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Kaon Condensation and Equation of State in the Relativistic Mean-Field Theory

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Abstract

Equation of state of the kaon-condensed phase is studied within a relativistic formalism. We find a *self-suppression* mechanism such that the kaon condensate decreases the nucleon effective mass by way of the relativistic effect, which reduces the interaction energy between nucleon and kaon in high density region, and in turn suppresses its development. To get more insight about this we extend a chiral symmetry approach to include the relativistic mean field theory. Then the σ mean-field enhances such self-suppression mechanism. Some features of kaon-condensed neutron stars are briefly discussed.

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Recently kaon condensation has been attracting much attention as a new phase of dense hadronic matter [1]. It strongly affects the nuclear equation of state (EOS) and gives one of very possible scenarios for the rapid cooling process of neutron stars [2, 3, 4]. We have investigated the EOS and chemical components of neutron-star matter involving the s-wave K^- condensate and studied the kaon-induced Urca and the direct Urca processes for the cooling mechanism, by way of a $SU(3)_L \times SU(3)_R$ chiral symmetry approach within the standard non-relativistic framework [3, 4]. In our formulation the condensed phase $|K\rangle$ is represented as a chirally rotated state with the amplitude $|K^-| = (f/\sqrt{2})\sin\theta$ (θ: the chiral angle and f: the meson decay constant). These studies suggest that the EOS may be considerably softened due to the too large attraction between kaon and nucleon in high density region, essentially through the KN sigma term Σ_{KN}. Then neutron stars cannot have larger masses than 1.5 times of the solar mass ($M_\odot$) in some cases; which leads some authors to mini black-hole scenario after supernova explosions [5].

In this letter we would like to point out the importance of relativistic effects, which have never been taken into account in the above context. The driving force of kaon condensation has been shown to be the s-wave KN interaction, the KN sigma term Σ_{KN} and the Tomozawa-Weinberg type vector interaction. The former is closely related to the $s\bar{s}$-content inside nucleon [6]. The effective interaction Lagrangian for the KN sigma term Σ_{KN} reads

$$\mathcal{L}_{KN} = \frac{\Sigma_{KN}}{f^2}\bar{\psi}\psi K^\dagger K \,, \tag{1}$$

where ψ and K are fields of nucleon and kaon, respectively.

In the non-relativistic framework the interaction energy coming from this term is linearly proportional to the baryon density ρ_B, so that the energy gain (ΔE) increases unlimitedly as density. In the relativistic picture, on the other hand, its density dependence is quite different: the interaction energy must be proportional to the Lorentz scalar density $\rho_s \equiv \langle\bar{\psi}\psi\rangle$ instead of baryon density ρ_B, whose density dependence is moderate in the high density region, $\rho_s \propto \rho_B^{2/3}$. Through the scalar interaction (1), kaon condensate reduces the nucleon effective mass, $M^* = M - \Sigma_{KN}\langle K^\dagger K\rangle/f^2$, and in turn the scalar density, $\rho_s = \int_F d\boldsymbol{k}/(2\pi)^3\, M^*/\sqrt{\boldsymbol{k}^2 + M^{*2}}$. Thus this effect reduces the kaon-nucleon scalar interaction energy. Namely, in the relativistic framework, kaon condensation has a *self-suppression* mechanism implicitly. Hence one may suppose that the trouble of too large energy gain may arise from the disadvantage of the non-relativistic framework.

We shall discuss the relativistic effects in a realistic way by extending our chiral symmetry approach by incorporating the relativistic mean-field (RMF) theory [7], which is considered to be endowed with other nuclear interactions besides KN one and allows us to study EOS and properties of neutron stars in a fully relativistic way. It is well known that there is another scalar interaction in the RMF theory which reduces the effective nucleon mass in medium, so that the above-mentioned effect should be enhanced: the σ mean-field suppresses the development of kaon condensate as well.

In the RMF theory, furthermore, ρ_s saturates to a finite value in the limit of the infinite density because the effective mass takes the limit value zero through the σ mean-field [7]. Therefore it is easily conceivable that the RMF theory extremely modifies the properties of the neutron-star matter with kaon condensate.

Our model-hamiltonian consists of three parts:

$$\mathcal{H} = \mathcal{H}_N + \mathcal{H}_K + \mathcal{H}_{KN} , \tag{2}$$

where $\mathcal{H}_N$, $\mathcal{H}_K$ and $\mathcal{H}_{KN}$ are the nucleon part given by the RMF theory, the free kaon part and the interaction part between nucleon and kaon taken from Ref. [4], respectively. The nucleon part includes σ- and ω-mesons, and the isovector contact interaction terms. The definite form of $\mathcal{H}_{KN}$ is given in Ref. [4] as

$$\mathcal{H}_{KN} = -\frac{\Sigma_{KN}}{f^2}\bar{\psi}\psi K^\dagger K - \frac{i}{f^2}\bar{\psi}\gamma_\mu\frac{1}{4}(3+\tau_3)\psi K^\dagger\partial^\mu K + O(|K|^4) . \tag{3}$$

It is to be noted that the second term comes from the Tomozawa-Weinberg term.

In order to get the physical state under the charge neutrality, we should treat the effective Hamiltonian $\mathcal{H}^{\rm eff} = \mathcal{H} + \mu Q$ with the Lagrange multiplier (the charge chemical potential) μ for charge Q. Then the total effective energy density $\epsilon^{\rm eff}$ is obtained by the expectation value of the above $\mathcal{H}^{\rm eff}$ as

$$\begin{aligned} E^{\rm eff}/V \equiv \epsilon^{\rm eff} \quad &= \frac{2}{(2\pi)^3}\int {\rm d}\boldsymbol{k} f_p(\boldsymbol{k})E_p^*(\boldsymbol{k}) + \frac{2}{(2\pi)^3}\int {\rm d}\boldsymbol{k} f_n(\boldsymbol{k})E_n^*(\boldsymbol{k}) \\ &+\tilde{U}[\langle\sigma\rangle] + \frac{g_\omega^2}{2m_\omega^2}\rho_B^2 + \frac{C_s^{IV}}{2M^2}(\rho_s(p)-\rho_s(n))^2 + \frac{C_v^{IV}}{2M^2}\rho_B^2(1-2x)^2 \\ &+\frac{\mu^4}{4\pi^2} + \mu(\rho_B x - \frac{\mu^3}{3\pi^2}) + (1-\cos\theta)[f^2 m_K^2 - \frac{\mu}{2}\rho_B(1+x)] \\ &-\frac{1}{2}\mu^2 f^2 \sin^2\theta \end{aligned} \tag{4}$$

with the proton mixing ratio $x = \rho(p)/\rho_B$, the momentum distribution $f_i(\boldsymbol{k}) = \theta(k_F^{(i)} - |\boldsymbol{k}|)$, and the effective kinetic energy

$$E_i^*(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + M_i^{*2}} \quad (i = p \text{ and } n) . \tag{5}$$

The scalar density $\rho_s(i)$ is given as

$$\rho_s(i) = \frac{2}{(2\pi)^3}\int {\rm d}\boldsymbol{k} f_i(\boldsymbol{k})\frac{M_i^*}{E_i^*(\boldsymbol{k})} , \tag{6}$$

and the effective mass M_i^* is defined by

$$M_i^* = M - g_\sigma\langle\sigma\rangle + C_s^{IV}\rho_s^{IV}\tau_3(i) - \Sigma_{KN}(1-\cos\theta) , \tag{7}$$

where M is the nucleon mass and $\langle\sigma\rangle$ the σ mean-field.

In Eq. (4), g_σ, g_ω, m_σ and m_ω are the coupling constants for σ- and ω-mesons and their masses, respectively, and C_s^{IV} and C_v^{IV} are the coupling constants of the contact interactions for the isovector-scalar and vector channels, respectively. The latter interaction (the symmetry energy term) is important to see the chemical components of the ground state, which is in turn related to the possibility of the direct Urca process [4, 8, 9]. On the other hand it has little effect on EOS. Here we set the isovector-scalar coupling $C_s^{IV} = 0$ for simplicity. The self-energy potential of the sigma field is taken here [10] as

$$\tilde{U}[\langle\sigma\rangle] = \frac{\frac{1}{2}m_s^2\langle\sigma\rangle^2 + \frac{1}{3}B_\sigma\langle\sigma\rangle^3 + \frac{1}{4}C_\sigma\langle\sigma\rangle^4}{1 + \frac{1}{2}A_\sigma\langle\sigma\rangle^2} \,. \tag{8}$$

We determine $\langle\sigma\rangle, \mu, x$ and θ and get the ground state from the following extremization conditions for the effective energy (4):

$$\frac{\partial}{\partial\langle\sigma\rangle}\epsilon^{\rm eff} = \frac{\partial}{\partial\mu}\epsilon^{\rm eff} = \frac{\partial}{\partial x}\epsilon^{\rm eff} = \frac{\partial}{\partial\theta}\epsilon^{\rm eff} = 0 \,. \tag{9}$$

The other parameters are determined by the RMF theory to satisfy the normal nuclear matter properties. Here we take into account the empirical values of the saturation density $\rho_0 = 0.17$ fm^{-3}, the binding energy $E_B = 16$ MeV, the incompressibility $K = 200$ MeV and the symmetry energy $a_{\rm sym} = 32$ MeV. As for the value of K we must admit some range, but here we take only a typical one.

The value of the effective mass at the saturation point is chosen for some suitable values ($(M^*/M)_0 = 0.70, 0.75, 0.83$). The analysis of the proton-nucleus elastic scattering suggests $(M^*/M)_0 \simeq 0.5 - 0.7$ [11], and the study of the isoscalar giant quadrupole resonance suggests $(M^*/M)_0 \simeq 0.75$ [12]. On the other hand non-relativistic calculation gives $(M^*/M)_0 \sim 0.8$ [13]. We may regard this value as an extreme one from the viewpoint of the heavy ion collisions since it has been shown that this value may be ruled out from the analysis of K^+ production in the Au+Au collision at $E_{\rm lab}$=1 GeV/u [10]. The parameter-sets we have used are summarized in Table 1. The value of Σ_{KN} is estimated as 300 – 500 MeV depending on the $\bar{s}s$-content inside nucleons [6]; here we choose 400 MeV as an example.

	g_σ	g_ω	B_σ (fm)	C_σ (fm^2)	A_σ (fm^2)	C_v^{IV}	$(M^*/M)_0$
PM1	9.94	9.99	23.5	0.0	5.65	20.3	0.70
PM2	8.87	8.91	30.3	0.0	5.56	21.6	0.75
PM3	7.42	6.80	0.0	912	59.4	23.5	0.83

Table 1 Parameter sets used in this paper. Also given is the effective mass $(M^*/M)_0$ at the saturation density. In all cases we have used C_s^{IV}=0, m_σ = 550 MeV and m_ω = 783 MeV.

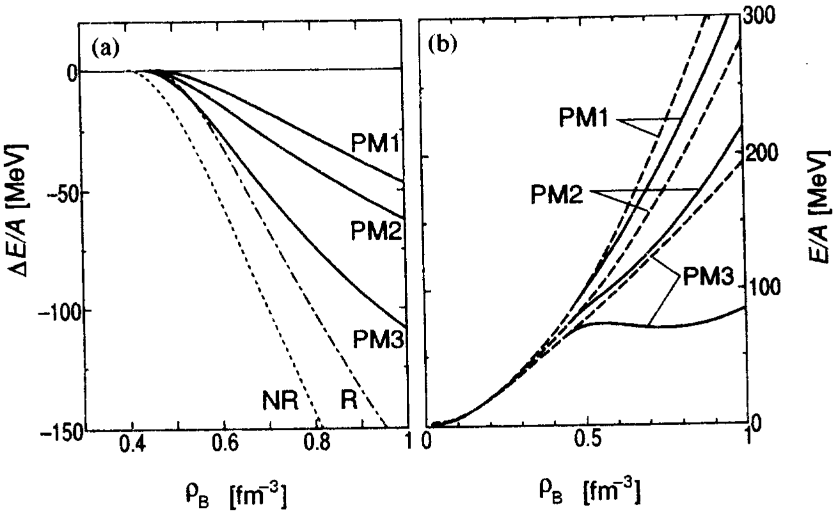

Fig. 1 (a) Density dependence of the energy gain between the kaon-condensed phase and the normal phase and (b) the equation of state for neutron-star-matter. The dotted (NR) and dash-dotted (R) lines correspond to the non-relativistic and simply relativistic results, respectively. The solid and dashed lines show the results of the kaon-condensed phase and the normal phase in the RMF theory with the parameter-sets PM1, PM2 and PM3, respectively.

In Fig. 1 we show (a) the energy difference (ΔE) between these two phases and (b) the EOS of the neutron star matter in the normal phase and the kaon-condensed phase. In Fig. 1(a), the dotted line (NR) shows our previous result with the non-relativistic calculation [4]. The dash-dotted line (R) is the simply relativistic result without any RMF, and the full results including the RMF theory are shown by the solid lines. Comparing them we can obviously see the relativistic effect: e.g. for PM2, the energy gain is reduced about 20 MeV at $\rho_B = 4\rho_0$ by a simple relativistic effect, and further reduced about 30 MeV by the RMF theory. Totally about 60 % of the energy gain is lost there! This effect becomes larger as density increases. In Fig. 1(b), the solid and dashed lines show the EOS of the kaon-condensed phase and the normal phase, respectively. With decrease of the effective mass the energy gain becomes smaller; i.e. the smaller effective mass through the σ mean-field weakens the EOS of neutron star matter in the kaon-condensed phase. Note that in the case PM3 there is thermodynamically unstable region with negative pressure in the kaon-condensed phase just above the threshold density. This means that the density gap appears inside neutron stars as discussed later [14].

In Fig. 2 we show the density dependence of (a) the proton mixing ratio x, (b) the charge chemical potential μ and (c) the chiral angle θ. The RMF theory moderates these values: x and θ do not exceed $1/2$ and $\pi/2$, respectively, and μ is always *positive* in the relevant density region, $\rho_B < 6\rho_0$, which would be achieved in the typical neutron star with $1.44\, M_\odot$. It is to be noted that we have seen a drastic situation in the well-developed phase within the non-relativistic framework: $x > 1/2$, $\theta > \pi/2$ and $\mu < 0$ [4, 9].

The two quantities x, μ are related to the possibility of the direct Urca process [4, 9]; when $x > 1/2$ ($\mu < 0$), positron participates in this process instead of electron. This critical density, which is independent of choice of the symmetry energy term and other parameters in the RMF theory, is given by the following equation,

$$\rho_s(\mu = 0) = \frac{f^2 m_K^2}{\Sigma_{KN}} . \qquad (10)$$

The scalar density ρ_s is much smaller than the baryon density, so that it cannot easily reach the critical value as the baryon density increases. Hence we can say that the possibility of the positron direct Urca process is *unlikely* in neutron stars.

Although we have not proved it analytically yet, numerical results strongly suggest that θ should be less than $\pi/2$ for *any* density. We can see this even when we turn off the RMF theory. Hence this is a result inherent in the relativistic formalism. We think that this result is favorable from the viewpoint of chiral rotation: in this picture $\theta = \pi/2$ is a maximal angle where kaon amplitude ($|K| \propto \sin\theta$) also becomes maximum [18][1]. We believe that only the relativistic formalism can give a theoretically consistent result for kaon condensation.

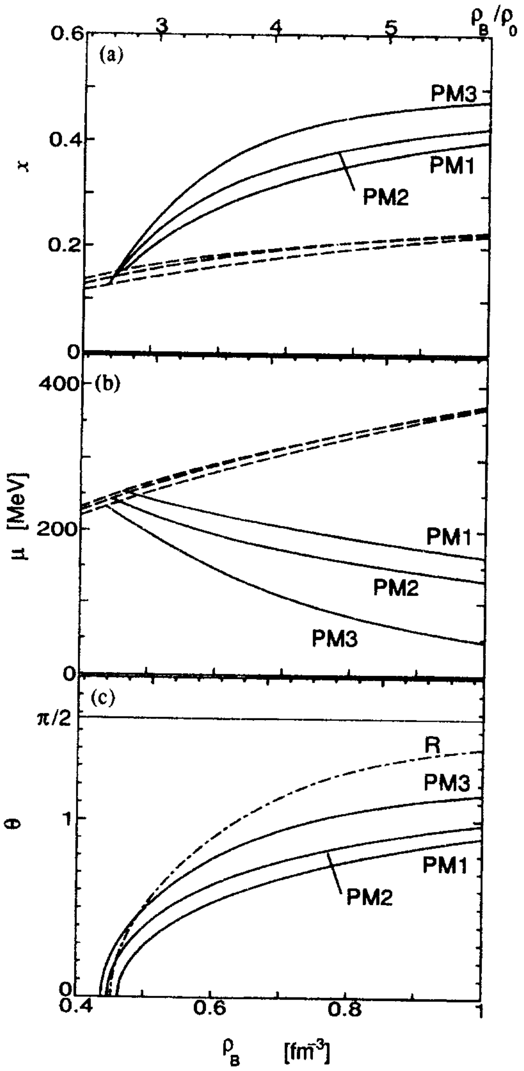

Fig. 2 (a) the proton mixing, (b) the charge chemical potential and (c) the chiral angle as functions of the density for neutron-star matter with the parameter-sets PM1, PM2 and PM3. The meaning of the lines is the same as in Fig. 1.

Finally we should study the neutron star properties in the relativistic picture. As a typical example we give the relation between gravitational mass (M_G) and radius (R) of neutron stars in Fig. 3. The result is sensitively dependent on the value of the effective mass. In the case PM3, there appears a gravitationally unstable branch which separates two types of stable neutron stars. Moreover, the maximum mass of neutron star cannot exceed 1.5 $M_\odot$. This situation is quite similar to the one given by Thorsson et al. [9]. For other cases kaon-condensed neutron stars are not so different from normal ones as far as

[1] For pion condensation, $\theta = \pi/2$ is the limit value even in the non-relativistic treatment.

their global aspects are concerned. These results show that it is marginal to expect kaon condensate inside observed neutron stars: the existence of kaon condensate depends on their mass[2].

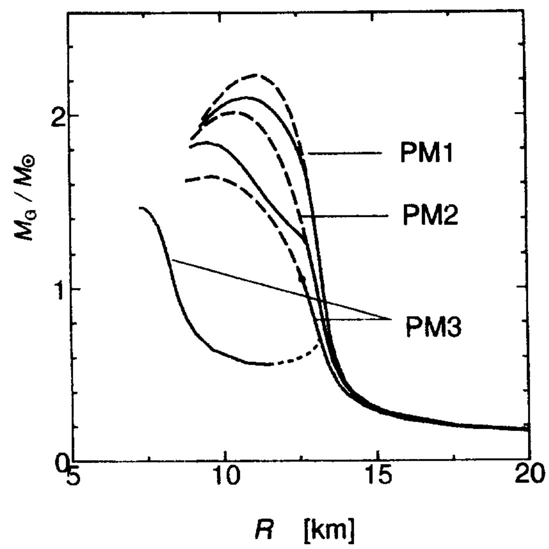

Fig.3 Mass–radius relation of neutron stars for the parameter-sets PM1, PM2 and PM3. The meanings of the lines is the same as in Fig. 1.

Even if the kaon condensation does not have so large effect on the structure of neutron stars, it may play a significant role in the neutron star cooling through the kaon-induced Urca or the direct Urca process [2, 3, 4]. This subject will be discussed in a separate paper [16].

Recently there are many discussions about the off-shell effect in the $\bar{K}N$ scattering amplitude [17], which implies the additional μ^2 dependent term in Eq. (4). In our formalism it appears through the second-order effect of the axial-vector current [18, 19], which is not taken into account here. If we include this effect, our results given here would be slightly changed quantitatively. However, main results about the relativistic effect remain valid at least qualitatively. In fact, the threshold density for kaon condensation is almost the same as that given by the simplified treatment here [19]. Anyway, we report the full results in the forthcoming paper [16].

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[2]It has been noted that one does not have to explain all the present data on neutron-star cooling in one type of matter [15].

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