A)

OITS-542
August 1994

AZIMUTHAL CORRELATION FOR COHERENT SOURCES IN PHASE TRANSITION

SCW 9439

Rudolph C. HWA[1] and C. S. LAM[2]

[1] Institute of Theoretical Science and Department of Physics
University of Oregon, Eugene, OR 97403

[2] Department of Physics, McGill University
Montreal, Quebec, Canada H3A 2T8

Abstract

We present a formulation of the correlation problem in the transverse plane of high-energy nuclear collisions. The correlation variable is the azimuthal angle between two transverse momenta. We assume that the effects of quark-hadron phase transition are manifested in the existence of a coherence angle χ and a parameter ϵ characterizing the irreducible correlation. We then derive the consequences on the observable two-particle correlation function.

Among the various observable effects of the formation of quark-gluon plasma in heavy-ion collisions, the ones that have been extensively studied are signals from the primordial state when the quark system has high temperature and high density, e.g., photons and dileptons. Less investigated are the possible phenomenological implications near the end of the plasma lifetime when the quarks turn into hadrons. Theoretical understanding of the quark-hadron phase transition process is still meager at this point, although there has been some small success recently in finding the observables that exhibit scaling behaviors as consequences of the transition [1, 2]. In this paper we look for another observable that may reveal the coherence aspect of the plasma as it undergoes a second-order phase transition (PT).

In statistical physics it is well known that a system at second-order PT has infinite coherence length ξ, a property that can be checked in experiments by taking the system to the critical point. In heavy-ion collisions even if the plasma that is formed can be described by the same physics during hadronization, the experiments cannot be controlled by the tuning of a parameter like the temperature. Hadrons may be produced under varying conditions having a range of values of ξ, which may all be of the order of the dimensions of the system. Even when hadronization occurs at the phase transition temperature, finite pion mass may prevent ξ from being too large. Furthermore, the geometrical shape of the system and its dynamical behavior introduce essential complications that must be sorted out from the features that characterize the PT.

We limit the degree of complication by focusing on the azimuthal dependence of the observables. There are several reasons for this choice. First, the longitudinal expansion is relativistic. How causality affects coherence is a complex issue that we can avoid by considering only a small rapidity interval. Secondly, the two-dimensional geometry in the transverse plane may provide interesting features that are absent in the longitudinal one-dimensional system that is usually studied. Thirdly, the coherence length ξ has a dimension, while the measure of correlation in the azimuthal angle is dimensionless; their relationship may contain some elements of physical significance.

On the subject of correlation and, more generally, multiparticle interference phenomena, there have been many investigations from various points of view and with different emphases [3-9]. A common approach is to use the classical source formalism to describe the pion emission process. Adopting the same approach, we use $j(\mathbf{r})$ to denote the source function so that $|j(\mathbf{r})|^2$ is the probability of emitting a pion from the spatial point $\mathbf{r}$. In applying the formalism to heavy-ion collisions, we consider the specific case of second-order quark-hadron PT, where hadronization is assumed to take place on the surface of the expanding cylindrical plasma. This corresponds to the simplest scenario for a plasma undergoing PT, where the temperature T in the interior of the plasma is greater than the critical temperature T_c, and is $\leq T_c$ at the surface. Assuming boost invariance along the longitudinal direction, we can focus our attention on a particular, small rapidity interval, so that $j(\mathbf{r})$ is nonvanishing only in a thin shell of constant radius $|\mathbf{r}| = R$ in the transverse plane. We further make the simplifying assumption that transverse expansion has a negligible effect on the azimuthal correlation which is therefore to be calculated for a fixed R at a given time t of evolution of the plasma. The t dependence of R can be considered later after the azimuthal correlation is determined for pions emitted from a given R. The usual procedure in the study of correlations is to integrate first the single- and two-particle distributions over space-time before calculating the correlation. Our procedure is different and can be justified phenomenologically only if experimental cuts can be made in the transverse momentum p_T that corresponds to narrow ranges of R. When that can be done, the theoretical description of fixed-R azimuthal correlation then becomes very transparent, being decoupled from the expansion problem.

Let the single- and two-particle distributions be denoted by $P_1(\mathbf{k})$ and $P_2(\mathbf{k}_1, \mathbf{k}_2)$, respectively, so that the correlation function is

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = P_2(\mathbf{k}_1, \mathbf{k}_2)/P_1(\mathbf{k}_1)P_1(\mathbf{k}_2). \tag{1}$$

If the pion-emitting sources are in a pure coherent state, then $P_1(\mathbf{k})$ is related to $J(\mathbf{k})$, the Fourier transform of $j(\mathbf{r})$, by [3, 5, 10]

$$P_1(\mathbf{k}) = \langle J|a^\dagger(\mathbf{k})a(\mathbf{k})|J\rangle = |J(\mathbf{k})|^2 \tag{2}$$

where $a(\mathbf{k})$ is the pion annihilation operator, of which $|J\rangle$ is the eigenstate with eigenvalue $J(\mathbf{k})$. If they are not in a pure coherent state, the problem can be treated by multiplying $j(\mathbf{r})$ by a phase factor $\gamma(\mathbf{r})$ and endowing the ensemble average $\langle\gamma(\mathbf{r})\gamma^*(\mathbf{r}')\rangle$ with information on the coherence of the state. For a totally chaotic source we would have

$$\langle\gamma(\mathbf{r})\gamma^*(\mathbf{r}')\rangle = \delta(\mathbf{r} - \mathbf{r}'). \tag{3}$$

The Fourier transform in the transverse plane becomes, in the notation $\mathbf{k} = (k, \phi)$ and $\mathbf{r} = (r, \alpha)$,

$$J(\mathbf{k}) = \int d^2r\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, j(\mathbf{r})\, \gamma(\mathbf{r}) = \int dr\, r\, j(r) \int d\alpha\, e^{-ikr\cos(\phi-\alpha)}\, \gamma(\alpha), \tag{4}$$

where, for the geometry of a thin circular shell, we have assumed that $\gamma(\mathbf{r})$ depends only on the spatial azimuthal angle α, and $j(\mathbf{r})$ on r only. Approximating $j(r)$ by $\delta(r - R)$, and ignoring the overall normalization factor of $J(\mathbf{k})$ that is cancelled in the ratio in (1), we have

$$J(\phi) = \int_{-\pi/2}^{\pi/2} d\alpha\, e^{-ikR\cos\alpha}\, \gamma(\alpha + \phi). \tag{5}$$

We set the limits of integration in (5) to $\pm\pi/2$, measured from the direction of $\mathbf{k}$, on the grounds that only the source in the same hemisphere as $\mathbf{k}$ can influence the pions emitted at ϕ.

Our main dynamical input that summarizes the behavior of the quark-gluon system undergoing phase transition is

$$\langle \gamma(\alpha)\, \gamma^*(\beta) \rangle = c_2 \exp(-|\alpha - \beta|/\chi) \tag{6}$$

where χ is the coherence angle. If a more detailed study of the quark-hadron PT problem yields a coherence length ξ (which may be a quantity averaged over a range of hadronization temperature), then we would relate it to χ by $\chi = \xi/R$. In that sense we may regard (6) as describing the correlation between two points in the plasma in PT, the distance between them being wrapped around the circle of radius R. To conform with the angular part of (3) the normalization factor c_2 can be determined, in the case where χ is not large, from the integral of (6) over β from $-\infty$ to $+\infty$, yielding $c_2 = (2\chi)^{-1}$. This will be used in the following with χ regarded as a free parameter, although the precise value of c_2 does not affect the correlation function C_2, since it gets cancelled in the ratio in (1). We now can generalize (2) to the case of finite coherence length, getting

$$P_1(\phi) = \langle |J(\phi)|^2 \rangle = (2\chi)^{-1} \int_{-\pi/2}^{\pi/2} d\alpha\, d\beta\, e^{-ikR(\cos\alpha - \cos\beta) - |\alpha-\beta|/\chi}, \tag{7}$$

which is, of course, independent of ϕ, as it should. If χ is small, P_1 has the normalization equal to π.

For $P_2(\phi_1, \phi_2)$ where ϕ_1 and ϕ_2 are the azimuthal angles of $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, we need to consider the ensemble average of four phase factors $\langle \gamma(\alpha_1)\gamma^*(\beta_1)\gamma(\alpha_2)\gamma^*(\beta_2) \rangle$. Naively, one would expect it to be $\langle \gamma(\alpha_1)\gamma^*(\beta_1)\rangle \langle \gamma(\alpha_2)\gamma^*(\beta_2)\rangle + \langle \gamma(\alpha_1)\gamma^*(\beta_2)\rangle \langle \gamma^*(\beta_1)\gamma(\alpha_2)\rangle$. However, that is only partially correct, as can be seen in the following example where $\gamma(\alpha_i)$ is replaced by $e^{i\theta_i}$; we have

$$\langle e^{i\theta_i} e^{-i\theta_j} e^{i\theta_k} e^{-i\theta_l} \rangle = \delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk} - \delta_{ijkl}. \tag{8}$$

The last term is necessary in order that when we take $i = j = k = l$ in the limit $\chi \to \infty$, (8) reduces to 1, not 2, i.e., there should be no correlation in the pure coherent case. In the diagrammatic language the three terms on the RHS of (8) correspond respectively to the direct, exchange and quartic terms. In general, we should therefore write

$$\begin{aligned} \langle \gamma(\alpha_1)\gamma^*(\beta_1)\gamma(\alpha_2)\gamma^*(\beta_2)\rangle &= \langle \gamma(\alpha_1)\gamma^*(\beta_1)\rangle \langle \gamma(\alpha_2)\gamma^*(\beta_2)\rangle \\ &+ \langle \gamma(\alpha_1)\gamma^*(\beta_2)\rangle \langle \gamma(\alpha_2)\gamma^*(\beta_1)\rangle - \langle \gamma(\alpha_1)\gamma^*(\beta_1)\gamma(\alpha_2)\gamma^*(\beta_2)\rangle_4 \end{aligned} \tag{9}$$

where the quartic term represents new physics that is not contained in what is known about $\langle \gamma(\alpha)\gamma^*(\beta)\rangle$. In the spirit of the form used in (6) and (8) we parametrize the quartic term by

$$\begin{aligned} &\langle \gamma(\alpha_1)\gamma^*(\beta_1)\gamma(\alpha_2)\gamma^*(\beta_2)\rangle_4 \\ &\quad = c_4 \exp[-(|\alpha_1 - \beta_1| + |\alpha_1 - \beta_2| + |\alpha_2 - \beta_1| + |\alpha_2 - \beta_2|)/(2\epsilon\chi)] \end{aligned} \tag{10}$$

where ϵ is a new parameter that is unknown except that it is not expected to be large, probably ≤ 1; for otherwise such a strong 4-particle clustering effect would have been revealed in the 2-particle correlation function already. For finite χ, perfect coherence occurs when $\alpha_1 = \alpha_2 = \beta_1 = \beta_2$, *viz.*, the rhs of (8) should be 1. This requires $c_4 = c_2^2$. Note that in the limit $\chi \to 0$ (10) becomes $\epsilon^2 \delta(\alpha_1 - \beta_1)\, \delta(\alpha_2 - \beta_2)\, \exp[-|\alpha_1 - \alpha_2|/(\epsilon\chi)]$, where the last exponential factor makes the whole expression vanish. That is the usual result for Gaussian random noise, corresponding to a totally incoherent source.

We now can obtain from the two-particle distribution

$$P_2(\phi_1, \phi_2) = \langle |J(\phi_1)|^2 |J(\phi_2)|^2 \rangle = \int d\alpha_1 d\beta_1 d\alpha_2 d\beta_2$$
$$\cdot \exp[-ikR(\cos(\alpha_1 - \phi_1) + \cos(\alpha_2 - \phi_2) - \cos(\beta_1 - \phi_1) - \cos(\beta_2 - \phi_2))]$$
$$\cdot \langle \gamma(\alpha_1)\gamma^*(\beta_1)\gamma(\alpha_2)\gamma^*(\beta_2) \rangle \tag{11}$$

the correlation function

$$C_2(\phi) = 1 + \frac{A_1(\phi) - A_2(\phi)}{P_1^2}, \tag{12}$$

where $\phi = \phi_2 - \phi_1$, and

$$A_1(\phi) = B(\phi)B(-\phi), \tag{13}$$

$$B(\phi) = (2\chi)^{-1} \int_{-\pi/2}^{\pi/2} d\alpha d\beta \exp[-ikR(\cos\alpha - \cos\beta)] \exp[-|\alpha - \beta - \phi|/\chi], \tag{14}$$

$$A_2(\phi) = (2\chi)^{-2} \int_{-\pi/2}^{\pi/2} d\alpha_1 d\beta_1 d\alpha_2 d\beta_2 \exp[-ikR(\cos\alpha_1 + \cos\alpha_2 - \cos\beta_1 - \cos\beta_2)]$$
$$\cdot \exp[-(|\alpha_1 - \beta_1| + |\alpha_2 - \beta_2| + |\alpha_1 - \beta_2 - \phi| + |\alpha_2 - \beta_1 + \phi|)/(2\epsilon\chi)]. \tag{15}$$

$A_2(\phi)$ is invariant under $\phi \to -\phi$, so $C_2(\phi)$ is an even function of ϕ.

It is useful to consider the limits $kR \to 0$ and ∞ as useful bounds of the more physical cases for intermediate values of kR. For $kR \to \infty$ one can use the stationary-phase approximation to pick out regions near $\alpha_i = \beta_i = 0$ and get

$$C_2(\phi) \simeq 1 + e^{-2|\phi|/\chi} - e^{-|\phi|/(\epsilon\chi)}. \tag{16}$$

For $kR \to 0, B(\phi)$ and $A_2(\phi)$ involve integrals of elementary functions and can be carried out analytically. However, the process is very long and tedious so we will not reproduce the formulas here. The results for both cases will be presented below.

Consider next the limit $\chi \to 0$ for which we use the superscript $^{(0)}$ on the functions in (12)-(15). Since $A_2^{(0)}(\phi) = 0,\ P_1^{(0)} = \pi$ and

$$B^{(0)}(\phi) = \int_{-\pi/2}^{\pi/2} d\alpha \exp\{-ikR[\cos\alpha - \cos(\alpha - \phi)]\} \tag{17}$$

we obtain for the chaotic sources

$$C_2^{(0)}(\phi) = 1 + |B^{(0)}(\phi)/\pi|^2. \tag{18}$$

This has the proper limit of 2 as $\phi \to 0$. In Fig. 1 is shown the ϕ dependences of $C_2^{(0)}(\phi)$ for a range of values of kR. This may be regarded as the result of the Hanbury-Brown-Twiss interferometry [11] for azimuthal correlation. The behavior in Fig. 1 looks different from the usual empirical parametrization of Bose-Einstein correlation [12]

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = 1 + \lambda \exp(-q^2 R^2/2), \tag{19}$$

where $q = |\mathbf{k}_1 - \mathbf{k}_2|$, and R is the size of the emitting source. There is oscillation in ϕ that cannot be parametrized by a Gaussian. However, the general features at very small ϕ are rather similar when q is identified with $2k\sin(\phi/2)$. Fig. 1 shows that, for kR not very large, the peak in ϕ can be quite wide, which is a rather striking phenomenon in the azimuthal correlation.

When there is some coherence in the system, we have two parameters, χ and ϵ, in addition to kR. In Figs. 2 and 3 we show our computed results for various combinations of the three parameters. It is clear from (14) and (15) that there are rapid oscillations at large kR, although they are suppressed if χ and/or $\epsilon\chi$ are small, a feature which is evident in Figs. 1-3. The curves shown can only give an indication of what is expected at large kR, given the finite accuracy used in the calculation. Those curves are sufficient for our purpose here, since in reality the system under study is not as ideal as we have formulated. Variation in k and R, etc., will imply substantial smearing of $C_2(\phi)$, rendering the curves shown to be fairly accurate averages of the theoretical results to be compared with the observed data. Figs. 2(a) and 3(a) are similar because χ is small in both cases, so they correspond to various kR sections of Fig. 1.

What is most notable about our result is that when χ is large, like around π, and ϵ not too small, $C_2(\phi)$ is nearly independent of kR and ϕ, and is roughly 1 even near $\phi = 0$. An observation of such features in the data should surely be a clear signal for an unusual phenomenon, highly suggestive of a phase transition. At the very least it would be hard to avoid the conclusion that there is strong coherence in the system undergoing hadronization.

In the less spectacular cases we would have the intermediate situation of finite, small, but nonvanishing values of χ. It is a general feature that the peak at small ϕ becomes narrower at larger kR and the zero-intercept $C_2(0)$ does not depend sensitively on moderate values of kR. As is usually recognized, $C_2(0)$ provides information on the degree of coherence. In our formulation that corresponds to values of χ and ϵ, which can be determined only by fitting $C_2(\phi)$ over the whole range of ϕ. Since our knowledge on ϵ is meager, data on $C_2(\phi)$ can give us valuable hint on the size of the quartic term. If ϵ is small, $C_2(\phi)$ can be approximated by the conventional formula (19), which for small ϕ becomes

$$C_2(\phi) = 1 + \lambda \exp(-\phi^2/\phi_0^2). \tag{20}$$

It involves two parameters λ and ϕ_0, which can be approximately related to our χ, ϵ and kR. However, if there is anticorrelation, *i.e.*, when $C_2(\phi)$ dips below 1, then

λ in (20) would have to assume an unconventional negative value; an experimental confirmation of that would by itself be interesting.

In this paper we have departed from the conventional approach to the study of correlations in a number of ways.

First, our emphasis is in azimuthal correlation in the transverse plane. This eliminates the complication arising from the relatisvistic longitudinal expansion. The transverse expansion is used to our advantage implicitly by requiring that the data be analyzed in various p_T cuts so that hadronization at different times and different radii of the plasma surface can be separately studied in the hope that information about the system that would otherwise be lost due to the usual integration over p_T can now be extracted.

Second, instead of the usual way of introducing coherence through the use of a parameter mixing the coherent and chaotic components, we formulate the problem at a more basic level through the use of a coherence length, which in the present azimuthal problem takes the form of a dimensionless coherence angle χ.

Third, instead of avoiding it, we confront the fact that there is a quartic term in the two-particle correlation function and parametrize its contribution by a coherence angle $\epsilon\chi$. Since the value of ϵ is largely unknown, any estimate of it by phenomenology would be of some scientific interest.

It is hoped that this work will stimulate the experimental effort to measure the azimuthal correlation. When the data become available and the dependence on p_T becomes known, specific suggestions on how best to do the analysis in determining the coherence parameters may then be made more concrete. It seeems at this point that any data on $C_2(\phi)$ would be highly interesting.

One of us (RCH) would like to thank R. Weiner and Yu. Sinyokov for interesting discussions. This work was supported, in part, by the U.S. Department of Energy under Grant No. DE-FG06-91ER40637, and by the Natural Sciences and Engineering Council of Canada and the Québec Department of Education.

References

[1] R.C. Hwa and M.T. Nazirov, Phys. Rev. Lett. **69** (1992) 741; R.C. Hwa, Phys. Rev. C**50** (1994) 383.

[2] R.C. Hwa, C.S. Lam, and J. Pan, Phys. Rev. Lett. **72** (1994) 820.

[3] M. Guylassy, S.K. Kauffmann, and L.W. Wilson, Phys. Rev. C**20** (1979) 2267.

[4] C.S. Lam and S.Y. Lo, Phys. Rev. Lett. **52** (1984) 1184; Phys. Rev. D**33** (1986) 1336; Int. J. Mod. Phys. A**1** (1986) 451.

[5] S. Pratt, Phys. Rev. Lett. **53** (1984) 1219; Phys. Lett. **B301** (1993) 159; S. Pratt and V. Zelevinsky, Phys. Rev. Lett. **72** (1994) 816.

[6] G. Bertsch, M. Gong, and Tohyama, Phys. Rev. C**37** (1988) 1896.

[7] R.M. Weiner, Phys. Lett. **B232** (1989) 278; **B242** (1990) 547; I. Andreev, M. Plümer and R. Weiner, Phys. Rev. Lett. **67** (1991) 3475; Int. J. Mod. Phys. **A8** (1993) 4577.

[8] D.K. Srivastava and J.I. Kapusta, Phys. Rev. C**48** (1993) 1335.

[9] Yu.M. Sinyokov and A.Yu. Tolstykh, Z. Phys. C**61** (1994) 593.

[10] J.C. Botke, D.J. Scalapino, and R.L. Sugar, Phys. Rev. D**9** (1974) 813.

[11] R. Hanbury-Brown adn R.Q. Twiss, Proc. Roy. Soc. London **A242** (1957) 300; **A243** (1957) 291.

[12] See the Proceedings of the Workshop on *Correlations and Multiparticle Production*, Marburg 1990, edited by M. Plümer, S. Raha and R.M. Weiner (World Scientific, Singapore, 1991).

FIGURE CAPTIONS

Fig. 1 Correlation function $C_2^{(0)}(\phi)$ for $\chi = 0$ as a function of ϕ and kR.

Fig. 2 Correlation function $C_2(\phi)$ for $\epsilon = 1$ and several values of kR with (a) $\chi = \pi/100$, (b) $\chi = \pi/8$, and (c) $\chi = \pi$.

Fig. 3 Correlation function $C_2(\phi)$ for $\epsilon = 0.5$ and several values of kR with (a) $\chi = \pi/100$, (b) $\chi = \pi/8$, and (c) $\chi = \pi$.

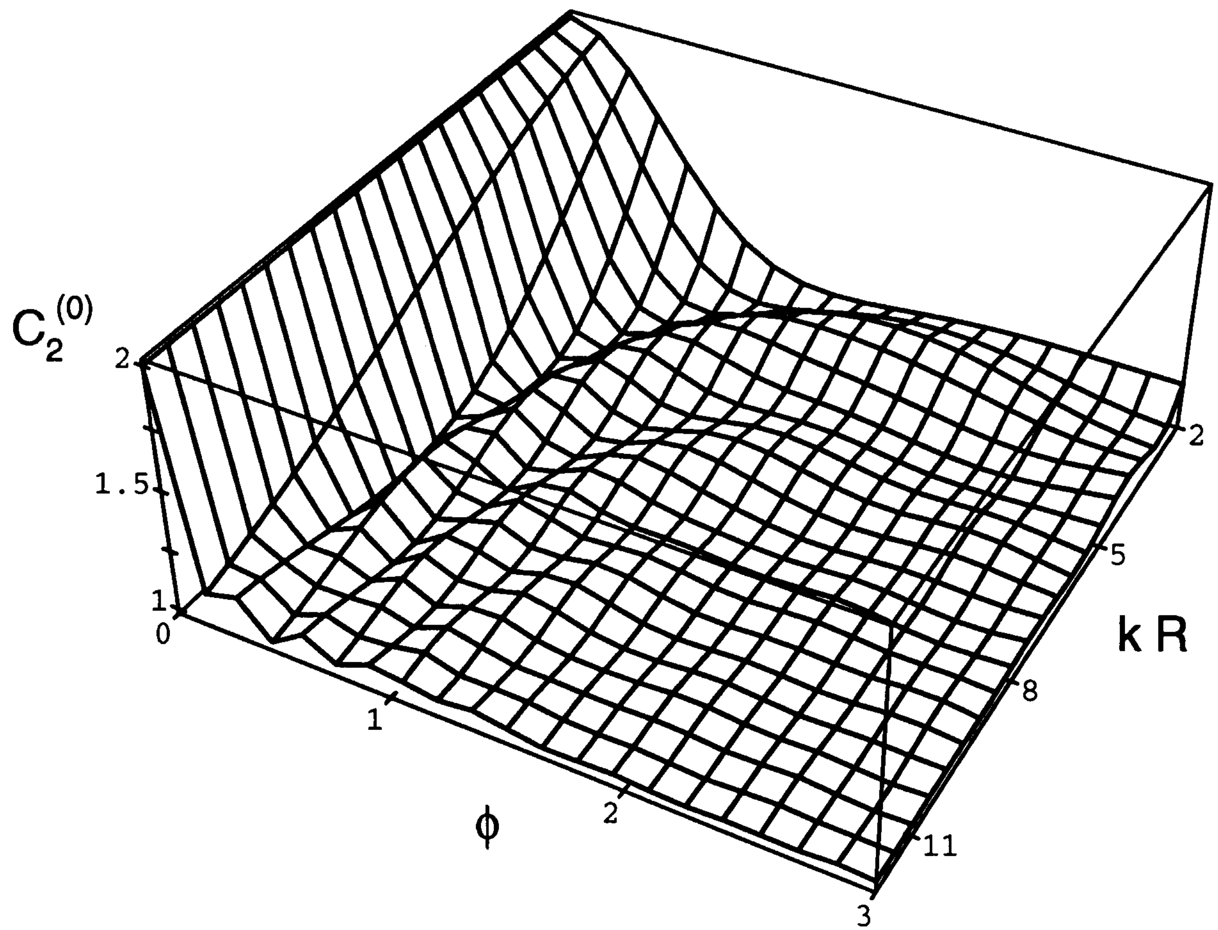

Fig. 1

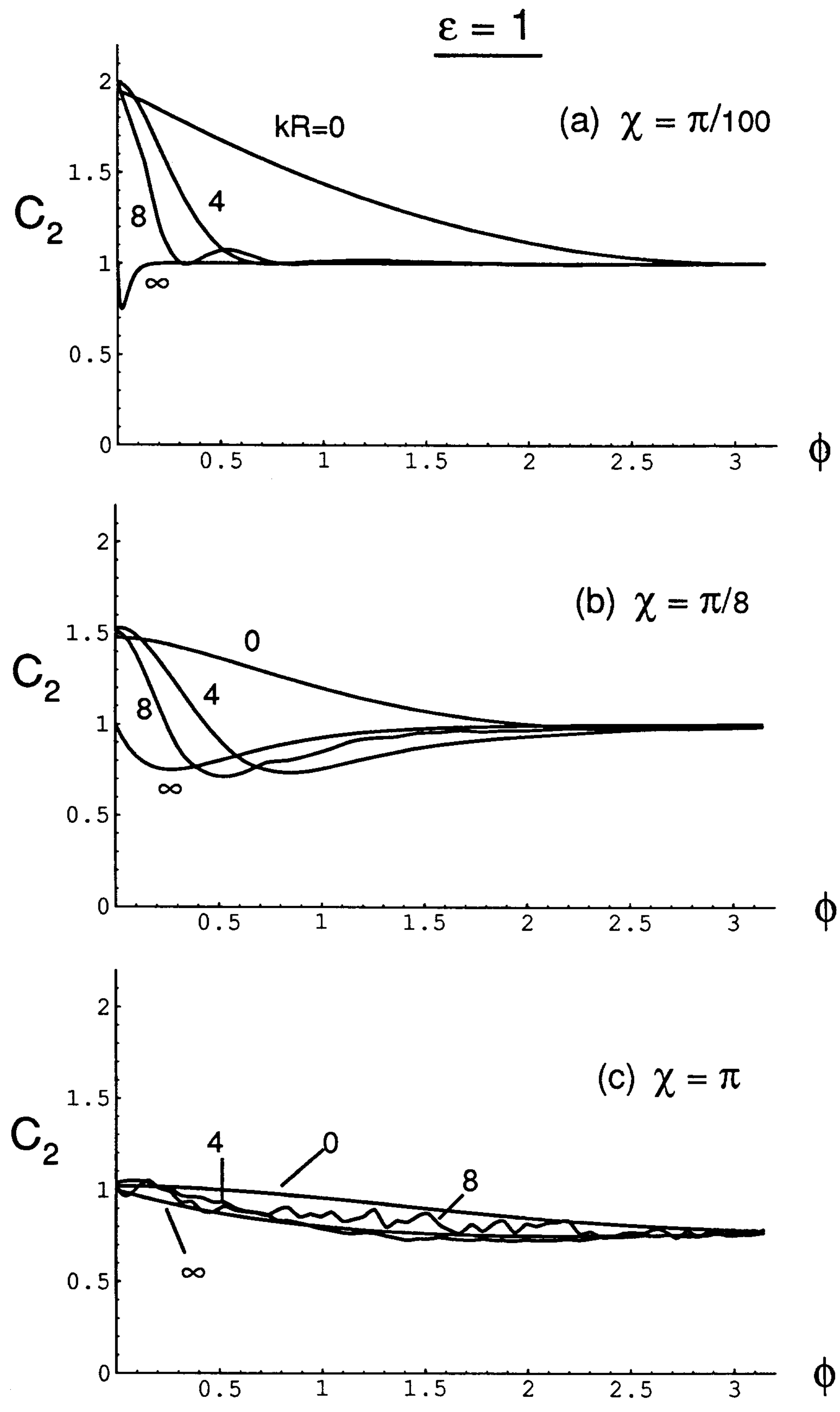

Fig. 2

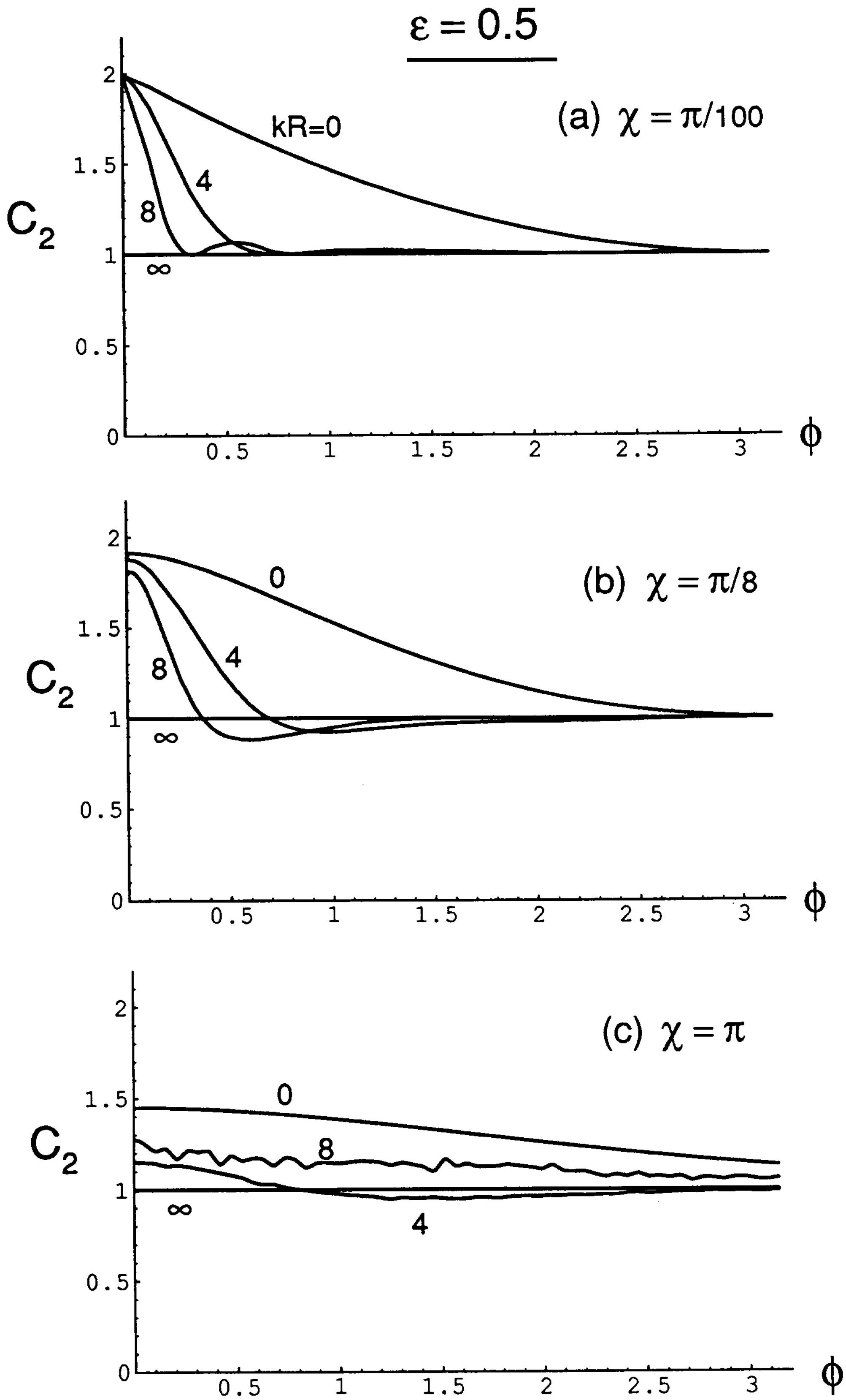

Fig. 3