IFT **Instituto de Física Teórica**
Universidade Estadual Paulista

October/94 IFT–P.038/94

Theoretical prevision for the low-energy $^3S_1 - ^3D_1$ mixing parameters *

Sadhan K. Adhikari, Carlos F. de Araújo

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona 145
01405-900 - São Paulo, S.P.
Brazil

and

T. Frederico

Instituto de Estudos Avançados
Centro Técnico Aeroespacial
12231-970 - São José dos Campos, S.P.
Brazil

*To appear in Physical Review C (Rapid Communications)

Theoretical prevision for the low-energy $^3S_1 - ^3D_1$ mixing parameters

Sadhan K. Adhikari[a], Carlos F. de Araujo, Jr.[a], and T. Frederico[b]

[a]*Instituto de Física Teórica, Universidade Estadual Paulista, 01405-900 São Paulo, São Paulo, Brasil*

[b]*Instituto de Estudos Avançados, Centro Técnico Aeroespacial, 12231-970 São José dos Campos, São Paulo, Brasil*

Using a recent shape-independent approximation for the $^3S_1 - ^3D_1$ mixing parameter, theoretical prevision for the low-energy mixing parameters is made. The present prevision is consistent with the deuteron binding energy, its asymptotic D state to S-state ratio, η_d, the triplet scattering length, and the meson exchange tail of the tensor nucleon-nucleon potential. The theoretical prevision upto an incident laboratory energy of 25 MeV is consistent with the recent multi-energy determination of mixing parameters, but is much higher than many single energy determinations of the same. The low single-energy values of the mixing parameter could be reproduced by meson-theoretic potentials only with a substantially reduced η_d.

PACS: 21.30.+y, 13.75.Cs

Accepted: Physical Review C (Rapid Communication)

The usual shape-independent effective-range expansion has been extremely useful for predicting model-independent low-energy phase shifts of the nucleon-nucleon (NN) system form a knowledge of the scattering length and effective range (alternatively, the bound- or virtual-state energy)[1, 2].

In the coupled $^3S_1 -^3D_1$ channel, apart from the phase shifts, one needs the mixing parameters for a complete analysis. The mixing parameter is extremely important because it is supposed to carry the informations about the NN tensor force[3, 4]. More than three decades ago it was conjectured[5] that a knowledge of the low-energy NN central observables (deuteron binding, scattering length etc.), the deuteron D-state to S-state asymptotic normalization parameter η_d, and the correct meson-exchange (mostly, pion) tail of the NN tensor force should suffice to determine the low-energy mixing parameters. Despite repeated recent claims to this effect, a systematic theoretical prevision of the mixing parameters using this idea has not been made. In this work, using a recently suggested shape-independent approximation for the mixing parameter we make such a prevision for both Stapp-Ypsilantis[6] and Blatt-Biedenharn[7] mixing parameters, ϵ_1 and ϵ_{BB}, respectively.

The recent shape-independent approximation for ϵ_{BB} is given by[4]

$$\frac{tan(2\epsilon_{BB})}{2k^2} = \frac{\eta_d}{\alpha^2} + B_0 \frac{m_\pi^2}{4\alpha^2}\frac{k^2+\alpha^2}{k^2+m_\pi^2/4}. \quad (1)$$

where the B_0, given by

$$B_0 = \left(\frac{a_{02}}{a_t} - \frac{\eta_d}{\alpha^2}\right). \quad (2)$$

has been shown to be uniquely determined by the long-range part of the NN potential. Here α^2 is the deuteron binding energy, m_π is the pion mass, a_t is the NN S wave triplet scattering length, and $a_{02} \equiv \lim_{k\to 0}(K_{02}/k^2)$, where $K_{ll'}$ is the on-shell NN K matrix element for the ll' channel.

Using the Lippmann-Schwinger equation for the coupled $S - D$ channel we derived the following approximate relation for B_0 correct to within few percent[4]:

$$B_0 \simeq \frac{1}{a_t}\lim_{k\to 0}\left(\frac{V_{02}(k,k)}{k^2}\right) - \frac{2}{\pi}\alpha^2\int_0^\infty dq \frac{K_{00}(0,q;0)}{a_t(\alpha^2+q^2)}\lim_{k\to 0}\left(\frac{V_{02}(q,k)}{k^2}\right). \quad (3)$$

In this equation $K_{00}(0,q;0)$ are the half-off-shell S-wave K matrix elements and $V_{02}(q,k)$ are the tensor potential elements. In Eq. (3) the low-q values of the tensor potential $V_{02}(q,k)$, and the K matrix elements dominate the integral because of additional factors of q^2 in the denominator for large q. The half-shell function $g(q) \equiv K(0,q;0)/a_t$ is a universal function independent of potential models[8] and any reasonable approximation to this quantity in Eq. (3) leads to the same result to less than an estimated error of 1 %.

It has been emphasized[4] that B_0 is reasonably model independent and can be fixed by only the tail of the NN tensor potential. It has been noted[4] that separable tensor Yamaguchi[9] and square-well potentials, which do not possess the one-pion-exchange tail, when fitted to reproduce the deuteron binding and the asymptotic normalization η_d, badly fails to reproduce the correct value of B_0, and yields $B_0 \simeq 0$ fm^2. Consequently, such potentials lead to a poor description of the low-energy mixing parameters. The correct long-range meson-exchange tail of the tensor potential is essential for a reproduction of the low-energy mixing parameters. The last term of Eq. (1) parametrizes the effect of the one-pion-exchange left-hand cut of the NN tensor potential. This is fundamental in reproducing the correct mixing parameters using approximation (1).

We find that approximation (1) accurately reproduces the mixing parameters of Reid[10] and a one-boson-exchange Bonn[11] potentials up to a laboratory energy of 25 MeV. Deviations are expected at higher energies where details of the tensor potential at medium ranges are eminent. Also, as various meson-theoretic potentials have essentially the same one-pion-exchange tail, and are fitted to same low-energy S-wave observables and η_d, it is not surprising that they yield essentially the same mixing parameters up to an energy of 25 MeV. As different meson-theoretic potentials have different medium range behaviors, the mixing parameters for various realistic potentials tend to be different at higher energies.

Hence, if the minimum ingradients such as the long-range behavior of the meson-theoretic tensor NN potential, the low-energy NN S-wave observables, and the correct η_d are known, the low-energy mixing parameters could be uniquely determined. However, the long-range behavior of the tensor potential is predominantly the one-pion-exchange tail with some small admixture of exchange of heavier mesons. Also, η_d is not uniquely known experimentally[12]. This means that the constants η_d and B_0 in Eq. (1) are not uniquely known and have some uncertainty. We shall include this uncertainty in predicting the model independent estimate for the low-energy mixing parameters. In case of doubt about the correct value of η_d and B_0, we chose to overestimate the uncertainty rather than underestimating it in the present theoretical evaluation of the mixing parameters.

Recently, there been lot of theoretical and experimental activities[12, 13] in measuring η_d. The recent and presumably the most accurate measurement of η_d is $\eta_d = 0.0256 \pm .0004$[13]. The meson-theoretic potentials yield for η_d values ranging from 0.0255 to 0.0266: Paris 0.0261, Bonn OBE A 0.0259, Bonn OBE B 0.0263, Nijmegen 0.0255, Urbana 0.0255, Super soft core 0.0263, Reid 0.0264. In view of this uncertainty in our theoretical estimate we take $\eta_d = 0.026 \pm 0.001$.

In a recent theoretical study we calculated[4] the constant B_0 using (2) for the Reid soft-core and the OBE Bonn NN potentials and found $B_0 = -0.190$ fm^2 and -0.182fm^2, respectively. The approximation (3) for B_0 underestimates these values by about 3%[4]. These potentials have the correct (phenomenological) long-range parts of the NN tensor potential V_{02}. It was verified[4], in calculations using Eq. (3), that the result for B_0 is essentially unchanged if the short-range parts (heavier meson exchanges) of the NN tensor potential are suppressed. However, B_0 is also (weakly) sensitive to the medium-range behavior of the tensor potential. If only the one pion-exchange tail of the NN tensor potential is used in Eq. (3) one obtains the limiting value $B_0 = -0.162$ fm^2. This is the extremum value of B_0 when all the short- and intermediate-range parts of the tensor potential are dropped. It was noted that the approximation (3) underestimates the exact result by about 3% and hence B_0 for one-pion-exchange potential is expected to be approximately -0.17 fm^2. Though the one-pion-exchange potential gives a good description of B_0, heavier meson exchanges at intermediate ranges are needed for a precise reproduction of this constant. The effect of the exchange of heavier mesons at intermediate distances is to reduce B_0 by about 0.01, or 0.02 fm^2[4]. In view of this in the present

theoretical estimate we set the variation of B_0 in the range -0.17 to -0.20 fm^2.

In Figs. 1 and 2 we plot the Blatt-Biedenharn mixing parameter ϵ_{BB} and the Stapp mixing parameter ϵ_1 versus energy, respectively. The experimental points are taken from Ref. [3, 14–17]. The theoretical curves correspond to the following values of the constants η_d and B_0; A: $\eta_d = 0.027$ and $B_0 = -0.17$ fm^2, B: $\eta_d = 0.025$ and $B_0 = -0.17$ fm^2, C: $\eta_d = 0.027$ and $B_0 = -0.20$ fm^2, and D: $\eta_d = 0.025$ and $B_0 = -0.20$ fm^2. The recent multi-energy determinations of Stoks et al. [14] are denoted by □, the single-energy determinations of Wilburn et al.[15] and the 25 MeV determination by Henneck [3] are denoted by ×, the single-energy determinations of Aarnt et al. are denoted by △ [16] and ○ [17].

The present theoretical estimates for ϵ_{BB} are transformed to ϵ_1 by using the experimental values of the S and D wave bar phase shifts at specific energies. It should be noted that in the energy range $E_{Lab} = 10-25$ MeV the numerical values of ϵ_1 and ϵ_{BB} are approximately equal. However, at higher and lower energies they are quite different. It is realized from Figs. 1 and 2 that the multi-energy determinations of Stoks et al. [14] and the 25 MeV determination of Henneck [3] are consistent with theory. However, it is not easy to predict η_d from these mixing parameter. The 1 and 5 MeV mixing parameters of Stoks et al. seem to suggest an η_d between 0.026 and 0.027, whereas the 10 and 25 MeV mixing parameters of Stoks et al. and the 25 MeV mixing parameter of Henneck seem to suggest an η_d between 0.025 and 0.026. It is noted that some of the single-energy determinations [15–17] yield much too low value for the mixing parameter.

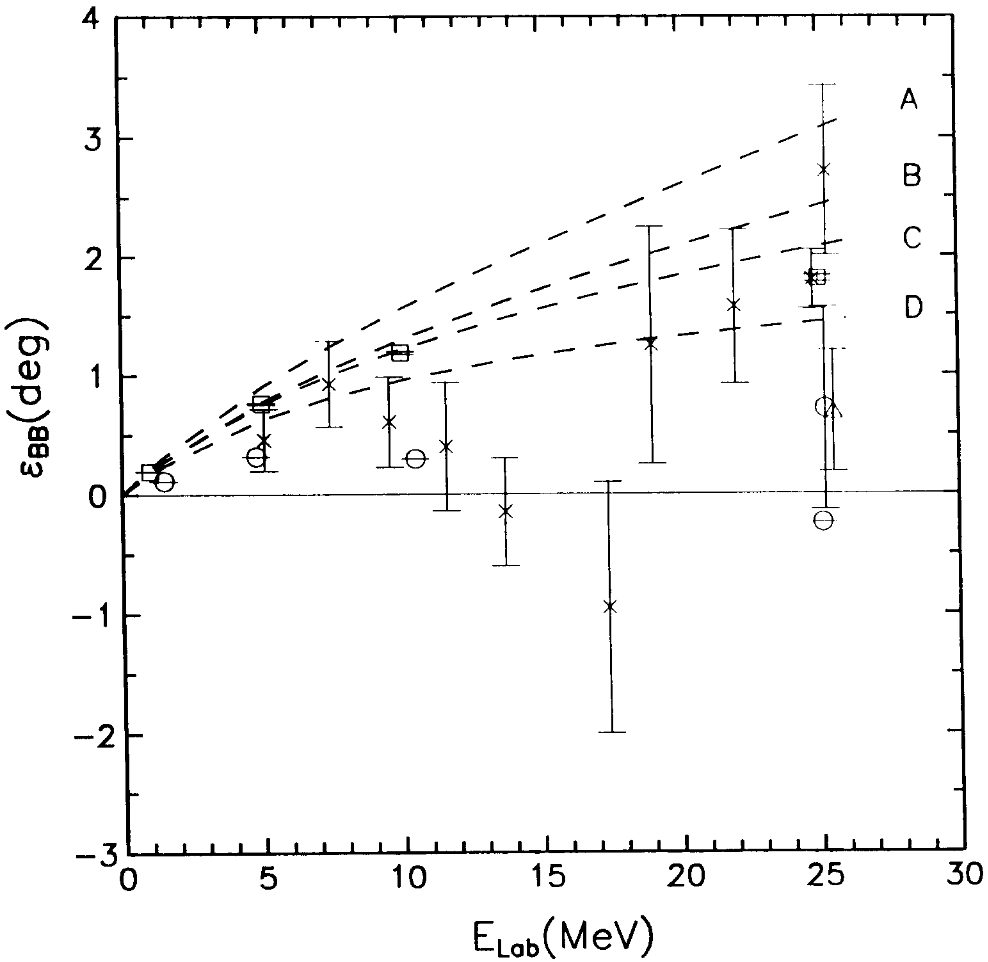
It seems obvious that some of the analyses of Refs. [16, 17] are rather old and unreliable and are inadequate to provide constraints on the mixing parameter. Some of these are single-energy analyses or even single-experiment analyses. A discussion of this point appears in Ref. [14] and should be taken into consideration while using these analyses in providing constraints on the mixing parameter.

The constant B_0 is determined principally by the long-range behavior of the tensor potential dominated mostly by the well established one-pion-exchange tail. Hence it appears that some of the single-energy determination of the low-energy experimental mixing parameters with the error bars are inconsistent with the long range part of meson-theoretic potentials and the current experimental η_d, taken to be 0.026 ± 0.001. In order to accomodate these experimental points with the meson-theoretic NN potentials η_d has to be much smaller than 0.025.

In conclusion, we have made for the first time theoretical prevision for the NN $^3S_1-^3D_1$ mixing parameters up to an incident laboratory energy of 25 MeV consistent with experimental η_d (taken to be 0.026 ± 0.001), the long-range tail of the meson-theoretic tensor potentials, and other low-energy NN observables in this channel. In this energy domain some of the the single-energy determinations for the mixing parameter are much too smaller than theoretical previsions. Theoretical meson-theoretic NN potentials will not yield such low values for mixing parameters without substantially reducing the current experimental value of η_d.

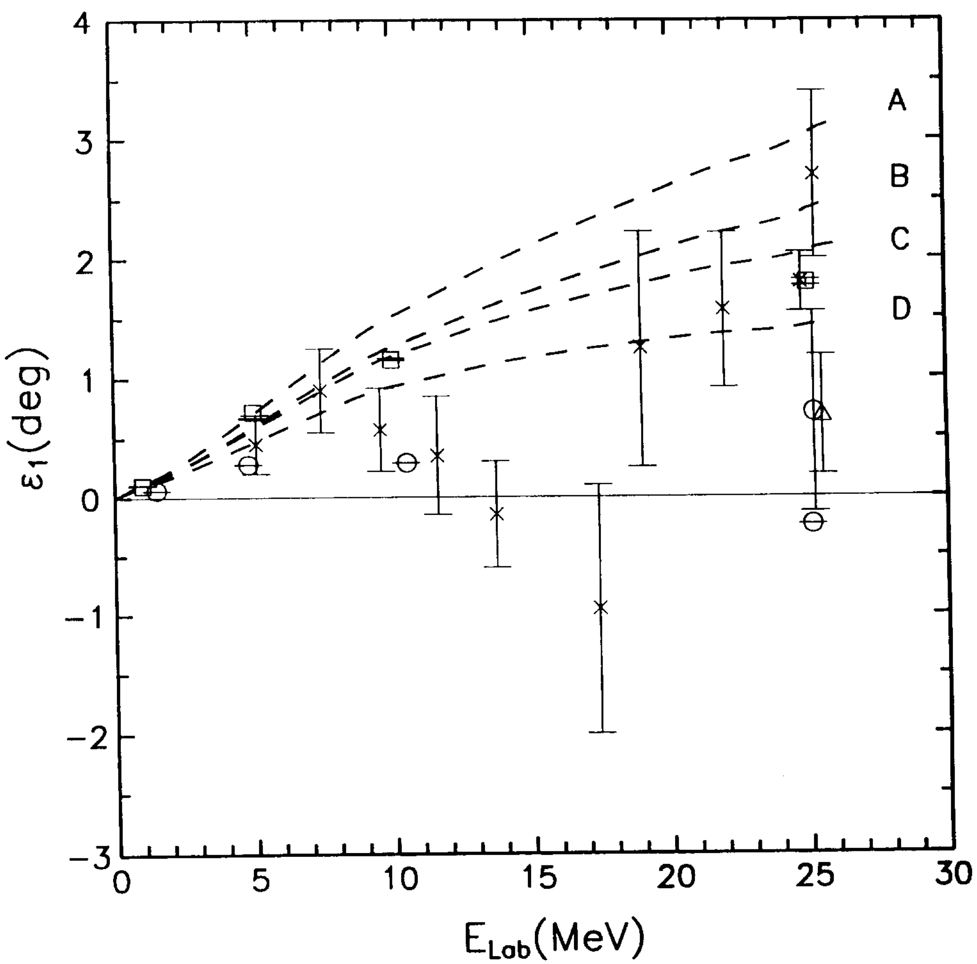
The work has been supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico, Coordenação de Aperfeiçoamento de Pessoal do Nivel Superior, and Financiadoras de Estudos e Projetos of Brazil.

FIG. 1



A
B
C
D
ε_{BB}(deg)
E_{Lab}(MeV)

FIG. 2



A
B
C
D
ε_1(deg)
E_{Lab}(MeV)

REFERENCES

[1] H. A. Bethe, Phys. Rev. 76, 38 (1949).

[2] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, (Wiley, New York, 1952).

[3] R. Henneck, Phys. Rev. C 47, 1859 (1993).

[4] S. K. Adhikari, L. Tomio, J. P. B. C. de Melo, and T. Frederico, Phys. Lett. 318B, 14 (1993).

[5] D. Y. Wong, Phys. Rev. Lett. 2, 406 (1959), L. Mathelitsch and B. J. Verwest, Phys. Rev. C 29, 739 (1984).

[6] H. P. Stapp, T. Ypsilantis, and N. Metropolis, Phys. Rev. 105, 302 (1957).

[7] J. M. Blatt and L. C. Biedenharn, Phys. Rev. 86, 399 (1952); L. C. Biedenharn and J. M. Blatt, ibid. 93, 1387 (1954).

[8] E. F. Redish and K. Stricker Bauer, Phys. Rev. C. 36, 513 (1987); K. Amos, L. Berge, F. A. Brieva, A. Katsogiannis, L. Petris, and L. Rikus, ibid. 37, 934 (1988).

[9] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. 95, 1635 (1954).

[10] S. C. Peiper, Phys. Rev. C 9, 883 (1974); R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[11] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. 149, 1 (1987).

[12] J. T. Londergan, C. E. Price, and E. J. Stephenson, Phys. Rev. C 35, 902 (1987); L. D. Knutson, P. C. Colby, and B. P. Hichwa, Phys. Rev. C 24, 411 (1981); L. D. Knutson, B. P. Hichwa, A. Barrosso, A. M. Eiro, F. D. Santos, and R. C. Johnson, Phys. Rev. Lett. 35, 1570 (1975).

[13] N. L. Rodning and L. D. Knutson, ibid. 57, 2248 (1986).

[14] V. G. J. Stoks et al., Phys. Rev. C 48, 792 (1993).

[15] W. S. Wilburn, Gould, Haase, Huffman, Keith, Koster, Roberson, and Tornow, Phys. Rev. Lett. 71, 1982 (1993).

[16] R. A. Arndt, J. S. Hyslop, and L. D. Roper, Phys. Rev. D 35, 128 (1987).

[17] R. A. Arndt, R. H. Hackman, and L. D. Roper, Phys. Rev. C 15, 1002 (1977);

Figure Caption

1. Blatt-Biedenharn mixing parameters in degrees. The experimental results are from Ref. ([14]) denoted (□), Ref. ([3, 15]) denoted (×), Ref. ([16]) denoted (△), and Ref. ([17]) denoted (○). The theoretical previsions using approximation (1) are denoted by A: $\eta_d = 0.027$ and $B_0 = -0.17$ fm^2, B: $\eta_d = 0.025$ and $B_0 = -0.17$ fm^2, C: $\eta_d = 0.027$ and $B_0 = -0.20$ fm^2, and D: $\eta_d = 0.025$ and $B_0 = -0.20$ fm^2. Note that some of the experimental points have almost zero error bar.

2. Same as Fig. 1 for the Stapp mixing parameters.