PSI-PR-94-27
August 1994

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Abstract

We study intensity correlations for pions from $\bar{N}N$ annihilation at rest. Pions coming from a coherent source with local isospin show strong Hanbury-Brown Twiss correlations for like charged pairs. Isospin projections, integration over the source and experimental binning leads to the emergence of these correlations without any thermal or related random phase assumptions.

[1] The Division de Physique Théorique is a Research Unit of the Universities of Paris 11 and 6 associated to CNRS.

Hanbury-Brown Twiss (H-BT) intensity correlations [1] between identical particles at small relative momentum arise when the radiating source has some spatial structure, and when there is some averaging mechanism among the parts of the source [2]. The correlations come about because in the two particle joint probability there are product terms that survive since the average of the product is not the product of the averages. If there is a single coherent source of the radiation, or if there is no averaging mechanism, there are no significant (H-BT) correlations between like particles at small relative momentum.

In this work we study correlations between like charge pions coming from nucleon-antinucleon annihilation at rest. Enhancements at small relative momentum in the number of like charge pion pairs compared with unlike pairs are a well studied feature of annihilation [3] [4]. These enhancements have been interpreted as arising from a finite size "fireball" source, with the required averaging coming naturally from the statistical nature of the thermal source. Recently we have argued that annihilation is not a statistical process at all, but rather that it takes place quickly with the emission of a coherent pion wave [5]. Quantizing this wave using the method of coherent states and imposing the constraints of energy-momentum and isospin conservation gives an excellent account of the global features of annihilation including the single pion spectrum, pion number distribution and charge ratios. Our first work modeled the pion wave as emerging coherently from a spherically symmetric source. Such an approach cannot yield the H-BT correlations seen in the data. We now examine the simplest extension of our coherent state picture required to account for the observed enhancements. Dynamical mechanisms, associated with pion resonances can also lead to these enhancements and have been studied elsewhere [6], but we will not discuss them further here.

To obtain the H-BT correlations we must replace the single coherent pion source by a structured source and find an averaging mechanism. We want this mechanism to maintain as much as possible of the notion of a rapid coherent source for the pion radiation, both because this is suggested by dynamical classical models of the annihilation [7] [8], and because there is no obvious randomizing or thermalizing mechanism. The central point of this paper is that a number of averaging mechanisms arise naturally, any one of which is sufficient to reveal the H-BT correlations. These mechanisms include averaging over the locations of the radiating points in the source, making the cuts usually made in experiment when only the relative momentum is kept fixed, and taking care of isospin projections. Because any one of these averages is enough, the data can tell us little about the details of which mechanism is actually dominating. The possibility of averaging mechanisms for coherent sources has been suggested before, [2], but here we give explicit implementations of this idea.

There are two kinds of pair correlations studied in the literature. The first is the usual pair correlation function for a pair of particles of momentum $\vec{p}$ and $\vec{q}$,

$$C_2(\vec{p},\vec{q}) = \frac{W_2(\vec{p},\vec{q})}{W_1(\vec{p})W_1(\vec{q})} - 1 \tag{1}$$

where $W_2(\vec{p},\vec{q})$ is the joint probability of finding one particle of momentum $\vec{p}$ and one of momentum $\vec{q}$, while $W_1(\vec{k})$ is the probability of finding a single particle of momentum $\vec{k}$. [2] The second form, more commonly studied for pions, is

$$R_{++/+-}(\vec{p},\vec{q}) = \frac{W_2(\vec{p}+,\vec{q}+)}{W_2(\vec{p}+,\vec{q}-)} \tag{2}$$

where $W_2(\vec{p}a,\vec{q}b)$ is the joint probability of finding two pions one of momentum $\vec{p}$ and charge type a and the other of momentum $\vec{q}$ and charge type b. Of course the closely related ratio $R_{--/+-}$ is also studied. What is observed experimentally is $R_{aa/ab}$ as a function of $\vec{Q} = \vec{p}-\vec{q}$ (or its covariant equivalent) with $\vec{p}+\vec{q}$ summed over some experimental range. As a function of Q^2, $R_{aa/ab}$ is typically 1 for large Q and shows a peak reaching 2 to 3 for $Q^2 = 0$, that peak starting near $Q^2 = .1(GeV/c)^2$. For coherent states with good isospin, ([5]), $W_2(\vec{p}a,\vec{q}b)$ factors into a part depending only on $\vec{p},\vec{q}$ and an isospin dependent part carrying the labels a, and b. Hence $R_{ab/cd}(\vec{p},\vec{q})$ will depend only on ab/cd and not on momenta, contrary to experiment. To obtain the observed small relative momentum peaking of $R_{aa/ab}$, we need to mix isospin and momentum or spatial variables in a more dynamical way.

Consider a classical pion wave emitted by a source of finite extent with that wave coming from N points in the source, $\vec{x}_i$. The most general amplitude for coherent emission of this wave with momentum $\vec{p}$ and charge

[2] For a pure coherent state, $W_2(\vec{p},\vec{q}) = W_1(\vec{p})W_1(\vec{q})$, so that $C_2(\vec{p},\vec{q}) = 0$.

type a from the N points is

$$A(\vec{p}, a) = \sum_{i=1}^{N} f(\vec{p}, a, \vec{x}_i) \tag{3}$$

The single particle probability, $W_1(\vec{p}a)$ is given by $|A|^2$ with the $\vec{x}_i$ integrated over the source density. Let us assume that each point in the source emits pions of charge type a and momentum $\vec{p}$ with the same probability, and that the only difference between the points is that they are translated one from the other. Let us also assume that the amplitude for emission of a pion of charge type a is independent of momentum. Then the most general form for the emission amplitude is given by

$$f(\vec{p}, a, \vec{x}_i) = g(\vec{p}) T_a(\vec{x}_i) e^{-i\vec{p}\cdot\vec{x}_i} \tag{4}$$

where $g(\vec{p})$ is the Fourier transform of the local density for making a pion of momentum $\vec{p}$ in the vicinity of the ith point and $T_a(\vec{x}_i)$ is the ath component of a unit vector in isospin space corresponding to the amplitude for creation of a pion of charge type a at that point. In writing this form for $f(\vec{p}, a, \vec{x}_i)$ we have dropped an overall factor of $e^{i\vec{p}\cdot\vec{D}}$ where $\vec{D}$ is the distance from the center of the entire source to the detector. This factor, of modulus one, disappears when we take probabilities.

Let us examine $W_1(\vec{p}a)$ in this picture, we find

$$W_1(\vec{p}a) = |A(\vec{p}, a)|^2 = |g(\vec{p})|^2 \sum_{i,j} T_a(\vec{x}_i)(T_a(\vec{x}_j))^\dagger e^{-i\vec{p}\cdot(\vec{x}_i - \vec{x}_j)} \tag{5}$$

The N terms with $i = j$ give the probability for finding a pion of charge type a and momentum $\vec{p}$ as a sum of the probabilites from each of the sources at points i. This is the purely classical incoherent result. The cross terms $(i \neq j)$ terms represent the interference between the sources. If we average over isospins at each point, the cross terms will vanish. If we average over directions of $\vec{x}_i$, the cross term will be small. If we average over the magnitude of $\vec{x}_i$ within a source of size R and if we make kinematic cuts in $\vec{p}$ so that pR is large compared with one, that will further reduce the magnitude of the cross term. Thus in the single particle spectrum there are many independent and different assumptions that average the cross terms away. It is the two particle correlations that we are concerned with, but the arguments we gave above for the one particle probability carry over into the two body case, where isospin, source position and momentum cut averages all contribute to give the same effect as thermal averages.

Consider in this classical picture the joint probability of finding a pion of type a and momentum $\vec{p}$ and one of type b and momentum $\vec{q}$:

$$\begin{aligned} W_2(\vec{p}a, \vec{q}b) &= |A(\vec{p}, a)|^2 |A(\vec{q}, b)|^2 \\ &= |g(\vec{p})|^2 |g(\vec{q})|^2 \sum_{i,j,k,l} T_a(\vec{x}_i) T_b(\vec{x}_k) (T_a(\vec{x}_j) T_b(\vec{x}_l))^\dagger \\ & \quad \exp(-i(\vec{p} \cdot (\vec{x}_i - \vec{x}_j) + \vec{q} \cdot (\vec{x}_k - \vec{x}_l)) \end{aligned} \tag{6}$$

First consider the effect of averaging over isospin directions independently at each point. Only two kinds of terms survive. The first are the N^2 terms with $i = j$, $k = l$ and no condition on a and b. This is just the completely incoherent sum of probabilities of radiation from the N sources. Note that for $i = j$, $k = l$, there is no exponential term in $\vec{p}$ or $\vec{q}$. The other surviving term has $i = l$ and $j = k$ (with $i \neq j$) but only if also $a = b$. This term carries the exponential factor

$$e^{-i\vec{Q}\cdot(\vec{x}_i - \vec{x}_j)} \tag{7}$$

where $\vec{Q} = \vec{p} - \vec{q}$. There are $N(N-1)$ such terms. All the remaining terms in the double sum average to zero under the average of isospin at each point. The surviving cross term has precisely the form of the H-BT enhancement at small $\vec{Q}$, and contributes only for identical pions [3] while the N^2 direct or incoherent terms contribute to all charge types. The assumption of separately averaging the isospin at each point may seem drastic. However in any picture some isospin projection must be made, often with some weight. Most of those projections retain the essential feature of greatly favoring the H-BT cross term over all the others. Other sums or averages also favor this term from among the cross terms. These include averaging over the directions and magnitudes (inside the source) of the emission positions and summing over some range in $\vec{p} + \vec{q}$ for fixed $\vec{Q} = \vec{p} - \vec{q}$. If we neglect the terms with small or zero average, $R_{aa/ab}$ has the structure

$$R_{aa/ab} = 1 + F((QR)^2) \tag{8}$$

[3] This enhancement occurs for like charged or neutral pions. Dynamical enhancements associated with pion resonances will, however, differ in such cases. [6]

where $F(x)$ is zero for large x and peaks at $x = 0$. The source is of size R. Hence we find just the H-BT enhancement for identical pions. The parameters that describe that peak depend on the dynamics and on how we implement the isospin and spatial source average.

The discussion above has been phrased completely in terms of classical fields. What of the quantum nature of the pions and of coherent states? Any classical field result can be obtained from a coherent state formalism. Define the coherent state $|f>$ by

$$|f>= \mathcal{N} \exp(\sum_{b=1,3} \int d^3k A(\vec{k}, b) a_b^\dagger(\vec{k}))|0> \tag{9}$$

where A is defined in Eqn.(3) and where $a_b^\dagger(\vec{k})$ is the usual pion creation operator for a pion of momentum $\vec{k}$ and charge type b. The factor $\mathcal{N}$ is the normalization. If one uses the form of A given in (3) and (4) and evaluates

$$< f|a_a^\dagger(\vec{p})a_b^\dagger(\vec{q})a_a(\vec{p}))a_b(\vec{q})|f > \tag{10}$$

one finds exactly the classical result (6). This shows, as is well known, that the classical field result and the coherent state result are completely equivalent.

Let us consider the effect of imposing various constraints and dynamical averages on our pion radiation, using quantum coherent states.

First we discuss imposing four momentum conservation on the state [9],[5]. This constraint affects one and two particle probabilities differently so that C_2 of (1) is affected by its imposition. Even for a single, simple coherent state, C_2 will not vanish if four momentum conservation is imposed. By contrast, four momentum conservation effects cancel in the ratio $R_{aa/ab}(\vec{p}, \vec{q})$ so that that ratio is still given in terms of the appropriate ratio of the W_2 of (6). The only effect of four momentum conservation being to connect and restrict the range of $\vec{p}$ and $\vec{q}$, but that restriction applies equally to like and unlike charge pions. We can introduce an emission time as well as a place for each pion source point i. Our choice above corresponds to simultaneous emission from all points, consistent with our view of annihilation as proceeding in a sudden coherent burst. The formalism could easily accomodate radiation spread over time.

Our discussion of the classical wave following Eqn.(6) drew heavily on assuming independent isospin averaging at each point. What happens if we weaken this assumption? One might consider, using our isospin projection methods [5], that annihilation proceeds from two sources each of fixed isospin and those coupled up to a definite total isospin for the system. This involves isospin averages on (6) and some SU(2) algebra, but one finds again that the like charge pions lead to a large H-BT cross term and the unlike to a small or zero cross term. The details of this calculation will be presented elsewhere. [10]

Finally let us consider the effects of averaging over points in the source or summing over $\vec{p}+\vec{q}$ for fixed relative momentum. For these purposes the effect of isospin adds nothing but algebraic complexity and hence we will consider the problem without isospin, for only one kind of boson. As a further simplification consider the case of only two sources. We will return to the general case below, but the two source case may make some physical sense since $\bar{N}N$ annihilation is naturally a system with two centers. For two sources and without isospin we find

$$\begin{aligned} W_2(\vec{p},\vec{q}) &= 4|g(\vec{p})|^2|g(\vec{q})|^2[1+\cos(\vec{p}\cdot\vec{r})+\cos(\vec{q}\cdot\vec{r}) \\ &\quad +\frac{1}{2}\cos((\vec{p}+\vec{q})\cdot\vec{r})+\frac{1}{2}\cos((\vec{p}-\vec{q})\cdot\vec{r})] \end{aligned} \quad (11)$$

where $\vec{r}=\vec{x}_1-\vec{x}_2$ is the distance between the two sources. If we average over the directions of $\vec{r}$ in the source, (Even if the sources are not spherically symmetric, for annihilation at rest they are randomly oriented in the laboratory.) we find

$$\bar{W}_2(\vec{p},\vec{q}) = 4|g(\vec{p})|^2|g(\vec{q})|^2(1+j_0(pr)+j_0(qr)+\frac{1}{2}(j_0(Pr)+j_0(Qr))) \quad (12)$$

where, we have introduced the relative and total momentum variables $\vec{Q}=\vec{p}-\vec{q}$ and $\vec{P}=\vec{p}+\vec{q}$, and where $j_0(x)=\sin(x)/x$ is the usual spherical Bessel function. For large x, it is of order $1/x$, and its average falls off much faster. We are interested in $\bar{W}_2$ for $P>>Q$. In that case p and q are both of order P. There are five terms in the parenthesis of Eqn(12). The first term, 1 is the classical coherent term that would be there even for non-identical particles. The last term is a function of Q only and is the H-BT term. For Q small it is of order 1. The remaining three terms are of order $1/P$ and are therefore small. Note that in order that the H-BT terms emerge prominently, it is sufficient to average only over the source directions. If we had averaged first over the directions of $\vec{P}$ and $\vec{Q}$ we would have obtained the same result.

This form for $\bar{W}_2$ oscillates for fixed r. Let us further average $\bar{W}_2$ over a range of r, we choose a smooth spherical averaging function $e^{-\beta r}$, (which must be normalized in the average). This will reduce the importance of the three middle terms. Integrating over some range of P for fixed Q would have the same effect. This is equivalent to binning data in an experiment. To compare with experiment we must divide $\bar{W}_2$ by the "unlike" pair from. We thus define $R_2(\vec{p},\vec{q}) = \bar{W}_2((\vec{p},\vec{q})/4|g(\vec{p})|^2|g(\vec{q})|^2$, where we now include in $\bar{W}_2$ the averaging over the range of r. In Figure 1 we plot R_2 as a function of the square of the relative four momentum between the pions for $p = q = 2.5fm^{-1}$, and for $\beta = 1fm^{-1}$. These values are typical of annihilation. We see the same qualitative features as in the data, a peak at small Q^2 and R_2 going to a constant near one at large Q. The upper curve in Figure 1 contains all terms in R_2 while the lower curve has the three "small" or inner terms set to zero. The plots are very similar, differing by about 3%. Thus we see that averaging over directions of the source or equivalently summing over total momentum directions makes the H-BT correlations dominate the boson pair intensity correlations for two sources. To compare directly with experiment, one needs to use the same variables and cuts as are used in the experiment, as well as to take into account dynamical correlations.

The discussion for N sources proceeds as in the case of two. Consider W_2 of Eqn(6) with no isospin. Introducing $\vec{P}$ and $\vec{Q}$ it can be written

$$W_2(\vec{P},\vec{Q}) = \sum_{i,j,k,l} \exp(-i/2(\vec{P}\cdot(\vec{x}_i-\vec{x}_j+\vec{x}_k-\vec{x}_l)+\vec{Q}\cdot(\vec{x}_i-\vec{x}_j+\vec{x}_l-\vec{x}_k)) \quad (13)$$

We want to sum over $\vec{P}$ for fixed $\vec{Q}$. As a first orientation assume that sum goes over all $\vec{P}$. Then W_2 vanishes unless $\vec{x}_i - \vec{x}_j + \vec{x}_k - \vec{x}_l = 0$. Using this condition to eliminate $\vec{x}_l$, and calling W_2 summed over all $\vec{P}$ $\tilde{W}_2(\vec{Q})$, we find

$$\tilde{W}_2(\vec{Q}) = \sum_{i,j,k} \exp(-i(\vec{Q}\cdot(\vec{x}_i - \vec{x}_j))) \quad (14)$$

Since the summand does not depend on $\vec{x}_k$ the sum on k gives a factor of N. There are N more terms with $i = j$. These give the classical, incoherent N^2. The $N-1$ terms with $i \neq j$ are the H-BT terms. When averaged over the directions and positions of the sources they will give a form just as we found in the two source case, but with a slightly different value for the height of the peak at $Q = 0$. Corrections to the assumption that we sum over all $\vec{P}$ will give terms of order $1/P$ for typical P in the experimental range. These terms will be small compared with the leading H-BT term. In this example, the dominance of the H-BT term comes about from a completely coherent source with no random elements and arises purely from the inclusive nature of the P sum.

In conclusion we have seen that Hanbury-Brown Twiss (H-BT) intensity correlations between identical pions coming from nucleon-antinucleon annihilation at rest can occur from coherent non-thermal sources. What is needed is that the pion source have finite size, that the isospin be local and that there be some averaging process. These processes could include isospin projections, averaging over the source, or summing over unobserved momenta. Since any one of these effects is enough to give the H-BT small relative momentum enhancement for identical pions seen in the data, and since all generally are present, it is difficult to disentangle details of reaction mechanisms from these correlations. But we should stress again that no thermal assumption is needed to give the pion correlations.

Similar considerations might well apply to heavy ion physics where some thermal averaging may be appropriate, but complete thermal equilibrium may not. Our results suggest that H-BT correlations among pions from heavy ion reactions can have many origins and are not a direct sign of thermal equilibrium. Our method for isospin averaging applied to energetic heavy ion reactions may make connection with isospin treatments of the disordered chiral condensate. [11]

RDA, FC, and J-PD thank the theory group of the Division of Nuclear and Particle Physics of the Paul Scherrer Institute for, once again, providing a stimulating environment for this work. The work of RDA is partially supported by the United States National Science Foundation.

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Figure Caption

The ratio R_2 of identical pions to unlike pions as a function of the square of their relative four momentum for pions from $\bar{N}N$ annihilation at rest treated as a coherent source. The solid curve is the full correlation function, the dashed curve contains only the Hanbury-Brown Twiss correlations. The parameters are given in the text.