \sigma^{bAB'}\psi_A\lambda^{(b)C})$$
$$+\frac{1}{8\sqrt{2}}\sigma^2 a^4\sigma^{(a)C}{}_{A'}[1-(f-1)^2]\lambda^{(a)}{}_C$$
$$+\frac{1}{2}\sigma^2 a^3\lambda^{(a)A}(-n_{AB'}\bar{\psi}_{A'}\bar{\lambda}^{(a)B'} + n_{BA'}\psi_A\lambda^{(a)B} - \frac{1}{2}n_{AA'}\psi_B\lambda^{(a)B} + \frac{1}{2}n_{AA'}\bar{\psi}_{B'}\bar{\lambda}^{(a)B'}) . \tag{9}$$

We have used $\epsilon_{AA'i} = \sigma^a_{AA'}\epsilon_{ai}$, where $\sigma^a_{AA'}(a = 1,2,3)$ are Infeld-van der Waerden symbols [1]. The contributions from the spin-2 field and spin-3/2 field to $\bar{S}_{A'}$ constraint are

$$\frac{i}{2\sqrt{2}}a\pi_a\bar{\psi}_{A'} - \frac{3}{\sqrt{2}}\sigma^2 a^2\bar{\psi}_{A'} + \frac{3}{8}\sigma^2 a^3 n^B{}_{A'}\bar{\psi}^{B'}\psi_B\bar{\psi}_{B'} . \tag{10}$$

The following terms are also present in the $\bar{S}_{A'}$ supersymmetry constraint:

$$-\frac{1}{\sqrt{2}}\sigma^2 a^3 gD^a n_{AA'}\lambda^{(a)A}$$
$$+\frac{\sigma^2 a^3}{(1+\phi\bar{\phi})^2}(-n_{BA'}\bar{\psi}_{B'} + \frac{1}{2}n_{BB'}\bar{\psi}_{A'})\chi^B\bar{\chi}^{B'}$$
$$-\frac{1}{4}\sigma^2 a^3(n_{AB'}\lambda^{(a)A}\bar{\lambda}^{(a)}{}_{A'}\bar{\psi}^{B'} + n_{AA'}\lambda^{(a)A}\bar{\lambda}^{(a)}{}_{B'}\bar{\psi}^{B'})$$
$$-\frac{1}{4(1+\phi\bar{\phi})^2}\sigma^2 a^3(n_{AB'}\chi^A\bar{\chi}_{A'}\bar{\psi}^{B'} + n_{AA'}\chi^A\bar{\chi}_{B'}\bar{\psi}^{B'}) . \tag{11}$$

The supersymmetry constraint $\bar{S}_{A'}$ is then the sum of the above expressions. The supersymmetry constraint S_A is just the complex conjugate of $\bar{S}_{A'}$. Notice that with our choice of gauge group $SU(2)$ and compact Kähler manifold, it directly follows that the analytical potential $P(\Phi^I)$ is zero [30]

Let us here solve explicitly the corresponding quantum supersymmetry constraints. First we need to redefine the fermionic fields. χ_A, ψ_A and λ_A in order to simplify the Dirac brackets [2], following the steps described in [22,28,29]:

$$\hat{\chi}_A = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}(1+\phi\bar{\phi})}\chi_A\ , \quad \hat{\bar{\chi}}_{A'} = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}(1+\phi\bar{\phi})}\bar{\chi}_{A'}\ . \tag{12}$$

The conjugate momenta become

$$\pi_{\chi_A} = -in_{AA'}\hat{\bar{\chi}}^{A'}\ , \quad \pi_{\bar{\chi}_{A'}} = -in_{AA'}\hat{\chi}^A\ . \tag{13}$$

This pair forms a set of second class constraints. The Dirac bracket ($[\ ]_*$) is

$$[\hat{\chi}_A, \hat{\bar{\chi}}_{A'}]_* = -in_{AA'}\ . \tag{14}$$

Similarly for the ψ_A field,

$$\hat{\psi}_A = \frac{\sqrt{3}}{2^{\frac{1}{4}}}\sigma a^{\frac{3}{2}}\psi_A\ , \quad \hat{\bar{\psi}}_{A'} = \frac{\sqrt{3}}{2^{\frac{1}{4}}}\sigma a^{\frac{3}{2}}\bar{\psi}_{A'}\ . \tag{15}$$

where the conjugate momenta are

$$\pi_{\psi_A} = in_{AA'}\hat{\bar{\psi}}^{A'}\ , \quad \pi_{\bar{\psi}_{A'}} = in_{AA'}\hat{\psi}^A\ . \tag{16}$$

and the Dirac bracket becomes

$$[\hat{\psi}_A, \hat{\bar{\psi}}_{A'}]_* = in_{AA'}\ . \tag{17}$$

and also for the λ_A field:

$$\hat{\lambda}^{(a)}{}_A = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}}\lambda^{(a)}{}_A, \quad \hat{\bar{\lambda}}^{(a)}{}_{A'} = \frac{\sigma a^{\frac{3}{2}}}{2^{\frac{1}{4}}}\bar{\lambda}^{(a)}{}_{A'} \tag{18}$$

giving

$$\pi_{\lambda^{(a)}{}_A} = -in_{AA'}\hat{\bar{\lambda}}^{(a)A'}\ , \quad \pi_{\bar{\lambda}^{(a)}{}_{A'}} = -in_{AA'}\hat{\lambda}^{(a)A} \tag{19}$$

with

$$[\hat{\lambda}^{(a)}{}_A, \hat{\bar{\lambda}}^{(a)}{}_{A'}]_* = -i\delta^{ab}n_{AA'} \tag{20}$$

Furthermore,

$$[a,\pi_a]_* = 1\ ,\ [\phi,\pi_\phi]_* = 1\ ,\ [\bar\phi,\pi_{\bar\phi}]_* = 1\ ,\ [f,\pi_f] = 1 \tag{21}$$

and the rest of the brackets are zero.

It is simpler to describe the theory using only (say) unprimed spinors, and, to this end, we define

$$\psi_A = 2n_A{}^{B'}\bar\psi_{B'}\ ,\ \bar\chi_A = 2n_A{}^{B'}\bar\chi_{B'}\ ,\ \bar\lambda^{(a)}_A = 2n_A{}^{B'}\bar\lambda^{(a)}_{B'} \tag{22}$$

with which the new Dirac brackets are

$$[\chi_A,\bar\chi_B]_* = -i\epsilon_{AB}\ ,\ [\psi_A,\bar\psi_B]_* = i\epsilon_{AB}\ ,[\lambda^{(a)}_A,\bar\lambda^{(a)}_{A'}]_* = -i\delta^{ab}\epsilon_{AB} \tag{23}$$

The rest of the brackets remain unchanged. Quantum mechanically, one replaces the Dirac brackets by anti-commutators if both arguments are odd (O) or commutators if otherwise (E):

$$[E_1,E_2] = i[E_1,E_2]_*\ ,\ [O,E] = i[O,E]_*\ ,\ \{O_1,O_2\} = i[O_1,O_2]_*\ . \tag{24}$$

Here, we take units with $\hbar = 1$. The only non-zero (anti-)commutator relations are:

$$\{\lambda^{(a)}_A,\bar\lambda^{(b)}_B\} = \delta^{ab}\epsilon_{AB}\ ,\ \{\chi_A,\bar\chi_B\} = \epsilon_{AB}\ ,\ \{\psi_A,\bar\psi_B\} = -\epsilon_{AB} \tag{25}$$

$$[a,\pi_a] = [\phi,\pi_\phi] = [\bar\phi,\pi_{\bar\phi}] = [f,\pi_f] = i$$

Here we choose $(\chi_A,\psi_A,a,\phi,\bar\phi)$ to be the coordinates of the configuration space, and $\bar\chi_A,\bar\psi_A,\pi_a,\pi_\phi,\pi_{\bar\phi}$ to be the momentum operators in this representation. Hence

$$\bar\lambda^a{}_A \to -\frac{\partial}{\partial\lambda^{(a)A}}\ ,\ \bar\chi^A \to -\frac{\partial}{\partial\chi^A},\ \bar\psi_A \to \frac{\partial}{\partial\psi^A}$$

$$\pi_a \to \frac{\partial}{\partial a},\ \pi_\phi \to -i\frac{\partial}{\partial\phi},\ \pi_{\bar\phi} \to -i\frac{\partial}{\partial\bar\phi}\ ,\ \pi_f \to -i\frac{\partial}{\partial f} \tag{26}$$

Following the ordering used in ref.[5], we put all the fermionic derivatives in S_A on the right. In $\bar S_A$, all the fermonic derivatives are on the left. Implementing all these redefinitions, the supersymmetry constraints have the differential operator form

$$S_A = -\frac{i}{\sqrt2}(1+\bar\phi\phi)\chi_A\frac{\partial}{\partial\phi} - \frac{1}{2\sqrt6}a\psi_A\frac{\partial}{\partial a}$$

$$-\sqrt{\frac{3}{2}}\sigma^2a^2\psi_A - \frac{5i}{4\sqrt2}\bar\phi\chi_A\chi^B\frac{\partial}{\partial\chi^B}$$

$$-\frac{1}{8\sqrt6}\psi_B\psi^B\frac{\partial}{\partial\psi^A} - \frac{i}{4\sqrt2}\bar\phi\chi_A\psi^B\frac{\partial}{\partial\psi^B}$$

$$-\frac{5}{4\sqrt6}\chi_A\psi^B\frac{\partial}{\partial\chi^B} + \frac{\sqrt3}{4\sqrt2}\chi^B\psi_B\frac{\partial}{\partial\chi^A}$$

$$+\frac{1}{2\sqrt6}\psi_A\chi^B\frac{\partial}{\partial\chi^B}$$

$$+\frac{1}{3\sqrt6}\sigma^a{}_{AB'}\sigma^{bCC'}n_D{}^{B'}n^B{}_{C'}\lambda^{(a)D}\psi_C\frac{\partial}{\partial\bar\lambda^{(b)B}}$$

$$+\frac{1}{6\sqrt6}\sigma^a{}_{AB'}\sigma^{bBA'}n_D{}^{B'}n^E{}_{A'}\lambda^{(a)D}\bar\lambda^{(b)}{}_B\frac{\partial}{\partial\psi^E}$$

$$-\frac{1}{2\sqrt6}\psi_A\lambda^{(a)C}\frac{\partial}{\lambda^{(a)C}} + \frac{3}{8\sqrt6}\bar\lambda^a{}_A\lambda^{(a)C}\frac{\partial}{\partial\psi^C}$$

$$+\frac{1}{2\sqrt2}\sigma^2a^3gD^{(a)}\lambda^a{}_A - \frac{1}{4\sqrt6}\psi^C\lambda^{(a)}_C\frac{\partial}{\partial\bar\lambda^{(a)A}}$$

$$+\frac{\sigma^2a^2gf}{\sqrt2(1+\phi\bar\phi)}\sigma^a{}_{AA'}n^{BA'}\bar X^{(a)}\chi_B$$

$$+\sigma^a{}_{AA'}n^{BA'}\bar\lambda^{(a)}_B\left(-\frac{\sqrt2}{3}\frac{\partial}{\partial f} + \frac{1}{8\sqrt2}(1-(f-1)^2)\sigma^2a^4\right) \tag{27}$$

and

$$\bar S_A = \frac{i}{\sqrt2}(1+\bar\phi\phi)\frac{\partial}{\partial\chi^A}\frac{\partial}{\partial\bar\phi} + \frac{1}{2\sqrt6}a\frac{\partial}{\partial a}\frac{\partial}{\partial\psi^A}$$

$$-\sqrt{\frac{3}{2}}\sigma^2a^2\frac{\partial}{\partial\psi^A} + \frac{5i}{4\sqrt2}\phi\frac{\partial}{\partial\chi^A}\frac{\partial}{\chi^B}\chi^B$$

$$-\frac{1}{8\sqrt6}\epsilon^{BC}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^C}\psi_A - \frac{i}{4\sqrt2}\phi\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^A}\psi^B$$

$$-\frac{5}{4\sqrt6}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^A}\chi^B - \frac{\sqrt3}{4\sqrt2}\epsilon^{BC}\frac{\partial}{\partial\psi^B}\frac{\partial}{\partial\chi^C}\chi_A$$

$$-\frac{1}{2\sqrt6}\frac{\partial}{\partial\psi^A}\frac{\partial}{\partial\chi^B}\chi^B$$

$$+\frac{1}{3\sqrt{6}}\sigma^{aB}{}_{A'}\sigma^{bCC'}n_{A}{}^{A'}n^{D}{}_{C'}\frac{\partial}{\partial\psi^{D}}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\bar{\lambda}^{(b)}{}_{C}$$

$$+\frac{2}{3\sqrt{6}}\sigma^{aB}{}_{A'}\sigma^{B'}_{D}n_{A}{}^{A'}n^{C}{}_{B'}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\frac{\partial}{\partial\bar{\lambda}^{(b)C}}\psi^{D}$$

$$+\frac{1}{2\sqrt{6}}\frac{\partial}{\partial\psi^{A}}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\lambda^{(a)B}+\frac{3}{2\sqrt{6}}\frac{\partial}{\partial\bar{\lambda}^{(a)B}}\frac{\partial}{\partial\bar{\lambda}^{(a)A}}\psi^{B}$$

$$-\frac{1}{\sqrt{2}}\sigma^{2}a^{3}gD^{(a)}\frac{\partial}{\partial\bar{\lambda}^{(a)A}}+\frac{1}{4\sqrt{6}}\epsilon^{BC}\frac{\partial}{\partial\psi^{B}}\frac{\partial}{\partial\bar{\lambda}^{(a)C}}\bar{\lambda}^{(a)}_{A}$$

$$+\frac{\sigma^{2}a^{2}gf}{\sqrt{2}(1+\varphi\bar{\phi})}n_{A}{}^{A'}\sigma^{aB}{}_{A'}X^{(a)}\frac{\partial}{\partial\chi^{B}}$$

$$+n_{A}{}^{A'}\sigma^{aB}{}_{A'}\left(\frac{2\sqrt{2}}{3}\frac{\partial}{\partial f}+\frac{1}{4\sqrt{2}}(1-(f-1)^{2})\sigma^{2}a^{4}\right)\frac{\partial}{\partial\bar{\lambda}^{(a)B}} \tag{28}$$

We now proceed to find the wavefunction of our model. The Lorentz constraint J_{AB} is easy to solve. It tells us that the wave function should be a Lorentz scalar. We can easily see that the most general form of the wave function which satisfies the Lorentz constraint is

$$\Psi=A+iB\psi^{C}\psi_{C}+C\psi^{C}\chi_{C}+iD\chi^{C}\chi_{C}+E\psi^{C}\psi_{C}\chi^{C}\chi_{C}$$

$$+c_{a}\lambda^{(a)C}\chi_{C}+d_{a}\lambda^{(a)C}\chi_{C}+c_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}+\epsilon_{a}\bar{\lambda}^{(a)C}\chi_{C}\psi^{D}\psi_{D}$$

$$+f_{a}\lambda^{(a)C}\psi_{C}\chi^{D}\chi_{D}+d_{ab}\lambda^{(a)C}\chi_{C}\bar{\lambda}^{(a)D}\chi_{D}+\epsilon_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\psi^{D}\psi_{D}$$

$$+f_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\chi^{D}\chi_{D}+g_{ab}\lambda^{(a)C}\bar{\lambda}^{(b)}_{C}\chi^{D}\psi_{D}+c_{abc}\lambda^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\psi_{D}$$

$$+d_{abc}\bar{\lambda}^{(a)C}\lambda^{(b)}_{C}\lambda^{(c)D}\chi_{D}+c_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_{D}+h_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\psi^{D}\psi_{D}\chi^{E}\chi_{E}$$

$$+\epsilon_{abc}\bar{\lambda}^{(a)C}\lambda^{(b)}_{C}\lambda^{(c)D}\chi_{D}\psi^{E}\psi_{E}+f_{abc}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\lambda^{(c)D}\psi_{D}\chi^{E}\chi_{E}+d_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_{D}\psi^{E}\psi_{E}$$

$$+\epsilon_{abcd}\lambda^{(a)C}\lambda^{(b)}_{C}\lambda^{(c)D}\lambda^{(d)}_{D}\chi^{E}\chi_{E}+f_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_{D}\psi^{E}\chi_{E}$$

$$+g_{abcd}\lambda^{(a)C}\lambda^{(b)}_{C}\lambda^{(c)D}\psi_{D}\bar{\lambda}^{(d)E}\chi_{E}$$

$$\mu_{1}\lambda^{(2)C}\bar{\lambda}^{(2)}_{C}\lambda^{(3)D}\lambda^{(3)}_{D}\lambda^{(1)E}\psi_{E}+\mu_{2}\lambda^{(1)C}\lambda^{(1)}_{C}\lambda^{(3)D}\bar{\lambda}^{(3)}_{D}\bar{\lambda}^{(2)E}\psi_{E}+\mu_{3}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_{D}\bar{\lambda}^{(3)E}\psi_{E}$$

$$+\nu_{1}\bar{\lambda}^{(2)C}\lambda^{(2)}_{C}\bar{\lambda}^{(3)D}\lambda^{(3)}_{D}\bar{\lambda}^{(1)E}\chi_{E}+\nu_{2}\lambda^{(1)C}\lambda^{(1)}_{C}\lambda^{(3)D}\bar{\lambda}^{(3)}_{D}\lambda^{(2)E}\chi_{E}+\nu_{3}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_{D}\lambda^{(3)E}\chi_{E}$$

$$+F\lambda^{(1)C}\lambda^{(1)}_{C}\lambda^{(2)D}\lambda^{(2)}_{D}\lambda^{(3)E}\lambda^{(3)}_{E}+h_{abcd}\bar{\lambda}^{(a)C}\lambda^{(b)}_{C}\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_{D}\psi^{E}\psi_{E}\chi^{F}\chi_{F}$$

$$+\delta_{1}\lambda^{(2)C}\lambda^{(2)}_{C}\lambda^{(3)D}\bar{\lambda}^{(3)}_{D}\lambda^{(1)E}\psi_{E}\chi^{F}\chi_{F}$$

$$+\delta_{2}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(3)D}\lambda^{(3)}_{D}\lambda^{(2)E}\psi_{E}\chi^{F}\chi_{F}$$

$$+\delta_{3}\lambda^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\lambda^{(2)}_{D}\lambda^{(3)E}\psi_{E}\chi^{F}\chi_{F}$$

$$+\gamma_{1}\lambda^{(2)C}\bar{\lambda}^{(2)}_{C}\bar{\lambda}^{(3)D}\lambda^{(3)}_{D}\bar{\lambda}^{(1)E}\chi_{E}\psi^{F}\psi_{F}$$

$$+\gamma_{2}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(3)D}\lambda^{(3)}_{D}\bar{\lambda}^{(2)E}\chi_{E}\psi^{F}\psi_{F}$$

$$+\gamma_{3}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_{D}\bar{\lambda}^{(3)E}\chi_{E}\psi^{F}\psi_{F}$$

$$+G\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\lambda^{(2)D}\bar{\lambda}^{(2)}_{D}\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)}_{E}\psi^{F}\psi_{F}$$

$$+H\lambda^{(1)C}\lambda^{(1)}_{C}\lambda^{(2)D}\lambda^{(2)}_{D}\lambda^{(3)E}\lambda^{(3)}_{E}\chi^{F}\chi_{F}$$

$$+I\lambda^{(1)C}\bar{\lambda}^{(1)}_{C}\lambda^{(2)D}\lambda^{(2)}_{D}\lambda^{(3)E}\bar{\lambda}^{(3)}_{E}\chi^{F}\psi_{F}$$

$$+K\lambda^{(1)C}\lambda^{(1)}_{C}\lambda^{(2)D}\bar{\lambda}^{(2)}_{D}\lambda^{(3)E}\lambda^{(3)}_{E}\psi^{F}\psi_{F}\chi^{G}\chi_{G} \tag{29}$$

where A, B, C, D, E etc are functions of a, φ and $\bar{\varphi}$ only. This Ansatz contains all allowed combinations of the fermionic fields and is the most general Lorentz invariant function we can write down.

The next step is to solve the supersymmetry constraints $S_A\Psi=0$ and $\bar{S}_{A'}\Psi=0$. Since the wave function (29) is of even order in fermionic variables, the equations $S_A\Psi=0$ and $\bar{S}_A\Psi=0$ will be of odd order in fermionic variables. Since each order in fermionic variables is independent, the number of constraint equations will be very high. Their full analysis is quite tedious and to write all the terms would overburden the reader. Let us show some examples of the calculations involved in solving the $S_A\Psi=0$ constraint.

Consider the terms linear in χ_A:

$$\left[-\frac{i}{\sqrt{2}}(1+\varphi\bar{\phi})\frac{\partial A}{\partial\phi}\right]\chi_{A}+\frac{\sigma^{2}a^{2}gf}{\sqrt{2}(1+\varphi\bar{\varphi})}\sigma^{a}{}_{AA'}n^{BA'}X^{(a)}A\chi_{B}=0. \tag{30}$$

Since this is true for all χ_A, the above equation becomes

$$\left[-\frac{i}{\sqrt{2}}(1+\varphi\bar{\phi})\frac{\partial A}{\partial\phi}\right]\epsilon_{A}{}^{B}+\frac{\sigma^{2}a^{2}gf}{\sqrt{2}(1+\varphi\bar{\phi})}\sigma^{a}{}_{AA'}n^{BA'}X^{(a)}=0. \tag{31}$$

Mutliplying the whole equation by $n_{BB'}$ and using the relation $n_{BB'}n^{BA'}=\frac{1}{2}\epsilon_{B'}{}^{A'}$, we can see that the two terms are independent of each other since the σ matrices are orthogonal to the n matrix. Thus, we conclude that

$$A=0. \tag{32}$$

Now consider,eg.the terms linear in $\chi_B\psi^C\psi_C$. We have

$$\left[(1+\phi\bar\phi)\frac{\partial B}{\partial\bar\phi}+\frac{1}{2}\phi B+i\frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a}-i\frac{7}{4\sqrt{3}}C+i\frac{\sqrt{3}}{2}\sigma^2a^2C\right]\chi_A\psi^C\psi_C$$

$$+i\frac{\sigma^2a^2gf}{\sqrt{2}(1+\phi\bar\phi)}\sigma^a{}_{AA'}n^{BA'}\bar\lambda^{(a)}B\chi_B\psi^C\psi_C=0. \qquad (33)$$

By the same argument as above, the first term is independent of the second one and we have the result

$$B=0. \qquad (34)$$

As we proceed, this pattern keeps repeating itself. Some equations show that the coefficients have some symmetry properties. For example, $d_{ab}=2g_{ab}$. But when these two terms are combined with each other, they become zero. This can be seen as follows,

$$d_{ab}\lambda^{(a)C}\chi_C\lambda^{(a)D}\chi_D+g_{ab}\lambda^{(a)C}\lambda^{(b)}_C\chi^D\psi_D$$

$$=2g_{ab}\lambda^{(a)C}\lambda^{(b)D}\chi_D\psi_C+g_{ab}\lambda^{(a)C}\lambda^{(b)}_C\chi^D\psi_D$$

$$=-g_{ab}\lambda^{(a)C}\lambda^{(b)}_C\chi^D\psi_D+g_{ab}\lambda^{(a)C}\lambda^{(b)}_C\chi^D\psi_D, \qquad (35)$$

using the property that $g_{ab}=g_{ba}$ and the spinor identity $\theta_{AB}=\frac{1}{2}\theta_C{}^C\epsilon_{AB}$ where θ_{AB} is anti-symetric in the two indices. The same property applies to the terms with coefficients f_{abcd} and g_{abcd}. Other equations imply that the coefficients c_{abc}, d_{abc}, c_{abcd}, e_{abc}, f_{abc}, d_{abcd}, e_{abcd}, h_{abcd} are totally symmetric in their indices. This then leads to the terms cancelling with each other, as can easily be shown. In the end, considering both the $S_A\Psi=0$ and $\bar S_{A'}\Psi=0$ constraints, we are left with the surprising result that the wave function (29) must be zero in order to satisfy the quantum constraints.

Now we briefly comment on the case when we set the multiplet consisting of f and λ_A to zero. The only coefficients left in the Ansatz of the wave fuction are A, B, C, D and E. We will get four equations from $S_A\Psi=0$ and another four equations from $\bar S_{A'}\Psi=0$.

$$-\frac{i}{\sqrt{2}}(1+\phi\bar\phi)\frac{\partial A}{\partial\phi}=0\ . \qquad (36a)$$

$$-\frac{a}{2\sqrt{6}}\frac{\partial A}{\partial a}-\sqrt{\frac{3}{2}}\sigma^2a^2A=0\ , \qquad (36b)$$

$$(1+\phi\bar\phi)\frac{\partial B}{\partial\phi}+\frac{1}{2}\bar\phi B+\frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a}-\frac{7}{4\sqrt{3}}C+\frac{\sqrt{3}}{2}\sigma^2a^2C=0\ , \qquad (36c)$$

$$\frac{a}{\sqrt{3}}\frac{\partial D}{\partial a}+2\sqrt{3}\sigma^2a^2D-\sqrt{3}D-(1+\phi\bar\phi)\frac{C}{\phi}-\frac{3}{2}\bar\phi C=0\ , \qquad (36d)$$

$$i\sqrt{2}(1+\phi\bar\phi)\frac{\partial E}{\partial\bar\phi}=0\ , \qquad (37a)$$

$$\frac{a}{\sqrt{6}}\frac{\partial E}{\partial a}-\sqrt{6}\sigma^2a^2E=0\ , \qquad (37b)$$

$$\frac{a}{\sqrt{3}}\frac{\partial B}{\partial a}-2\sqrt{3}\sigma^2a^2B-\sqrt{3}B+(1+\phi\bar\phi)\frac{\partial C}{\partial\phi}+\frac{3}{2}\bar\phi C=0\ , \qquad (37c)$$

$$(1+\phi\bar\phi)\frac{\partial D}{\partial\bar\phi}+\frac{1}{2}\phi D-\frac{a}{4\sqrt{3}}\frac{\partial C}{\partial a}+\frac{7}{4\sqrt{3}}C+\frac{\sqrt{3}}{2}\sigma^2a^2C=0\ . \qquad (37d)$$

We can see that (36a), (36b) and (37a), (37b) constitute decoupled equations for A and E, respectively. They have the general solution.

$$A=f(\bar\phi)\exp(-3\sigma^2a^2)\ ,\ E=g(\phi)\exp(3\sigma^2a^2) \qquad (38)$$

where f,g are arbitrary anti-holomorphic and holomorphic functions of ϕ, respectively. Eq. (36c) and (36d) are coupled equations between B and C and eq. (37c) and (37d) are coupled equations between C and D. The first step to decouple these equations is as follows. Let $B=\tilde B(1+\phi\bar\phi)^{-\frac{1}{2}}$, $C=\frac{\tilde C}{\sqrt{3}}(1+\phi\bar\phi)^{-\frac{3}{2}}$, $D=\tilde D(1+\phi\bar\phi)^{-\frac{1}{2}}$. Equations (36c), (36d), (37c) and (37d) then become

$$(1+\phi\bar\phi)^2\frac{\partial\tilde B}{\partial\phi}+\frac{a}{12}\frac{\partial\tilde C}{\partial a}-\frac{7}{12}\tilde C+\frac{1}{2}\sigma^2a^2\tilde C=0\ , \qquad (39a)$$

$$(1+\phi\bar\phi)^2\frac{\partial\tilde D}{\partial\bar\phi}-\frac{a}{12}\frac{\partial\tilde C}{\partial a}+\frac{7}{12}\tilde C+\frac{1}{2}\sigma^2a^2\tilde C=0\ , \qquad (39b)$$

$$\frac{\partial\tilde C}{\partial\phi}-a\frac{\partial\tilde D}{\partial a}-6\sigma^2a^2\tilde D+3\tilde D=0\ , \qquad (39c)$$

$$\frac{\partial\tilde C}{\partial\bar\phi}+a\frac{\partial\tilde B}{\partial a}-6\sigma^2a^2\tilde B-3\tilde B=0\ . \qquad (39d)$$

From (39a) and (39d), we can eliminate $\tilde B$ to get a partial differential equation for $\tilde C$:

$$(1+\phi\bar\phi)^2\frac{\partial\tilde C}{\partial\bar\phi\partial\phi}-\frac{a}{12}\frac{\partial}{\partial a}\left(a\frac{\partial\tilde C}{\partial a}\right)+\frac{5}{6}a\frac{\partial\tilde C}{\partial a}+\left[3\sigma^4a^4+3\sigma^2a^2-\frac{7}{4}\right]\tilde C=0\ , \qquad (40)$$

and from (39b) and (39c), we will get another partial differential equation for $\tilde{C}$:

$$(1+\phi\bar{\phi})^2 \frac{\partial \tilde{C}}{\partial\phi\partial\bar{\phi}} - \frac{a}{12}\frac{\partial}{\partial a}\left(a\frac{\partial \tilde{C}}{\partial a}\right) + \frac{5}{6}a\frac{\partial \tilde{C}}{\partial a} + \left[3\sigma^4 a^4 - 3\sigma^2 a^2 - \frac{7}{4}\right]\tilde{C} = 0 . \quad (41)$$

We can see immediately that $\tilde{C} = 0$ because the coefficients of $\sigma^2 a^2 \tilde{C}$ are different for these two equations. Using this result, we find

$$B = h(\bar{\phi})(1+\phi\bar{\phi})^{-\frac{1}{2}} a^3 \exp(3\sigma^2 a^2) , \; C = 0 , \; D = k(\phi)(1+\phi\bar{\phi})^{-\frac{1}{2}} a^3 \exp(-3\sigma^2 a^2) . \quad (42)$$

We can also check that when the Kähler manifold is replaced by $\mathbf{R}^2$, $\tilde{C}$ remains zero. This should be compared with the result of [17]

To summarise, we have applied the canonical formulation of the more general theory of $\mathcal{N} = 1$ supergravity with supermatter [22] to a $k = +1$ FRW mini-superspace model, subject to suitable Ansätze for the the gravitational field, gravitino field and the gauge vector field A^a_μ as well as the scalar fields and corresponding fermionic partners. After a dimensional reduction, we derived the supersymmetric constraints for our one-dimensional model. We then solved the Lorentz and supersymmetry constraints for the case of a two-dimensional spherically symmetric Kähler manifold. When the model is truncated by setting the spin-1, spin-$\frac{1}{2}$ multiplet to zero, a very restricted set of quantum states is obtained. However for the case when all supermatter fields are included, we found that there are *no* physical states in this model. A similar conclusion was also obtained in ref. [7-8] where no matter but a cosmological constant term was present. All this seems to suggest that as one introduces more terms in a locally supersymmetric action,giving more field modes with associated mixing, then the constraints impose severe restrictions on the possible allowed states, assuming homogeneity and isotropy. This is not to say that there might not be many inhomogenous states.

In the future, the framework presented in this paper will be extended to the case, for example, of a Bianchi-I universe. We would like to see if the same type of results occur there. As more gravitino modes are present [8], we could consider a non-zero analytic potential $P(\Phi)$. The potential term in the supersymmetry constraints is similar to that induced by a cosmological constant. It will be interesting to check in the case of $P(\Phi) \neq 0$ (possibly restricting ourselves to zero spin-1 and fermionic partner fields) whether no physical states can be found as solutions of the quantum constraints. In that case the analytic potential terms could not be present in a realistic N=1 supergravity theory with supermatter.

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