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Abstract

Based on the very general properties of the rotational band of axially symmetric nucleus, five rules of the I variation of the kinematic and dynamic moments of inertia are obtained, which may serve as the effective criteria of the spin assignment of rotational band. Several illustrative examples of SD bands are analyzed. For the SD band ^{152}Dy(1), the spin assignments $I_0 \leq 24$ are ruled out.

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Since the discovery of the superdeformed (SD) band in ^{152}Dy [1] a great number of SD bands were discovered and rich experimental information was obtained. However, the experimental data on SD bands consist in a series of γ ray energies linking levels of unknown spin. So in most analyses of experimental data are concerned with only the dynamic moments of inertia $J^{(2)}$, rather than the kinematic moments of inertia $J^{(1)}$. Obviously, the level spin determination is fundamental to understanding the physics of the new regime of deformation. Up to now, several approaches to determine the level spin of SD bands were developed [2,3,4] and the same spin assignments were obtained for the SD bands in the A $\sim$ 190 region (except for a very few cases). However, there have been some comments on the uncertainty in these spin assignments [5] and some people still take a skeptical attitude.

Usually the kinematic moment of inertia is extracted from the experimental intraband transition energy by the difference quotient

$$J^{(1)}(I-1)/\hbar^2 = (2I-1)/E_\gamma(I \to I-2), \tag{1}$$

and the dynamic moment of inertia by

$$J^{(2)}(I)/\hbar^2 = \frac{4}{\Delta E_\gamma(I)} = \frac{4}{E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)}. \tag{2}$$

As the functions of I the extracted $J^{(1)}$ increases with the assigned level spin, but the pattern of the extracted $J^{(2)}$ is independent of the spin assignment. This extraction is model-independent and is generally considered to be reliable provided the moments of inertia vary smoothly with the angular momentum. Of course, while the relative errors in the E_γ's of SD bands are rather small ($\delta E_\gamma/E_\gamma \sim 10^{-3}, \delta E_\gamma \leq 1$ keV), the relative errors in the ΔE_γ's is larger by an order of magnitude ($\delta(\Delta E_\gamma)/\Delta E_\gamma > 10^{-2}$), so the uncertainty in the dynamic moments of inertia thus extracted is usually rather large. However, careful investigation shows that there exists close connection between the features of both kinds of moment of inertia of the same band, which seems not to attract attention. In this paper we will show that, based on the very general properties of rotational spectra, some simple, illustrative, but very useful criteria of the spin assignment may be derived from the investigation of the I variations of $J^{(1)}$ and $J^{(2)}$ and their relations, and may be used effectively to determine the spins of SD bands.

According to the famous work by Bohr and Mottelson [6], the $K = 0$ rotational spectra of axially symmetric nuclei, under the adiabatic approximation, can be expressed as the function of $I(I+1)$ and expanded in power of $I(I+1)$. Let $\xi = \sqrt{I(I+1)}$, the rotational energy can be expressed as

$$E = A\xi^2 + B\xi^4 + C\xi^6 + D\xi^8 + \cdots \quad (3)$$

The expression for the rotational energy of $K \neq 0$ band [6] takes a form similar to eq(3), but includes a band-head energy, and $I(I+1)$ is replaced by $I(I+1) - K^2$. It was well established that the extensive experimental data on nuclear rotational bands (below bandcrossing) are described very well by eq(3). Systematic analyses of the large amount of data on the rotational spectra of rare-earth and actinide nuclei showed [6,7] that $|B/A| \sim 10^{-3}$, $|C/A| \sim 10^{-6}$, $|D/A| \sim 10^{-9}$, etc.; i.e. the convergence of the $I(I+1)$ expansion is satisfactory, and the two-parameter AB expression (putting $C = D = \cdots = 0$ in eq. (3)) is widely used for the description of rotational spectra. For the SD bands, the convergence is even better [2] ($|B/A| \sim 10^{-4}$, $|C/A| \sim 10^{-8}$), i.e. compared to the normally deformed nuclei, the SD nucleus appears to be a more rigid rotator with axial symmetry. From eq.(3) the kinematic and dynamic moments of inertia [8], $J^{(1)}/\hbar^2 = \dfrac{\xi d\xi}{dE}$ and $J^{(2)}/\hbar^2 = \left(\dfrac{d^2E}{d\xi^2}\right)^{-1}$, are expressed as

$$J^{(1)}/\hbar^2 = \frac{1}{2A}\left(1 + 2\frac{B}{A}\xi^2 + 3\frac{C}{A}\xi^4 + \cdots\right)^{-1} \quad (4)$$

$$J^{(2)}/\hbar^2 = \frac{1}{2A}\left(1 + 6\frac{B}{A}\xi^2 + 15\frac{C}{A}\xi^4 + \cdots\right)^{-1} \quad (5)$$

$$J^{(2)}/J^{(1)} = \left\{1 - 4\frac{B}{A}\xi^2 - 12\left[\left(\frac{B}{A}\right)^2 + \frac{C}{A}\right]\xi^4 + \cdots\right\} \quad (6)$$

$$\lim_{I\to 0} J^{(1)}/\hbar^2 = \lim_{I\to 0} J^{(2)}/\hbar^2 = 1/2A \quad (7)$$

It is easily verified that

$$\frac{1}{\hbar^2}\frac{dJ^{(1)}}{d\xi} = -2\frac{B}{A^2}\xi\left[1 - \left(\frac{4B}{A} - \frac{3C}{B}\right)\xi^2 + \cdots\right] \quad (8)$$

$$\frac{1}{\hbar^2}\frac{dJ^{(2)}}{d\xi} = -6\frac{B}{A^2}\xi\left[1 - \left(\frac{12B}{A} - \frac{5C}{B}\right)\xi^2 + \cdots\right] \quad (9)$$

$$\frac{dJ^{(2)}}{d\xi}\bigg/\frac{dJ^{(1)}}{d\xi} = 3\left[1 - 2\left(4\frac{B}{A} - \frac{C}{B}\right)\xi^2 + \cdots\right] \quad (10)$$

$$\lim_{I\to 0}\frac{dJ^{(1)}}{d\xi} = \lim_{I\to 0}\frac{dJ^{(2)}}{d\xi} = 0 \quad (11)$$

$$\frac{1}{\hbar^2}\frac{d^2J^{(1)}}{d\xi^2} = -\frac{2B}{A^2}\left[1 - 3\left(\frac{4B}{A} - \frac{3C}{B}\right)\xi^2 + \cdots\right] \quad (12)$$

$$\frac{1}{\hbar^2}\frac{d^2J^{(2)}}{d\xi^2} = -\frac{6B}{A^2}\left[1 - 3\left(\frac{12B}{A} - \frac{5C}{B}\right)\xi^2 + \cdots\right] \quad (13)$$

$$\frac{d^2J^{(2)}}{d\xi^2}\bigg/\frac{d^2J^{(1)}}{d\xi^2} = 3\left[1 - 6\left(\frac{4B}{A} - \frac{C}{B}\right)\xi^2 + \cdots\right] \quad (14)$$

From eqs. (4)—(14), several rules can be observed:

(a) As the functions of I or $\xi = \sqrt{I(I+1)}$, both $J^{(1)}$ and $J^{(2)}$ of the same band tend to the same limiting value as $I \to 0$.

(b) Both $J^{(1)}$ and $J^{(2)}$ monotonously increase with I (for $B < 0$), or monotonously decrease with I (for $B > 0$), but the slope of $J^{(2)}$ is much steeper than that of $J^{(1)}$ ($dJ^{(2)}/d\xi \approx 3dJ^{(1)}/d\xi$ in the low spin range).

(c) $J^{(1)}$-ξ and $J^{(2)}$-ξ plots never cross each other at any nonzero spin value.

(d) Both the slopes of $J^{(1)}$ and $J^{(2)}$ tend to zero as $I \to 0$, i.e., both $J^{(1)}$-ξ and $J^{(2)}$-ξ plots become horizontal as $I \to 0$.

(e) Both $J^{(1)}$-ξ and $J^{(2)}$-ξ plots concave upwards (for $B < 0$), or concave downwards (for $B > 0$).

The overall analyses of the large amount of available rotational bands (below band-crossing) of normally deformed nuclei whose spins were established shows that these five rules do hold without exception. As an illustrative example, the analysis of the ground band of the well-deformed nucleus ^{174}Yb [9] is given in Fig. 1. In Fig. 1(a), are displayed the $J^{(1)}$ and $J^{(2)}$ extracted by eqs. (1) and (2) using the measured spin sequence 0, 2, 4, $\cdots$. For all the other rotational bands in normally deformed nuclei, the situation is quite similar. However, if one artificially changes the spin of each level, some of these rules may fail in certain spin range. For example, if the spin of each level is artificially increased by 1 $\hbar$ (Fig. 1(b)) or 2 $\hbar$ (Fig. 1(c)), i.e., the measured spin sequence 0, 2, 4, $\cdots$ is replaced by 1, 3, 5, $\cdots$ or 2, 4, 6, $\cdots$, it is found that, though the pattern of $J^{(2)}$-ξ is not influenced by the spin change, the kinematic moments of inertia is shifted upwards to various extents (except for the very high spin states), hence the relations between $J^{(1)}$ and $J^{(2)}$ change. From Figs. 1(b) and (c), it is seen that: (1) $J^{(1)}$-ξ and $J^{(2)}$-ξ plots cross each other at certain spin I_c. (2) For $I < I_c$, $J^{(1)}$ increases with decreasing I, i.e., its monotonousness is broken. (3) As $I \to 0$, $J^{(1)}$-ξ plot does not become horizontal, and $J^{(1)}$ does not tend to the same limiting value as that of $J^{(2)}$. On the other hand, if the spin of each level is artificially decreased by 1 $\hbar$ (Fig. 1(d)) or 2 $\hbar$ (Fig. 1(e)), i.e., the measured spin sequence 0, 2, 4, 6, $\cdots$ is replaced by -1 (unphysical), 1, 3, 5, $\cdots$, or -2 (unphysical), 0, 2, 4, $\cdots$, it is seen that: (1) In the low spin range, the slope of $J^{(1)}$ increase with decreasing I and does not tend to zero as $I \to 0$. (2) As $I \to 0$, $J^{(1)}$ does not tend to the same limiting value as $J^{(2)}$. (3) In the low spin range, $J^{(1)}$-ξ plot becomes concave downwards.

Therefore, for a rotational band whose level spins are unknown, we may assume a spin sequence, I_0, I_0+2, I_0+4, $\cdots$, and then construct the $J^{(1)}$-ξ and $J^{(2)}$-ξ plots using eqs. (1) and (2) from the measured intraband γ ray energies E_γ's. In this case, violation of one (or more) of the five rules implies an unreasonable spin assignment is made, hence such spin assignment must be ruled out.

Now we use the five rules mentioned above as the criteria of the spin assignment of SD bands. As an illustrative example, several spin assignments for the SD band in ^{196}Pb [10] are displayed in Fig. 2. $E_\gamma(I_0 + 2 \to I_0) = 169.6$ keV, I_0 is the spin of the lowest level observed. Obviously, the spin assignment $I_0 \geq 7$ (Figs. 2(c), (d)) are forbidden by the rules (a), (c) and (d). On the other hand, $I_0 \leq 5$ (Fig. 2(a)) should be ruled out, because the rules (a), (d) and (e) are violated. Therefore, only the spin assignment $I_0 = 6$ (Fig. 2(b)) is allowed.

In Fig. 3 the analyses for the two identical SD bands [11], ^{192}Hg [2,12] and ^{194}Hg(2) [13] are shown. $E_\gamma(I_0 + 2 \to I_0) = 214.6$ keV for ^{192}Hg, and $E_\gamma(I_0 + 2 \to I_0) = 203.3$ keV for ^{194}Hg(2). It is seen that for $I_0 \geq 9$ the rules (a), (c) and (d) are obviously violated. For $I_0 \leq 7$, the rules (a), (d) and (e) are violated. So, only the spin assignment $I_0 = 8$ is allowed, which is the same as that given in refs. [2--4,11].

Finally, the yrast SD band in ^{152}Dy is addressed in Fig. 4. $E_\gamma(I_0 + 2 \to I_0) = 602.4$

keV [1,14,15]. In ref. [1], I_0 is assigned to be 22. In ref. [14], it was pointed out that it was unlikely that I_0 is smaller than 22. In ref. [3], the spin assignment $I_0 = 25$ was made. From Figs. 4(a), (b) and (c), it is obviously seen that the assignments $I_0 \leq 24$ are definitely forbidden by the rules (b), (c), (d) and (e). On the other hand, the spin assignments $I_0 \geq 27$ are obviously ruled out by the rules (a) and (d). It is seen (Fig. 4(e)) that the spin assignment $I_0 = 26$ is the most plausible candidate. However, due to the large relative errors in the extracted $J^{(2)}$'s, $I_0 = 25$ cannot be ruled out decisively.

By the way, it should be noted that the five rules drawn above remain valid in the Harris ω expansion formalism [6,16]

$$E = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 + \delta\omega^8 + \cdots, \quad (\omega = dE/d\xi) \tag{15}$$

of which the convergence is believed [6] to be superior to that of the $I(I+1)$ expansion (3) and the two-parameter $\alpha\beta$ expansion (putting $\gamma = \delta = \cdots = 0$ in eq. (15)) is widely used in the high-spin nuclear physics. It is easily verified that

$$J^{(1)}/\hbar^2 = 2\alpha + \frac{4}{3}\beta\omega^2 + \frac{6}{5}\gamma\omega^4 + \cdots \tag{16}$$

$$J^{(2)}/\hbar^2 = 2\alpha + 4\beta\omega^2 + 6\gamma\omega^4 + \cdots \tag{17}$$

and a series equations similar to eqs. (7), (10), (11) and (14) can be obtained.

In summary, based on the very general properties of rotational spectra of axially symmetric nuclei, demonstrated by Bohr and Mottelson, some useful rules for the I variation of the kinematic and dynamic moments of inertia may be obtained, which may serve as the effective criteria of the spin assignment of a rotational band whose spins are unknown. These criteria do not invoke the least squares fitting of the experimental data ($J^{(2)}$'s, or E_γ's) with some model-dependent formulae [2—4]. Using these criteria, the level spins of the SD bands available in the $A \sim 190$ region have been determined unambiguously and

will be published elsewhere. As for the SD bands in the $A \sim 150$ region, the situation is a little more complicated, i.e., while the spins of some SD bands can be determined unambiguously, for the spin assignments of the other SD bands, there may be two (or more) potential candidates, which do not obviously violate these criteria. This is partly because the spin of the lowest level observed (I_0) is rather high, and partly because the behavior of the SD bands in the $A \sim 150$ region is not so regular as that in the $A \sim 190$ region, where the bandcrossing is rarely found.

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Figure Captions

Fig. 1 The kinematic (solid circle) and dynamic (open circle) moments of inertia of the ground band of ^{174}Yb extracted by eqs. (1) and (2) from the experimental γ transition energies [9]. In Fig. 1(a), the measured spin sequence 0, 2, 4, 6, $\cdots$ is adopted. In Figs 1(b), (c), (d) and (e), the spin sequence 0, 2, 4, 6, $\cdots$ is artificially replaced by 1, 3, 5, 7, $\cdots$, 2, 4, 6, 8, $\cdots$, and -1 (unphysical), 1, 3, 5, $\cdots$, and -2 (unphysical), 0, 2, 4, $\cdots$, respectively. $J^{(1)}$ and $J^{(2)}$ are in units of $\hbar^2$MeV^{-1}.

Fig. 2 The kinematic (solid circle) and dynamic (open circle) moments of inertia of the SD band in ^{196}Pb, extracted by eqs. (1) and (2) from the experiment γ transition energies [10] for various spin assignments. $E_\gamma(I_0+2\rightarrow I_0) = 169.9$ keV. In Figs. 2(a), (b), (c) and (d), the spin of the lowest level observed I_0 is assigned to be $I_0 = 5$, 6, 7, and 8, respectively. Obviously, only the spin assignment $I_0 = 6$ is allowed (Fig. 2(b)).

Fig. 3 The same as Fig. 2, but for the two identical SD bands ^{192}Hg [2,12] and ^{194}Hg(2) [13]. $E_\gamma(I_0+2\rightarrow I_0) = 214.6$ keV for ^{192}Hg and $E_\gamma(I_0+2\rightarrow I_0) = 201.3$ keV for ^{194}Hg(2). Obviously, only the spin assignment $I_0 = 8$ is allowed (Fig. 3(b)).

Fig. 4 The same as Fig.2, but for the yrast SD band in ^{152}Dy [1,14,15]. $E_\gamma(I_0+2\rightarrow I_0) = 602.4$ keV.

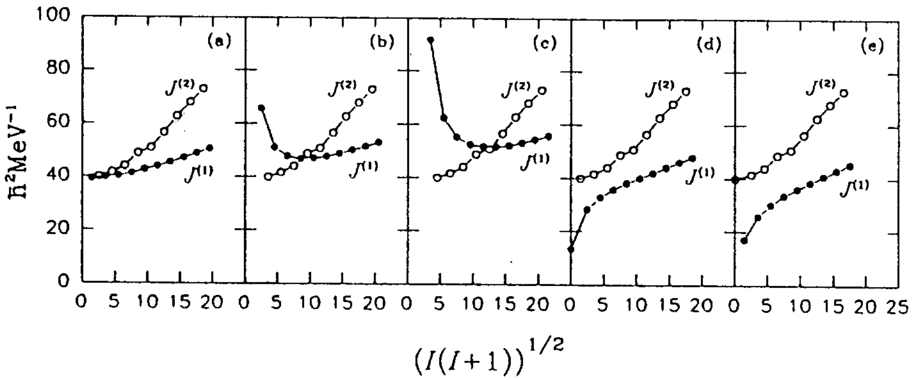


Fig. 1

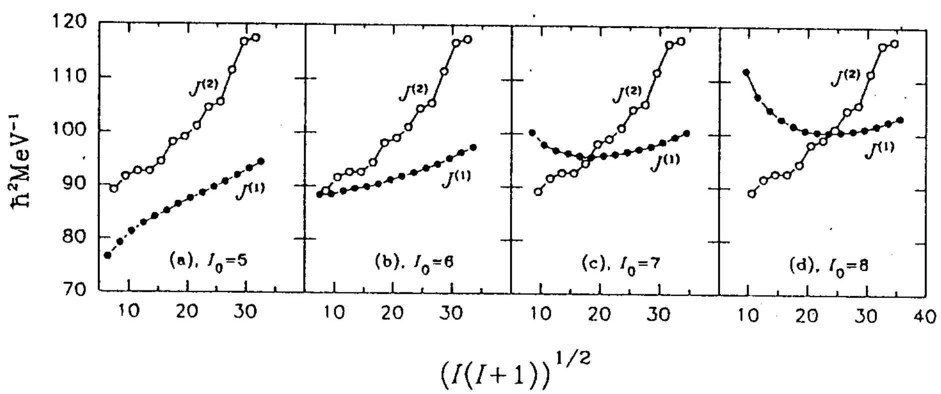


Fig. 2

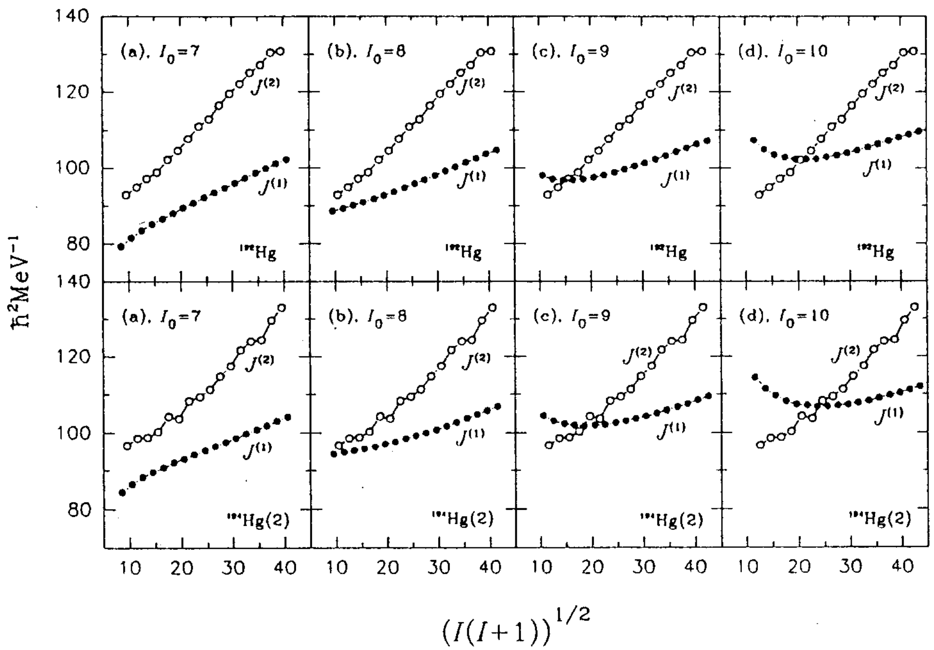
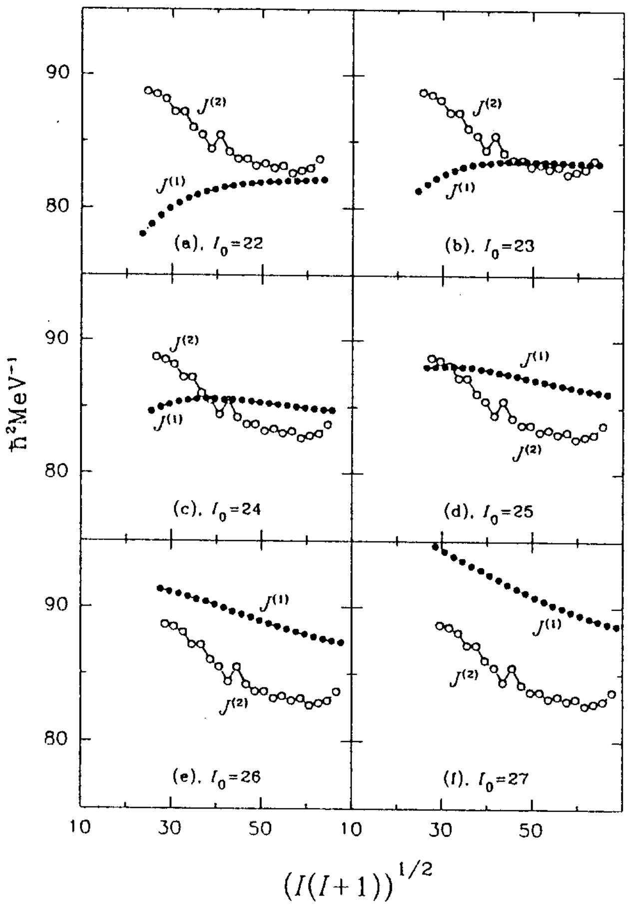


Fig. 3



90
80
$\hbar^2 MeV^{-1}$
$J^{(1)}$
$J^{(2)}$
(a). I_0=22
(b). I_0=23
(c). I_0=24
(d). I_0=25
(e). I_0=26
(f). I_0=27
10
30
50
$(I(I+1))^{1/2}$

Fig. 4