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Spin assignment of the superdeformed bands in ^{152}Dy and neighboring nuclei

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Abstract

Using the criteria for the spin assignment of rotational band, based on the very general properties of the rotational band of axially symmetric nuclei, the spins of nine SD bands in ^{152}Dy and neighboring nuclei (^{152}Dy(1), ^{151}Tb(2), ^{153}Dy(2,3), ^{153}Dy(1), ^{152}Dy(4,5), ^{151}Dy(1) and ^{152}Dy(6)) are determined. The close connections between these SD bands (transition energy, $J^{(1)}$ and $J^{(2)}$, difference in the angular momentum alignment, etc.) are investigated.

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I. INTRODUCTION

Since the first observation of the discrete line superdeformed (SD) transitions in ^{152}Dy [1], this nucleus has been the focus of many experimental [2—6] and theoretical [7—9] studies. However, as yet no linking transitions have been confirmed between the yrast SD band and the normal deformed states, which makes the definite assignment of spin, parity and excitation energy very difficult for the SD band. In ref. 1 the spin of the lowest level observed in the yrast SD band of ^{152}Dy was assigned to be $I_0 = 22$. This assignment is clearly not unambiguous. However, it was pointed out in ref. [3] that it is unlikely that the I_0 value is less than 22. In ref. [10], the spin assignment $I_0 = 25$ was made. Recently, in addition to the yrast SD band ^{152}Dy(1), seven new rotational bands have been observed [4] in ^{152}Dy, which are referred to as bands 2—8. These bands each have between 14 and 17 regularly spaced transitions. Bands 1—6 are classified as SD bands and bands 7 and 8 as low deformation bands. The known low deformation prolate band [11] (referred to as band 9) is also observed. For the SD bands 1—6, spin assignment were made in [4]; particularly, the spin of the lowest level observed in ^{152}Dy(1) is assigned to be $I_0 = 24$ in place of the previously assigned $I_0 = 22$.

Obviously, the level spin determination is fundamental to understanding the physics of the new regime of deformation. Recently, a new approach to determine the spin sequence of a rotational band whose spin is unknown was proposed [12,13]. This new approach is based on the very general properties of rotational band in axially symmetric nuclei, demonstrated by A. Bohr and B. R. Mottelson [14]. In contrast to the previous approaches [15—18] to determine the spin of SD band, this approach is mode-independent and does not invoke the least squares fit of experimental data (on the dynamic moment of inertia $J^{(2)}$, or the transition energy) to certain formulae for rotational spectra. This approach has turned out to be very effective to determine the spins of SD bands. Using this new approach the 37 SD bands observed in the $A \sim 190$ region have been determined unambiguously [13]. In the present paper, the spins of the SD bands in ^{152}Dy and neighboring nuclei are addressed. A sketch of the new approach to determine the spin of rotational band is given in Sect. II. The spin assignments of nine SD bands in ^{152}Dy and neighboring nuclei are made in Sect. III. Based on these spin assignments, similarities and differences between these SD bands are discussed and a series interesting relations between these SD bands are presented in Sect IV. A brief summary is given in Sect. V.

II SKETCH OF THE NEW APPROACH TO DETERMINE THE SPIN OF ROTATIONAL BAND

Usually, the kinematic moment of inertia is extracted from the experimental intra-band transition energy by the difference quotient

$$J^{(1)}(I-1)/\hbar^2 = (2I-1)/E_\gamma(I \to I-2), \tag{1}$$

and the dynamic moment of inertia by

$$J^{(2)}(I)/\hbar^2 = \frac{4}{\Delta E_\gamma(I)} = \frac{4}{E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)}. \tag{2}$$

As a function of I the extracted $J^{(1)}$ increases with the assigned level spin, but the pattern of the extracted $J^{(2)}$ is independent of the spin assignment. This extraction is mode-independent and meaningful provided the moments of inertia vary smoothly with the angular momentum (i.e., the local $I(I+1)$ rule for the rotational spectra holds). For SD bands, because the level spins are unknown, the experimentalists have

no choice but to address $J^{(2)}$ rather than $J^{(1)}$. However, careful investigation shows that there exists close connection between the features of $J^{(1)}$ and $J^{(2)}$ moments of inertia of the same band, which seems not to attract attention.

According to the famous work by Bohr and Mottelson [14], the $K=0$ rotational spectra of an axially symmetric nucleus can be expressed as a function of $I(I+1)$ and expanded in powers of $\xi = \sqrt{I(I+1)}$, i.e.

$$E = A\xi^2 + B\xi^4 + C\xi^6 + D\xi^8 + \cdots \tag{3}$$

The expression for the rotational energy of $K \neq 0$ band [14] takes a form similar to eq(3), but includes a band-head energy, and $I(I+1)$ is replaced by $I(I+1) - K^2$. It is well known that the extensive experimental data on nuclear rotational spectra (below bandcrossing) can be described very well by eq. (3). Systematic analysis of the large amount of data on the rotational spectra of normally deformed rare-earth and actinide nuclei showed [14,19] that $|B/A| \sim 10^{-3}$, $|C/A| \sim 10^{-6}$, $|D/A| \sim 10^{-9}$, etc.; i.e. the convergence of the $I(I+1)$ expansion is satisfactory. For the SD bands, the convergence is even better [16] ($|B/A| \sim 10^{-4}$, $|C/A| \sim 10^{-8}$, etc.), i.e., compared to the normally deformed nuclei, the SD nucleus appears to be a more rigid rotator with axial symmetry. From eq. (3) the kinematic and dynamic moments of inertia [20],

$$J^{(1)}/\hbar^2 = \xi \Big/ \frac{dE}{d\xi}, \quad J^{(2)}/\hbar^2 = \left(\frac{d^2E}{d\xi^2}\right)^{-1} \tag{4}$$

can be expressed as

$$J^{(1)}/\hbar^2 = \frac{1}{2A}(1 + 2\frac{B}{A}\xi^2 + 3\frac{C}{A}\xi^4 + \cdots)^{-1} \tag{5}$$

$$J^{(2)}/\hbar^2 = \frac{1}{2A}(1 + 6\frac{B}{A}\xi^2 + 15\frac{C}{A}\xi^4 + \cdots)^{-1}. \tag{6}$$

From this, several rules for a rotational band can be observed (for details see refs. [12,13]):

(a) As $I \to 0$, both $J^{(1)}$ and $J^{(2)}$ of the same band tend to the same limiting value (bandhead moment of inertia).

(b) *Both* $J^{(1)}$ *and* $J^{(2)}$ *monotonously increase with* I (for $B < 0$), or *monotonously decrease with* I (for $B > 0$), but the slope of $J^{(2)}$ is much steeper than that of $J^{(1)}$ ($dJ^{(2)}/d\xi \approx 3dJ^{(1)}/d\xi$ in the low spin range).

(c) Hence, $J^{(1)}$-ξ *and* $J^{(2)}$-ξ *plots never cross each other.*

(d) *As* $I \to 0$, $\frac{dJ^{(1)}}{d\xi}$ and $\frac{J^{(2)}}{d\xi}$ tend to zero, i.e., *both* $J^{(1)}$-ξ *and* $J^{(2)}$-ξ *plots become horizontal.*

(e) *Both* $J^{(1)}$-ξ *and* $J^{(2)}$-ξ *plots concave upwards* ($B < 0$)*, or downwards* ($B > 0$)*.*

(f) $\sqrt{J^{(1)3}/J^{(2)}}$ remains constant (bandhead moment of inertia) at lower spins, and only for very high spin states it decreases gradually with increasing I.

Extensive analysis of large amount of rotational bands of normally deformed nuclei, whose spins were known, shows that these six rules do hold without exception. An illustrative example (the ground bands of ^{162}Dy) is presented in Fig. 1. In fact, all the $J^{(1)}$-ξ and $J^{(2)}$-ξ plots of normally deformed bands are of the standard form shown in Fig. 1(c). However, if one artificially changes the spin sequence of a rotational band, these rules may fail, particularly in the low spin range. For example, if the spin of each level is increases by 1 $\hbar$ (Fig. 1(d)) or 2 $\hbar$ (Fig. 1(e)), i.e., the measured spin sequence 0, 2, 4, $\cdots$ is artificially replaced by 1, 3, 5, $\cdots$ or 2, 4, 6, $\cdots$, the pattern of $J^{(1)}$-ξ will change. Particularly, in the low spin range the extracted $J^{(1)}$ increases significantly (the lower the level spin, the larger the increase of $J^{(1)}$ value), and the $J^{(1)}$-ξ plot

concaves upwards. However, the pattern of the extracted $J^{(2)}$-ξ is not influenced by the change of the spin sequence, and for different assignments of the spin sequence the $J^{(2)}$-ξ plot is displaced as a whole. Therefore, the relations between $J^{(1)}$ and $J^{(2)}$ will change. From Figs. 1(d) and (e), it is seen that: (1) $J^{(1)}$-ξ and $J^{(2)}$-ξ plots cross each other at certain spin I_c. (2) As $I \to 0$, $J^{(1)}$ increases with decreasing I, i.e., the monotonousness is broken. (3) As $I \to 0$, $J^{(1)}$-ξ plot does not become horizontal, and $J^{(1)}$ does not tend to the same limiting value as that of $J^{(2)}$. On the other hand, if the spin of each level is artificially decreased by 1 $\hbar$ (Fig. 1(b)) or 2 $\hbar$ (Fig. 1(a)), i.e., the measured spin sequence 0, 2, 4, 6, $\cdots$ is artificially replaced by -1 (unphysical), 1, 3, 5, $\cdots$, or -2 (unphysical), 0, 2, 4, $\cdots$, the extracted $J^{(1)}$ decreases to various extents (the lower the level spin, the larger the decrease of $J^{(1)}$ value). It is seen from Figs. 1(b) and (a) that: (i) In the low spin range, the slope of $J^{(1)}$ increase with decreasing I and does not tend to zero as $I \to 0$. (ii) As $I \to 0$, $J^{(1)}$ does not tend to the same limiting value as $J^{(2)}$. (iii) In the low spin range, $J^{(1)}$-ξ plot concaves downwards.

Therefore, for a rotational band whose level spins are unknown, we may assume a spin sequence, I_0, I_0+2, I_0+4, $\cdots$, and then construct the $J^{(1)}$-ξ and $J^{(2)}$-ξ plots using eqs. (1) and (2) from the measured intraband γ ray energies. In this case, violation of one (or more) of the six rules implies an unreasonable spin assignment is made, hence such spin assignment must be ruled out. Analysis shows that using this procedure we can find out, in most cases, the reasonable spin assignment of a rotational band whose spins are unknown.

III SPIN ASSIGNMENT OF THE SD BANDS IN ^{152}Dy AND NEIGHBORING NUCLEI

Now we use the rules mentioned above as the criteria of spin assignment of the SD bands in ^{152}Dy and related SD bands in neighboring nuclei.

Firstly, let us address the yrast SD band ^{152}Dy(1). The lowest transition energy observed is $E_\gamma(I_0+2 \to I_0) = 602.4$ keV. In Fig. 2, eight trial values of $I_0 = 22$, 23, 24, 25, 26, 27, 28, and 29 are adopted. It is seen that $I_0 = 22$ is definitely forbidden by the six criteria mentioned above, particularly the criteria (b) (the same monotonousness for $J^{(1)}$ and $J^{(2)}$) and (f) ($\sqrt{J^{(1)3}/J^{(2)}} \approx$ const) are obviously violated. $I_0 = 23$, 24, and 25 are also not allowed by these criteria, particularly because $J^{(1)}$-ξ and $J^{(2)}$-ξ plots cross each other. On the other hand, $I_0 \geq 27$ are ruled out, because $\sqrt{J^{(1)3}/J^{(2)}}$ increases monotonously with decreasing spin, which in turn implies the criterion (a) is clearly violated. Therefore, the unique plausible candidate is $I_0 = 26$.

It was pointed out by Byrski et al. [21] that the yrast SD band ^{152}Dy(1) is identical to the excited SD band ^{151}Tb(2) in neighboring nuclei. The analysis of the SD band ^{151}Tb(2) is presented in Fig. 3. $E_\gamma(I_0 + 2 \to I_0) = 647.0$ keV, $I_0 = 25.5$, 26.5, 27.5, 28.5 and 29.5 are tried. Obviously, $I_0 \leq 26.5$ are ruled out, because the $J^{(1)}$-ξ and $J^{(2)}$-ξ plots cross each other. On the other hand, $I_0 \geq 28.5$ are forbidden, because $\sqrt{J^{(1)3}/J^{(2)}}$ decreases significantly with increasing spin I. Therefore, we have no choice but $I_0 = 27.5$.

Similar arguments can be applied to determine the level spins of the excited signature partner SD bands ^{153}Dy(2,3) [22] (see Figs. 4 and 5). $E_\gamma(I_0+2 \to I_0) = 816.5$ keV for ^{153}Dy(2), and 935.7 kev for ^{153}Dy(3). The most plausible candidates are, $I_0 = 34.5$ for ^{153}Dy(2), and $I_0 = 39.5$ for ^{153}Dy(3).

It is very interesting to note that for the spin assignment made above, the features of the $J^{(1)}$ and $J^{(2)}$ moments of inertia are quite similar for these four SD bands,

i.e., (i) $J^{(2)} < J^{(1)}$, (ii) *both $J^{(1)}$ and $J^{(2)}$ decrease very slowly and monotonously with increasing I (for $I < 60$)*. The variation of $J^{(1)}$ within the spin range observed is less than 4%. So it is not surprising that the same high-N configuration [7—9] $\pi 6^4 \nu 7^2$ has been assigned to these SD bands [4,21,22].

For comparison, the analysis of the yrast SD band in ^{153}Dy, ^{153}Dy(1), is given in Fig. 6. $E_\gamma(I_0 + 2 \to I_0) = 809.6$ keV, $I_0 = 35.5$ is obtained. The I variation of the $J^{(1)}$ and $J^{(2)}$ moments of inertia for ^{153}Dy(1) is also similar to that of ^{153}Dy(2,3), ^{151}Tb(2) and ^{152}Dy(1), though the high-N configuration for ^{153}Dy(1) is assigned [22] to be $\pi 6^4 \nu 7^3$.

Now we address another group of SD bands, the signature partner SD bands ^{153}Dy(4) and ^{152}Dy(5), and the related yrast SD band in neighboring odd-A nucleus, ^{151}Dy(1). For the SD band ^{152}Dy(4), $E_\gamma(I_0 + 2 \to I_0) = 669.6$ keV, $I_0^\pi = 27^-$ was assigned in ref. [4]. Similarly, for ^{152}Dy(5), $E_\gamma(I_0+2 \to I_0) = 642.1$ keV, $I_0^\pi = 26^-$ was adopted. However, the present analysis presented in Figs. 7 and 8. shows that these assignments are not allowed, because in this case the $J^{(1)}$-ξ and $J^{(2)}$-ξ plots cross each other, and in addition, while $J^{(1)}$ decreases with I, $J^{(2)}$ increases, i.e., they have opposite monotonousness. From Fig. 7 and 8, the most plausible I_0 value should be 3($\hbar$) smaller, i.e., $I_0 = 24$ for ^{152}Dy(4) and $I_0 = 23$ for ^{152}Dy(5). In this case, *both* $J^{(1)}$ and $J^{(2)}$ moments of inertia *increase very slowly* with increasing I, (the variations in $J^{(1)}$ is less than 2% in the spin range observed), which is consistent with the requirement of the same monotonousness.

As has been pointed out in ref. [4] the behavior of the $J^{(2)}$ of the SD bands ^{152}Dy(4) and ^{152}Dy(5) is very similar to that of the yrast SD band ^{151}Dy(1). In Fig. 9 is presented the analysis of ^{151}Dy(1), $E_\gamma(I_0+2 \to I_0) = 577.1$ keV. The reasonable spin assignment is $I_0 = 20.5$, and in this case, the behavior of the $J^{(1)}$ and $J^{(2)}$ moments is quite similar to that of ^{152}Dy(4) and ^{152}Dy(5). Therefore, it is not surprising that the same high-N configuration $\pi 6^4 \nu 7^1$ is assigned to these three bands [4,23]. In ref. [23], for the SD band ^{151}Dy(1), I_0 is assigned to be 25.5 rather than 20.5. It was emphasized [23] that:

> "For ^{151}Dy(1), the value of $J^{(1)}$ exceeds that of $J^{(2)}$ (for $I_0 = 25.5$). In contrast, $J^{(2)} > J^{(1)}$ in ^{152}Dy(1) (for $I_0 = 22$), a fact for which there is, to our knowledge, no explanation. Furthermore, while $J^{(1)}$ is seen to decrease and $J^{(2)}$ to increase with spin in ^{151}Dy(1), the opposite happens in ^{152}Dy(2)" (see Fig. 3 of ref. [23]).

It is interesting to note that for the present spin assignments ($I_0 = 26$ for ^{152}Dy(1) and $I_0 = 20.5$ for ^{151}Dy(1)), this puzzle disappears. In contrast to the first group of SD bands (^{152}Dy(1), ^{151}Tb(2) and ^{153}Dy(1,2,3)), the features of this group of SD bands (^{152}Dy(4,5) and ^{151}Dy(1)) is, more or less, similar to that of the SD bands in the $A \sim 190$ region, i.e., $J^{(2)} > J^{(1)}$ and *both* $J^{(1)}$ and $J^{(2)}$ *increase very slowly with* I, and *both* the $J^{(1)}$-ξ and $J^{(2)}$-ξ plots *concave upwards.*

In ref. [4], for the new SD band ^{152}Dy(6), $E_\gamma(I_0 + 2 \to I_0) = 761.5$ keV, $I_0 = 32$ is assigned, and ^{152}Dy(6) is considered to be similar to the yrast SD band in neighboring nucleus, ^{153}Dy(1). However, from Fig. 10 the assignment $I_0 = 32$ seems unlikely, because in this case $J^{(1)}$-ξ and $J^{(2)}$-ξ plots cross each other. In fact $I_0 \geq 32$ should be ruled out by the criteria (c). On the other hand, $I_0 \leq 30$ are not allowed, because $\sqrt{J^{(1)3}/J^{(2)}}$ clearly increases with I. Therefore, the most plausible choice is $I_0 = 31$. In this case, both $J^{(1)}$ and $J^{(2)}$ *increase very slowly* with I, like the behavior of the SD bands ^{152}Dy(4), ^{152}Dy(5) and ^{151}Dy(1), but unlike the SD band ^{153}Dy(1), of which both $J^{(1)}$ and $J^{(2)}$ *decrease very slowly* with I.

It has been pointed out [4] that there exists bandmixing between the SD bands ^{152}Dy(2) and ^{152}Dy(3), like that observed in the SD bands ^{193}Hg(1) and ^{193}Hg(4) in the $A \sim 190$ region. The spin assignment for these bands is more complicated and will be investigated in another paper.

The spin assignments made by the present approach for the nine SD bands in ^{152}Dy and neighboring nuclei are summarized in Table I.

IV. DISCUSSIONS

Using the spin assignments made in Sect III. (see Table I), a series of interesting relations between the SD bands in ^{152}Dy and neighboring nuclei will be found, which reminds us the similar relations between the SD bands observed in the $A \sim 190$ region.

Firstly, let us address the group of SD bands: the yrast SD band ^{151}Dy(1) ($\alpha = 1/2$), and the excited signature partner SD bands ^{152}Dy(4) ($\alpha = 0$) and ^{152}Dy(5) ($\alpha = 1$). It is found that, on the one hand, the $\alpha = 1/2$ band extracted from ^{152}Dy(4) and ^{152}Dy(5) (see Table II(d)) is almost identical to the observed SD band ^{151}Dy(1) ($\alpha = 1/2$); (hereafter, $E_\gamma(I) \equiv E_\gamma(I \to I-2)$)), i.e.

$$\begin{aligned} &\frac{1}{2}\left[E_\gamma(I - \frac{1}{2}, {}^{152}\text{Dy}(4), \alpha = 0) + E_\gamma(I + \frac{1}{2}, {}^{152}\text{Dy}(5), \alpha = 1)\right] \\ = \;& E_\gamma(I, {}^{151}\text{Dy}(1), \alpha = \frac{1}{2}), \end{aligned} \tag{7}$$

On the other hand, though the $\alpha = -1/2$ band has not yet been observed in ^{151}Dy, the $\alpha = -1/2$ band extracted from ^{152}Dy(4) and ^{152}Dy(5) (see Table II(e)) is almost identical to the $\alpha = -1/2$ band extracted from the observed SD band ^{151}Dy(1), $\alpha = 1/2$ (see Table II(f)); i.e.

$$\frac{1}{2}\left[E_\gamma(I + \frac{1}{2}, {}^{152}\text{Dy}(4), \alpha = 0) + E_\gamma(I - \frac{1}{2}, {}^{152}\text{Dy}(5), \alpha = 1)\right]$$

$$= \frac{1}{2}\left[E_\gamma(I-1,{}^{151}\mathrm{Dy}(1),\alpha=\frac{1}{2})+E_\gamma(I+1,{}^{151}\mathrm{Dy}(1),\alpha=\frac{1}{2})\right]. \quad (8)$$

It has been well established [24] that there exist a signature partner SD bands ^{194}Hg(2), $\alpha = 1$ and ^{194}Hg(3), $\alpha = 0$, and the extracted $\alpha = 1/2$ and $\alpha = -1/2$ bands are almost identical to the excited SD bands observed in ^{193}Hg and ^{191}Hg; i.e.

$$\begin{aligned} &\frac{1}{2}\left[E_\gamma(I-\frac{1}{2},{}^{194}\mathrm{Hg}(3),\alpha=0)+E_\gamma(I+\frac{1}{2},{}^{194}\mathrm{Hg}(2),\alpha=1)\right] \\ =\ &E_\gamma(I,{}^{193}\mathrm{Hg}(2),\alpha=\frac{1}{2})=E_\gamma(I,{}^{191}\mathrm{Hg}(2),\alpha=\frac{1}{2}), \quad (9) \\ &\frac{1}{2}\left[E_\gamma(I+\frac{1}{2},{}^{194}\mathrm{Hg}(3),\alpha=0)+E_\gamma(I-\frac{1}{2},{}^{194}\mathrm{Hg}(2),\alpha=1)\right] \\ =\ &E_\gamma(I,{}^{193}\mathrm{Hg}(3),\alpha=-\frac{1}{2})=E_\gamma(I,{}^{191}\mathrm{Hg}(3),\alpha=-\frac{1}{2}), \quad (10)\end{aligned}$$

which are similar to eqs. (7) and (8). The relations (7) and (8) imply that these SD bands have the same angular momentum alignment, i.e.

$$I_x({}^{152}\mathrm{Dy}(4,5)) - I_x({}^{151}\mathrm{Dy}(1)) \approx 0 \quad \text{in the frequency range observed,} \quad (11)$$

just like

$$I_x({}^{194}\mathrm{Hg}(2,3)) - I_x({}^{193}\mathrm{Hg}(2,3)) \approx 0 \quad \text{in the frequency range observed.} \quad (12)$$

Secondly, let us consider the group of SD bands: ^{152}Dy(1), ^{151}Tb(2), and ^{153}Dy(2) and ^{153}Dy(3). It has been noted by Byrski et al., [21] that the experimental E_γ spectra of ^{152}Dy(1) and ^{151}Tb(2) are almost identical in the observed spin range. From the spin assignment made in Sect III (see Table I), it is clearly seen that

$$E_\gamma(I,{}^{152}\mathrm{Dy}(1),\alpha=0)=E_\gamma(I-\frac{1}{2},{}^{151}\mathrm{Tb}(2),\alpha=-\frac{1}{2}), \quad (13)$$

which implies that the difference in the angular momentum alignment between the two SD bands is nearly 1/2, i.e.

$$I_x({}^{152}\mathrm{Dy}(1)) - I_x({}^{151}\mathrm{Tb}(2)) \approx \frac{1}{2}, \quad (14)$$

which seems to be consistent with the assignment configuration [21] $\pi 6^4 \nu 7^2$ for ^{152}Dy(1) and $\pi 6^4([301|1/2)^{-1}\nu 7^2$ (or "^{152}Dy(1) core"$\otimes(\pi[301|1/2)^{-1})$ for ^{151}Tb(2).

From the experimental SD bands ^{153}Dy(2,3), we can extract the $\alpha = 1$ and $\alpha = 0$ bands (see Table III (e) and (f)), and it is found that the extracted $\alpha = 1$ band is almost identical to the observed SD bands ^{152}Dy(1) and ^{151}Tb(2),

$$\begin{aligned} &\frac{1}{2}\left[E_\gamma(I-\frac{1}{2},{}^{153}\mathrm{Dy}(2),\alpha=\frac{1}{2})+E_\gamma(I+\frac{1}{2},{}^{153}\mathrm{Dy}(3),\alpha=-\frac{1}{2})\right] \\ =\ &E_\gamma(I+1,{}^{152}\mathrm{Dy}(1),\alpha=0) \quad (15) \\ =\ &E_\gamma(I+\frac{1}{2},{}^{151}\mathrm{Tb}(2),\alpha=-\frac{1}{2}),\end{aligned}$$

but for the extracted $\alpha = 0$ band (see Table III (f))

$$\frac{1}{2}\left[E_\gamma(I+\frac{1}{2},{}^{153}\mathrm{Dy}(2),\alpha=\frac{1}{2})+E_\gamma(I-\frac{1}{2},{}^{153}\mathrm{Dy}(3),\alpha=-\frac{1}{2})\right] \quad (I \text{ even}),$$

the corresponding counterpart $\alpha = 1$ band in ^{152}Dy has not yet been found. However, this extracted $\alpha = 0$ band is nearly identical to the $\alpha = 1$ band (see Table III (g)) extracted from the observed SD band ^{152}Dy(1), $\alpha = 0$ and the $\alpha = 1/2$ band extracted from the observed SD band ^{151}Tb(2), $\alpha = -1/2$, i.e.

$$\begin{aligned} &\frac{1}{2}\left[E_\gamma(I+\frac{1}{2},{}^{153}\mathrm{Dy}(2),\alpha=\frac{1}{2})+E_\gamma(I-\frac{1}{2},{}^{153}\mathrm{Dy}(3),\alpha=-\frac{1}{2})\right] \\ =\ &\frac{1}{2}\left[E_\gamma(I-1,{}^{152}\mathrm{Dy}(1),\alpha=0)+E_\gamma(I+1,{}^{152}\mathrm{Dy}(1),\alpha=0)\right] \\ =\ &\frac{1}{2}\left[E_\gamma(I-\frac{3}{2},{}^{151}\mathrm{Tb}(2),\alpha=-\frac{1}{2})+E_\gamma(I+\frac{1}{2},{}^{151}\mathrm{Tb}(2),\alpha=-\frac{1}{2})\right]. \quad (16)\end{aligned}$$

Also this reminds us the similar relations established in the $A \sim 190$ region. It is well known that the $\alpha = 1$ band extracted from the observed SD bands ^{193}Hg(2,3) is almost identical to the observed excited SD band ^{194}Hg(2), $\alpha = 1$, and the observed yrast SD band ^{192}Hg(1), i.e.

$$\frac{1}{2}\left[E_\gamma(I-\frac{1}{2},{}^{193}\mathrm{Hg}(2),\alpha=\frac{1}{2})+E_\gamma(I+\frac{1}{2},{}^{193}\mathrm{Hg}(3),\alpha=-\frac{1}{2})\right]$$

$$= E_\gamma(I, {}^{194}\text{Hg}(2), \alpha = 1) \quad (17)$$

$$= E_\gamma(I-1, {}^{192}\text{Hg}(1), \alpha = 0).$$

Similarly, the $\alpha = 0$ band extracted from ^{193}Hg(2,3) is almost identical to the observed SD band ^{194}Hg(3), $\alpha = 0$,

$$\frac{1}{2}\left[E_\gamma(I+\frac{1}{2}, {}^{193}\text{Hg}(2), \alpha = \frac{1}{2}) + E_\gamma(I-\frac{1}{2}, {}^{193}\text{Hg}(3), \alpha = -\frac{1}{2})\right]$$

$$= E_\gamma(I, {}^{194}\text{Hg}(3), \alpha = 0), \quad (18)$$

but the expected $\alpha = 1$ SD band in ^{192}Hg has not been observed.

The relations (15) and (16) imply

$$I_x({}^{151}\text{Tb}(2)) - I_x({}^{153}\text{Dy}(2,3)) \approx \frac{1}{2}, \quad (19)$$

$$I_x({}^{152}\text{Dy}(1)) - I_x({}^{153}\text{Dy}(2,3)) \approx 1, \quad (20)$$

V. SUMMARY

Using the new approach based on the very general properties of the rotational spectra of axially symmetric nuclei, the spins of the nine SD bands in ^{152}Dy and neighboring nuclei (^{152}Dy(1), (4), (5), (6), ^{151}Tb(2), ^{151}Dy(1) and ^{153}Dy(1), (2), (3) are determined and summarized in Table I. Particularly, the spin of the lowest level observed in the yrast SD band ^{152}Dy(1) is determined to be $I_0 = 26$ ($E_\gamma(I_0 + 2 \to I_0) = 602.4$ keV) rather than the previously assigned $I_0 = 22$ or 24.

Based on the present spin assignment, several observations can be made:

(i) The group of SD bands (the signature partners ^{152}Dy(4), $\alpha = 0$ and ^{152}Dy(5), $\alpha = 1$, and the yrast SD band ^{151}Dy(1)) are almost identical to each other in the spin range observed and have the same angular momentum alignment

$I_x({}^{152}\text{Dy}(4,5)) \approx I_x({}^{151}\text{Dy}(1))$, which seems consistent with the same high-N configuration $\pi 6^4 \nu 7^1$ assigned to these SD bands. For these SD bands and ^{152}Dy(6), (a) $J^{(2)} > J^{(1)}$, (b) both $J^{(1)}$ and $J^{(2)}$ monotonously, but very slowly *increase* with spin.

(ii) The group of SD bands (^{152}Dy(1), $\alpha = 0$, ^{151}Tb(2), $\alpha = -1/2$, ^{153}Dy(2), $\alpha = 1/2$, and ^{153}Dy(3), $\alpha = -1/2$) are almost identical to each other in the spin range observed, but have different angular momentum alignment, i.e., in the observed spin range $I_x({}^{152}\text{Dy}(1)) - I_x({}^{151}\text{Tb}(2)) \approx 1/2$, $I_x({}^{151}\text{Tb}(2)) - I_x({}^{153}\text{Dy}(2,3)) \approx 1/2$, $I_x({}^{152}\text{Dy}(1)) - I_x({}^{153}\text{Dy}(2,3)) \approx 1$. For these SD bands and ^{153}Dy(1), (a) $J^{(2)} < J^{(1)}$, (b) both $J^{(1)}$ and $J^{(2)}$ monotonously, but very slowly *decrease* with spin.

References

[1] P. J. Twin et al., Phys. Rev. Lett. 57, 811(1986).

[2] M. A. Bentley et al., Phys. Rev. Lett. 59, 2141(1987).

[3] M. A. Bentley et al., J. Phys. G. 17, 481(1991).

[4] P. J. Dagnall, Ph. D. thesis, University of Liverpool (1994).

[5] For an experimental review, see, e.g., P. J. Twin, Nucl. Phys. A520, 17c(1990); A522, 13c(1991).

[6] R. V. F. Janssens and T. L. Khoo, Annu. Rev. Part. Nucl. Sci. 41, 321 (1991).

[7] T. Bengtsson, I. Ragnarsson and S. Åberg, Phys. Lett. B208, 39(1988).

[8] W. Nazarewicz, R. Wyss and A. Johnson, Nucl. Phys. A503, 285(1989), Phys. Lett. B225, 208(1989).

[9] S. Åberg, Nucl. Phys. A520, 35c(1990).

[10] J. Y. Zeng et al., Phys. Rev. C44, R1745(1991).

[11] B. M. Nyako, J. Simpson, P. J. Twin, D. Howe, P. D. Forsyth, and J. F. Sharpey-Schafer, Phys. Rev. Lett. 56, 2680(1986).

[12] J. Y. Zeng, W. Q. Wu, Y. A. Lei, and E. G. Zhao, preprint, AS-ITP-94-33 (Institute of Theoretical Physics, Academia Sinica).

[13] J. Y. Zeng, Y. A. Lei and W. Q. Wu, preprint, AS-ITP-94-46

[14] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. 2, (1975, Benjamin).

[15] J. E. Draper et al., Phys. Rev. C42, R1791(1991).

[16] J. A. Becker et al., Phys. Rev. C 46, 889(1992); Nucl. Phys. A 520, 187c(1990).

[17] C. S. Wu et al., Phys. Rev. C45, 261(1992).

[18] R. Piepenbring and K. V. Protasov, Z. Phys. A 345, 7(1993).

[19] F. X. Xu, C. S. Wu and J. Y. Zeng, Phys. Rev. C40, 2337(1989).

[20] I. Hamamoto, Treatise on Heavy-Ion Science, 3, 313(1985, Plenum Publishing Co.)

[21] T. Byrski et al., Phys. Rev. Lett. 64, 1650(1990).

[22] J. K. Johansson et al., Phys. Rev. Lett. 63, 2200(1989).

[23] G-E. Rathke et al., Phys. Lett. B209, 177(1988).

[24] F. S. Stephens, et al., Phys. Rev. Lett. 64, 2623(1990); 65, 301(1990).

Table Captions

Table I Spin assignment of the nine SD bands in ^{152}Dy and neighboring nuclei. I_0 is the spin of the lowest level observed.

Table II Transition energies (in units of keV) and spin assignments of the group of SD bands: ^{151}Dy(1), ^{152}Dy(4) and ^{152}Dy(5). $E_\gamma(I) \equiv E_\gamma(I \to I-2)$. Columns 2, 3 and 4 are the experimental transition energies of the SD bands ^{151}Dy(1), ^{152}Dy(4), and ^{152}Dy(5), respectively. Columns 5 and 6 are the transition energies of the $\alpha = 1/2$ and $\alpha = -1/2$ bands extracted from the experimental transition energies of ^{152}Dy(4) and ^{152}Dy(5), i.e.

$$E_\gamma(I, \alpha = \frac{1}{2}) = \frac{1}{2}\left[E_\gamma(I - \frac{1}{2}, {}^{152}\mathrm{Dy}(4), \alpha = 0) + E_\gamma(I + \frac{1}{2}, {}^{152}\mathrm{Dy}(5), \alpha = 1)\right],$$

$$E_\gamma(I, \alpha = -\frac{1}{2}) = \frac{1}{2}\left[E_\gamma(I + \frac{1}{2}, {}^{152}\mathrm{Dy}(4), \alpha = 0) + E_\gamma(I - \frac{1}{2}, {}^{152}\mathrm{Dy}(5), \alpha = 1)\right].$$

Column 7 is the transition energies of the $\alpha = -1/2$ band extracted from the experimental transition energies of ^{151}Dy(1), i.e.

$$E_\gamma(I, \alpha = -\frac{1}{2}) = \frac{1}{2}\left[E_\gamma(I - 1, {}^{151}\mathrm{Dy}(1), \alpha = \frac{1}{2}) + E_\gamma(I + 1, {}^{151}\mathrm{Dy}(1), \alpha = \frac{1}{2})\right].$$

Table III Transition energies (in units of keV) and spin assignments of the group of SD bands: ^{152}Dy(1), ^{151}Tb(2), ^{153}Dy(2), and ^{153}Dy(3). $E_\gamma(I) \equiv E_\gamma(I \to I-2)$. Columns 2, 3, 4, and 5 are the transition energies of the SD bands ^{152}Dy(1), ^{151}Tb(2), ^{153}Dy(2), and ^{153}Dy(3), respectively. Columns 6 and 7 are the transition energies of the $\alpha = 1$ and $\alpha = 0$ bands extracted from the experimental transition energies of ^{153}Dy(2) and ^{153}Dy(3), i.e.

$$E_\gamma(I, \alpha = 1) = \frac{1}{2}\left[E_\gamma(I - \frac{1}{2}, {}^{153}\mathrm{Dy}(2), \alpha = \frac{1}{2}) + E_\gamma(I + \frac{1}{2}, {}^{153}\mathrm{Dy}(3), \alpha = -\frac{1}{2})\right],$$

$$E_\gamma(I, \alpha = 0) = \frac{1}{2}\left[E_\gamma(I + \frac{1}{2}, {}^{153}\mathrm{Dy}(2), \alpha = \frac{1}{2}) + E_\gamma(I - \frac{1}{2}, {}^{153}\mathrm{Dy}(3), \alpha = -\frac{1}{2})\right].$$

Column 8 is the transition energies of the $\alpha = 1$ band extracted from the experimental transition energies of ^{152}Dy(1), i.e.

$$E_\gamma(I, \alpha = 1) = \frac{1}{2}\left[E_\gamma(I - 1, {}^{152}\mathrm{Dy}(1), \alpha = 0) + E_\gamma(I + 1, {}^{152}\mathrm{Dy}(1), \alpha = 0)\right].$$

Table I

SD band	$E_\gamma(I_0+2 \rightarrow I_0)$, keV	I_0	signature α
^{152}Dy(1)	602.4	26	0
^{152}Dy(4)	669.6	24	0
^{152}Dy(5)	642.1	23	1
^{152}Dy(6)	761.5	31	1
^{151}Tb(2)	647.0	27.5	−1/2
^{153}Dy(2)	816.5	34.5	1/2
^{153}Dy(3)	935.7	39.5	−1/2
^{153}Dy(1)	809.6	35.5	−1/2
^{151}Dy(1)	577.1	20.5	1/2

Table II

I	a ^{151}Dy(1) $\alpha=1/2$ $E_\gamma(I+\frac{1}{2})$ expt.	b ^{152}Dy(4) $\alpha=0$ $E_\gamma(I)$ expt.	c ^{152}Dy(5) $\alpha=1$ $E_\gamma(I+1)$ expt.	d $\alpha=1/2$ $E_\gamma(I+\frac{1}{2})$ extr.	e $\alpha=-1/2$ $E_\gamma(I-\frac{1}{2})$ extr.	f $\alpha=-1/2$ $E_\gamma(I-\frac{1}{2})$ extr.
58	1490.3					
56	1442.4					
54	1393.4	1375.9				
52	1343.4	1327.7			1316.0	1318.4
50	1293.3	1281.0	1304.2	1292.6	1267.4	1269.0
48	1244.6	1228.0	1253.8	1240.9	1216.9	1219.7
46	1194.7	1180.3	1205.8	1193.1	1168.1	1169.5
44	1144.3	1130.6	1155.8	1143.2	1117.8	1119.2
42	1094.1	1080.8	1105.0	1092.9	1067.9	1068.4
40	1042.6	1029.8	1055.0	1042.4	1017.5	1017.0
38	991.4	978.7	1005.1	991.9	966.3	965.9
36	940.4	929.0	953.8	941.4	916.1	914.8
34	889.1	876.8	901.3	889.1	864.0	863.5
32	837.8	825.0	851.1	838.1	811.9	811.8
30	785.8	772.8	798.8	785.8	759.9	760.1
28	734.4	721.0	747.0	734.0	708.0	708.1
26	681.8	669.6	694.9	682.3	655.9	655.0
24	628.1		642.1			
22	577.1					

Table III

	a	b	c	d	e	f	g
	^{152}Dy(1)	^{151}Tb(2)	^{153}Dy(2)	^{153}Dy(3)			
I	$\alpha=0$	$\alpha=-1/2$	$\alpha=1/2$	$\alpha=-1/2$	$\alpha=1$	$\alpha=0$	$\alpha=1$
	$E_\gamma(I)$	$E_\gamma(I-\frac{1}{2})$	$E_\gamma(I+\frac{1}{2})$	$E_\gamma(I-\frac{1}{2})$	$E_\gamma(I+1)$	$E_\gamma(I)$	$E_\gamma(I+1)$
	expt.	expt.	expt.	expt.	extr.	extr.	extr.
66	1497.8						
64	1449.6						
62	1401.3			1410.0			
60	1352.9	1353	1387.9	1362.3	1399.0	1375.1	1377.1
58	1304.8	1305	1340.7	1314.7	1351.4	1327.7	1328.9
56	1256.6	1256	1291.0	1266.3	1302.9	1278.7	1280.7
54	1208.6	1207	1244.1	1219.9	1255.2	1232.0	1232.6
52	1160.5	1158	1196.2	1170.5	1208.1	1183.4	1184.6
50	1112.7	1112	1148.6	1123.7	1159.6	1136.2	1136.6
48	1064.9	1063	1100.0	1075.1	1111.9	1087.6	1088.8
46	1017.4	1016	1052.2	1028.7	1063.7	1040.5	1041.2
44	970.6	970	1006.1	980.1	1017.4	993.1	994.0
42	923.2	922	958.6	935.7	969.4	947.2	946.9
40	876.4	876	909.9		922.8		
38	829.9	828	862.1				
36	784.0	783	816.5				
34	738.1	738					
32	692.7	692					
30	647.5	647					
28	602.4						

Figure Captions

Fig. 1 $J^{(1)}$ and $J^{(2)}$ of the ground rotational band of the normally deformed nucleus ^{162}Dy. $J^{(1)}$ and $J^{(2)}$ are extracted directly from the experimental γ-ray energy using eqs. (1) and (2). In Fig. 1(c), the observed spin sequence $I = 0, 2, 4, 6, \cdots$ is used. In Figs. 1(d) and (e), the spin of each level is artificially increased by 1 and 2, i.e., the spin sequence 0, 2, 4, 6, $\cdots$ is replaced by 1, 3, 5, 7, $\cdots$ in Fig. 1(d) and by 2, 4, 6, 8, $\cdots$ in Fig. 1(e). Similarly, the spin sequence adopted in Fig. 1(b) is -1 (unphysical), 1, 3, 5, $\cdots$, and in Fig. 1(a) is -2 (unphysical), 0, 2, 4, $\cdots$. $J^{(1)}$ and $J^{(2)}$ are in units of $\hbar^2$MeV^{-1}.

Fig. 2 $J^{(1)}$ and $J^{(2)}$ of the yrast SD band ^{152}Dy. $J^{(1)}$ and $J^{(2)}$ are extracted directly from the experimental γ-ray energy using eqs. (1) and (2). $E_\gamma(I_0+2 \to I_0) =$ 602.4 keV is the transition energy between the levels with spins I_0+2 and I_0. I_0 is the spin of the lowest level observed in this SD band. The $J^{(1)}$-ξ and $J^{(2)}$-ξ plots for eight spin assignments $I_0 = 22, 23, 24, 25, 26, 27, 28$, and 29 are presented in the upper panel of Fig. 2. The corresponding $\sqrt{J^{(1)3}/J^{(2)}}$-ξ plots are presented in the lower panel of Fig. 2.

Fig. 3 The same as Fig. 2, but for the SD band ^{151}Tb(2) and five trial values of $I_0 = 25.5, 26.5, 27.5, 28.5$, and 29.5 are adopted, $E_\gamma(I_0+2 \to I_0) = 647.0$ keV.

Fig. 4 The same as Fig. 2, but for the SD band ^{153}Dy(2) and five trial values of $I_0 = 32.5, 33.5, 34.5, 35.5$, and 36.5 are adopted, $E_\gamma(I_0+2 \to I_0) = 816.5$ keV.

Fig. 5 The same as Fig. 2, but for the SD band ^{153}Dy(3) and five trial values of $I_0 = 37.5, 38.5, 39.5, 40.5$, and 41.5 are adopted, $E_\gamma(I_0+2 \to I_0) = 935.7$ keV.

Fig. 6 The same as Fig. 2, but for the SD band ^{153}Dy(1) and five trial values of

$I_0 = 33.5, 34.5, 35.5, 36.5$, and 37.5 are adopted, $E_\gamma(I_0+2 \rightarrow I_0) = 809.6$ keV.

Fig. 7 The same as Fig. 2, but for the SD band ^{152}Dy(4) and five trial values of $I_0 = 22, 23, 24, 25$, and 26 are adopted, $E_\gamma(I_0+2 \rightarrow I_0) = 669.6$ keV.

Fig. 8 The same as Fig. 2, but for the SD band ^{152}Dy(5) and five trial values of $I_0 = 21, 22, 23, 24$, and 25 are adopted, $E_\gamma(I_0+2 \rightarrow I_0) = 642.1$ keV.

Fig. 9 The same as Fig. 2, but for the SD band ^{151}Dy(1) and five trial values of $I_0 = 18.5, 19.5, 20.5, 21.5$, and 22.5 are adopted, $E_\gamma(I_0+2 \rightarrow I_0) = 577.1$ keV.

Fig. 10 The same as Fig. 2, but for the SD band ^{152}Dy(6) and five trial values of $I_0 = 29, 30, 31, 32$, and 33 are adopted, $E_\gamma(I_0+2 \rightarrow I_0) = 761.5$ keV.

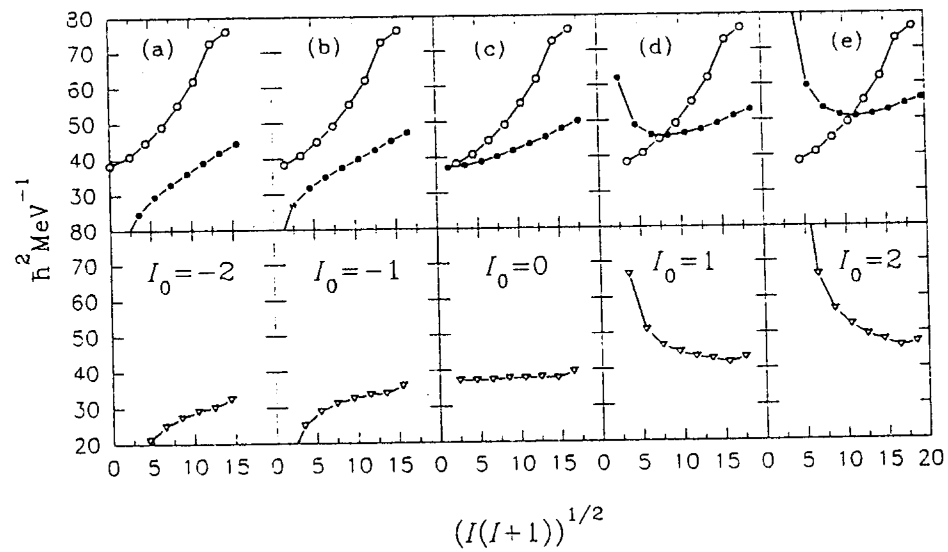
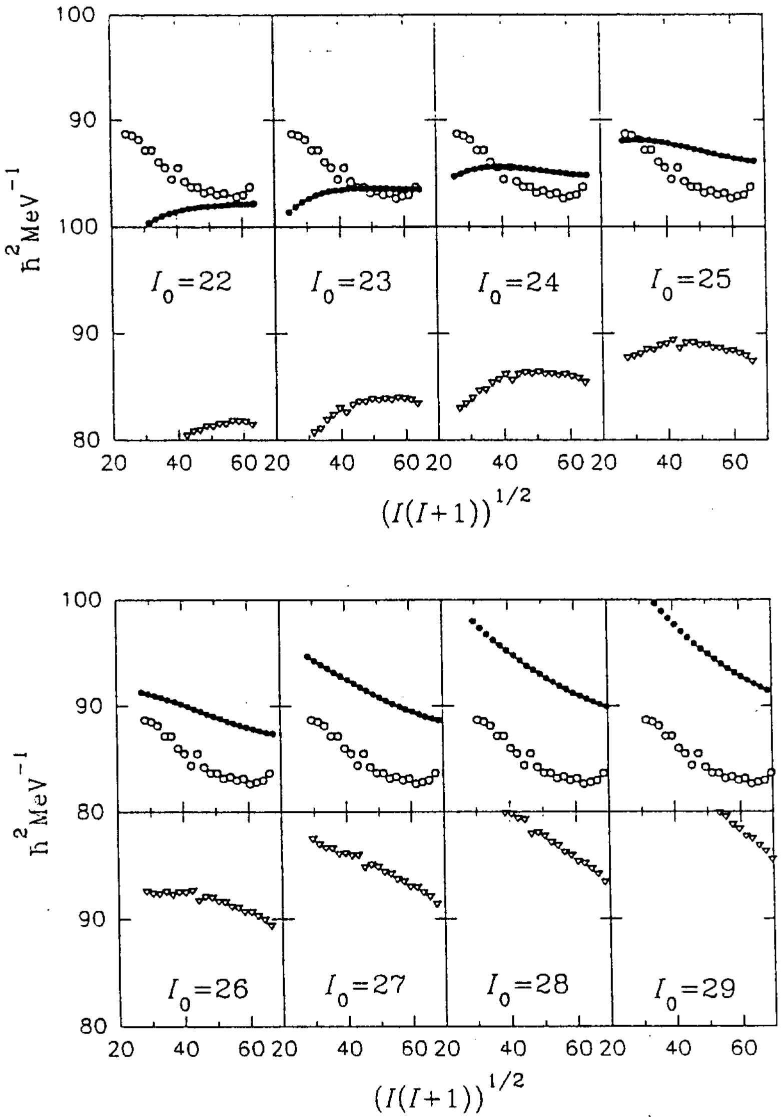
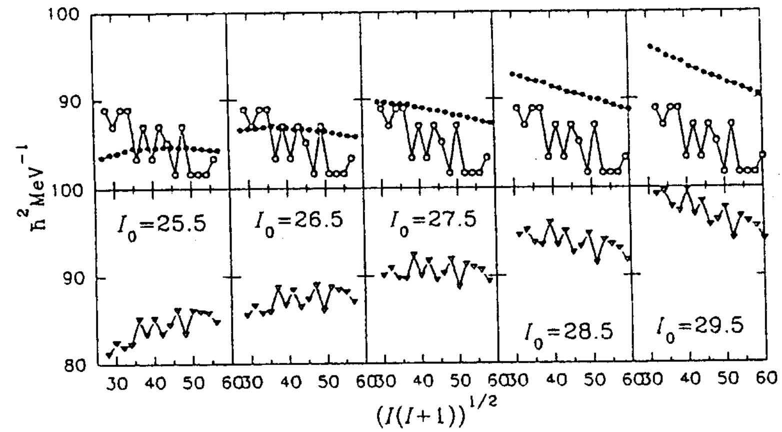


Fig. 1



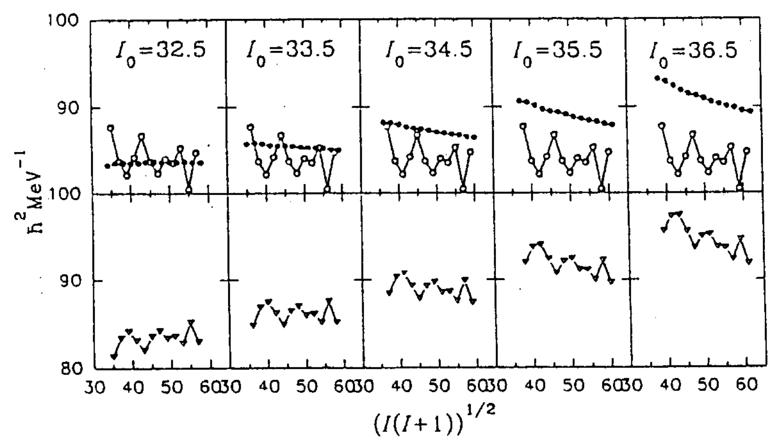
^{152}Dy(1) $E_\gamma(I_0+2->I_0)$=602.4 keV

Fig. 2



^{151}Tb(2)* $E_\gamma(I_0+2->I_0)$=647.0 keV

Fig. 3



^{153}Dy(2) $E_\gamma(I_0+2->I_0)$=816.5 keV

Fig. 4

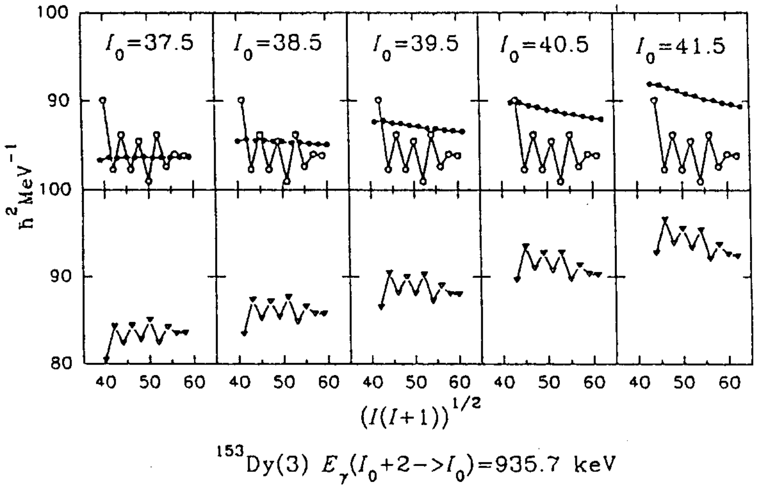


Fig. 5

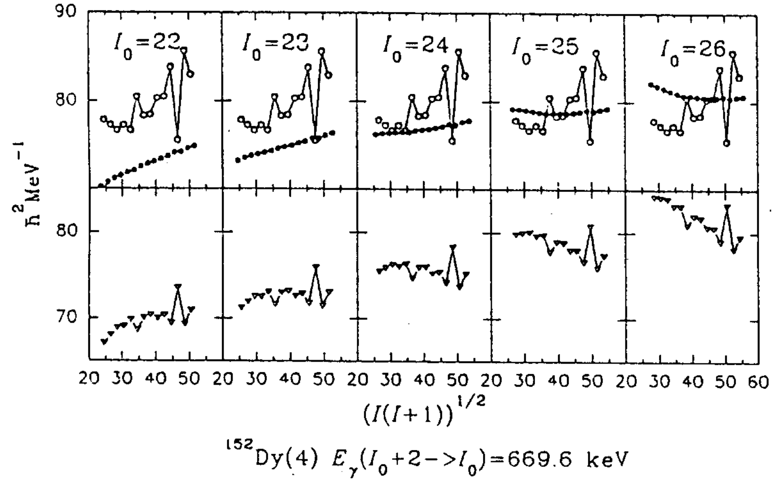


Fig. 7

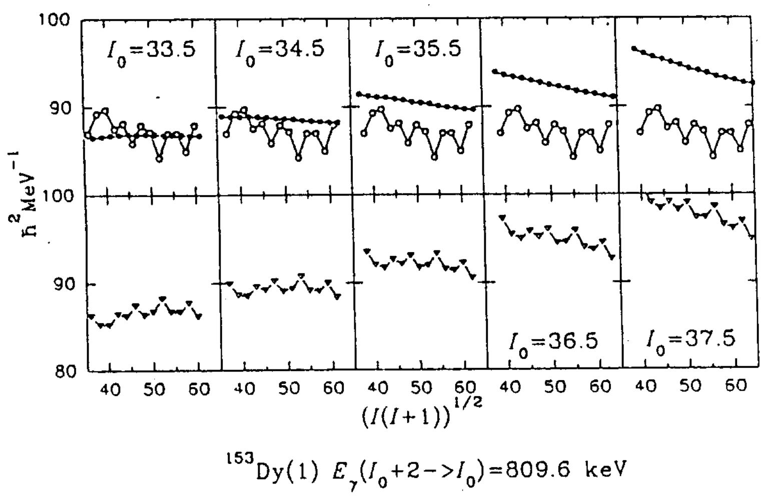


Fig. 6

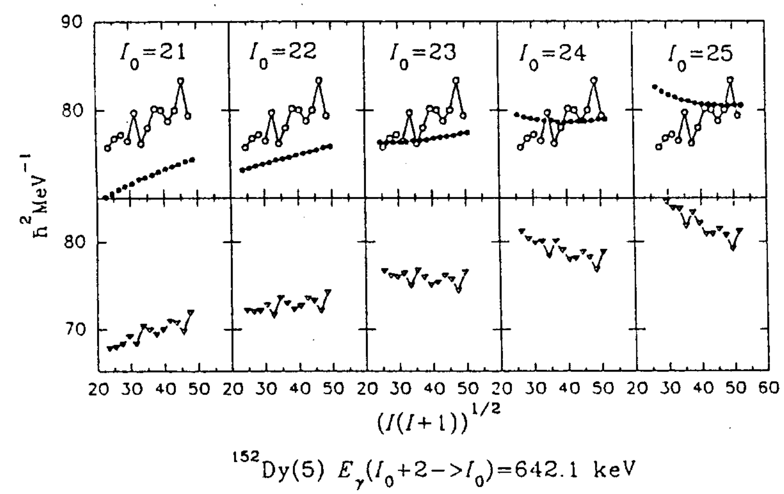
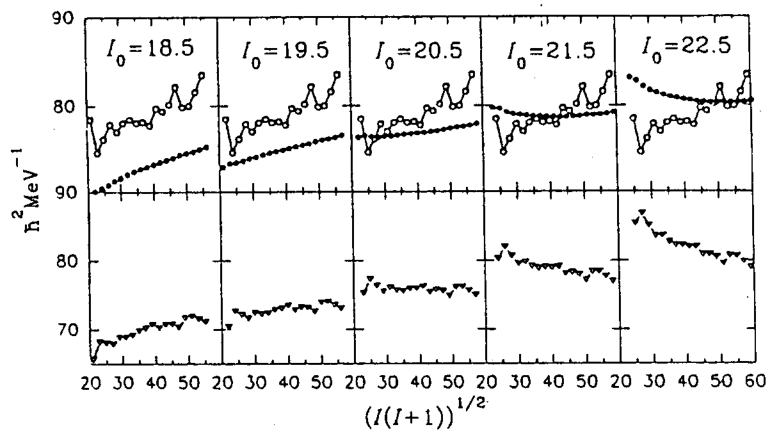
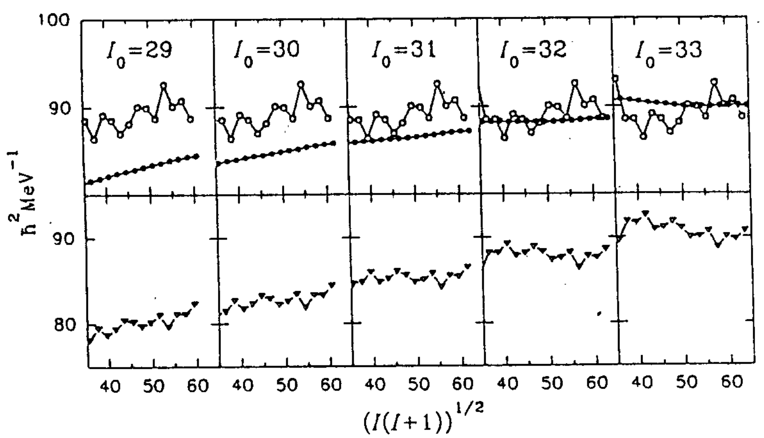


Fig. 8



^{151}Dy(1) $E_\gamma(I_0+2->I_0)$=577.1 keV

Fig. 9



^{152}Dy(6) $E_\gamma(I_0+2->I_0)$=761.5 keV

Fig. 10