LABORATORIO INTERDISCIPLINARE PER
LE SCIENZE NATURALI ED UMANISTICHE

PC

$N = 1$ SUPERGRAVITY AS A NON-LINEAR REALIZATION II. THE GENERAL CASE*

D. Maxera†

and

J. Niederle†

Interdisciplinary Laboratory for Natural and Humanistic Sciences of the International School for Advanced Studies (SISSA/ISAS), 34014 Trieste, Italy

ABSTRACT

In the paper the non-linear realization approach based on Cartan forms and the inverse Higgs effect is extended to the general $N = 1$ supergravity. Thus all $N = 1$ supergravities are reformulated as generalized σ-models analogously to the simplest case of the minimal Einstein $N = 1$ supergravity treated in [1]. This gives new geometrical insights into the supergravitation theories which together with possible implications of the proposed formulation are briefly discussed in the last section of the paper.

* This publication is based on work sponsored by the US-Czech Science and Technological Joint Fund under Project Number 92067 and by the Grant ASCR under Project Number 19085.

† Permanent address: Institute of Physics, Academy of Sciences of the Czech Republic, Na Slovance 2, 180 40 Prague 8.

ILAS/EP - 8/1994

SISSA
34014 TRIESTE
Strada Costiera, 11

TRIESTE

1 INTRODUCTION

It was shown in [1] that the minimal Einstein N=1 supergravity can be consistently reformulated as a non-linear σ−model describing a spontaneous breaking of its infinite-dimensional gauge supergroup down to the rigid N=1 Poincaré supergroup. More precisely it was shown that the minimal Einstein N=1 supergravity can be derived by means of simultaneous non-linear realizations of its two complex finite-dimensional gauge subgroups. They generate via their closure the whole infinite-dimensional gauge supergroup of the theory and their intersection contains the rigid N=1 Poincaré supergroup which is chosen as a stability group of the vacuum . It appeared that the only independent Goldstone superfield accompanying the above mentioned spontaneous breaking of the symmetry is an axial-vector superfield $H^{\mu\dot{\mu}}(x,\theta,\bar{\theta})$ identified with the prepotential of the considered supergravity. All the other Goldstone superfields can be expressed in terms of $H^{\mu\dot{\mu}}$ and its derivatives by imposing appropriate covariant constraints on the corresponding Cartan superforms, i.e. by applying the so-called inverse Higgs effect [2]. This non-linear realization technique based on Cartan forms and exploiting the inverse Higgs effect gave new geometrical insights into the minimal Einstein supergravity and indicated profound relations with topological fields theories, (super)p−branes etc.

In this paper we shall extend the non-linear realization approach to the general N=1 supergravity. In other words, we shall reformulate the N=1 supergravity as a generalised σ−model analogously to the simplest case of the minimal Einstein N=1 supergravity treated in [1].

Thus, in Section 2, first the gauge supergroup of the N=1 supergravity will be specified and then it will be shown how this supergroup can be represented as a closure of its two appropriate finite-dimensional subgroups. The non-linear realization of these finite-dimensional subgroups and eliminations of various Goldstone superfields by the inverse Higgs effect will be treated in Section 3 and 4. Finally, in Section 5, the derived results for non-minimal Einstein N=1 supergravities with different values of the Siegel-Gates parameter n will be compared with those treated in the literature and the main conclusions summarized.

2 GAUGE SUPERGROUP OF N=1 NON–MINIMAL EINSTEIN SUPERGRAVITY AND ITS STRUCTURE

The N=1 non-minimal Einstein supergravity (further N=1 supergravity) can be formulated on a $(4+4)$–dimensional complex superspace

$$C^{4/4} = \{(x_L^{\mu\dot{\mu}}, \theta_L^{\alpha}, \bar{\phi}_L^{\dot{\alpha}})\} = \{(x_R^{\mu\dot{\mu}}, \bar{\theta}_R^{\dot{\alpha}}, \phi_R^{\alpha})\}, \tag{2.1}$$

(with $(x_L^{\mu\dot{\mu}}, \theta_L^{\alpha}, \bar{\phi}_L^{\dot{\alpha}})$ and $(x_R^{\mu\dot{\mu}}, \bar{\theta}_R^{\dot{\alpha}}, \phi_R^{\alpha}) = (x_L^{\mu\dot{\mu}}, \theta_L^{\alpha}, \bar{\phi}_L^{\dot{\alpha}})^+$ being its left– and right–handed parametrizations) or, more precisely, on a $(4+4)$-dimensional real, physical superspace $R^{4/4}$ that forms a hypersurface in $C^{4/4}$:

$$R^{4/4} = \{(x^{\mu\dot{\mu}}, \theta^{\alpha}, \bar{\theta}^{\dot{\alpha}}); x^{\mu\dot{\mu}} = \mathrm{Re}(x_L^{\mu\dot{\mu}}),\ \theta^{\mu} = \theta_L^{\mu},\ \bar{\theta}^{\dot{\mu}} = \bar{\theta}_R^{\dot{\mu}})\}, \tag{2.2}$$

the shape of which is determined by the folloving superfunctions

$$\begin{aligned} H^{\mu\dot\mu}(x,\theta,\bar\theta) &= \mathrm{Im}(x_L^{\mu\dot\mu}), \\ H^{\mu}(x,\theta,\bar\theta) &= \phi_R^{\mu} - \theta_L^{\mu}, \\ \text{and} \qquad \bar H^{\dot\mu}(x,\theta,\bar\theta) &= \bar\phi_L^{\dot\mu} - \bar\theta_R^{\dot\mu}. \end{aligned} \tag{2.3}$$

(For details see [3,4,5] and for our notation the Appendix).

The N=1 supergravity is a theory invariant with respect to an infinite-parameter complex gauge supergroup acting on $C^{4/4}$ and leaving its subspace

$$C^{4/4}/C^{0/2} = C^{4/2} = \{(x_L^{\mu\dot\mu}, \theta_L^{\mu})\} = \{(x_R^{\mu\dot\mu}, \bar\theta_R^{\dot\mu}) = (x_L^{\mu\dot\mu}, \theta_L^{\mu})^+\} \tag{2.4}$$

invariant. The infinitesimal action of the gauge supergroup on $C^{4/4}$ is given by

$$\begin{aligned} \delta x_L^{\mu\dot\mu} &= \lambda^{\mu\dot\mu}(x_L, \theta_L), \\ \delta\theta_L^{\mu} &= \lambda^{\mu}(x_L, \theta_L), \\ \delta\bar\phi_L^{\dot\mu} &= \bar\varrho^{\dot\mu}(x_L, \theta_L, \bar\phi_L) \end{aligned} \tag{2.5}$$

with $\lambda^{\mu\dot\mu}$, λ^{μ} and $\bar\varrho^{\dot\mu}$ being arbitrary superfunction parameters satisfying the constraint

$$(3n+1)\frac{\partial\bar\varrho^{\dot\mu}}{\partial\bar\phi_L^{\dot\mu}} = (n+1)\left(\frac{\partial\lambda^{\mu\dot\mu}}{\partial x_L^{\mu\dot\mu}} - \frac{\partial\lambda^{\dot\mu}}{\partial\theta_L^{\mu}}\right). \tag{2.6}$$

Here n is the Gates-Siegel parameter [6] which is complex in general and fulfills the condition $|3n+1| \neq |n+1|$ unless $n = 0$. Following [6] we shall consider n to be real and different from $0, \infty$. For any particular value of n we obtain a different subgroup of the gauge supergroup (defined by (2.5)) leaving (2.6) invariant and thus one of the non-minimal Einstein supergravities. Actually (2.6) expresses infinitesimally a correlation of the transformations of supervolumes in $C^{4/4}$ and $C^{4/2}$. For $n = -\frac{1}{3}$ we obtain the N=1 minimal Einstein supergravity studied in detail in [1].

It can be proved analogously to [5] that **the superalgebra A of infinite-dimensional gauge supergroup G which is determined by (2.5) and (2.6) with fixed n, can be expressed as a closure of its two finite-dimensional subalgebras A_I and A_{II}.** The superalgebras A_I and A_{II} correspond to the appropriate subgroups G_I and G_{II} respectively of supergroup G and are specified as follows.

First let us specify A_0 – the superalgebra corresponding to the biggest common subgroup G_0 of G_I and G_{II}. The generators of superalgebra A_0 are of the form:

$$\begin{aligned} &P_{\mu\dot\mu} = -i\partial_{\mu\dot\mu}, && Q_{\mu} = -i\partial_{\mu}, \\ &F_{\dot\mu}^{-\frac{1}{2}} = -i\bar\partial_{\dot\mu}, && Q^{\mu}_{\alpha\dot\alpha} = -i\theta^{\mu}\partial_{\alpha\dot\alpha}, \\ &R^{\alpha\dot\alpha}{}_{\beta\dot\beta} = -i(x_L^{\alpha\dot\alpha}\partial_{\beta\dot\beta} + a\delta^{\alpha}_{\beta}\delta^{\dot\alpha}_{\dot\beta}\bar\phi^{\dot\varrho}\bar\partial_{\dot\varrho}), && \\ &T^{\alpha}_{\beta} = -i(\theta^{\alpha}\partial_{\beta} - a\delta^{\alpha}_{\beta}\bar\phi^{\dot\varrho}\bar\partial_{\dot\varrho}), && F^{0}_{\dot\alpha\dot\beta} = -i(\bar\phi_{\dot\alpha}\bar\partial_{\dot\beta} + \bar\phi_{\dot\beta}\bar\partial_{\dot\alpha}), \end{aligned} \tag{2.7}$$

where $a \equiv \frac{(n+1)}{2(3n+1)}$, n is the Gates-Siegel parameter [6] and $\partial_{\alpha\dot\alpha} \equiv \frac{\partial}{\partial x_L^{\alpha\dot\alpha}}$, $\partial_{\alpha} \equiv \frac{\partial}{\partial\theta^{\alpha}}$ and $\bar\partial_{\dot\alpha} \equiv \frac{\partial}{\partial\bar\phi^{\dot\alpha}}$. Superalgebra A_0 contains generators of the Lorentz group, namely

$$M_{\alpha\beta} = R_{(\alpha\beta)} + T_{(\alpha\beta)} \quad \text{and} \quad \mathrm{M}_{\dot\alpha\dot\beta} \;=\; \mathrm{R}_{(\dot\alpha\dot\beta)} + \mathrm{F}^0_{\dot\alpha\dot\beta}, \tag{2.8}$$

where $X_{(\alpha\beta)} \equiv X_{\alpha\beta} + X_{\beta\alpha}$ and $R_{(\alpha\beta)}$, $R_{(\dot\alpha\dot\beta)}$ are determined by the decomposition

$$R^{\alpha\dot\alpha}{}_{\beta\dot\beta} = \frac{1}{4}(R^{(\alpha(\dot\alpha}{}_{\beta)\dot\beta)} + \delta^\alpha_\beta R^{(\dot\alpha}{}_{\dot\beta)} + \delta^{\dot\alpha}_{\dot\beta} R^{(\alpha}{}_{\beta)} + \delta^\alpha_\beta \delta^{\dot\alpha}_{\dot\beta} R). \tag{2.9}$$

The superalgebra A_I contains, in addition to the generators from A_0, the generators

$$I^{\alpha\dot\alpha}_\mu = -i x^{\alpha\dot\alpha}_L \partial_\mu, \tag{2.10}$$

whereas the superalgebra A_{II} the generators

$$\begin{array}{ll} K_{\alpha\dot\alpha} = -i\theta^2 \partial_{\alpha\dot\alpha}, & G_\mu = -i(\theta^2 \partial_\mu + 2a\theta_\mu \bar\phi^{\dot\varrho} \bar\partial_{\dot\varrho}), \\ F^{\frac{1}{2}}_{\dot\mu} = -i\theta^2 \bar\partial_{\dot\mu}, & F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta} = -i\theta^\mu(\bar\phi_{\dot\alpha}\bar\partial_{\dot\beta} + \bar\phi_{\dot\beta}\bar\partial_{\dot\alpha}), \\ F^1_{\dot\alpha\dot\beta} = -i\theta^2(\bar\phi_{\dot\alpha}\bar\partial_{\dot\beta} + \bar\phi_{\dot\beta}\bar\partial_{\dot\alpha}), & F^{0\alpha}_{\dot\mu} = -i\theta^\alpha \bar\partial_{\dot\mu}. \end{array} \tag{2.11}$$

The non-zero binary relations of superalgebra A_0 are of the form

$$\begin{array}{ll} \{Q^\beta_{\alpha\dot\alpha}, Q_\mu\} = -i\delta^\beta_\mu P_{\alpha\dot\alpha}, & [R^{\alpha\dot\alpha}{}_{\beta\dot\beta}, P_{\gamma\dot\gamma}] = i\delta^\alpha_\gamma \delta^{\dot\alpha}_{\dot\gamma} P_{\beta\dot\beta}, \\ [T^\alpha{}_\beta, Q_\mu] = i\delta^\alpha_\mu Q_\beta, & [F^0_{\dot\alpha\dot\beta}, F^{-\frac{1}{2}}_{\dot\mu}] = i(\epsilon_{\dot\alpha\dot\mu} F^{-\frac{1}{2}}_{\dot\beta} + \epsilon_{\dot\beta\dot\mu} F^{-\frac{1}{2}}_{\dot\alpha}), \\ [T^\alpha{}_\beta, F^{-\frac{1}{2}}_{\dot\mu}] = -ia\delta^\alpha_\beta F^{-\frac{1}{2}}_{\dot\mu}, & [T^\alpha{}_\beta, Q^\mu_{\gamma\dot\gamma}] = -i\delta^\mu_\beta Q^\alpha_{\gamma\dot\gamma}, \\ [R^{\alpha\dot\alpha}{}_{\beta\dot\beta}, F^{-\frac{1}{2}}_{\dot\mu}] = ia\delta^\alpha_\beta \delta^{\dot\alpha}_{\dot\beta} F^{-\frac{1}{2}}_{\dot\mu}, & [T^\alpha{}_\beta, T^\gamma{}_\delta] = -i(\delta^\gamma_\beta T^\alpha{}_\delta - \delta^\alpha_\delta T^\gamma{}_\beta), \\ [R^{\alpha\dot\alpha}{}_{\beta\dot\beta}, R^{\gamma\dot\gamma}{}_{\delta\dot\delta}] = -i(\delta^\gamma_\beta \delta^{\dot\gamma}_{\dot\beta} R^{\alpha\dot\alpha}{}_{\delta\dot\delta} - \delta^\alpha_\delta \delta^{\dot\alpha}_{\dot\delta} R^{\gamma\dot\gamma}{}_{\beta\dot\beta}), & [R^{\alpha\dot\alpha}{}_{\beta\dot\beta}, Q^\mu_{\gamma\dot\gamma}] = i\delta^\alpha_\gamma \delta^{\dot\alpha}_{\dot\gamma} Q^\mu_{\beta\dot\beta}, \\ [F^0_{\dot\alpha\dot\beta}, F^0_{\dot\gamma\dot\delta}] = i(\epsilon_{\dot\alpha\dot\delta} F^0_{\dot\beta\dot\gamma} + \epsilon_{\dot\beta\dot\delta} F^0_{\dot\alpha\dot\gamma} + \epsilon_{\dot\alpha\dot\gamma} F^0_{\dot\beta\dot\delta} + \epsilon_{\dot\beta\dot\gamma} F^0_{\dot\alpha\dot\delta}). & \end{array} \tag{2.12}$$

The remaining non-zero binary relations of superalgebra A_I are given by

$$\begin{array}{ll} [I^{\alpha\dot\alpha}_\mu, P_{\beta\dot\beta}] = i\delta^\alpha_\beta \delta^{\dot\alpha}_{\dot\beta} Q_\mu, & \{I^{\alpha\dot\alpha}_\mu, Q^\nu_{\beta\dot\beta}\} = -i\delta^\nu_\mu R^{\alpha\dot\alpha}{}_{\beta\dot\beta} - i\delta^\alpha_\beta \delta^{\dot\alpha}_{\dot\beta} T^\nu{}_\mu, \\ [I^{\alpha\dot\alpha}_\mu, T^\beta{}_\gamma] = -i\delta^\beta_\mu I^{\alpha\dot\alpha}_\gamma, & [I^{\alpha\dot\alpha}_\mu, R^{\beta\dot\beta}{}_{\gamma\dot\gamma}] = i\delta^\alpha_\gamma \delta^{\dot\alpha}_{\dot\gamma} I^{\beta\dot\beta}_\mu \end{array} \tag{2.13}$$

and those of superalgebra A_{II} can be written in the form

$$\begin{array}{ll} [K_{\alpha\dot\alpha}, Q_\nu] = 2i\epsilon_{\nu\mu} Q^\mu_{\alpha\dot\alpha}, & [K_{\alpha\dot\alpha}, R^{\beta\dot\beta}{}_{\gamma\dot\gamma}] = -i\delta^\beta_\alpha \delta^{\dot\beta}_{\dot\alpha} K_{\gamma\dot\gamma}, \\ [K_{\alpha\dot\alpha}, T^\mu{}_\nu] = i\delta^\mu_\nu K_{\alpha\dot\alpha}, & \{G_\mu, Q_\nu\} = -2i\epsilon_{\nu\alpha} T^\alpha{}_\mu, \\ \{G_\mu, F^{-\frac{1}{2}}_{\dot\mu}\} = 2ia\epsilon_{\mu\alpha} F^{0\alpha}_{\dot\mu}, & \{G_\mu, Q^\nu_{\alpha\dot\alpha}\} = -i\delta^\nu_\mu K_{\alpha\dot\alpha}, \\ [F^{0\alpha}_{\dot\mu}, T^\beta{}_\gamma] = ia\delta^\beta_\gamma F^{0\alpha}_{\dot\mu} + i\delta^\alpha_\gamma F^{0\beta}_{\dot\mu}, & [F^{0\alpha}_{\dot\mu}, Q_\mu] = i\delta^\alpha_\mu F^{-\frac{1}{2}}_{\dot\mu}, \\ [F^{0\alpha}_{\dot\mu}, R^{\beta\dot\beta}{}_{\gamma\dot\gamma}] = -ia\delta^\beta_\gamma \delta^{\dot\beta}_{\dot\gamma} F^{0\alpha}_{\dot\mu}, & [F^{0\alpha}_{\dot\mu}, F^0_{\dot\alpha\dot\beta}] = -i(\epsilon_{\dot\alpha\dot\mu} F^{0\alpha}_{\dot\beta} + \epsilon_{\dot\beta\dot\mu} F^{0\alpha}_{\dot\alpha}), \\ [F^{\frac{1}{2}}_{\dot\mu}, R^{\alpha\dot\alpha}{}_{\beta\dot\beta}] = -ia\delta^\alpha_\beta \delta^{\dot\alpha}_{\dot\beta} F^{\frac{1}{2}}_{\dot\mu}, & [F^{\frac{1}{2}}_{\dot\mu}, T^\alpha{}_\beta] = i(a+1)\delta^\alpha_\beta F^{\frac{1}{2}}_{\dot\mu}, \\ [G_\mu, T^\alpha{}_\beta] = i\epsilon_{\mu\beta}\epsilon^{\alpha\gamma} G_\gamma, & [F^{\frac{1}{2}}_{\dot\mu}, F^0_{\dot\alpha\dot\beta}] = -i(\epsilon_{\dot\alpha\dot\mu} F^{\frac{1}{2}}_{\dot\beta} + \epsilon_{\dot\beta\dot\mu} F^{\frac{1}{2}}_{\dot\alpha}), \\ \{F^{\frac{1}{2}}_{\dot\mu}, Q_\mu\} = -2i\epsilon_{\mu\alpha} F^{0\alpha}_{\dot\mu}, & \{F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta}, Q_\nu\} = -i\delta^\mu_\nu F^0_{\dot\alpha\dot\beta}, \\ \{F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta}, F^{-\frac{1}{2}}_{\dot\mu}\} = i(\epsilon_{\dot\alpha\dot\mu} F^{0\mu}_{\dot\beta} + \epsilon_{\dot\beta\dot\mu} F^{0\mu}_{\dot\alpha}), & [F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta}, T^\alpha{}_\beta] = i\delta^\mu_\beta F^{\frac{1}{2}\ \alpha}_{\dot\alpha\dot\beta}, \end{array}$$

$$
\begin{aligned}
&[F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta}, F^{0}_{\dot\gamma\dot\delta}] = i(\epsilon_{\dot\alpha\dot\delta}F^{\frac{1}{2}\ \mu}_{\dot\beta\dot\gamma} + \epsilon_{\dot\beta\dot\delta}F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\gamma} + \epsilon_{\dot\alpha\dot\gamma}F^{\frac{1}{2}\ \mu}_{\dot\beta\dot\delta} + \epsilon_{\dot\beta\dot\gamma}F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\delta}), && (2.14)\\
&[F^{1}_{\dot\alpha\dot\beta}, Q_\mu] = 2i\epsilon_{\mu\nu}F^{\frac{1}{2}\ \nu}_{\dot\alpha\dot\beta}, \qquad [F^{1}_{\dot\alpha\dot\beta}, F^{-\frac{1}{2}}_{\dot\mu}] = i\epsilon_{\dot\alpha\dot\mu}F^{\frac{1}{2}}_{\dot\beta} + i\epsilon_{\dot\beta\dot\mu}F^{\frac{1}{2}}_{\dot\alpha},\\
&[F^{1}_{\dot\alpha\dot\beta}, F^{0}_{\dot\gamma\dot\delta}] = i(\epsilon_{\dot\alpha\dot\delta}F^{1}_{\dot\beta\dot\gamma} + \epsilon_{\dot\beta\dot\delta}F^{1}_{\dot\alpha\dot\gamma} + \epsilon_{\dot\alpha\dot\gamma}F^{1}_{\dot\beta\dot\delta} + \epsilon_{\dot\beta\dot\gamma}F^{1}_{\dot\alpha\dot\delta}),\\
&[G_\mu, F^{0\alpha}_{\dot\mu}] = -i(a+1)\delta^{\alpha}_{\mu}F^{\frac{1}{2}}_{\dot\mu}, \qquad [F^{0\alpha}_{\dot\mu}, F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta}] = \tfrac{i}{2}\epsilon^{\mu\alpha}(\epsilon_{\dot\alpha\dot\mu}F^{\frac{1}{2}}_{\dot\beta} + \epsilon_{\dot\beta\dot\mu}F^{\frac{1}{2}}_{\dot\alpha}),\\
&[F^{\frac{1}{2}\ \mu}_{\dot\alpha\dot\beta}, F^{\frac{1}{2}\ \nu}_{\dot\gamma\dot\delta}] = -\tfrac{i}{2}\epsilon^{\mu\nu}(\epsilon_{\dot\alpha\dot\delta}F^{1}_{\dot\beta\dot\gamma} + \epsilon_{\dot\beta\dot\delta}F^{1}_{\dot\alpha\dot\gamma} + \epsilon_{\dot\alpha\dot\gamma}F^{1}_{\dot\beta\dot\delta} + \epsilon_{\dot\beta\dot\gamma}F^{1}_{\dot\alpha\dot\delta}),\\
&[F^{1}_{\dot\alpha\dot\beta}, T^{\alpha}_{\beta}] = i\delta^{\alpha}_{\beta}F^{1}_{\dot\alpha\dot\beta}, \qquad [G_\mu, F^{\frac{1}{2}\ \nu}_{\dot\alpha\dot\beta}] = -i\delta^{\mu}_{\nu}F^{1}_{\dot\alpha\dot\beta}.
\end{aligned}
$$

3 NON-LINEAR REALIZATION OF G

3.1 Non-linear realization of G_0

In what follows we shall not need the generators of A_0 in the explicit coordinate realization (2.7). They will be regarded as abstract and subjected to relations (2.12).

Each element g_0 of supergroup G_0 can be parametrized in the following way convenient for constructing our non-linear realization of G_0:

$$
g_0 = g_0' e^{i(l^{\alpha\beta}M_{\alpha\beta} + \bar l^{\dot\alpha\dot\beta}M_{\dot\alpha\dot\beta})}. \tag{3.1}
$$

Here

$$
g_0' \equiv e^{ix_L^{\mu\dot\mu}P_{L\mu\dot\mu}}\ e^{i\theta^\mu Q_\mu}\ e^{i\bar\phi^{\dot\mu}F^{-\frac{1}{2}}_{\dot\mu}}\ e^{iq^{\alpha\dot\alpha}_{\mu}Q^{\mu}_{\alpha\dot\alpha}}\ e^{ir_{\alpha\dot\alpha}{}^{\beta\dot\beta}\ R^{\alpha\dot\alpha}{}_{\beta\dot\beta}}\ e^{it_\alpha{}^\beta T^\alpha_\beta}\ e^{if^{\dot\alpha\dot\beta}F^0_{\dot\alpha\dot\beta}}, \tag{3.2}
$$

and the group parameters $r_{\alpha\dot\alpha}{}^{\beta\dot\beta}$, t^{β}_{α} and $f^{\dot\alpha\dot\beta}$ are restricted by a condition that g_0' should not contain any generator of the Lorentz group (2.8), i. e. by the condition

$$
\frac{1}{4}r^{(\alpha\beta)} + \frac{1}{2}t^{(\alpha\beta)} + \frac{1}{4}\bar r^{(\alpha\beta)} - \bar f^{(\alpha\beta)} = 0, \tag{3.3}
$$

in which $r^{(\alpha\beta)}$ and $\bar r^{(\alpha\beta)} \equiv (r^{(\dot\alpha\dot\beta)})^+$ are defined by the decomposition

$$
r_{\alpha\dot\alpha}{}^{\beta\dot\beta} = \frac{1}{4}(r^{(\alpha(\dot\alpha}{}_{\beta)\dot\beta)} + \delta^{\beta}_{\alpha}r_{(\dot\alpha}{}^{\dot\beta)} + \delta^{\dot\beta}_{\dot\alpha}\bar r_{(\alpha}{}^{\beta)} + \delta^{\beta}_{\alpha}\delta^{\dot\beta}_{\dot\alpha}r). \tag{3.4}
$$

The transformation properties of the group parameters $x_L^{\mu\dot\mu}$, θ^μ, $\bar\phi^{\dot\mu}$, $q^{\alpha\dot\alpha}_{\mu}$, $r_{\alpha\dot\alpha}{}^{\beta\dot\beta}$, $t_\alpha{}^\beta$, $f^{\dot\alpha\dot\beta}$ with respect to G_0 follow from the group multiplication law

$$
g_0'' g_0' = g_0''', \qquad g_0', g_0'', g_0''' \in G_0, \tag{3.5}
$$

in which element g_0' given by (3.2) is multiplied by a fixed element g_0'' and both g_0' and the resulting element g_0''' are expressed in parametrization (3.1).

Now we shall introduce Cartan 1–forms $\omega_{x_L}{}^{\mu\dot\mu}$, $\omega_\theta{}^\mu$, $\omega_{\bar\phi}{}^{\dot\mu}$, $\omega_q{}^{\alpha\dot\alpha}_{\ \mu}$, $\omega_r{}_{\alpha\dot\alpha}^{\ \beta\dot\beta}$, $\omega_t{}_{\alpha}^{\ \beta}$ and $\omega_f{}^{\dot\alpha\dot\beta}$, which are left-invariant w.r.t. G_0 following the general routine [1,7,8, 9], i.e. as projections of $g_0'^{-1}dg_0'$ into the complete set of the corresponding generators

$$
\begin{aligned}
g_0'^{-1}dg_0' \ &=\ i\omega_{x_L}{}^{\mu\dot\mu}P_{\mu\dot\mu} + i\omega_\theta{}^\mu Q_\mu + i\omega_{\bar\phi}{}^{\dot\mu}F^{-\frac{1}{2}}_{\dot\mu} + i\omega_q{}^{\alpha\dot\alpha}_{\ \mu}Q^{\mu}_{\alpha\dot\alpha} + \\
&+\ i\omega_r{}_{\alpha\dot\alpha}^{\ \beta\dot\beta}R^{\alpha\dot\alpha}{}_{\beta\dot\beta} + i\omega_t{}_{\alpha}^{\ \beta}T^{\alpha}_{\beta} + i\omega_f{}^{\dot\alpha\dot\beta}F^{0}_{\dot\alpha\dot\beta}.
\end{aligned} \tag{3.6}
$$

Their explicit expressions can be found more easily by using the formulae

$$e^{-izZ}de^{izZ} = \frac{1-e^{-i\mathrm{Ad}_{zZ}}}{\mathrm{Ad}_{zZ}}(dzZ) \tag{3.7}$$

and

$$e^{-izZ}dxXe^{izZ} = e^{-i\mathrm{Ad}_{zZ}}(dxX), \tag{3.8}$$

where the action of operator Ad_{zZ} on an arbitrary element Y of the superalgebra (containing the element zZ) is defined by

$$\mathrm{Ad}_{zZ}(Y) \equiv [zZ, Y].$$

We obtain

$$\begin{aligned}
\omega_{x_L}{}^{\nu\dot\nu} &= (e^r)_{\mu\dot\mu}{}^{\nu\dot\nu}(dx^{\mu\dot\mu} - q^{\mu\dot\mu}_{\alpha}d\theta^{\alpha}), \\
\omega_{\theta}{}^{\nu} &= (e^t)_{\mu}{}^{\nu}d\theta^{\mu}, \\
\omega_{\bar\phi}{}^{\dot\mu} &= e^{ar_{\alpha\dot\alpha}{}^{\alpha\dot\alpha}}e^{-at_{\alpha}{}^{\alpha}}(e^{-f_{()}})^{\dot\mu}{}_{\dot\omega}d\bar\phi^{\dot\omega}, \\
\omega_{q}{}^{\alpha\dot\alpha}{}_{\mu} &= (e^r)_{\beta\dot\beta}{}^{\alpha\dot\alpha}(e^{-t})_{\mu}{}^{\nu}dq^{\beta\dot\beta}_{\nu}, \\
\omega_{r}{}^{\beta\dot\beta}{}_{\alpha\dot\alpha} &= -(\delta^{\beta}_{\gamma}\delta^{\dot\beta}_{\dot\gamma}\delta^{\delta}_{\alpha}\delta^{\dot\delta}_{\dot\alpha} - (e^r)_{\gamma\dot\gamma}{}^{\beta\dot\beta}(e^{-r})_{\alpha\dot\alpha}{}^{\delta\dot\delta}) \times \\
&\times\ (\delta^{\omega}_{\delta}\delta^{\dot\omega}_{\dot\delta}r_{\kappa\dot\kappa}{}^{\gamma\dot\gamma} - r_{\delta\dot\delta}{}^{\omega\dot\omega}\delta^{\gamma}_{\kappa}\delta^{\dot\gamma}_{\dot\kappa})^{-1}dr_{\omega\dot\omega}{}^{\kappa\dot\kappa}, \\
\omega_{t}{}^{\beta}{}_{\alpha} &= -(\delta^{\beta}_{\gamma}\delta^{\delta}_{\alpha} - (e^t)_{\gamma}{}^{\beta}(e^{-t})_{\alpha}{}^{\delta})(t_{\delta}{}^{\omega}\delta^{\gamma}_{\kappa} - \delta^{\omega}_{\delta}t_{\kappa}{}^{\gamma})^{-1}dt_{\omega}{}^{\kappa}, \\
\omega_{f}{}^{\dot\alpha\dot\beta} &= -(\delta^{\dot\alpha}_{\dot\gamma}\delta^{\dot\beta}_{\dot\delta} - (e^{f_{()}})^{\dot\alpha}{}_{\dot\gamma}(e^{f_{()}})^{\dot\beta}{}_{\dot\delta})(f^{(\dot\delta\dot\sigma)}\epsilon_{\dot\sigma\dot\mu}\delta^{\dot\gamma}_{\dot\nu} + f^{(\dot\gamma\dot\omega)}\epsilon_{\dot\omega\dot\nu}\delta^{\dot\delta}_{\dot\mu})^{-1}df^{\dot\mu\dot\nu}.
\end{aligned} \tag{3.9}$$

Here we have used the abbreviations

$$\begin{aligned}
(e^t)_{\mu}{}^{\nu} &\equiv \sum_n \frac{1}{n!}t_{\mu}{}^{\alpha}t_{\alpha}{}^{\beta}\dots t_{\omega}{}^{\nu}, \\
(e^r)_{\mu\dot\mu}{}^{\nu\dot\nu} &\equiv \sum_n \frac{1}{n!}r_{\mu\dot\mu}{}^{\alpha\dot\alpha}r_{\alpha\dot\alpha}{}^{\beta\dot\beta}\dots r_{\omega\dot\omega}{}^{\nu\dot\nu} \\
\text{and}\quad (e^{f_{()}})^{\dot\mu}{}_{\dot\omega} &\equiv \sum_n \frac{1}{n!}f^{(\dot\mu\dot\alpha)}\epsilon_{\dot\alpha\dot\beta}f^{(\dot\beta\dot\gamma)}\epsilon_{\dot\gamma\dot\delta}\dots f^{(\dot\tau\dot\sigma)}\epsilon_{\dot\sigma\dot\omega}.
\end{aligned} \tag{3.10}$$

The expressions $(\delta^{\omega}_{\delta}\delta^{\dot\omega}_{\dot\delta}r_{\kappa\dot\kappa}{}^{\gamma\dot\gamma} - r_{\delta\dot\delta}{}^{\omega\dot\omega}\delta^{\gamma}_{\kappa}\delta^{\dot\gamma}_{\dot\kappa})^{-1}$, $(\delta^{\omega}_{\delta}t_{\kappa}{}^{\gamma} - t_{\delta}{}^{\omega}\delta^{\gamma}_{\kappa})^{-1}$ and $(f^{(\dot\delta\dot\sigma)}\epsilon_{\dot\sigma\dot\mu}\delta^{\dot\gamma}_{\dot\nu} + f^{(\dot\gamma\dot\omega)}\epsilon_{\dot\omega\dot\nu}\delta^{\dot\delta}_{\dot\mu})^{-1}$ denote matrices inverse to the matrices $(\delta^{\omega}_{\delta}\delta^{\dot\omega}_{\dot\delta}r_{\kappa\dot\kappa}{}^{\gamma\dot\gamma} - r_{\delta\dot\delta}{}^{\omega\dot\omega}\delta^{\gamma}_{\kappa}\delta^{\dot\gamma}_{\dot\kappa})$, $(\delta^{\omega}_{\delta}t_{\kappa}{}^{\gamma} - t_{\delta}{}^{\omega}\delta^{\gamma}_{\kappa})$ and $(f^{(\dot\delta\dot\sigma)}\epsilon_{\dot\sigma\dot\mu}\delta^{\dot\gamma}_{\dot\nu} + f^{(\dot\gamma\dot\omega)}\epsilon_{\dot\omega\dot\nu}\delta^{\dot\delta}_{\dot\mu})$, respectively.

Let as note that the Cartan forms $\omega_{r}{}^{\beta\dot\beta}{}_{\alpha\dot\alpha}$, $\omega_{t}{}^{\beta}{}_{\alpha}$ and $\omega_{f}{}^{\dot\alpha\dot\beta}$ in (3.9) contain the Cartan forms corresponding to the Lorentz transformations, which transform inhomogeneously w.r.t. G_0. Their homogeneously transforming combinations are

$$\frac{1}{4}\omega_{r}{}^{(\alpha\dot\varrho}{}_{\beta)\dot\varrho} - \frac{1}{2}\omega_{t}{}^{(\alpha}{}_{\beta)}, \tag{3.11}$$

$$\frac{1}{4}\omega_{r}{}^{\varrho(\dot\alpha}{}_{\varrho\dot\beta)} + \omega_{f}{}^{(\dot\alpha}{}_{\dot\beta)} \tag{3.12}$$

and

$$\frac{1}{4}\omega_{r}{}^{(\alpha\dot\varrho}{}_{\beta)\dot\varrho} + \frac{1}{2}\omega_{t}{}^{(\alpha}{}_{\beta)} - (\frac{1}{4}\omega_{r}{}^{\varrho(\dot\alpha}{}_{\varrho\dot\beta)})^{+} + (\omega_{f}{}^{(\dot\alpha}{}_{\dot\beta)})^{+}. \tag{3.13}$$

In what follows the left–invariant Cartan 1–forms (3.9) will be used to eliminate some Goldstone superfields associated with various group generators. More precisely, the group parameters associated with $P^{\mu\dot{\mu}} = \frac{1}{2}(P_L^{\mu\dot{\mu}} + P_R^{\mu\dot{\mu}}) = \frac{1}{2}(P_L^{\mu\dot{\mu}} + \bar{P}_L^{\mu\dot{\mu}})$ will be identified with the "bosonic" coordinates $x^{\mu\dot{\mu}}$ of the real superspace $R^{4/4}$ defined in (2.2), while the parameters related to $\frac{i}{2}(P_L^{\mu\dot{\mu}} - P_R^{\mu\dot{\mu}})$ will be identified with the Goldstone superfield $H^{\mu\dot{\mu}}(x,\theta,\bar{\theta})$. Moreover, the group parameters $\theta^{\mu} = \theta_L^{\mu}$ and $\theta^{\dot{\mu}} = \bar{\theta}_R^{\dot{\mu}} = (\theta_L^{\mu})^{+}$ will be interpreted as the Grassmannian coordinates of superspace $R^{4/4}$. (It is worth mentioning that $x_L^{\alpha\dot{\alpha}}$, θ^{μ} and $\bar{\phi}^{\mu}$ transform w.r.t. G_0 in the same way as w.r.t. representation (2.7)). The remaining group parameters $\bar{\phi}^{\mu}$, $q_{\mu}^{\alpha\dot{\alpha}}$, $r_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}}$, $t_{\alpha}{}^{\beta}$ and $f^{\dot{\alpha}\beta}$ will be identified with the corresponding Goldstone superfields $\bar{\phi}^{\mu}(x,\theta,\bar{\theta})$, $q_{\mu}^{\alpha\dot{\alpha}}(x,\theta,\bar{\theta})$, $r_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}}(x,\theta,\bar{\theta})$, $t_{\alpha}{}^{\beta}(x,\theta,\bar{\theta})$ and $f^{\dot{\alpha}\beta}(x,\theta,\bar{\theta})$ on $R^{4/4}$, so that their transformation properties w.r.t. supergroup G will be the same as the corresponding group parameters. It will be shown in Section 4, that the superfields can be expressed in terms of $H^{\mu\dot{\mu}}$, H^{μ}, $H^{\dot{\mu}}$ by exploiting the inverse Higgs effect and thus eliminated from the theory. However, before doing that let us first specify transformation properties of Cartan forms (3.9) with respect to G_I and to G_{II} and then enumerate from them those, which are simultaneously invariant w.r.t. both G_I and G_{II}.

3.2 Non-linear realization of G_I

Any element g_I of supergroup G_I can be parametrized in the following form:

$$g_I = g_0' \, e^{i \, i_{\alpha\dot{\alpha}}^{\mu} I_{\mu}^{\alpha\dot{\alpha}}} e^{i(l^{\alpha\beta}M_{\alpha\beta} + \bar{l}^{\dot{\alpha}\dot{\beta}}M_{\dot{\alpha}\dot{\beta}})}, \tag{3.14}$$

with g_0' defined in (3.2). Since generators $I_{\mu}^{\alpha\dot{\alpha}}$ together with the Lorentz generators $M_{\alpha\beta}$, $M_{\dot{\alpha}\dot{\beta}}$ form a subalgebra of A_I, it is possible to realize them on the coordinates $x^{\alpha\dot{\alpha}}$, θ^{μ}, $\bar{\theta}^{\dot{\mu}}$ and the Goldstone superfields $\bar{\phi}^{\mu}$, $q_{\mu}^{\alpha\dot{\alpha}}$, $r_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}}$, $t_{\alpha}{}^{\beta}$ and $f^{\dot{\alpha}\beta}$ via transformations which follow from the group law in parametrization (3.14):

$$g_I g_0' = \Big(e^{ix'^{\mu\dot{\mu}}_L P_{L\,\mu\dot{\mu}}} e^{i\theta'^{\mu}Q_{\mu}} e^{i\bar{\phi}'^{\mu}F_{\mu}^{-\frac{1}{2}}} e^{iq'^{\alpha\dot{\alpha}}_{\mu}Q^{\mu}_{\alpha\dot{\alpha}}} e^{ir'_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}} R^{\alpha\dot{\alpha}}{}_{\beta\dot{\beta}}} \times \tag{3.15}$$

$$\times \quad e^{it'_{\alpha}{}^{\beta}T^{\alpha}{}_{\beta}} e^{if'^{\dot{\alpha}\beta}F^{0}_{\dot{\alpha}\beta}} \Big) e^{i \, i'^{\mu}_{\alpha\dot{\alpha}} I^{\alpha\dot{\alpha}}_{\mu}} e^{i(l'^{\alpha\beta}M_{\alpha\beta} + \bar{l}'^{\dot{\alpha}\dot{\beta}}M_{\dot{\alpha}\dot{\beta}})}, \tag{3.16}$$

where $g_I \in G_I$ and g_0' is defined in (3.2).

Thus we can easily see that the coordinates and the Goldstone superfields transform w.r.t. infinitesimal transformations $i_{\alpha\dot{\alpha}}^{\mu} I_{\mu}^{\alpha\dot{\alpha}}$ in the following way:

$$\begin{aligned}
\delta x_L^{\gamma\dot{\gamma}} &= 0 \\
\delta\theta^{\mu} &= -x_L^{\alpha\dot{\alpha}} i^{\mu}_{\alpha\dot{\alpha}} \\
\delta\bar{\phi}^{\mu} &= 0 \\
\delta q_{\mu}^{\alpha\dot{\alpha}} &= q_{\mu}^{\beta\dot{\beta}} q_{\nu}^{\alpha\dot{\alpha}} i^{\nu}_{\beta\dot{\beta}} \\
\delta r_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}} &= \rho_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} - \frac{1}{2}\mathrm{tgh}[\frac{1}{2}(r_{\alpha\dot{\alpha}}{}^{\gamma\dot{\gamma}}\delta_{\varsigma}^{\beta}\delta_{\dot{\delta}}^{\dot{\beta}} - r_{\delta\dot{\delta}}{}^{\beta\dot{\beta}}\delta_{\alpha}^{\gamma}\delta_{\dot{\alpha}}^{\dot{\gamma}})] \times
\end{aligned} \tag{3.17}$$

$$\times \left[\delta^{\dot\delta}_{\dot\gamma}\left(\frac{1}{8}\rho^{\delta)\dot\rho}_{(\gamma\dot\rho} + \frac{1}{4}\tau^{\delta)}_{(\gamma} + \frac{1}{8}\bar\rho^{\delta)\dot\rho}_{(\gamma\dot\rho}\right) + \delta^{\delta}_{\gamma}\left(\frac{1}{8}\bar\rho^{\rho\dot\delta)}_{\rho(\dot\gamma} + \frac{1}{4}\bar\tau^{\dot\delta)}_{(\dot\gamma} + \frac{1}{8}\rho^{\rho\dot\delta)}_{\rho(\dot\gamma}\right)\right],$$

$$\begin{aligned}
\delta t_{\alpha}{}^{\beta} &= \tau^{\beta}_{\alpha} - \frac{1}{2}\mathrm{tgh}[\frac{1}{2}(t_{\alpha}{}^{\gamma}\delta^{\beta}_{\delta} - t_{\delta}{}^{\beta}\delta^{\gamma}_{\alpha})]\left(\frac{1}{8}\rho^{\delta)\dot\rho}_{(\gamma\dot\rho} + \frac{1}{4}\tau^{\delta)}_{(\gamma} + \frac{1}{8}\bar\rho^{\delta)\dot\rho}_{(\gamma\dot\rho}\right), \\
\delta f^{\dot\beta\dot\gamma} &= -\frac{1}{4}\mathrm{tgh}[\frac{1}{2}(f^{(\dot\beta\dot\rho)}\epsilon_{\dot\rho\dot\delta}\delta^{\dot\gamma}_{\dot\epsilon} + f^{(\dot\gamma\dot\rho)}\epsilon_{\dot\rho\dot\epsilon}\delta^{\dot\beta}_{\dot\delta})]\epsilon^{\dot\epsilon\dot\alpha}\left(\frac{1}{8}\bar\rho^{\rho\dot\delta)}_{\rho(\dot\alpha} + \frac{1}{4}\bar\tau^{\dot\delta)}_{(\dot\alpha} + \frac{1}{8}\rho^{\rho\dot\delta)}_{\rho(\dot\alpha}\right),
\end{aligned}$$

where

$$\begin{aligned}
\rho^{\beta\dot\beta}_{\alpha\dot\alpha} &\equiv \left((e^{-r})_{\gamma\dot\gamma}{}^{\beta\dot\beta}(e^{r})_{\alpha\dot\alpha}{}^{\delta\dot\delta} - \delta^{\beta}_{\gamma}\delta^{\dot\beta}_{\dot\gamma}\delta^{\delta}_{\alpha}\delta^{\dot\delta}_{\dot\alpha}\right)^{-1}\left(\delta^{\kappa}_{\delta}\delta^{\dot\kappa}_{\dot\delta}r_{\omega\dot\omega}{}^{\gamma\dot\gamma} - \delta^{\gamma}_{\omega}\delta^{\dot\gamma}_{\dot\omega}r_{\delta\dot\delta}{}^{\kappa\dot\kappa}\right)q^{\omega\dot\omega}_{\mu}i^{\mu}_{\kappa\dot\kappa} \\
\text{and}\quad \tau^{\beta}_{\alpha} &\equiv \left((e^{-t})_{\gamma}{}^{\beta}(e^{t})_{\alpha}{}^{\delta} - \delta^{\beta}_{\gamma}\delta^{\delta}_{\alpha}\right)^{-1}(\delta^{\kappa}_{\delta}t_{\omega}{}^{\gamma} - \delta^{\gamma}_{\omega}t_{\delta}{}^{\kappa})\,q^{\mu\dot\mu}_{\kappa}i^{\omega}_{\mu\dot\mu}.
\end{aligned}$$

Except for $x^{\mu\dot\mu}_{L}$, θ^{μ} and $\bar\phi^{\dot\mu}$ these are in fact non-linear transformations because of the comutators $[I^{\alpha\dot\alpha}_{\mu}, T^{\beta}{}_{\gamma}] = -i\delta^{\beta}_{\mu}I^{\alpha\dot\alpha}_{\gamma}$ and $[I^{\alpha\dot\alpha}_{\mu}, R^{\beta\dot\beta}{}_{\gamma\dot\gamma}] = i\delta^{\alpha}_{\gamma}\delta^{\dot\alpha}_{\dot\gamma}I^{\beta\dot\beta}_{\mu}$.

Cartan forms (3.9) are transforming according to the adjoint representation of $I^{\alpha\dot\alpha}_{\mu}$, $M_{\alpha\beta}$ and $M_{\dot\alpha\dot\beta}$, i.e.

$$\begin{aligned}
\delta\omega_{x_L}{}^{\mu\dot\mu} &= l'^{(\mu}{}_{\nu)}\omega_{x_L}{}^{\nu\dot\mu} + \bar l'^{(\dot\mu}{}_{\dot\nu)}\omega_{x_L}{}^{\mu\dot\nu}, \\
\delta\omega_{\theta}{}^{\mu} &= -i'^{\mu}{}_{\alpha\dot\alpha}\omega_{x_L}{}^{\alpha\dot\alpha} + l'^{(\mu}{}_{\nu)}\omega_{\theta}{}^{\nu}, \\
\delta\omega_{\bar\phi}{}^{\dot\mu} &= \bar l'^{(\dot\mu}{}_{\dot\nu)}\omega_{\bar\phi}{}^{\dot\nu}, \\
\delta\omega_{q}{}^{\alpha\dot\alpha}{}_{\mu} &= l'^{(\alpha}{}_{\beta)}\omega_{q}{}^{\beta\dot\alpha}{}_{\mu} + \bar l'^{(\dot\alpha}{}_{\dot\beta)}\omega_{q}{}^{\alpha\dot\beta}{}_{\mu} - l'^{(\nu}{}_{\mu)}\omega_{q}{}^{\alpha\dot\alpha}{}_{\nu}, \qquad (3.18) \\
\delta\omega_{r}{}_{\alpha\dot\alpha}{}^{\beta\dot\beta} &= +i'^{\mu}{}_{\alpha\dot\alpha}\omega_{q}{}^{\beta\dot\beta}{}_{\mu} + l'^{(\beta}{}_{\gamma)}\omega_{r}{}_{\alpha\dot\alpha}{}^{\gamma\dot\beta} + \bar l'^{(\dot\beta}{}_{\dot\gamma)}\omega_{r}{}_{\alpha\dot\alpha}{}^{\beta\dot\gamma} - l'^{(\gamma}{}_{\alpha)}\omega_{r}{}_{\gamma\dot\alpha}{}^{\beta\dot\beta} - \bar l'^{(\dot\gamma}{}_{\dot\alpha)}\omega_{r}{}_{\alpha\dot\gamma}{}^{\beta\dot\beta}, \\
\delta\omega_{t}{}_{\alpha}{}^{\beta} &= -i'^{\beta}{}_{\mu\dot\mu}\omega_{q}{}^{\mu\dot\mu}{}_{\alpha} + l'^{(\beta}{}_{\gamma)}\omega_{t}{}_{\alpha}{}^{\gamma} - l'^{(\gamma}{}_{\alpha)}\omega_{t}{}_{\gamma}{}^{\beta}, \\
\delta\omega_{f}{}^{\dot\alpha\dot\beta} &= +\bar l'^{(\dot\alpha}{}_{\dot\gamma)}\omega_{f}{}^{\dot\gamma\dot\beta} + \bar l'^{(\dot\beta}{}_{\dot\gamma)}\omega_{f}{}^{\dot\alpha\dot\gamma}.
\end{aligned}$$

3.3 Non–linear realization of G_{II}

Any element g_{II} of the supergroup G_{II} can be parametrized in the form

$$g_{II} = g'_0\, e^{in_{\alpha}{}^{\dot\mu}F^{0\alpha}_{\dot\mu}}\, e^{ik^{\alpha\dot\alpha}K_{\alpha\dot\alpha}}\, e^{ig^{\mu}G_{\mu}}\, e^{io^{\dot\mu}F^{\frac{1}{2}}_{\dot\mu}}\, e^{ip^{\dot\mu\dot\nu}_{\mu}F^{\frac{1}{2}}{}_{\dot\alpha\dot\beta}^{\mu}}\, e^{is^{\dot\mu\dot\nu}F^{1}_{\dot\alpha\dot\beta}}e^{i(l^{\alpha\beta}M_{\alpha\beta}+\bar l^{\dot\alpha\dot\beta}M_{\dot\alpha\dot\beta})}, \qquad (3.19)$$

where g'_0 is defined in (3.2). Since the generators $F^{0\alpha}_{\dot\mu}$, $K_{\alpha\dot\alpha}$, G_{μ}, $F^{\frac{1}{2}}_{\dot\mu}$, $F^{\frac{1}{2}}{}^{\mu}_{\dot\alpha\dot\beta}$, $F^{1}_{\dot\alpha\dot\beta}$, $M_{\alpha\beta}$ and $M_{\dot\alpha\dot\beta}$ form a subalgebra of A_{II}, their realization and transformational properties can be obtained analogously as in the previous case, i.e. from the group multiplication law

$$g_{II}\;\cdot\left(e^{ix^{\mu\dot\mu}_{L}P_{L\,\mu\dot\mu}}\, e^{i\theta^{\mu}Q_{\mu}}\, e^{i\bar\phi^{\dot\mu}F^{-\frac{1}{2}}_{\dot\mu}}\, e^{iq^{\alpha\dot\alpha}_{\mu}Q^{\mu}_{\alpha\dot\alpha}}\, e^{ir_{\alpha\dot\alpha}{}^{\beta\dot\beta}R^{\alpha\dot\alpha}{}_{\beta\dot\beta}}\, e^{it_{\alpha}{}^{\beta}T^{\alpha}{}_{\beta}}\, e^{if^{\dot\alpha\dot\beta}F^{0}_{\dot\alpha\dot\beta}}\right) =$$

$$= \left(e^{ix'^{\mu\dot\mu}_L P_{L\,\mu\dot\mu}} e^{i\theta'^{\mu} Q_\mu} e^{i\bar\phi'^{\dot\mu} F^{-\frac{1}{2}}_{\dot\mu}} e^{iq'^{\alpha\dot\alpha}_{\mu} Q^{\mu}_{\alpha\dot\alpha}} e^{ir'_{\alpha\dot\alpha}{}^{\beta\dot\beta} R^{\alpha\dot\alpha}{}_{\beta\dot\beta}} e^{it'_{\alpha}{}^{\beta} T^{\alpha}{}_{\beta}} e^{if'^{\dot\alpha\dot\beta} F^{0}_{\dot\alpha\dot\beta}} \right) \times \quad (3.20)$$

$$\times \left(e^{in'_{\alpha}{}^{\dot\mu} F^{0\alpha}_{\dot\mu}} e^{ik'^{\alpha\dot\alpha} K_{\alpha\dot\alpha}} e^{ig'^{\mu} G_\mu} e^{io'^{\dot\mu} F^{\frac{1}{2}}_{\dot\mu}} e^{ip'^{\dot\mu\dot\nu}_{\mu} F^{\frac{1}{2}}{}^{\mu}_{\dot\alpha\dot\beta}} e^{is'^{\dot\mu\dot\nu} F^{1}_{\dot\alpha\dot\beta}} \right) e^{i(l'^{\alpha\beta} M_{\alpha\beta} + \bar l'^{\dot\alpha\dot\beta} M_{\dot\alpha\dot\beta})}.$$

From here we obtain the transformation properties of coordinates $x^{\alpha\dot\alpha}$, θ^μ and $\bar\phi^{\dot\mu}$ and of the Goldstone superfields $\bar\phi^{\dot\mu}$, $q^{\alpha\dot\alpha}_{\mu}$, $r_{\alpha\dot\alpha}{}^{\beta\dot\beta}$, $t_\alpha{}^{\beta}$ and $f^{\dot\alpha\dot\beta}$ with respect to infinitisimal transformations $n_\alpha{}^{\dot\mu}$, $k^{\alpha\dot\alpha}$, g^μ, $o^{\dot\mu}$, $p^{\dot\mu\dot\nu}_{\mu}$ and $s^{\dot\mu\dot\nu}$. They are of the form

$$\begin{aligned}
\delta x^{\gamma\dot\gamma}_L &= -\theta^2 k^{\gamma\dot\gamma} \\
\delta\theta^\mu &= -\theta^2 g^\mu \\
\delta\bar\phi^{\dot\mu} &= -\theta^\nu n_\nu{}^{\dot\mu} + \theta^2 o^{\dot\mu} + \epsilon_{\dot\alpha\dot\beta}\bar\phi^{\dot\beta}(\theta^\nu p^{(\dot\alpha\dot\mu)}_{\nu} + \theta^2 s^{(\dot\alpha\dot\mu)}) \\
\delta q^{\alpha\dot\alpha}_{\mu} &= 2\epsilon_{\mu\nu}\theta^\nu k^{\alpha\dot\alpha} + 2q^{\alpha\dot\alpha}_{\nu}\epsilon_{\mu\beta}\theta^\beta g^\nu \\
\delta r_{\alpha\dot\alpha}{}^{\beta\dot\beta} &= -\frac{1}{2}\mathrm{tgh}[\frac{1}{2}(r_{\alpha\dot\alpha}{}^{\gamma\dot\gamma}\delta^\beta_\delta\delta^{\dot\beta}_{\dot\delta} - r_{\delta\dot\delta}{}^{\beta\dot\beta}\delta^\gamma_\alpha\delta^{\dot\gamma}_{\dot\alpha})] \times \\
&\quad \times \left[\delta^{\dot\delta}_{\dot\gamma}\left(\frac{1}{4}\tau^{\delta)}_{(\gamma} - \frac{1}{2}\bar\xi^{\delta)}_{(\gamma}\right) + \delta^{\delta}_{\gamma}\left(\frac{1}{4}\bar\tau^{\dot\delta)}_{(\dot\gamma} - \frac{1}{2}\xi^{\dot\delta}_{\dot\gamma}\right)\right], \\
\delta t_\alpha{}^{\beta} &= \tau^\beta_\alpha - \frac{1}{2}\mathrm{tgh}[\frac{1}{2}(t_\alpha{}^{\gamma}\delta^\beta_\delta - t_\delta{}^{\beta}\delta^\gamma_\alpha)]\left(\frac{1}{4}\tau^{\delta)}_{(\gamma} - \frac{1}{2}\bar\xi^{\delta)}_{(\gamma}\right), \\
\delta f^{\dot\beta\dot\gamma} &= -\frac{1}{4}\mathrm{tgh}[\frac{1}{2}(f^{(\dot\beta\dot\rho)}\epsilon_{\dot\rho\dot\delta}\delta^{\dot\gamma}_{\dot\epsilon} + f^{(\dot\gamma\dot\rho)}\epsilon_{\dot\rho\dot\epsilon}\delta^{\dot\beta}_{\dot\delta})]\epsilon^{\dot\epsilon\dot\alpha}\left(\frac{1}{4}\bar\tau^{\dot\delta)}_{(\dot\alpha} - \frac{1}{2}\xi^{\dot\delta}_{\dot\alpha}\right),
\end{aligned} \quad (3.21)$$

where

$$\begin{aligned}
\xi^{\dot\alpha\dot\beta} &\equiv \left((e^{-f()})_{\dot\gamma}{}^{\dot\alpha}(e^{-f()})_{\dot\delta}{}^{\dot\beta} - \delta^{\dot\alpha}_{\dot\gamma}\delta^{\dot\beta}_{\dot\delta}\right)^{-1}\left(\delta^{\dot\gamma}_{\dot\omega}f^{(\dot\delta\dot\rho)}\epsilon_{\dot\rho\dot\epsilon} + \delta^{\dot\delta}_{\dot\epsilon}f^{(\dot\gamma\dot\rho)}\epsilon_{\dot\rho\dot\omega}\right)(\theta^\nu p^{\dot\epsilon\dot\omega}_{\nu} + \theta^2 s^{\dot\epsilon\dot\omega}), \\
\text{and } \tau^\beta_\alpha &\equiv -\left((e^{-t})_{\gamma}{}^{\beta}(e^{t})_{\alpha}{}^{\delta} - \delta^\beta_\gamma\delta^\delta_\alpha\right)^{-1}(\delta^\kappa_\delta t_\omega{}^{\gamma} - \delta^\gamma_\omega t_\delta{}^{\kappa})\,2\epsilon_{\kappa\nu}\theta^\nu g^\omega.
\end{aligned}$$

The corresponding transformation properties of the Cartan forms (3.9) are given by

$$\begin{aligned}
\delta\omega_{x_L}{}^{\mu\dot\mu} &= l'^{(\mu}_{\ \nu)}\omega_{x_L}{}^{\nu\dot\mu} + \bar l'^{(\dot\mu}_{\ \dot\nu)}\omega_{x_L}{}^{\mu\dot\nu}, \\
\delta\omega_\theta{}^{\mu} &= +l'^{(\mu}_{\ \nu)}\omega_\theta{}^{\nu}, \\
\delta\omega_{\bar\phi}{}^{\dot\mu} &= -n'_\alpha{}^{\dot\mu}\omega_\theta{}^{\alpha} + \bar l'^{(\dot\mu}_{\ \dot\nu)}\omega_{\bar\phi}{}^{\dot\nu}, \\
\delta\omega_{q}{}^{\alpha\dot\alpha}_{\mu} &= +2\epsilon_{\mu\nu}k'^{\alpha\dot\alpha}\omega_\theta{}^{\nu} + l'^{(\alpha}_{\ \beta)}\omega_q{}^{\beta\dot\alpha}_{\mu} + \bar l'^{(\dot\alpha}_{\ \dot\beta)}\omega_q{}^{\alpha\dot\beta}_{\mu} - l'^{(\nu}_{\ \mu)}\omega_q{}^{\alpha\dot\alpha}_{\nu}, \\
\delta\omega_{r}{}_{\alpha\dot\alpha}{}^{\beta\dot\beta} &= +l'^{(\beta}_{\ \gamma)}\omega_r{}_{\alpha\dot\alpha}{}^{\gamma\dot\beta} + \bar l'^{(\dot\beta}_{\ \dot\gamma)}\omega_r{}_{\alpha\dot\alpha}{}^{\beta\dot\gamma} - l'^{(\gamma}_{\ \alpha)}\omega_r{}_{\gamma\dot\alpha}{}^{\beta\dot\beta} - \bar l'^{(\dot\gamma}_{\ \dot\alpha)}\omega_r{}_{\alpha\dot\gamma}{}^{\beta\dot\beta}, \\
\delta\omega_{t}{}_{\alpha}{}^{\beta} &= -2\epsilon_{\alpha\mu}g'^{\beta}\omega_\theta{}^{\mu} + l'^{(\beta}_{\ \gamma)}\omega_t{}_{\alpha}{}^{\gamma} - l'^{(\gamma}_{\ \alpha)}\omega_t{}_{\gamma}{}^{\beta}, \\
\delta\omega_f{}^{\dot\alpha\dot\beta} &= +p'^{\dot\alpha\dot\beta}_{\mu}\omega_\theta{}^{\mu} + \bar l'^{(\dot\alpha}_{\ \dot\gamma)}\omega_f{}^{\dot\gamma\dot\beta} + \bar l'^{(\dot\beta}_{\ \dot\gamma)}\omega_f{}^{\dot\alpha\dot\gamma}.
\end{aligned} \quad (3.22)$$

4 ELIMINATION OF SUPERFLUOUS GOLDSTONE SUPERFIELDS

4.1 Covariant elimination of Goldstone superfields associated with A_0

Now we shall express all Goldstone superfields introduced in the previous section in terms of those defined in (2.3) by imposing appropriate constraints invariant under G on the corresponding Cartan superforms (the inverse Higgs effect). Thus we shall eliminate all the Goldstone superfields (except $H^{\alpha\dot\alpha}(x,\theta,\bar\theta)$, $H^{\alpha}(x,\theta,\bar\theta)$ and $\bar H^{\dot\alpha}(x,\theta,\bar\theta)$) from the theory. Further, we shall use superfield $\bar H^{\dot\alpha} = \bar\phi^{\dot\alpha} - \bar\theta^{\dot\alpha}$ instead of $\bar\phi^{\dot\alpha}$.

Denoting the real and imaginary part of Cartan form $\omega_{x_L}^{\ \alpha\dot\alpha}$ by $\omega_x^{\ \alpha\dot\alpha}$ and $\omega_H^{\alpha\dot\alpha}$ respectively, the complex conjugate of $\omega_\theta^{\ \alpha}$ by $\bar\omega_{\bar\theta}^{\ \dot\alpha}$, the covariant projection of Cartan form ω_X onto ω_Y by symbol $\frac{\omega_X}{\omega_Y}$ and using the explicit expressions of the Cartan forms given in (3.9) we can figure out all invariants with respect to both G_I and G_{II}. We can find that there are four invariants w.r.t. G_0, which are simultaneously invariant with respect to G_I and G_{II} namely

$$\frac{\omega_H^{\alpha\dot\alpha}}{\omega_\theta^{\ \mu}},\ \frac{\omega_{\rm H}^{\ \alpha\dot\alpha}}{\bar\omega_{\bar\theta}^{\ \dot\mu}},\ \frac{\omega_{\rm H}^{\ \dot\alpha}}{\bar\omega_{\bar\theta}^{\ \dot\mu}} \text{ and } \frac{\omega_{{\rm q}\ \nu}^{\ \alpha\dot\alpha}}{\bar\omega_{\bar\theta}^{\ \dot\mu}}.$$

They will be used to eliminate Goldstone superfields $q_\mu^{\alpha\dot\alpha}$, $r_{\beta\dot\beta}^{\alpha\dot\alpha}$, t_β^α and $f^{\dot\alpha\beta}$ from the theory.

First we eliminate superfield $q_\mu^{\alpha\dot\alpha}(x,\theta,\bar\theta)$ by putting $\frac{\omega_H^{\alpha\dot\alpha}}{\omega_\theta^{\ \mu}} = 0$. We introduce the symbols $\Delta H^{\alpha\dot\alpha}$ and $\Delta x^{\alpha\dot\alpha}$, the covariant differentials, by the formulæ

$$\Delta H^{\alpha\dot\alpha} \equiv dH^{\alpha\dot\alpha} - \frac{1}{2i}(q_\mu^{\alpha\dot\alpha} d\theta^\mu + \bar q_{\dot\mu}^{\alpha\dot\alpha} d\bar\theta^{\dot\mu}), \tag{4.1}$$

$$\Delta x^{\alpha\dot\alpha} \equiv dx^{\alpha\dot\alpha} - \frac{1}{2}(q_\mu^{\alpha\dot\alpha} d\theta^\mu - \bar q_{\dot\mu}^{\alpha\dot\alpha} d\bar\theta^{\dot\mu}). \tag{4.2}$$

Then the Cartan forms

$$\omega_H^{\alpha\dot\alpha} = \frac{1}{2i}(\omega_{x_L}^{\ \alpha\dot\alpha} - \bar\omega_{x_L}^{\alpha\dot\alpha}) \text{ and} \tag{4.3}$$

$$\omega_x^{\ \alpha\dot\alpha} = \frac{1}{2}(\omega_{x_L}^{\ \alpha\dot\alpha} + \bar\omega_{x_L}^{\alpha\dot\alpha}) \tag{4.4}$$

can be written as

$$\omega_H^{\alpha\dot\alpha} = A_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta H^{\beta\dot\beta} + B_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta x^{\beta\dot\beta}, \tag{4.5}$$

$$\omega_x^{\ \alpha\dot\alpha} = C_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta H^{\beta\dot\beta} + D_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta x^{\beta\dot\beta}, \tag{4.6}$$

where A, B, C a D are matrices. In the neigbourhood of $r_{\alpha\dot\alpha}^{\ \ \beta\dot\beta} = 0$ the matrices A and D are regular (~ 1) and B and C close to zero. We can, therefore, write

$$\begin{aligned}\omega_H^{\alpha\dot\alpha} &= A_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta H^{\beta\dot\beta} + B_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta x^{\beta\dot\beta} = \\ &= (A - BD^{-1}C)_{\beta\dot\beta}^{\alpha\dot\alpha}\Delta H^{\beta\dot\beta} + (BD^{-1})_{\beta\dot\beta}^{\alpha\dot\alpha}\omega_x^{\ \beta\dot\beta},\end{aligned}$$

from which it follows that $\frac{\omega_H^{\alpha\dot\alpha}}{\omega_\theta{}^\beta} = 0$ if and only if $\frac{\Delta H^{\alpha\dot\alpha}}{\omega_\theta{}^\beta} = 0$. However,

$$\begin{aligned}
\Delta H^{\alpha\dot\alpha} &= dx^{\beta\dot\beta}\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}} + d\theta^\beta\frac{\partial H^{\alpha\dot\alpha}}{\partial\theta^\beta} + d\bar\theta^{\dot\beta}\frac{\partial H^{\alpha\dot\alpha}}{\partial\bar\theta^{\dot\beta}} - \\
&- \frac{1}{2i}\left(q_\mu^{\alpha\dot\alpha}d\theta^\mu + \bar q_{\dot\mu}^{\alpha\dot\alpha}d\bar\theta^{\dot\mu}\right) = \\
&= \frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}\Delta x^{\beta\dot\beta} + \left[\frac{1}{2}\left(+\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}q_\mu^{\beta\dot\beta} + iq_\mu^{\alpha\dot\alpha}\right) - \frac{\partial H^{\alpha\dot\alpha}}{\partial\theta^\mu}\right]d\theta^\mu + \\
&+ \left[\frac{1}{2}\left(-\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}\bar q_{\dot\mu}^{\beta\dot\beta} + i\bar q_{\dot\mu}^{\alpha\dot\alpha}\right) - \frac{\partial H^{\alpha\dot\alpha}}{\partial\bar\theta^{\dot\mu}}\right]d\bar\theta^{\dot\mu},
\end{aligned}$$

so that

$$\frac{\omega_H^{\alpha\dot\alpha}}{\omega_\theta{}^\mu} = E\left(\frac{1}{2}\left(\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}q_\mu^{\beta\dot\beta} + iq_\mu^{\alpha\dot\alpha}\right) - \frac{\partial H^{\alpha\dot\alpha}}{\partial\theta^\mu}\right), \tag{4.7}$$

with E being a regular matrix.

Thus, putting

$$\frac{\omega_H^{\alpha\dot\alpha}}{\omega_\theta{}^\beta} = 0, \tag{4.8}$$

we obtain

$$q_\mu^{\alpha\dot\alpha} = -2i\left(\delta_\beta^\alpha\delta_{\dot\beta}^{\dot\alpha} - i\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}\right)^{-1}\frac{\partial H^{\beta\dot\beta}}{\partial\theta^\mu}. \tag{4.9}$$

By complex conjugation we obtain

$$\frac{\omega_H^{\alpha\dot\alpha}}{\bar\omega_{\bar\theta}{}^{\dot\mu}} = 0, \tag{4.10}$$

$$\bar q_{\dot\mu}^{\alpha\dot\alpha} = -2i\left(\delta_\beta^\alpha\delta_{\dot\beta}^{\dot\alpha} + i\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}\right)^{-1}\frac{\partial H^{\beta\dot\beta}}{\partial\bar\theta^{\dot\mu}}. \tag{4.11}$$

Denoting

$$\begin{aligned}
\nabla_\mu &\equiv \frac{\partial}{\partial\theta^\mu} - \frac{1}{2}q_\mu^{\gamma\dot\gamma}\frac{\partial}{\partial x^{\gamma\dot\gamma}} \\
\text{and}\ \ \nabla_{\dot\mu} &\equiv \frac{\partial}{\partial\bar\theta^{\dot\mu}} + \frac{1}{2}\bar q_{\dot\mu}^{\gamma\dot\gamma}\frac{\partial}{\partial x^{\gamma\dot\gamma}},
\end{aligned} \tag{4.12}$$

where q and $\bar q$ are assumed to be functions of $H^{\alpha\dot\alpha}$ and of its derivatives according to (4.9) and (4.11), we can rewrite

$$q_\mu^{\alpha\dot\alpha} = -2i\nabla_\mu H^{\alpha\dot\alpha} \ \text{ and } \ \bar q_{\dot\mu}^{\alpha\dot\alpha} = -2i\nabla_{\dot\mu}H^{\alpha\dot\alpha}, \tag{4.13}$$

so that Goldstone superfield $q_\mu^{\alpha\dot\alpha}$ is expressed in terms of derivatives of superfield $H^{\alpha\dot\alpha}(x,\theta,\bar\theta)$.

Now we eliminate superfields $r_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}}$, $f^{\dot{\alpha}\dot{\beta}}$ and $t_{\alpha}{}^{\beta}$. For that purpose we shall use

$$\frac{\omega_H{}^{\dot{\alpha}}}{\bar{\omega}_{\bar{\theta}}{}^{\dot{\beta}}} = 0, \quad (4.14)$$

$$\frac{\omega_q{}^{\alpha\dot{\alpha}}{}_{\beta}}{\bar{\omega}_{\bar{\theta}}{}^{\dot{\beta}}} = -i\delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}} \quad (4.15)$$

and the fact that superfields r, f and t fulfill (3.3). Notice that $\frac{\omega_q{}^{\alpha\dot{\alpha}}{}_{\beta}}{\bar{\omega}_{\bar{\theta}}{}^{\dot{\beta}}}$ in (4.15) was not taken to be equal to zero but to a proper Lorentz–invariant constant matrix in order to be in harmony with the right flat–superspace limit:

$$H^{\alpha\dot{\alpha}} = \frac{1}{2}\theta^{\alpha}\bar{\theta}^{\dot{\alpha}}, \quad \bar{H}^{\dot{\mu}} = 0, \quad H^{\mu} = 0 \quad (4.16)$$

and consequently $q^{\alpha\dot{\alpha}}_{\mu} = -i\delta^{\alpha}_{\mu}\bar{\theta}^{\dot{\alpha}}$.

Since superfields $r_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}}$, $f^{\dot{\alpha}\dot{\beta}}$ and $t_{\alpha}{}^{\beta}$ have $16+3+4$ components which are restricted by $3+4+16$ conditions from (3.3), (4.14) and (4.15), they can be completely eliminated from the theory.

5 INVARIANT ACTION

Now we are going to construct a Lagrange density invariant w.r.t both G_I and G_{II}. First, it seems that projection $\frac{\omega_H{}^{\alpha\dot{\alpha}}}{\omega_x{}^{\beta\dot{\beta}}}$ can be such a suitable invariant, since it is obviously invariant w.r.t. G_{II} (c.w. (3.22)) and it is invariant w.r.t. G_I due to (4.8) and (4.10). Moreover $\frac{\omega_H{}^{\alpha\dot{\alpha}}}{\omega_x{}^{\beta\dot{\beta}}} = 0$ because of (4.8), (4.10) and (4.15). This can be easily seen by taking into account the following facts. All superfields can be transformed by $e^{ix^{\mu\dot{\mu}}P_{L\mu\dot{\mu}}}\, e^{i\theta^{\mu}Q_{\mu}}g_0'^{-1}$ to zero in any point of superspace $R^{4/4}$. Then expression (4.15) takes the form

$$\frac{\partial q^{\alpha\dot{\alpha}}_{\beta}}{\partial\bar{\theta}^{\dot{\beta}}} = -i\delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}}. \quad (5.1)$$

Thus, comparing the expression obtained by substituting formulae (4.9) into (5.1) with that obtained by its complex conjugation, we get

$$2(\delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}} - i\frac{\partial H^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}})^{-1}\frac{\partial^2 H^{\beta\dot{\beta}}}{\partial\bar{\theta}^{\dot{\mu}}\partial\theta^{\mu}} = 2(\delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}} + i\frac{\partial H^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}})^{-1}\frac{\partial^2 H^{\beta\dot{\beta}}}{\partial\bar{\theta}^{\dot{\mu}}\partial\theta^{\mu}} = \delta^{\alpha}_{\mu}\delta^{\dot{\alpha}}_{\dot{\mu}}. \quad (5.2)$$

From here we see that $\frac{\partial H^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}} = 0$ and consequently

$$\frac{\omega_H{}^{\alpha\dot{\alpha}}}{\omega_x{}^{\beta\dot{\beta}}} = 0 \quad (5.3)$$

Q.E.D. Then (5.3) together with (4.8) and (4.10) gives $\omega_H{}^{\alpha\dot{\alpha}} = 0$.

Since there are no invariants w.r.t both G_I and G_{II} containing at most a second derivative, the invariant action can only be constructed via integrating over an invariant supervolume made of invariant forms $\omega_x{}^{\alpha\dot\alpha}$ and $\omega_\theta{}^\mu$ instead of noninvariant expressions $dx^{\alpha\dot\alpha}$ and $d\theta^\mu$. In fact, as follows from (3.18), Cartan forms $\omega_x{}^{\alpha\dot\alpha}$ and $\omega_\theta{}^\mu$ are not invariant w.r.t. transformations $e^{i\, i'^{\alpha\dot\alpha}_{\mu} I^{\mu}_{\alpha\dot\alpha}}$, but since these transformations are triangular, i.e. $\omega_x{}^{\alpha\dot\alpha} \to \omega_x{}^{\alpha\dot\alpha}$, $\omega_\theta{}^\mu \to \omega_\theta{}^\mu + i'^{\mu}_{\alpha\dot\alpha}\omega_x{}^{\alpha\dot\alpha}$, their superdeterminant is equal to 1 and therefore they need not be considered.

Writing the action in the form

$$I = \int E\, d^2\theta d^2\bar\theta d^4x, \tag{5.4}$$

the Lagrange density E is given by

$$\begin{aligned} E &= \mathrm{Ber}\mathrm{E}^{\mathrm{N}}_{\mathrm{M}} = \det[\mathrm{E}^{\alpha\dot\alpha}_{\beta\dot\beta} + \mathrm{E}^{\alpha\dot\alpha}_{\mu}(\mathrm{E}^{-1})^{\mu}_{\nu}\mathrm{E}^{\nu}_{\beta\dot\beta} + \mathrm{E}^{\alpha\dot\alpha}_{\dot\mu}(\mathrm{E}^{-1})^{\dot\mu}_{\dot\nu}\mathrm{E}^{\dot\nu}_{\beta\dot\beta}] \times \\ &\times \det{}^{-1}(\mathrm{E}^{\mu}_{\nu})\det{}^{-1}(\mathrm{E}^{\dot\mu}_{\dot\nu}), \end{aligned} \tag{5.5}$$

where E^N_M is defined via the decomposition

$$\omega_Z{}^N \equiv (\omega_x{}^{\alpha\dot\alpha},\, \omega_\theta{}^\mu,\, \bar\omega_{\bar\theta}{}^{\dot\mu}) \equiv dx^{\beta\dot\beta}E^N_{\beta\dot\beta} + d\theta^\mu E^N_\mu + d\bar\theta^{\dot\mu}E^N_{\dot\mu}. \tag{5.6}$$

It is not difficult but rather tedious to express E^N_M in terms of $H^{\alpha\dot\alpha}$, $\bar H^{\dot\mu}$ and $\bar H^\mu$. Thus using (3.9) and definitions (4.1), (4.2), we finally get

$$\begin{aligned} \omega_x{}^{\alpha\dot\alpha} &= \mathrm{Re}\left((\mathrm{e}^{\mathrm{r}})^{\alpha\dot\alpha}_{\beta\dot\beta}\right)\Delta\mathrm{x}^{\beta\dot\beta} - \mathrm{Im}\left((\mathrm{e}^{\mathrm{r}})^{\alpha\dot\alpha}_{\beta\dot\beta}\right)\Delta\mathrm{H}^{\beta\dot\beta} = \\ &= \left[\mathrm{Re}\left((\mathrm{e}^{\mathrm{r}})^{\alpha\dot\alpha}_{\gamma\dot\gamma}\right) - \mathrm{Im}\left((\mathrm{e}^{\mathrm{r}})^{\alpha\dot\alpha}_{\beta\dot\beta}\right)\frac{\partial\mathrm{H}^{\beta\dot\beta}}{\partial\mathrm{x}^{\gamma\dot\gamma}}\right]\Delta x^{\gamma\dot\gamma} = E^{\alpha\dot\alpha}_{\gamma\dot\gamma}\Delta x^{\gamma\dot\gamma}. \end{aligned} \tag{5.7}$$

It is consistent with (5.6), since $\Delta x^{\gamma\dot\gamma} = dx^{\gamma\dot\gamma} + \ldots d\theta + \ldots d\bar\theta$. Expressing (4.15) in terms of superfields we get

$$-\frac{1}{2}\frac{\partial q^{\alpha\dot\alpha}_{\beta}}{\partial x^{\gamma\dot\gamma}}\bar q^{\gamma\dot\gamma}_{\dot\beta} + \frac{\partial q^{\alpha\dot\alpha}_{\beta}}{\partial\bar\theta^{\dot\beta}} = -i(e^{-r})_{\gamma\dot\gamma}{}^{\alpha\dot\alpha}(e^t)_\beta{}^\gamma(e^{\bar t})_{\dot\beta}{}^{\dot\gamma}. \tag{5.8}$$

Denoting

$$\begin{aligned} A^{\alpha\dot\alpha}_{\beta\dot\beta} &\equiv (\delta^\alpha_\beta\delta^{\dot\alpha}_{\dot\beta} - i\frac{\partial H^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}}), \\ \bar A^{\alpha\dot\alpha}_{\beta\dot\beta} &\equiv (A^{\alpha\dot\alpha}_{\beta\dot\beta})^+ \\ \text{and}\quad L^{\alpha\dot\alpha}_{\beta\dot\beta} &\equiv i\left(-\frac{1}{2}\frac{\partial q^{\alpha\dot\alpha}_{\beta}}{\partial x^{\gamma\dot\gamma}}\bar q^{\gamma\dot\gamma}_{\dot\beta} + \frac{\partial q^{\alpha\dot\alpha}_{\beta}}{\partial\bar\theta^{\dot\beta}}\right) = 2\nabla_{\dot\beta}\nabla_\beta H^{\alpha\dot\alpha} = [\nabla_{\dot\beta},\nabla_\beta]H^{\alpha\dot\alpha} + \{\nabla_{\dot\beta},\nabla_\beta\}H^{\alpha\dot\alpha} \end{aligned} \tag{5.9}$$

and using the fact that

$$\{\nabla_{\dot\beta},\nabla_\beta\} = \frac{1}{2}(\nabla_\beta\bar q^{\gamma\dot\gamma}_{\dot\beta})\partial_{\gamma\dot\gamma} - \frac{1}{2}(\nabla_{\dot\beta}q^{\gamma\dot\gamma}_{\beta})\partial_{\gamma\dot\gamma} = i\left([\nabla_{\dot\beta},\nabla_\beta]H^{\gamma\dot\gamma}\right)\partial_{\gamma\dot\gamma}, \tag{5.10}$$

we can write

$$L^{\alpha\dot\alpha}_{\beta\dot\beta} = \bar A^{\alpha\dot\alpha}_{\gamma\dot\gamma}[\nabla_{\dot\beta}, \nabla_\beta] H^{\gamma\dot\gamma}. \tag{5.11}$$

From here it follows that

$$\left(A^{\alpha\dot\alpha}_{\gamma\dot\gamma} L^{\gamma\dot\gamma}_{\beta\dot\beta}\right) = \left(A^{\alpha\dot\alpha}_{\gamma\dot\gamma} L^{\gamma\dot\gamma}_{\beta\dot\beta}\right)^+. \tag{5.12}$$

Expression (5.8) yields

$$(e^r)^{\alpha\dot\alpha}_{\beta\dot\beta} = (e^t)^\alpha_\gamma (e^{\bar t})^{\dot\alpha}_{\dot\gamma} (L^{-1})^{\gamma\dot\gamma}_{\delta\dot\delta} (A^{-1})^{\delta\dot\delta}_{\epsilon\dot\epsilon} A^{\epsilon\dot\epsilon}_{\beta\dot\beta}. \tag{5.13}$$

Thus we obtain

$$\begin{aligned} \mathrm{Re}((\mathrm{e^r})^{\alpha\dot\alpha}_{\beta\dot\beta}) &= (e^t)^\alpha_\gamma (e^{\bar t})^{\dot\alpha}_{\dot\gamma} (L^{-1})^{\gamma\dot\gamma}_{\delta\dot\delta} (A^{-1})^{\delta\dot\delta}_{\beta\dot\beta} = \\ &= (e^r)^{\alpha\dot\alpha}_{\gamma\dot\gamma} (A^{-1})^{\gamma\dot\gamma}_{\beta\dot\beta}, \end{aligned} \tag{5.14}$$

$$\begin{aligned} \mathrm{Im}((\mathrm{e^r})^{\alpha\dot\alpha}_{\beta\dot\beta}) &= -(e^t)^\alpha_\gamma (e^{\bar t})^{\dot\alpha}_{\dot\gamma} (L^{-1})^{\gamma\dot\gamma}_{\delta\dot\delta} (A^{-1})^{\delta\dot\delta}_{\beta\dot\beta} \frac{\partial H^{\epsilon\dot\epsilon}}{\partial x^{\beta\dot\beta}} = \\ &= -(e^r)^{\alpha\dot\alpha}_{\gamma\dot\gamma} (A^{-1})^{\gamma\dot\gamma}_{\delta\dot\delta} \frac{\partial H^{\delta\dot\delta}}{\partial x^{\beta\dot\beta}} \end{aligned} \tag{5.15}$$

and consequently

$$\begin{aligned} E^{\alpha\dot\alpha}_{\epsilon\dot\epsilon} &= \mathrm{Re}((\mathrm{e^r})^{\alpha\dot\alpha}_{\epsilon\dot\epsilon}) - \mathrm{Im}((\mathrm{e^r})^{\alpha\dot\alpha}_{\beta\dot\beta}) \frac{\partial \mathrm{H}^{\beta\dot\beta}}{\partial \mathrm{x}^{\epsilon\dot\epsilon}} = \\ &= (e^r)^{\alpha\dot\alpha}_{\beta\dot\beta} (A^{-1})^{\beta\dot\beta}_{\gamma\dot\gamma} (\delta^\gamma_\epsilon \delta^{\dot\gamma}_{\dot\epsilon} + \frac{\partial H^{\gamma\dot\gamma}}{\partial x^{\delta\dot\delta}} \frac{\partial H^{\delta\dot\delta}}{\partial x^{\epsilon\dot\epsilon}}) = \\ &= (e^r)^{\alpha\dot\alpha}_{\beta\dot\beta} (A^{-1})^{\beta\dot\beta}_{\gamma\dot\gamma} A^{\gamma\dot\gamma}_{\delta\dot\delta} \bar A^{\delta\dot\delta}_{\epsilon\dot\epsilon} = (e^r)^{\alpha\dot\alpha}_{\delta\dot\delta} \bar A^{\delta\dot\delta}_{\epsilon\dot\epsilon}. \end{aligned} \tag{5.16}$$

Denoting $\varrho \equiv e^{r\,\alpha\dot\alpha}_{\alpha\dot\alpha}$, $\tau \equiv e^{t\,\alpha}_{\alpha}$ and $A \equiv \det(\mathrm{A}^{\beta\dot\beta}_{\gamma\dot\gamma})$ we get

$$\det \mathrm{E}^{\alpha\dot\alpha}_{\beta\dot\beta} = |\varrho||\mathrm{A}|. \tag{5.17}$$

Finally, using (3.9), we can write $\omega_\theta{}^\mu$ and $\bar\omega_{\bar\theta}{}^{\dot\mu}$ in the following way:

$$\begin{aligned} \omega_\theta{}^\mu &= d\theta^\nu E^\mu_\nu + dx^{\alpha\dot\alpha} E^\mu_{\alpha\dot\alpha} = d\theta^\nu (e^t)_\nu{}^\mu, & (5.18)\\ \bar\omega_{\bar\theta}{}^{\dot\mu} &= d\bar\theta^{\dot\nu} E^{\dot\mu}_{\dot\nu} + dx^{\alpha\dot\alpha} E^{\dot\mu}_{\alpha\dot\alpha} = d\bar\theta^{\dot\nu} (e^{\bar t})_{\dot\nu}{}^{\dot\mu} & (5.19) \end{aligned}$$

so that

$$E^\mu_\nu = (e^t)_\nu{}^\mu, \quad \det \mathrm{E}^\mu_\nu = \tau, \quad \mathrm{E}^{\dot\mu}_{\dot\nu} = (\mathrm{e}^{\bar t})_{\dot\nu}{}^{\dot\mu}, \quad \det \mathrm{E}^{\dot\mu}_{\dot\nu} = \bar\tau. \tag{5.20}$$

Since $E^\mu_{\alpha\dot\alpha} = 0 = E^{\dot\mu}_{\alpha\dot\alpha}$ due to (5.18) and (5.19), the Lagrange density E (5.5) is of the form

$$E = \mathrm{Ber} \mathrm{E}^{\mathrm{N}}_{\mathrm{M}} = \det(\mathrm{E}^{\alpha\dot\alpha}_{\beta\dot\beta}) \det{}^{-1}(\mathrm{E}^\mu_\nu) \det{}^{-1}(\mathrm{E}^{\dot\mu}_{\dot\nu}) = |\varrho||\mathrm{A}||\tau|^{-2}. \tag{5.21}$$

Now we are going to express E respectively ρ and τ in terms of $H^{\alpha\dot\alpha}$, $\bar H^{\dot\mu}$, H^μ and their derivatives.

For that purpose first consider the determinant of both sides of equation (5.8). We obtain

$$L \equiv \det L^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} = \varrho^{-1}|\tau|^4. \tag{5.22}$$

Expressing (4.14) in terms of superfields r, t, f we get

$$M^{\dot{\alpha}}_{\dot{\beta}} = e^{-a(r_{\rho\dot{\rho}}{}^{\rho\dot{\rho}} - t_\rho{}^\rho)}(e^{f()})_{\dot{\gamma}}{}^{\dot{\alpha}}(e^{\bar{t}})_{\dot{\beta}}{}^{\dot{\gamma}} \tag{5.23}$$

where we have used the abreviation

$$M^{\dot{\alpha}}_{\dot{\beta}} \equiv \delta^{\dot{\alpha}}_{\dot{\beta}} + \nabla_{\dot{\beta}}\bar{H}^{\dot{\beta}} = -\frac{1}{2}\frac{\partial \bar{H}^{\dot{\alpha}}}{\partial x^{\gamma\dot{\gamma}}}\bar{q}^{\gamma\dot{\gamma}}_{\dot{\beta}} + \frac{\partial \bar{H}^{\dot{\alpha}}}{\partial \bar{\theta}^{\dot{\beta}}} + \delta^{\dot{\alpha}}_{\dot{\beta}}. \tag{5.24}$$

Taking the determinant of (5.23)

$$M \equiv \det M^{\dot{\alpha}}_{\dot{\beta}} = \varrho^{-2a}\tau^{2a}\bar{\tau}, \quad a = \frac{n+1}{2(3n+1)}, \tag{5.25}$$

we can find ϱ and τ from (5.22) and (5.25). Substituting them to (5.21) we finally obtain Lagrange density E in the form

$$E = |A||L|^n|M|^{-(3n+1)}, \tag{5.26}$$

where n is the ordinary Gates-Siegel parameter [6].

5.1 The case $n = -\frac{1}{3}$

Let us discuss now a particular case of supergravity for which $n = -\frac{1}{3}$, i.e. the N=1 minimal Einstein supergravity. In this case Lagrange density E is of the form

$$E = |A||L|^{-\frac{1}{3}} \tag{5.27}$$

thus independent of superfields H^μ and $\bar{H}^{\dot{\mu}}$.

It is easy to show that the Lagrangian arising from (5.27) is equivalent to that derived in [1], the density of which can be written as

$$\mathrm{Ber}E_{IN}{}^N{}_M = 2^{-\frac{8}{3}}(\det\hat{e}_{IN})^{-\frac{1}{6}}(\det\hat{r}_{IN})^{-\frac{1}{6}}(\det A_{IN})^{\frac{1}{2}}(\det\bar{A}_{IN})^{\frac{1}{2}}. \tag{5.28}$$

Here all quantities from [1] are denoted by index IN.

Taking into account that $A^{\alpha\dot{\alpha}}_{IN\ \beta\dot{\beta}} = A^{\alpha\dot{\alpha}}_{\beta\dot{\beta}}$, $\nabla_{IN\ \alpha} = \nabla_\alpha$, $\bar{\nabla}_{IN\ \dot{\alpha}} = -\bar{\nabla}_{\dot{\alpha}}$, $\hat{e}^{\alpha\dot{\alpha}}_{IN\ \beta\dot{\beta}} = \bar{\nabla}_{IN\ \dot{\beta}}\nabla_{IN\ \beta}H^{\alpha\dot{\alpha}} = -\frac{1}{2}L^{\alpha\dot{\alpha}}_{\beta\dot{\beta}}$ and $\hat{r}^{\alpha\dot{\alpha}}_{IN\ \beta\dot{\beta}} = \nabla_{IN\ \beta}\bar{\nabla}_{IN\ \dot{\beta}}H^{\alpha\dot{\alpha}} = \frac{1}{2}\bar{L}^{\alpha\dot{\alpha}}_{\beta\dot{\beta}}$, we find that

$$\mathrm{Ber}E_{IN}{}^N{}_M = 2^{-\frac{4}{3}}|A||L|^{-\frac{1}{3}} = 2^{-\frac{4}{3}}E. \tag{5.29}$$

Q.E.D.

5.2 The general case

Now we are going to compare the Lagrange density (5.26) with that in [6]. For that purpose we expand E up to $O(H^2)$ in the neigbourhood of the flat superspace defined in (4.16), i.e. $\hat{H}^{\alpha\dot{\alpha}} \equiv H^{\alpha\dot{\alpha}} - \frac{1}{2}\theta^{\alpha}\theta^{\dot{\alpha}}$, H^{μ} and $\bar{H}^{\dot{\mu}}$. Let us denote $G^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} \equiv [\nabla_{\dot{\beta}}, \nabla_{\beta}]\hat{H}^{\alpha\dot{\alpha}} = \bar{A}^{-1\alpha\dot{\alpha}}_{\gamma\dot{\gamma}} L^{\gamma\dot{\gamma}}_{\beta\dot{\beta}} - \delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}}$, $D_{\mu} \equiv \frac{\partial}{\partial\theta^{\mu}} + \frac{i}{2}\bar{\theta}^{\dot{\mu}}\frac{\partial}{\partial x^{\mu\dot{\mu}}}$ and $\bar{D}_{\dot{\mu}} \equiv \frac{\partial}{\partial\bar{\theta}^{\dot{\mu}}} + \frac{i}{2}\theta^{\mu}\frac{\partial}{\partial x^{\mu\dot{\mu}}}$. Then using $(\det(1+A))^n = 1 + n\mathrm{Tr}A - \frac{n}{2}\mathrm{Tr}(A^2) + \frac{n^2}{2}(\mathrm{Tr}A)^2 + O(A^3)$ with A being a matrix we can expand E in the following way

$$
\begin{aligned}
E &= |A||L|^n|M|^{-(3n+1)} = \\
&= |\det(\delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}} - i\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}})|^{(n+1)}|\det(\delta^{\gamma}_{\delta}\delta^{\dot{\gamma}}_{\dot{\delta}} + G^{\gamma\dot{\gamma}}_{\delta\dot{\delta}})|^n|\det(\delta^{\dot{\mu}}_{\dot{\nu}} + \nabla_{\dot{\nu}}\bar{H}^{\dot{\mu}}|^{-(3n+1)} = \\
&= \left[1 + \frac{n+1}{2}\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}}\frac{\partial\hat{H}^{\beta\dot{\beta}}}{\partial x^{\alpha\dot{\alpha}}}\right]\left[1 + nG^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}} - \frac{n}{2}G^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}G^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} + \frac{n^2}{2}G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}G^{\beta\dot{\beta}}_{\beta\dot{\beta}}\right] \times \\
&\times \left[1 - \frac{3n+1}{2}(\nabla_{\mu}H^{\mu} + \nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}}) + \frac{3n+1}{4}(\nabla_{\dot{\nu}}\bar{H}^{\dot{\mu}}\nabla_{\dot{\mu}}\bar{H}^{\dot{\nu}} + \nabla_{\nu}H^{\mu}\nabla_{\mu}H^{\nu}) + \right.\\
&+ \left.\frac{(3n+1)^2}{8}(\nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}}\nabla_{\dot{\nu}}\bar{H}^{\dot{\nu}} + \nabla_{\mu}H^{\mu}\nabla_{\nu}H^{\nu}) + \frac{(3n+1)^2}{4}\nabla_{\mu}H^{\mu}\nabla_{\dot{\nu}}\bar{H}^{\dot{\nu}}\right] + O(H^3) = \\
&= 1 + \frac{n+1}{2}\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}}\frac{\partial\hat{H}^{\beta\dot{\beta}}}{\partial x^{\alpha\dot{\alpha}}} + nG^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}} - \frac{n}{2}G^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}G^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} + \frac{n^2}{2}G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}G^{\beta\dot{\beta}}_{\beta\dot{\beta}} - \\
&- \frac{3n+1}{2}(\nabla_{\mu}H^{\mu} + \nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}}) + \frac{3n+1}{4}(\nabla_{\dot{\nu}}\bar{H}^{\dot{\mu}}\nabla_{\dot{\mu}}\bar{H}^{\dot{\nu}} + \nabla_{\nu}H^{\mu}\nabla_{\mu}H^{\nu}) + \\
&+ \frac{(3n+1)^2}{8}(\nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}}\nabla_{\dot{\nu}}\bar{H}^{\dot{\nu}} + \nabla_{\mu}H^{\mu}\nabla_{\nu}H^{\nu}) + \frac{(3n+1)^2}{4}\nabla_{\mu}H^{\mu}\nabla_{\dot{\nu}}\bar{H}^{\dot{\nu}} - \\
&- \frac{n(3n+1)}{2}G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}(\nabla_{\mu}H^{\mu} + \nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}}) + O(H^3).
\end{aligned} \tag{5.30}
$$

Then by using the identities

$$\nabla_{\nu}H^{\mu} = D_{\nu}H^{\mu} + O(H^2), \tag{5.31}$$

$$\nabla_{\dot{\nu}}\bar{H}^{\dot{\mu}} = \bar{D}_{\dot{\nu}}\bar{H}^{\dot{\mu}} + O(H^2), \tag{5.32}$$

$$\nabla_{\mu}H^{\mu} = i\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}D_{\mu}H^{\mu} + O(H^3) + \mathrm{div}, \tag{5.33}$$

$$\nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}} = -i\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}\bar{D}_{\dot{\mu}}\bar{H}^{\dot{\mu}} + O(H^3) + \mathrm{div}, \tag{5.34}$$

$$G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}} = \frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial\hat{H}^{\beta\dot{\beta}}}{\partial x^{\beta\dot{\beta}}} + O(H^3) + \mathrm{div}, \tag{5.35}$$

$$\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\beta\dot{\beta}}}\frac{\partial\hat{H}^{\beta\dot{\beta}}}{\partial x^{\alpha\dot{\alpha}}} = \frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial\hat{H}^{\beta\dot{\beta}}}{\partial x^{\beta\dot{\beta}}} + \mathrm{div}, \tag{5.36}$$

$$\bar{D}_{\dot{\nu}}\bar{H}^{\dot{\mu}}\bar{D}_{\dot{\mu}}\bar{H}^{\dot{\nu}} = -\bar{D}_{\dot{\mu}}\bar{H}^{\dot{\mu}}\bar{D}_{\dot{\nu}}\bar{H}^{\dot{\nu}} + \mathrm{div}, \tag{5.37}$$

$$D_{\nu}H^{\mu}D_{\mu}H^{\nu} = -D_{\mu}H^{\mu}D_{\nu}H^{\nu} + \mathrm{div}, \tag{5.38}$$

we obtain

$$E = \frac{3n+1}{2}\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial\hat{H}^{\beta\dot{\beta}}}{\partial x^{\beta\dot{\beta}}} - \frac{n}{2}G^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}G^{\alpha\dot{\alpha}}_{\beta\dot{\beta}} + \frac{n^2}{2}G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}G^{\beta\dot{\beta}}_{\beta\dot{\beta}} - \frac{3n+1}{2}i\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}(D_{\mu}H^{\mu} - \bar{D}_{\dot{\mu}}\bar{H}^{\dot{\mu}}) +$$

$$+\frac{1}{8}(3n+1)(3n-1)(\bar{D}_{\dot{\mu}}\bar{H}^{\dot{\mu}}\bar{D}_{\dot{\nu}}\bar{H}^{\dot{\nu}}+D_\mu H^\mu D_\nu H^\nu)+\frac{(3n+1)^2}{4}D_\mu H^\mu \bar{D}_{\dot{\nu}}\bar{H}^{\dot{\nu}}-$$

$$-\frac{n(3n+1)}{2}G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}(\nabla_\mu H^\mu+\nabla_{\dot{\mu}}\bar{H}^{\dot{\mu}})+O(H^3)= \tag{5.39}$$

$$=-\frac{n}{6}(G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}G^{\beta\dot{\beta}}_{\beta\dot{\beta}}+3G^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}G^{\alpha\dot{\alpha}}_{\beta\dot{\beta}})+(3n+1)n\{\frac{1}{6}[-G^{\alpha\dot{\alpha}}_{\alpha\dot{\alpha}}+\frac{3}{2}(D_\mu H^\mu+\bar{D}_{\dot{\mu}}\bar{H}^{\dot{\mu}})]^2+$$

$$+\frac{1}{2n}[\frac{\partial\hat{H}^{\alpha\dot{\alpha}}}{\partial x^{\alpha\dot{\alpha}}}-\frac{i}{2}(D_\mu H^\mu-\bar{D}_{\dot{\mu}}\bar{H}^{\dot{\mu}})]^2\}+O(H^3) \tag{5.40}$$

This expansion is equivalent (up to a scaling factor of the superfields) to expression (4.7) introduced by Siegel and Gates in [6].

5.3 Conclusion

In the paper the $N=1$ supergravity is consistently reformulated as a simultaneous non-linear realization of two complex finite-dimensional supergroups G_I and G_{II} generating via their closure the whole infinite-dimensional $N=1$ supergravity gauge group. Thus, the $N=1$ supergravity is found to be a kind of the non-linear σ-model describing a partial spontaneous breaking of the underlying infinite-dimensional supersymmetry down to the rigid $N=1$ supersymmetry. This kind of reformulation of $N=1$ supergravity has several advantages.

First of all the nonlinear realization approach allows an algorithmic construction of $N=1$ supergravity based on the universal method of Cartan forms augmented with the inverse Higgs phenomena. The only independent Goldstone superfields actualy needed to accomplish the above mentioned supersymmetry breaking appear to be axial vector superfield $H^{\mu\dot{\mu}}(x,\theta,\bar{\theta})$ and spinor superfields $H^\mu(x,\theta,\bar{\theta})$ and $\bar{H}^{\dot{\mu}}(x,\theta,\bar{\theta})$ identified with the $N=1$ supergravity prepotentials. They contain the fields mediating interactions. The other Goldstone superfields are expressed in terms of $H^{\mu\dot{\mu}}$, H^μ and $\bar{H}^{\dot{\mu}}$ and their derivatives. It is worth mentioning that the inverse-Higgs-effect constraints presented here are purely algebraic, in contradistinction to the standart $N=1$ supergravity constraints that are reduced to certain differential equations (vanishing of some components of the torsion), solutions of which are the prepotentials. In the present formulation these differential equations appear to be a consequence of the Maurer-Cartan structure equations for the complex finite-dimensional supergroups G_I and G_{II}.

Second many object and relations introduced "by hand" or postulated in the Ogievetsky-Sokatchev approach [3,4] acquire a clear group-theoretical meaning. For instance, the objects F and $\bar{F}$ playing the crucial role in the Ogievetsky-Sokatchev formulation turn out to be related to the Goldstone superfield associated with the spontaneously broken generator D_{II} of G_{II} introduced in [1].

Third the complex geometry of $N=1$ supergravity reappears here very clearly. Primarily, it manifests that one deals with the complex supergroups G_I and G_{II} in a holomorphic parametrization (cf. the $N=1$ super Young-Mills theory which can be interpreted as a realization of complex extension of local internal symmetry [11]). The $C^{4/4}$ coordinates $x^{\mu\dot{\mu}}_L$, θ^μ_L, $\bar{\phi}^{\dot{\mu}}_L$ naturally arise as the parameters of the relevant complex coset spaces. The constraints arising from the inverse Higgs effect in the present formulation can also be interpreted as a kind of covariant chirality conditions (i.e. in the case $n=-\frac{1}{3}$ as the absence of the $d\bar{\theta}$ projections in the corresponding Cartan forms).

Let us also mention that the paper deals with a treatement of the inverse Higgs phenomena for elimination of some Goldstone and gauge superfields which is quite general and universal. Finaly we hope that this formulation of supergravity theories alows to answer the question whether $N = 1$ supergravity can be reproduced as an effective "low-energy" limit of some higher dimensional superfield supersymmetric theories (by analogy with condensation of super-p-branes in field theory) or the related question of existence of theories with a lineary realized $N = 1$ supergravity group as well as to obtain the geometric prepotential formulations of supergravities with $N \geq 3$ which are unknown at present.

Appendix: Notation, conventions and identities

In the paper the following notations and conventions are used:

$$
\begin{array}{lll}
m,n,\ldots \in \{0,1,2,3\}, & \alpha,\dot\alpha,\beta,\dot\beta,\ldots \in \{1,2\}, & \\
\eta_{mn} = \mathrm{diag}(+,-,-,-), & \theta^\alpha\theta^\beta = -\theta^\beta\theta^\alpha, & \bar\theta^{\dot\alpha}\theta^\beta = -\theta^\beta\bar\theta^{\dot\alpha}, \\
(\bar\theta_{\dot\alpha})^+ = \theta_\alpha, & \theta^\beta = \epsilon^{\beta\alpha}\theta_\alpha, & \bar\theta^{\dot\beta} = \epsilon^{\dot\beta\dot\alpha}\bar\theta_{\dot\alpha}, \\
\epsilon^{\beta\alpha} = -\epsilon^{\alpha\beta}, & \epsilon^{\dot\beta\dot\alpha} = -\epsilon^{\dot\alpha\dot\beta}, & \epsilon^{12} = -\epsilon_{12} = \epsilon^{\dot1\dot2} = -\epsilon_{\dot1\dot2} = 1, \\
\theta^2 = \theta^\alpha\theta_\alpha, & \bar\theta^2 = \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}, & \frac{\partial\theta^\alpha}{\partial\theta^\beta} = \delta^\alpha_\beta, \\
(\theta^\alpha\bar\theta^{\dot\beta})^+ = \theta^\beta\bar\theta^{\dot\alpha}, & & \\
(\sigma_m)_{\alpha\dot\alpha} = (1,\vec\sigma)_{\alpha\dot\alpha}, & (\sigma_m)^{\beta\dot\beta} = \epsilon^{\beta\alpha}\epsilon^{\dot\beta\dot\alpha}(1,\vec\sigma)_{\alpha\dot\alpha} = (1,-\vec\sigma)^{\beta\dot\beta}, &
\end{array}
$$

where $\vec\sigma$ are the Pauli matrices

$$
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
$$

$$
x^{\alpha\dot\alpha} = x^m(\sigma_m)^{\alpha\dot\alpha}, \qquad \frac{\partial}{\partial x^{\beta\dot\beta}} = \frac{1}{2}(\sigma_m)_{\alpha\dot\alpha}\eta^{mn}\frac{\partial}{\partial x^n}.
$$

Consequently $\frac{\partial x^{\alpha\dot\alpha}}{\partial x^{\beta\dot\beta}} = (\sigma_m)^{\alpha\dot\alpha}\frac{1}{2}(\sigma_k)_{\alpha\dot\alpha}\eta^{kn}\frac{\partial x^m}{\partial x^n} = \frac{1}{2}(\sigma_m)^{\alpha\dot\alpha}\eta^{mk}(\sigma_k)_{\alpha\dot\alpha} = \delta^\alpha_\beta\delta^{\dot\alpha}_{\dot\beta}$. $(\alpha\beta)$ denotes symetrization over indices α, β, i.e. $X_{(\alpha\beta)} = X_{\alpha\beta} + X_{\beta\alpha}$ and $X^{(\alpha}{}_{\beta)} = \epsilon^{\alpha\gamma}X_{(\gamma\beta)}$.

The Poincaré algebra is formed by the generators $M_{\alpha\beta} = M_{\beta\alpha}$, $M_{\dot\alpha\dot\beta} = M_{\dot\beta\dot\alpha}$ and $P_{\alpha\dot\alpha}$ satisfying the relations

$$
\begin{aligned}
[M_{\dot\alpha\dot\beta}, M_{\dot\gamma\dot\delta}] &= i\epsilon_{\dot\beta\dot\gamma}M_{\dot\delta\dot\alpha} + i\epsilon_{\dot\alpha\dot\gamma}M_{\dot\delta\dot\beta} + i\epsilon_{\dot\alpha\dot\delta}M_{\dot\gamma\dot\beta} + i\epsilon_{\dot\beta\dot\delta}M_{\dot\gamma\dot\alpha}, \\
[M_{\alpha\beta}, M_{\gamma\delta}] &= i\epsilon_{\beta\gamma}M_{\delta\alpha} + i\epsilon_{\alpha\gamma}M_{\delta\beta} + i\epsilon_{\alpha\delta}M_{\gamma\beta} + i\epsilon_{\beta\delta}M_{\gamma\alpha}, \\
[M_{\alpha\beta}, P_{\gamma\dot\gamma}] &= i\epsilon_{\beta\gamma}P_{\alpha\dot\gamma} + i\epsilon_{\alpha\gamma}P_{\beta\dot\gamma}, \\
[M_{\dot\alpha\dot\beta}, P_{\gamma\dot\gamma}] &= i\epsilon_{\dot\beta\dot\gamma}P_{\gamma\dot\alpha} + i\epsilon_{\dot\alpha\dot\gamma}P_{\gamma\dot\beta}, \\
\text{others} &= 0.
\end{aligned}
$$

Acknowledgments

The authors would like to thank Professor S. Fantoni at the Interdisciplinary Laboratory for Natural and Humanistic Sciences of the International School for Advanced Studies in Trieste for the hospitality extended to them.

References

[1] E. A. Ivanov and J. Niederle, Phys.Rev. **D45** (1992) 4545.

[2] E. A. Ivanov and V. I. Ogievetsky, Theor. Mat. Fiz. **25** (1975) 164.

[3] V. I. Ogievetsky and E. S. Sokatchev, Phys. Let. **B79** (1978) 222; Yadern. Fiz. **28** (1978) 1631; **31** (1980) 264 (in Russian), [Sov. J.Nucl.Phys. **28** (1978) 840; **31** (1980) 140].

[4] V. I. Ogievetsky and E. S. Sokatchev, Yadern. Fiz. **31** (1980) 821; **32** (1980) 862, 870, 1142 (in Russian), [Sov. J. Nucl. Phys. **31** (1980) 424; **32** (1980) 443, 447, 589].

[5] E. A. Ivanov and J. Niederle, Class. Quantum Grav. **2** (1985) 631.

[6] W. Siegel and S. J. Gates Jr., Nucl. Phys. **B147** (1979) 77.

[7] C. J. Isham, Nuovo Cim., **LIX A** (1969) 356.

[8] S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** (1969) 2239.

[9] C. L. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177** (1969) 2247.

[10] E. Cartan, Geometry of Lie Groups and symmetrical spaces, Mir, Moskva 1949 (in Russian).

[11] E. A. Ivanov J. Phys. A: Math. Gen. 16 (1983) 2571.