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Abstract

A new version of the program CFPSIB is introduced. Using the same recurrent formula as that used in the program CFPSIB, but different calculation procedure, the fractional parentage coefficients (CFP's) of identical boson system with well-defined seniority can be evaluated by this new program. It is shown that this new version (named RCFPSIB) is more efficient than the program CFPSIB, especially for the system including a relatively large number of bosons, each with higher angular momentum l.

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NEW VERSION SUMMARY

Title of program: RCFPSIB

Catalogue number:

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland

Licensing provisions: none

Computer for which the new version is designed and others on which it has been tested: IBM RISC/6000 320H / VAX 8550; Department of Physics, Peking University, Beijing / CCAST(World Laboratory), Beijing

Operating system under which the new version has been tested : AIX 3.2.4 / VMS version 4.7

Programming language used in the new version: FORTRAN 77

Memory required to execute with typical data: 7,534,592 words

No. of bits in a word: 32

No. of lines in distributed program, including test data, etc: 910

Keywords:

identical bosons, angular momentum, seniority, multiplicity, coefficient of fractional parentage, isoscalar factor, reduced matrix element, interacting boson model, unitary group, orthogonal group

Nature of physical problem

The program calculates all the coefficients of fractional parentage (CFP)[1] of identical boson system, by using the recurrent relations[2] with well-defined seniority. CFP's are the expansion coefficients of symmetrized n-body wave functions in terms of the symmetrized $(n-1)$-body wave functions and the wave function of the nth boson. It is of fundamental importance in constructing a many-boson wave function with well-defined permutational symmetry and total angular momentum, and in calculating the reduced matrix elements(RME's) of a physical operator. The evaluation of the RME's is the basis of the calculation in nuclear, atomic and molecular physics. Moreover, CFP's of a system with a large number of identical bosons are important in investigating the physical mechanism of nuclear high spin states and superdeformed states[3] and examining the chaotic behavior in many-boson system[4].

Method of solution

The program RCFPSIB uses the approach developed in Refs.[2] to get all the CFP's of an identical boson system. In the approach, the CFP is factorized as a product of the isoscalar factor(ISF) of the reduction $U(N) \supset O(N)$ and that of $O(N) \supset O(3)$. The ISF of the reduction $U(N) \supset O(N)$ has been given analytically. The ISF of the reduction $O(N) \supset O(3)$ is evaluated by a recurrent relation. The recurrent relation is presented with well-defined seniority, and the recurrent process is controlled by the multiplicity of an irreducible representation (IRREP) of O(3) in an IRREP of group O(N). It provides an efficient algorithm for computation and is numerically stable for relatively large system.

Reasons for the new version

Practical calculations show that the FORTRAN program CFPSIB[5] is not efficient enough for the system including a large number of bosons. After a careful examination on the code, we find out that a lot of repetitious calculations are involved in the program. We then modify it, so that this new version is accomplished.

Restriction on the complexity of the problem

The program RCFPSIB can evaluate the CFP's of a system with identical bosons. At present, with the control parameters MNU=50, NFS=442, NTL=100, MAL=9, MM=4, the program can handle the system with maximal seniority 50 for d-bosons. With the control parameters MNU=16, NFS=895, NTL=48, MAL=37, MM=6, the program can handle the system with maximal seniority 16 for f-bosons. With the control parameters MNU=10, NFS=988, NTL=40, MAL=50, MM=8, the program can handle the system with maximal seniority 10 for g-bosons. With the control parameters MNU=7, NFS=593, NTL=35, MAL=35, MM=10, the program can handle the system with maximal seniority 7 for h-bosons. With the control parameters MNU=6, NFS=590, NTL=36, MAL=34, MM=12, the program can handle the system with maximal seniority 6 for i-bosons. After these control parameters are enlarged, the system can be enlarged.

Typical running time

This depends strongly on the number of the bosons and the angular momentum of each boson. For example, it takes about 56 Minutes on VAX 8550 to get all the ISF's, RME's and CFP's of a system including 50 d-bosons, about 5 hours for the 16 f-boson system, about 4 hours and 7 minutes for 10 g-bosons system, 31 minutes for 7 h-boson system, 19 minutes for 6 i-boson system.

References

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LONG WRITE-UP

1. Introduction

It is well known that the method of the coefficient of fractional parentage (CFP)[1-2] is one of the most efficient method for the construction of many particle wave functions. Moreover, the state vectors expressed in terms of CFP's make the calculation of matrix elements of physical operator simple. Since then several algorithm have been developed to evaluate the CFP's more and more efficiently[3-7], and many computer codes, such as JJCFP[8], GENESIS[9,10], CFPGEN[11], etc have also been set up. However, all these codes are not efficient enough for the system including a large number of identical particles. On the other hand, in order to investigate the physical mechanism of high spin states and superdeformed states in nucleus[12] and to examine the chaotic behavior in many boson system[13], it is imperative to develop a computer code for evaluating the CFP's of identical particle system including a large number of bosons, each with higher angular momentum(e.g. $l = 2,\ 3,\ 4, \cdots$).

Recently, Hong-Zhou Sun and the collaborators have developed new recurrent relations for determining the CFP's of the system with identical particles[6,7]. In the new approach, the CFP's of a identical particle system are well-defined with seniority and factorized into a product of isoscalar factors (ISF's) of the special unitary group and orthogonal group(for bosons) or symplectic group (for fermions) . Then the evaluation of CFP's is reduced to evaluate the ISF's of the group chains $U(N) \supset O(N)$ and $O(N) \otimes O(N) \supset O(N) \supset O(3)$ (for bosons) or $U(N) \supset SP(N)$ and $SP(N) \otimes SP(N) \supset SP(N) \supset O(3)$ (for fermions). It has been shown that this approach is quite efficient for calculating the CFP's of the system including a relatively large number of identical particles, and the method is numerically stable for relatively large system. Thereafter FORTRAN programs CFPSIB[14] (for boson system) and CFPSIF[15] (for fermion system) have been set up in the scheme of this new approach. For the program CFPSIB, because a lot of repetitious calculations are involved in the code, it is still not efficient enough for a system with a large number of bosons. To solve this problems, we rewrite the program CFPSIB, and a new version of code named RCFPSIB is then set up, with which the CFP's of a system including a large number

of bosons, each with any physically meaningful angular momentum, can be determined much more efficiently.

In this paper, the program is introduced in details. In section 2, we give briefly the mathematical formulae of the recurrent relations and the method of calculation. In section 3, the program structure is described. Finally, we show the results of test runs and give a brief discussion on the program.

2. Mathematical Formulae and Calculation Method

In the program RCFPSIB, the new recurrent relations of CFP's[7] and the formulae of multiplicity (MUL)[7,14-17] are used. The CFP's of the system of bosons each with angular momentum l have been factorized as

$$\begin{aligned}&\langle n\ \nu\ \alpha\ L\ \}\ n-1\ \nu_1\ \alpha_1\ L_1\ l\ L)\\ &=\left\langle n\ \nu\ \alpha\ L\ \|b_l^\dagger\|n-1\ \nu_1\ \alpha_1\ L_1\right\rangle/\sqrt{n}\\ &=\left\langle\begin{matrix}[1] & [n-1]\\ (1) & (\nu_1)\end{matrix}\middle|\begin{matrix}[n]\\ (\nu)\end{matrix}\right\rangle\left\langle\begin{matrix}(1) & (\nu_1)\\ l & \alpha_1\ L_1\end{matrix}\middle|\begin{matrix}(\nu)\\ \alpha\ L\end{matrix}\right\rangle,\end{aligned}\tag{1}$$

where $|n\ \nu\ \alpha\ L\rangle$ refers to the wave function of the state with boson number n, seniority ν, total angular momentum L and additional quantum number α, $\left\langle\begin{matrix}[1] & [n-1]\\ (1) & (\nu_1)\end{matrix}\middle|\begin{matrix}[n]\\ (\nu)\end{matrix}\right\rangle$ and $\left\langle\begin{matrix}(1) & (\nu_1)\\ l & \alpha_1\ L_1\end{matrix}\middle|\begin{matrix}(\nu)\\ \alpha\ L\end{matrix}\right\rangle$ are the ISF's of group reductions $U(N)\supset O(N)$ and $O(N)\supset O(3)$, in which $[n]$ reads the irreducible representation(IRREP) of group $U(N)$ $(N=2l+1)$ and (ν) refers to the IRREP of group $O(N)$. The seniority ν_1 of the parent state can be $\nu_1=\nu\pm1$. The ISF's of $U(N)\supset O(N)$ in the two cases $\nu_1=\nu-1$ and $\nu_1=\nu+1$ have been obtained analytically[7]. The ISF's of $O(N)\supset O(3)$ with $\nu_1=\nu+1$ and those with $\nu_1=\nu-1$ hold reciprocal relation

$$\left\langle\begin{matrix}(1) & (\nu+1)\\ l & \alpha_1\ L_1\end{matrix}\middle|\begin{matrix}(\nu)\\ \alpha\ L\end{matrix}\right\rangle=(-)^{L-l-L_1}\sqrt{\frac{(2L_1+1)d(N,\nu)}{(2L+1)d(N,\nu+1)}}\left\langle\begin{matrix}(1) & (\nu)\\ l & \alpha\ L\end{matrix}\middle|\begin{matrix}(\nu+1)\\ \alpha_1\ L_1\end{matrix}\right\rangle,\tag{2}$$

where

$$d(N,\ \nu)=\frac{(N-2+2\nu)(N-3+\nu)!}{\nu!(N-2)!}.$$

Then the ISF's of $O(N)\supset O(3)$ needed to be calculated explicitly are the ones with $\nu_1=\nu-1$. It has been shown that these ISF's can be obtained with the following recurrent relation

$$\left\langle\begin{matrix}(1) & (\nu-1)\\ l & \alpha_1\ L_1\end{matrix}\middle|\begin{matrix}(\nu)\\ (\alpha_1'\ L_1')\ L\end{matrix}\right\rangle=\frac{P(\alpha_1'\ L_1'\ \alpha_1\ L_1\ L)}{\sqrt{\nu\ P(\alpha_1'\ L_1'\ \alpha_1'\ L_1'\ L)}},\tag{3}$$

where $P(\alpha_1'\ L_1'\ \alpha_1\ L_1\ L)$ is given as

$$\begin{aligned}P(\alpha_1'\ L_1'\ \alpha_1\ L_1\ L)&=\delta_{\alpha_1'\ \alpha_1}\ \delta_{L_1'\ L_1}\\ &+(-)^{L+L_1'}(\nu-1)\sqrt{(2L_1'+1)(2L_1+1)}\sum_{\alpha_2,L_2}\left[\left\{\begin{matrix}l & L_2 & L_1'\\ l & L & L_1\end{matrix}\right\}-\frac{2\delta_{L_2L}}{(2L+1)(N-4+2\nu)}\right]\\ &\left\langle\begin{matrix}(1) & (\nu-2)\\ l & \alpha_2\ L_2\end{matrix}\middle|\begin{matrix}(\nu-1)\\ \alpha_1\ L_1\end{matrix}\right\rangle\left\langle\begin{matrix}(1) & (\nu-2)\\ l & \alpha_2\ L_2\end{matrix}\middle|\begin{matrix}(\nu-1)\\ \alpha_1'\ L_1'\end{matrix}\right\rangle.\end{aligned}\tag{4}$$

With eqs. (2-4) and the initial value $\left\langle\begin{matrix}(1) & (1)\\ l & l\end{matrix}\middle|\begin{matrix}(2)\\ L\end{matrix}\right\rangle=1$ $(L=0,2,4,\cdots,2l)$ all the ISF's with $\nu\le n$ of $O(N)\supset O(3)$ can be obtained. Of course the states $|\nu\ \nu\ (\alpha_1'\ L_1')\ L\ M)$ constructed in this way are over-complete. We then have to find out the linear independent and orthonormal states $|\nu\ \nu\ \alpha\ L\ M)$ and the corresponding ISF's $\left\langle\begin{matrix}(1) & (\nu-1)\\ l & \alpha_1\ L_1\end{matrix}\middle|\begin{matrix}(\nu)\\ \alpha\ L\end{matrix}\right\rangle$ by taking the advantage of the orthogonalization method[14].

In order to make the orthogonal process more efficient, we use the MUL of states with the same seniority ν and the same angular momentum L to control the calculation. The MUL of the total angular momentum $L=L_{max}-\xi$ of a boson system with single angular momentum l and seniority ν $(L_{max}=\nu\ l)$ is[14-17]

$$\gamma(\nu\ L)=\beta(\nu\ L)\ -\ \beta(\nu-2\ L),\tag{5}$$

in which

$$\beta(\nu\ L)=\sum_{\alpha=1}^{\nu}P_{\alpha\ m}(\xi)\ -\ \sum_{\alpha=1}^{\nu}P_{\alpha\ m}(\xi-1),\tag{6}$$

where $P_{\alpha\ m}(\xi)$ is the α-row partition of an integer $\xi\ge0$ with $m=2l$. It is the number of states with angular momentum L and all the possible seniority $\nu,\ \nu-2,\ \nu-4,\cdots\cdots$.

The $P_{\alpha\ m}(\xi)$ can be calculated by a recurrent formula[14-17]

$$P_{\alpha\ m}(\xi) = \sum_{i=1}^{\alpha-1} \sum_{j=1}^{F(\xi)} P_{i\ m-j}(\xi - \alpha\ j)\ , \tag{7}$$

with

$$F(\xi) = \begin{cases} [\xi/\alpha], & \xi \geq 2\alpha\ , \\ 1, & \xi < 2\alpha\ , \end{cases}$$

where $[\xi]$ indicates the integer part of number ξ (e.g., $[3/2] = 1$). $P_{1\ m}(\xi)$ and $P_{2\ m}(\xi)$ are initial values of $P_{\alpha\ m}(\xi)$[14-17].

3. Program Structure

RCFPSIB is a revised version on the FORTRAN program CFPSIB. With this new version all the ISF's, RME's and CFP's of a system with n bosons, each with angular momentum l can be evaluated efficiently. The new version code RCFPSIB consists of the main program RCFPSIB, subroutines MULT, INISF, GEN, GRME, QUIT and some functions.

3.1 Main Program RCFPSIB

At the beginning of the main program, the values of the control parameters MNU, NTL, NFS, MAL, MM and the others are given. Their meanings are listed in the following. MM is the double value of the single angular momentum l of the bosons. MNU refers to the maximal seniority of the system. NTL is the number of the possible total angular momentum L of the system. NFS indicates the largest value of the order index of the states with any possible total angular momentum and additional quantum number. MAL is the maximal additional quantum number of all the possible total angular momentum. Then the angular momentum l of a single boson and the number n of the bosons are read in. With the subroutine MULT being called, all the multiplicities of the states with seniority $\nu = n,\ n-1,\ n-2,\ \cdots, 2,\ 1,\ 0$ can be obtained. Thereafter, all the ISF's $\left\langle \begin{array}{cc|c} (1) & (\nu-1) & (\nu) \\ l & \alpha_1\ L_1 & \alpha\ L \end{array} \right\rangle$ with $\nu = n,\ n-1,\ n-2,\ \cdots, 2,\ 1$ are evaluated by calling the subroutine INISF and calling the subroutine GEN(NU, KDJ) within the DO loops with arguments NU and KDJ. The arguments NU, KDJ refer to the seniority ν and the sequence

index of the angular momentum L. At last all the reduced matrix elements (RME's) $\langle n\ \nu\ \alpha\ L \| b_l^{\dagger} \| n-1\ \nu_1\ \alpha_1\ L_1 \rangle$ and the CFP's $\langle n\ \nu\ \alpha\ L | \} n-1\ \nu_1\ \alpha_1\ L_1\ l\ L \rangle$ with $\nu = n,\ n-2,\ n-4,\ \cdots, 1$ or $0,\ \nu_1 = \nu \mp 1 \leq n-1$ are given simultaneously by calling the subroutine GRME.

3.2 Subroutine MULT

Subroutine MULT employs eq. (5) to get all the multiplicities of the states of a system of bosons, each with angular momentum l and maximal seniority ν_{max}. The arguments l and ν_{max} are input by the variables SIL, MAXNU in the common block PAME1. The states can be the ones with seniority $\nu = \nu_{max},\ \nu_{max}-1,\ \nu_{max}-2, \cdots, 2,\ 1,\ 0$ and angular momentum $L = \nu l - \xi\ (0 \leq \xi \leq \nu l)$. The calculation is achieved with the DO loop for variable ν and the embodied DO loop for the variable ξ. Within the inner DO loop, eq. (5) is employed with the function BETA(N, L, SIL). The results are written in the common blocks MUL1, MUL2 and RMES, in order to be transferred easily.

Function BETA(N, L, SIL) performs the calculation expressed in eq. (6). The arguments N, L, SIL correspond to the ν, L and the single boson angular momentum l with the restriction MM=$m_{max} = 2l$. The results of BETA(N, L, SIL) are transferred by the array BE(0;MNU, 0:NTL) in the common block BET. At the beginning of the function, we examine whether the array BE(N, L) is 0 or not. If BE(N, L)$\neq$ 0, it means that the value of BE has been given previously, we then employ the value directly. If BE(N, L) =0, we evaluate it by using the function PEFE(N, X, SIL). Designing the program in this way, a lot of repetitious calculations are avoided.

Function PEFE(N, X, SIL) takes the same way as that in function BETA to get the value of $\sum_{k=1}^{N} P_{k\,m}(X)$. The $P_{k\,m}(X)$ is given by another function PF(K, M, X). The data are transferred by the array PAT(MNU, 0:MM, 0:NTL) in the common block PAT1. Function PF(K, M, X) carries out the recurrent relation shown in eq. (7) with initial values given by function P2(M, X) and that of $P_{1\,M}(X) = \begin{cases} 0, & M < X, \\ 1, & M \geq X. \end{cases}$

3.3 Subroutine INISF(NUP)

Subroutine INISF(NUP) gives the initial values of the ISF's $\left\langle \begin{matrix} (1) & (\nu-2) \\ l & \alpha_2 L_2 \end{matrix} \middle| \begin{matrix} (\nu-1) \\ \alpha_1 L_1 \end{matrix} \right\rangle$ with all the possible α_2, L_2, α_1, L_1. The argument NUP refers to the seniority $\nu-1$. The results are written in the array PISF(NFS, NFS) of the common block PISFS so that the data can be transferred quickly.

3.4 Subroutine GEN(NU, KDJ)

Subroutine GEN(NU, KDJ) gives all the orthonormalized ISF's $\left\langle \begin{matrix} (1) & (\nu-1) \\ l & \alpha_1 L_1 \end{matrix} \middle| \begin{matrix} (\nu) \\ \alpha L \end{matrix} \right\rangle$ with all the possible α_1, L_1 and definite α, L by taking the Schmit orthogonalization procedure. The arguments NU, KDJ correspond to the seniority ν and the sequence index of the angular momentum L. The results are written in the SCRATCH FILE 11 and the common block PISFS. The supercomplete set of ISF's are given by employing the function PGISF(NU, K0, KP, JD), which carries out the calculation shown in eq. (3). The variables NU, JD represent the arguments ν, L in eq. (3). The arguments K0, KP indicate the order index of the states $|(\nu-1)(\nu-1)\alpha_1' L_1'\rangle$ and $|(\nu-1)(\nu-1)\alpha_1 L_1\rangle$ in the array INUS respectively. The variables L_1' and L_1 are determined by the array elements JS(NU-1, K0), JS(NU-1, KP) in the common block MUL2 separately. To avoid useless calculations, a logical function NOTRI(J1, J2, J3) is employed to examine whether the angular momenta J1, J2 J3 satisfy the triangle condition or not. The factor $P(\alpha_1', L_1', \alpha_1, L_1, L)$ is obtained by taking the function P(NU, K0, KP, JD) to perform the recursion of eq. (4), in which functions R6J(J1, J2, J3, J4, J5, J6) and DELT(I, J) are called. With the function R6J, the value of the 6-j symbol $\left\{ \begin{matrix} l & L_2 & L_1' \\ l & L & L_1 \end{matrix} \right\}$ is given. And the function DELT gives the value of $\delta_{\alpha_1' \alpha_1}$, etc. The values of the ISF's of the parent states and the values of $P(\alpha_1', L_1', \alpha_1', L_1', L)$ and $\frac{1}{(2L+1)(N-4+2\nu)}$ are transferred by the common block PISFS. Meanwhile, the rounding error is controlled by choosing the appropriate ISF's from the over-complete set and the orthogonality is also checked at every step of the recurrent procedure. If some kind of running errors take place in the running process, the program will stop by calling the subroutine QUIT and the running error will be listed.

3.5 Subroutine GRME

Subroutine GRME gives all the RME's $(n\nu\alpha L\|b_l^\dagger\|n-1\nu_1\alpha_1 L_1)$ and the CFP's $(n\nu\alpha L\}n-1\nu_1\alpha_1 L_1\ l\ L)$ by carrying out the calculation shown in eq. (1) and (2). The ISF's $\left\langle \begin{matrix} (1) & (\nu-1) \\ l & \alpha_1 L_1 \end{matrix} \middle| \begin{matrix} (\nu) \\ \alpha L \end{matrix} \right\rangle$ are read in from the SCRATCH FILE 11. The ISF's $\left\langle \begin{matrix} [1] & [n-1] \\ (1) & (\nu_1) \end{matrix} \middle| \begin{matrix} [n] \\ (\nu) \end{matrix} \right\rangle$ are given analytically. The calculated results (represented as RME, CFP respectively) are written in data files PRME.DAT and PCFP.DAT separately. If some running errors are encountered in the running process, the subroutine stop also and give the information about running error by calling the subroutine QUIT.

3.6 Subroutine QUIT

Subroutine QUIT terminates the program as any kind of running error takes place and gives the error information, such as, error in orthogonalizing, error in normalizing, IDR is inconsistent, and so on.

4. Results and Discussion

It has been shown in last section that the input card and the operation of this new version program are quite simple. After the command RUN RCFPSIB, one can input the single boson angular momentum l and the number n of the bosons along the line suggested by the test run information about the input data (see the first part of the TEST RUN OUTPUT), for instance, 2,36. All the ISF's, RME's $(n\nu\alpha L\|b_l^\dagger\|n-1\nu_1\alpha_1 L_1)$ and CFP's $(n\nu\alpha L\}n-1\nu_1\alpha_1 L_1\ l\ L)$ can be obtained. The part 2 and part 3 of the TEST RUN OUTPUT are examples of the calculated RME's and CFP's. These two cards show that the output data of this new version are physically meaningful and explicit, which can be used directly for performing other calculations.

As mentioned above we take the same calculation algorithm in the new version as that in the original program CFPSIB[14], but the program structure is quite different. The improvements are mainly on the following point: (1) in the subroutine MULT, all the evaluations are carried out by functions but not subroutines, and the calculated data at

every step are stored for being used in the next step. (2) all the initial values of the ISF's for a definite ν are given in a subroutine INISF and their values are replaced by the calculated ones at every step of the evaluation. (3) in subroutine GEN, which corresponds to the subroutine RMTSEN in CFPSIB, the orthonormalizing procedure is performed after the super-complete ISF's are given by calling the function PGISF, and the intermediate data are transferred by direct I/Os or common blocks. Moreover the orthogonalizing procedure is controlled by the multiplicity and only the appropriate ones are chosen from the over-complete set of the nonorthogonalized ISF's, so that the orthogonalization is accomplished quite quickly and the rounding error is controlled in an acceptable limit. (4) all the order indices labelling the ISF's are given when the multiplicities are calculated, and are transferred by a common block. With these measures being taken, a lot of repetitious calculations are avoided, and the required data for accomplishing the evaluation are transferred fast. Then this new version is much more efficient than the original version. For example, To get all the ISF's, RME's and CFP's of the system including 9 f-bosons on the computer VAX 8550, it takes about 150 minutes with the original program CFPSIB[14], however it takes only a lettle more than 1 minutes with this new version code RCFPSIB.

Because the multiplicity of an IRREP of group $O(3)$ in an IRREP of group $O(N)$ is used to control the calculation, and the intermediate data are stored and transferred by direct I/Os or common blocks, it saves the space occupied by the code efficiently. Therefore, this code can be used to treat relatively large system of bosons, each with a meaningful angular momentum l. At present, with the control parameters MNU=50, NFS=442, NTL=100, MAL=9, MM=4, the system can be the one with maximal seniority 50 for d-bosons. With the control parameters MNU=16, NFS=895, NTL=48, MAL=37, MM=6, the system can be the one with maximal seniority 16 for f-bosons. With the control parameters MNU=10, NFS=988, NTL=40, MAL=50, MM=8, the system can be the one with maximal seniority 10 for g-bosons. With the control parameters MNU=7, NFS=593, NTL=35, MAL=35, MM=10, the system can be the one with maximal seniority 7 for h-bosons. With the control parameters MNU=6, NFS=590, NTL=36, MAL=34, MM=12, the system can be the one with maximal seniority 6 for i-bosons. The capability can be strengthened by enlarging the control parameters MNU, NTL, NFS, MAL and MM, if it is needed. The

values MNU, NTL, and MM can be given as what is required. The economic value of NFS and MAL can be determined by running the subroutine MULT separately with a quite large trying NFS. For instance, with a trying NFS=2000, one can get all the appropriate values of the f-boson system with MNU $\leq$ 20. With a trying date NFS=4000, one can get all the suitable NFSs and MALs for g-boson system with seniority MNU $\leq$ 13, h-boson system with MNU $\leq$ 10 and i-boson system with MNU $\leq$ 8.

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TEST RUN OUTPUT

Part 1. An example of the test run output about the input data and running process

```
$RUN RCFPSIB
Please input the single angular momentum SIL and the maximal seniority
MAXNU, which will be calculated at this time
3,9

GRME is finished at NU =  1
GRME is finished at NU =  3
GRME is finished at NU =  5
GRME is finished at NU =  7
GRME is finished at NU =  9
NORMAL END OF THE PROGRAM
$
  LYX          job terminated at 28-APR-1994 20:45:41.03

  Accounting information:
  Buffered I/O count:           164      Peak working set size:  4522
  Direct I/O count:            1929      Peak page file size:   40242
  Page faults:                28471      Mounted volumes:           0
  Charged CPU time:     0 00:01:04.93    Elapsed time:     0 00:02:32.39
```

Part 2. A part of the test run output about the calculated reduced matrix elements

```
< 50 50  1  2 | D+ | 49  1  2> =    5.42481072744214
< 50 50  1  2 | D+ | 49  1  4> =   -4.53557367611072
< 50 50  1  4 | D+ | 49  1  2> =    3.62101363507555
< 50 50  1  4 | D+ | 49  1  4> =    2.06545831242740
< 50 50  1  4 | D+ | 49  1  5> =    4.77949415277763
< 50 50  1  4 | D+ | 49  1  6> =    3.12707177048844
< 50 50  1  5 | D+ | 49  1  4> =    4.54072481198561
< 50 50  1  5 | D+ | 49  1  5> =   -3.08345460606967
< 50 50  1  5 | D+ | 49  1  6> =    2.24476302512337
< 50 50  1  5 | D+ | 49  1  7> =   -3.85164443259821
< 50 50  1  6 | D+ | 49  1  4> =    2.89847601219026
< 50 50  1  6 | D+ | 49  1  5> =    2.19007563601826
< 50 50  1  6 | D+ | 49  1  6> =   -4.38382241117844
< 50 50  1  6 | D+ | 49  1  7> =   -2.07894852820315
< 50 50  1  6 | D+ | 49  1  8> =    1.20283109569296
< 50 50  1  6 | D+ | 49  2  8> =   -3.43739391906751
< 50 50  1  7 | D+ | 49  1  5> =    3.74874584620403
< 50 50  1  7 | D+ | 49  1  6> =    2.07393400644366
< 50 50  1  7 | D+ | 49  1  7> =   -0.72479106381360
< 50 50  1  7 | D+ | 49  1  8> =   -4.55377447296782
< 50 50  1  7 | D+ | 49  2  8> =   -1.98930613481256
< 50 50  1  7 | D+ | 49  1  9> =   -2.53499100247795
< 50 50  1  8 | D+ | 49  1  6> =    3.69100374036187
< 50 50  1  8 | D+ | 49  1  7> =   -0.38112327565423
< 50 50  1  8 | D+ | 49  1  8> =    3.57923393448407
< 50 50  1  8 | D+ | 49  2  8> =   -1.59272693221536
< 50 50  1  8 | D+ | 49  1  9> =   -3.36559987819391
< 50 50  1  8 | D+ | 49  1 10> =   -3.09132322136070
< 50 50  1  8 | D+ | 49  2 10> =    0.00000000000000
< 50 50  2  8 | D+ | 49  1  6> =    0.00000000000000
< 50 50  2  8 | D+ | 49  1  7> =    5.03539498525593
< 50 50  2  8 | D+ | 49  1  8> =    0.00000000000000
< 50 50  2  8 | D+ | 49  2  8> =   -3.02217276487653
< 50 50  2  8 | D+ | 49  1  9> =    1.17781543766131
< 50 50  2  8 | D+ | 49  1 10> =   -0.34602197554223
< 50 50  2  8 | D+ | 49  2 10> =   -3.74223044572970
< 50 50  1  9 | D+ | 49  1  7> =    2.66245602827621
< 50 50  1  9 | D+ | 49  1  8> =    0.00000000000001
< 50 50  1  9 | D+ | 49  2  8> =    3.68573413246105
< 50 50  1  9 | D+ | 49  1  9> =   -3.39322733327845
< 50 50  1  9 | D+ | 49  1 10> =    1.92871302076149
< 50 50  1  9 | D+ | 49  2 10> =   -1.66182478363963
< 50 50  1  9 | D+ | 49  1 11> =   -0.39265998180500
< 50 50  1  9 | D+ | 49  2 11> =   -3.34319049378198
< 50 50  1 10 | D+ | 49  1  8> =    4.10492168228365
< 50 50  1 10 | D+ | 49  2  8> =    0.17145158865271
< 50 50  1 10 | D+ | 49  1  9> =    0.91181426710353
< 50 50  1 10 | D+ | 49  1 10> =    2.11268520976446
< 50 50  1 10 | D+ | 49  2 10> =    0.00000000000000
< 50 50  1 10 | D+ | 49  1 11> =    4.75178801672820
< 50 50  1 10 | D+ | 49  2 11> =    0.00000000000002
< 50 50  1 10 | D+ | 49  1 12> =    2.29039056186779
< 50 50  1 10 | D+ | 49  2 12> =    0.00000000000003
```


Part 3. A part of the test run output about the calculated coefficient of fractional parentage

```
< 50 50  1  2 } 49 49  1  2  2  2> =    0.76718407869339
< 50 50  1  2 } 49 49  1  4  2  2> =   -0.64142698049545
< 50 50  1  4 } 49 49  1  2  2  4> =    0.51208865642548
< 50 50  1  4 } 49 49  1  4  2  4> =    0.29209992289543
< 50 50  1  4 } 49 49  1  5  2  4> =    0.67592257261276
< 50 50  1  4 } 49 49  1  6  2  4> =    0.44223472476006
< 50 50  1  4 } 49 49  1  6  2  5> =    0.64215546846390
< 50 50  1  5 } 49 49  1  4  2  5> =   -0.43606632947922
< 50 50  1  5 } 49 49  1  5  2  5> =    0.31745743751526
< 50 50  1  5 } 49 49  1  6  2  5> =   -0.54470479488373
< 50 50  1  5 } 49 49  1  7  2  5> =    0.40990641713142
< 50 50  1  6 } 49 49  1  4  2  6> =    0.30972346663475
< 50 50  1  6 } 49 49  1  5  2  6> =   -0.61996608972549
< 50 50  1  6 } 49 49  1  6  2  6> =   -0.29400771856308
< 50 50  1  6 } 49 49  1  7  2  6> =    0.17010600864887
< 50 50  1  6 } 49 49  1  8  2  6> =   -0.48612090945244
< 50 50  1  6 } 49 49  2  8  2  6> =    0.53015273809433
< 50 50  1  7 } 49 49  1  5  2  7> =    0.29329857230186
< 50 50  1  7 } 49 49  1  6  2  7> =   -0.10250093787909
< 50 50  1  7 } 49 49  1  7  2  7> =   -0.64400094747543
< 50 50  1  7 } 49 49  1  8  2  7> =   -0.28133037686348
< 50 50  1  7 } 49 49  2  8  2  7> =   -0.35850185155869
< 50 50  1  7 } 49 49  1  9  2  7> =    0.52198678255081
< 50 50  1  8 } 49 49  1  6  2  8> =   -0.05389897152781
< 50 50  1  8 } 49 49  1  7  2  8> =    0.50618010759354
< 50 50  1  8 } 49 49  1  8  2  8> =   -0.22524560987949
< 50 50  1  8 } 49 49  2  8  2  8> =   -0.47596770524979
< 50 50  1  8 } 49 49  1  9  2  8> =   -0.43717911839485
< 50 50  1  8 } 49 49  1 10  2  8> =    0.00000000000000
< 50 50  1  8 } 49 49  2 10  2  8> =    0.00000000000000
< 50 50  2  8 } 49 49  1  6  2  8> =    0.71211236715317
< 50 50  2  8 } 49 49  1  7  2  8> =    0.00000000000000
< 50 50  2  8 } 49 49  1  8  2  8> =   -0.42739978432655
< 50 50  2  8 } 49 49  2  8  2  8> =    0.16656824946404
< 50 50  2  8 } 49 49  1  9  2  8> =   -0.04893489554524
< 50 50  2  8 } 49 49  1 10  2  8> =   -0.52923130989075
< 50 50  2  8 } 49 49  2 10  2  8> =    0.37652814388275
< 50 50  1  9 } 49 49  1  7  2  9> =    0.00000000000000
< 50 50  1  9 } 49 49  1  8  2  9> =    0.52124154567719
< 50 50  1  9 } 49 49  2  8  2  9> =   -0.47987481951714
< 50 50  1  9 } 49 49  1  9  2  9> =    0.27276122570038
< 50 50  1  9 } 49 49  1 10  2  9> =   -0.23501750826836
< 50 50  1  9 } 49 49  2 10  2  9> =   -0.05553050711751
< 50 50  1  9 } 49 49  1 11  2  9> =   -0.47279852628708
< 50 50  1  9 } 49 49  2 11  2  9> =    0.58052361011505
< 50 50  1 10 } 49 49  1  8  2 10> =    0.02424691617489
< 50 50  1 10 } 49 49  2  8  2 10> =    0.12895001471043
< 50 50  1 10 } 49 49  1  9  2 10> =    0.29877880215645
< 50 50  1 10 } 49 49  1 10  2 10> =    0.00000000000000
< 50 50  1 10 } 49 49  2 10  2 10> =    0.67209428247452
< 50 50  1 10 } 49 49  1 11  2 10> =    0.00000000000000
< 50 50  1 10 } 49 49  2 11  2 10> =    0.32391014695168
< 50 50  1 10 } 49 49  1 12  2 10> =    0.00000000000000
< 50 50  1 10 } 49 49  2 12  2 10> =    0.00000000000000
```