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MAX–PLANCK–INSTITUT FÜR PHYSIK

WERNER–HEISENBERG–INSTITUT

MPI-PhE/94-12
May 1994

Density and Correlation Integrals in Deep-Inelastic Muon-Nucleon Scattering at 490 GeV

THE FERMILAB E665 COLLABORATION

80805 München · Föhringer Ring 6

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25 May, 1994

Submitted to Physics Letters B

Abstract

Results on density integrals $F_q(Q^2)$ and correlation integrals $K_q(Q^2)$ are presented for the first time in muon-nucleon scattering at $\sim$ 490 GeV, using data from the E665 experiment at the Tevatron of Fermilab. A clear rise of the F_q integrals with decreasing size of the phase-space cells ("intermittency") is observed for pairs and triplets of negative hadrons whereas the effect is much weaker for mixed charge combinations. From these findings it is concluded that the observed intermittency signal is mainly caused by Bose-Einstein interference. Furthermore, no energy (W) dependence of $F_2(Q^2)$ is observed within the W range of the E665 experiment. Finally, the third-order correlation integrals $K_3(Q^2)$ are found to be significantly different from zero which implies the presence of genuine three-particle correlations in muon-nucleon interactions.

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The study of correlations and fluctuations in hadron production at high energies has found considerable interest in recent years. In particular, the concept of "intermittency" has been introduced [1] into particle physics in order to describe the non-statistical fluctuations observed for individual events in the phase-space density distributions of hadrons produced in high energy reactions. Since then, intermittency has been investigated in all kinds of multi-particle production processes, from e^+e^- annihilation to nucleus-nucleus collisions [2], by measuring the so-called normalized factorial moments (and factorial cumulants [3, 2]) in one, two or three phase-space dimensions, as functions of the number M of cells into which the overall phase-space region Δ considered is subdivided. Intermittency in its strict sense (i.e. self-similarity of the hadron-production process at various phase-space scales) implies an increase of the factorial moments with decreasing cell size $\delta = \Delta/M$ according to a power law. A somewhat wider notion has become customary by calling "intermittency"any increase of the factorial moments with decreasing cell size.

In deep-inelastic muon-nucleon scattering intermittency has been studied in [4, 5], using data from the NA9 experiment of the European Muon Collaboration (EMC). In particular, it was found [5] that the second-order factorial moment shows a strong intermittent behaviour for pairs $(--)$ of negative hadrons, but not for pairs $(+-)$ of oppositely charged hadrons. From this result it was concluded that the observed intermittency signal was (mainly) due to Bose-Einstein (BE) interference, i.e. to BE correlations between negative pions.

More recently, a considerable improvement in the method of analysis has been achieved by replacing the factorial moments and factorial cumulants by the more general so-called density integrals and correlation integrals, respectively [6, 7, 8, 9, 2]*. The density and correlation integrals have several important advantages, coming partly from the fact that for their determination the overall phase space is no longer subdivided into cells with fixed boundaries (see below). The advantages are: an accidental and artificial separation of hadrons, that are nearby in phase space, by a fixed cell boundary is avoided; artificial fluctuations that are often observed in the cell-size dependence of factorial moments due to the fixed cell boundaries, are removed; and the statistical accuracy is considerably better for the density integrals than for the factorial moments due to the larger number of q-tuples of particles included (see below) which is particularly important for integrals of higher order q **.

Taking one phase-space dimension (e.g. rapidity y) as a simple example, the general definition of the normalized density integral $F_q(\delta y)$ of order q is

$$F_q(\delta y) = \frac{\int_\Omega dy_1 \dots dy_q \rho_q(y_1, \dots, y_q)}{\int_\Omega dy_1 \dots dy_q \rho_1(y_1) \dots \rho_1(y_q)} \equiv \frac{\int \rho_q}{\int \rho_1^q} \tag{1}$$

where $\rho_q(y_1, \dots, y_q)$ is the q-particle density distribution. The integration domain Ω is, for instance, a q-dimensional subspace which is defined such that for a point $(y_1, \dots, y_q)$

*In the literature, the density integrals are often called correlation integrals, and the correlation integrals are called integrated cumulants.

**It is worth mentioning that the integrals can be regarded as generalisations of the vertical factorial moments which, in contrast to the horizontal factorial moments, do not depend on the shape of the inclusive single particle distributions so that corrections [10] are not necessary.

in the subspace the closeness condition $|y_i - y_j| < \delta y$ holds for any pair (i, j) amongst the q particles. (For $q = 2$, Ω is a strip of width $\sqrt{2}\delta y$ around the diagonal $y_1 = y_2$ in the y_1, y_2 plane, limited by the boundaries of the overall y region considered).

To evaluate the integral in the numerator of eq. (1) one has to count for each event all possible ordered q-tuples of particles (i.e. q-tuples where the order of the particles in a q-tuple is relevant) which fulfill the closeness condition, i.e. fall inside the volume Ω, and then average the numbers of q-tuples over all events. This algorithm follows from the fact, that (e.g. for identical particles) in the relation

$$\int dy_1 \ldots dy_q \rho_q(y_1, \ldots, y_q) = \langle n(n-1) \cdots (n-q+1) \rangle \equiv \langle n^{[q]} \rangle \tag{2}$$

$n^{[q]}$ is the number of ordered q-tuples amongst n particles, according to combinatorics. $\langle\ \rangle$ means averaging over all events.

As usual, also in this paper the distance of particles is not considered in one phase-space dimension (y), but rather in three dimensions. A suitable variable to measure this 3-dimensional distance is the Lorentz-invariant four-momentum difference Q_{ij} defined as $Q_{ij}^2 = -(p_i - p_j)^2$ where p_i, p_j are the four-momenta $p = (\vec{p}, E)$ of particles i and j in a q-tuple. The closeness condition is then $Q_{ij}^2 < Q^2$ and F_q is plotted vs. $1/Q^2$ (or $1/Q$).

There are various ways to define the closeness condition for q-tuples of particles (i.e. to compute the correlation integrals), namely the "GHP", "Snake" and "Star" methods [6, 7]. For $q = 2$, the three methods are identical; for $q = 3$, the Snake and Star methods are the same. In this paper the Star method is applied. For the Star integral, the domain Ω is given by the sum of all "spheres" of radius Q^2 centered around each particle in an event [8]. In this case, the numerator in (1) can be written as

$$\int \rho_q(\vec{p}_1, \cdots \vec{p}_q)\ \theta_{12} \cdots \theta_{1q}\ d\vec{p}_1 \cdots d\vec{p}_q\ , \tag{3}$$

where the step functions $\theta_{1j} = \theta(Q^2 - Q_{1j}^2)$ (with $\theta\,(x) = 0\,(1)$ for $x < 0\,(x > 0)$) in the integral have the effect that only those q-tuples are counted, for which all $q - 1$ particles have a distance from particle 1 – the center of the "star" – which is smaller than Q^2.

The actual numerical computations of $F_2(Q^2)$ and $F_3(Q^2)$ in this paper were performed according to the "sphere counting" and "event mixing" algorithms given by eqs. (11) and (26) of [7] and eqs. (44) and (52) of [8]. The normalisation is then such that for full phase space (i.e. $Q_{\max}^2$) the integrals $F_q(Q_{\max}^2)$ are the same as those obtained from the full multiplicity distribution, e.g.

$$F_2(Q_{\max}^2) = \langle n(n-1) \rangle / \langle n \rangle^2 \tag{4}$$

where $\langle n \rangle$ and $\langle n(n-1) \rangle$ are moments of the full multiplicity distribution.

Higher-order density integrals ($q \geq 3$) contain contributions from lower-order correlations. We have therefore, in addition to the normalised density integrals $F_q = \int \rho_q / \int \rho_1^q$ (eq. (1)), also computed the normalized correlation integrals $K_q(Q^2)$ which correspond to the factorial cumulants and which reflect the genuine q-particle correlations. In the abbreviated notation of eq. (1) they are defined as $K_q = \int C_q / \int \rho_1^q$, where $C_q(1, \dots q)$ is the q-particle correlation function, e.g.

$$C_2(1,2) = \rho_2(1,2) - \rho_1(1)\rho_1(2) \rightarrow K_2 = F_2 - 1 \,, \tag{5}$$

$$C_3(1,2,3) = \rho_3(1,2,3) - \rho_2(1,2)\rho_1(3) - \text{permutations} + 2\rho_1(1)\rho_1(2)\rho_1(3) \,. \tag{6}$$

The relation

$$K_3 = F_3 - 3F_2 + 2 \,, \tag{7}$$

following from eq. (6) by integration and normalisation, holds only for the full phase space, i.e. for Q^2_{max}.

The data for this analysis come from the sample of μD and μp interactions obtained by E665 during the 1987/88 fixed target run at the Tevatron of Fermilab. The apparatus consists of a vertex detector with a streamer chamber and a forward spectrometer, providing a 80–90% acceptance of charged particles with momenta greater than 200 MeV/c over the full solid angle. Details on the set-up and on the data analysis of the experiment, in particular the processing of the streamer-chamber and forward-spectrometer tracks, can be found in [11] and [12].

Apart from slight differences in the definition of the kinematic region, the event selection is identical to the one described in [12]. The selection includes in particular the trigger requirements and the removal of radiative events using the information from the electromagnetic calorimeter.

The kinematic region considered in the present analysis is defined by

$$\begin{array}{cc} -q^2 > 1 \text{ GeV}^2 & 8 < W < 32 \text{ GeV} \\ x_{Bj} > 0.001 & \theta > 3.5 \text{ mrad} \\ 0.08 < y_{Bj} < 0.9 & 30 < \nu < 500 \text{ GeV} \,. \end{array}$$

Here q^2 is the leptonic four-momentum transfer squared, W is the effective mass of the total hadronic system, $\nu = E_\mu - E'_\mu$ is the energy transfer between incoming and outgoing muon with laboratory energies E_μ and E'_μ respectively, θ is the laboratory scattering angle of the muon, $x_{Bj} = -q^2/(2M\nu)$ (with the nucleon mass M) and $y_{Bj} = \nu/E_\mu$.

In the present analysis only charged hadrons are considered. Special care has been taken to remove e^+e^- pairs from photon conversions near the primary vertex. Furthermore, the track selection criteria were adjusted in order to avoid double counting of the same track. No acceptance corrections have been applied to the data, since Monte Carlo calculations have shown that they do not change the conclusions of the analysis. As particle identification for charged hadrons was not available, all particles were assigned the pion mass. Only particles with a center of mass rapidity between -3 and 3 and a transverse momentum (with respect to the virtual-photon direction) less than 2 GeV/c were included in the analysis. In order to exclude the kinematic region where proton production contributes strongly, the Feynman-x (as calculated using the pion mass) for positive particles was required to be greater than -0.2. An event with no accepted charged hadron was rejected, i.e. it was not included in the normalisation.

The sample of accepted events consists of $\sim$ 3000 events on H_2 and $\sim$ 9000 events on D_2. The sample is characterized by the following average values of kinematic variables: $\langle -q^2 \rangle = 8.15$ GeV^2, $\langle W \rangle = 17.5$ GeV, $\langle \nu \rangle = 186$ GeV, $\langle x_{Bj} \rangle = 0.04$ and $\langle y_{Bj} \rangle = 0.40$.

We now turn to the results of this analysis. Fig. 1a shows a log-log plot of the second-order density integral $F_2(Q^2)$ for $(--)$ and $(+-)$ pairs vs. $1/Q^2$. For both charge combinations $F_2(Q^2)$ rises with increasing $1/Q^2$, the points falling roughly on straight lines. This power-law dependence of $F_2(Q^2)$ on Q^2 is expected for genuine intermittency, e.g. for .. self-similar cascading process. The rise is much stronger for $(--)$ than for $(+-)$ pairs. From this it can be concluded that the intermittent behaviour is mainly due to BE correlations in the $(--)$ pairs (mainly pions) as observed in this experiment [13] for $Q^2 < 1$ GeV^2.

The same qualitative features were observed for the second-order factorial moments of $(--)$ and $(+-)$ pairs in μN interactions at the lower energy ($E_\mu = 280$ GeV, $\langle W \rangle = 13.4$ GeV) of the EMC NA9 experiment [5]. In fact, the slopes $\mathrm{d}\ln F_2(Q^2)/\mathrm{d}(1/Q^2)$ of the $(--)$ and $(+-)$ density integrals show very close agreement for the NA9 and E665 experiments [14], inspite of the somewhat different $\langle W \rangle$ values and the experimental differences of the two experiments.

Fig. 1b shows a log-log plot of the third-order density integral $F_3(Q^2)$ for triplets $(---)$ of negative hadrons and (ccc) of charged hadrons, where in the latter case it does not matter whether the charge of a hadron in the triplet is positive or negative (i.e. the three charged hadrons are treated as identical particles). Again, the rise is steeper for $(---)$ than for (ccc) triplets as expected, since BE correlations amongst like-sign pions influence the $(---)$ triplets more strongly than the (ccc) triplets. Furthermore, the rise is rather linear in both cases as expected for intermittency.

Because of limited statistics it is not meaningful to present any higher-order moments.

For the energy (W) dependence of F_2 the following behaviour is expected: For $(+-)$ pairs the slope of $F_2(Q^2)$ should increase with W due to the gradual appearance of hadron jets which introduce correlations amongst the hadrons. For $(--)$ pairs the situation is more complicated due to the additional presence of the BE interference. In order to obtain experimental information on the W dependence, the data are split into two subsamples: events with $W < 20$ GeV ($\langle W \rangle = 13$ GeV, low W) and events with $W > 20$ GeV ($\langle W \rangle = 23$ GeV, high W). Fig. 2 shows $F_2(Q^2)$ of $(+-)$ and $(--)$ pairs for the two subsamples. To facilitate the comparison, the data points are shifted such that $F_2(Q^2 = 1\ \mathrm{GeV}^2) = 1$. No significant energy dependence of the F_2 slope is observed for $Q^2 \gtrsim 0.01$ GeV^2; for smaller Q^2, the F_2 slope of $(--)$ pairs seems to be somewhat larger for the high-W sample than for the low-W sample.

We now turn to the second and third-order correlation integrals. $K_2(Q^2)$ is shown for $(--)$ and $(+-)$ pairs in Fig. 3a; it is trivially related to $F_2(Q^2)$ (Fig. 1a) by $K_2 = F_2 - 1$. Fig. 3b shows a log-log plot of $K_3(Q^2)$ for $(---)$ and (ccc) triplets. $K_3(Q^2)$ is definitely different from zero and rises with $1/Q^2$ both for $(---)$ and (ccc) triplets which implies that genuine three-particle correlations are present in μN scattering. This is in contrast to e.g. nucleus-nucleus collisions where three-particle correlations were found to be practically absent [15, 14]. The curve in Fig. 3b represents the expression $F_3 - 3F_2 + 2$ for charged hadrons as determined from the data. From a comparison with the K_3 data points for (ccc) triplets it is seen that in the Q^2 range considered relation (7) is only approximately fulfilled for charged hadrons.

In order to find the origin of the three-particle correlations in Fig. 3b μN Monte Carlo (MC) events were generated according to the Lund model (version LEPTO 6.1 [16]) *without* BE correlations. The MC predictions for $F_3(Q^2)$ and $K_3(Q^2)$ are shown in Fig. 4. For $(---)$ triplets, $K_3(Q^2)$ of the MC events is rather independent of $1/Q^2$, in contrast to the data (Fig. 3b). This shows that the rise of $K_3(Q^2)$ in the data is very likely due to three-particle BE correlations which were not incorporated into the Lund MC used. For (ccc) triplets the situation is more complicated, since in the data both BE correlations (weaker than in $(---)$) and resonance decays (e.g. η' decays, absent in $(---)$) contribute. In the MC (without BE, but with resonance decays) (Fig. 4b), $K_3(Q^2)$ is smaller than in the data but rises due to resonance decays.

In summary, we investigate in this paper the two and three particle correlations of hadrons produced in deep-inelastic muon-nucleon interactions by measuring the second and third-order density and correlation integrals. The following results were obtained:

- In a log-log plot, both $F_2(Q^2)$ and $F_3(Q^2)$ show an approximately linear rise with $1/Q^2$ which is characteristic of intermittency.
- $F_2(Q^2)$ rises more steeply with $1/Q^2$ for $(--)$ than for $(+-)$ pairs. The same is true for $F_3(Q^2)$ for $(---)$ triplets as compared to (ccc) triplets. This different behaviour is due to Bose-Einstein correlations between identical bosons (negative pions). The intermittency signal is thus mainly caused by Bose-Einstein interference.
- No significant energy (W) dependence of the F_2 slope for $(+-)$ pairs is observed between $\langle W \rangle \approx 13$ GeV and 23 GeV.
- $K_3(Q^2)$ for $(---)$ and (ccc) triplets is definitely different from zero and rises with $1/Q^2$; this implies the presence of genuine three-particle correlations in μN interactions.

Acknowledgements

We wish to thank all those persons, both at Fermilab and at the participating institutions, who have contributed to this experiment. This work was supported in part by the National Science Foundation, the U.S. Department of Energy, Nuclear Physics and High Energy Physics Divisions, the Bundesministerium für Forschung und Technologie, and the Polish Committee for Scientific Research.

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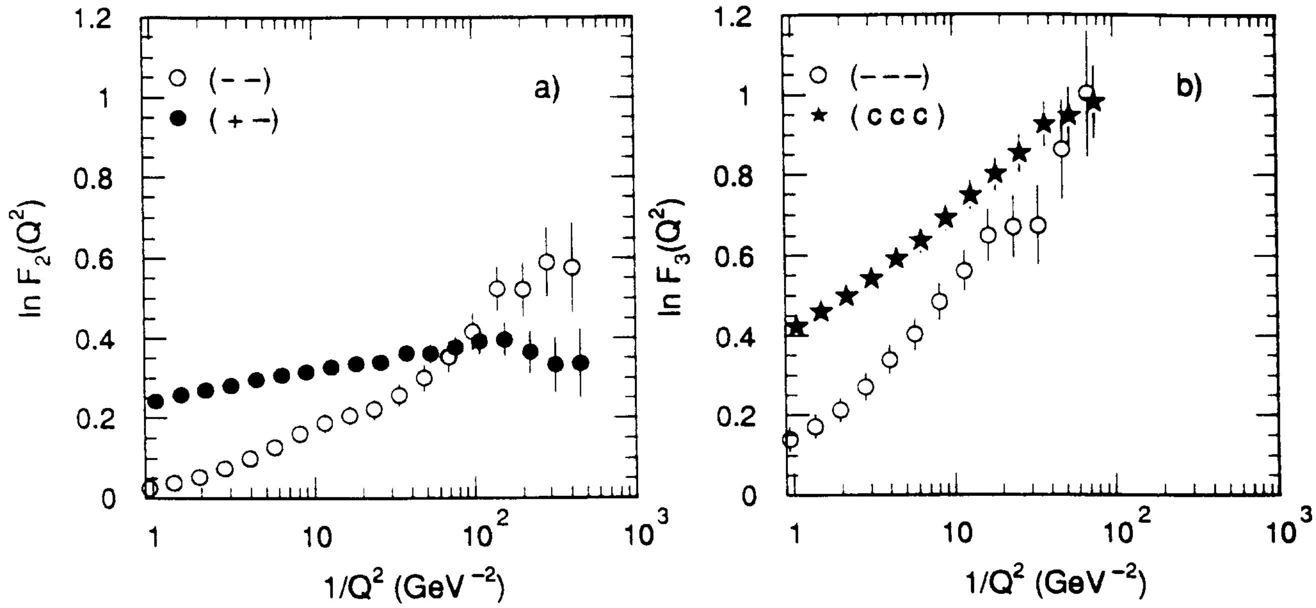

Fig. 1: Log-log plot of a) $F_2(Q^2)$ for $(--)$ and $(+-)$ pairs and b) $F_3(Q^2)$ for $(---)$ and (ccc) triplets, vs. $1/Q^2$

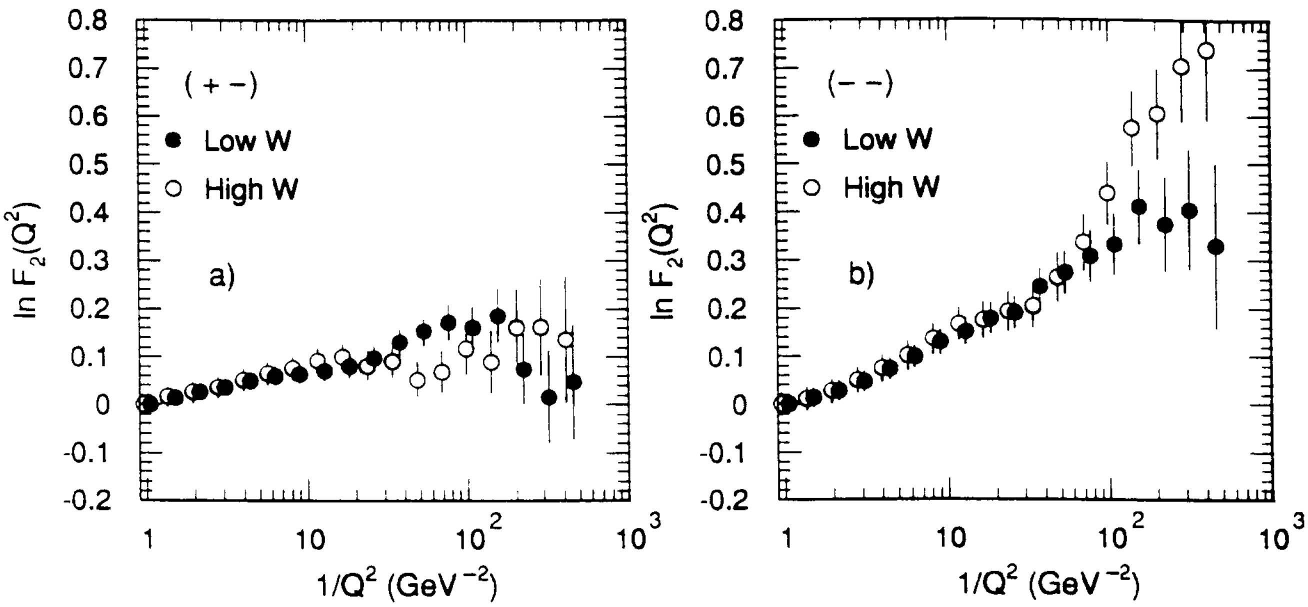

Fig. 2: Log-log plot of $F_2(Q^2)$ vs. $1/Q^2$ for events with low W ($W < 20$ GeV) and high W ($W > 20$ GeV), for a) $(+-)$ and b) $(--)$ pairs. The normalisation is such that $F_2\,(Q^2 = 1\ \mathrm{GeV}^2) = 1$

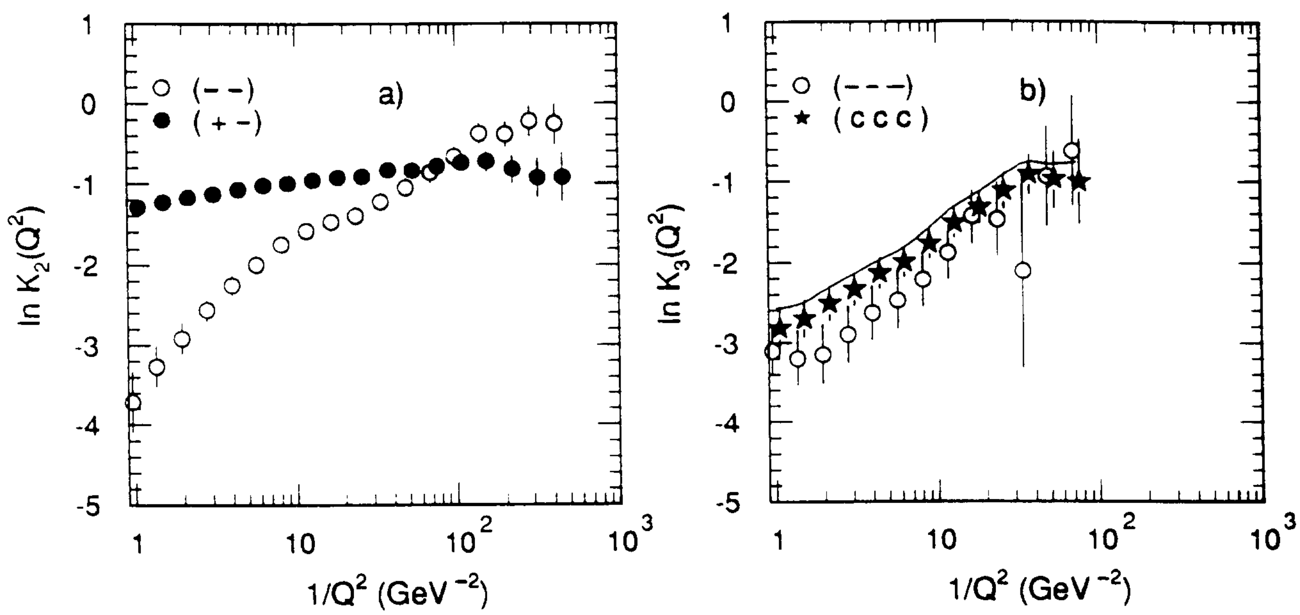

Fig. 3: Log-log plot of a) $K_2(Q^2)$ for $(--)$ and $(+-)$ pairs, and b) $K_3(Q^2)$ for $(---)$ and (ccc) triplets, vs. $1/Q^2$. The curve in b) shows $F_3 - 3F_2 + 2$ for charged hadrons.

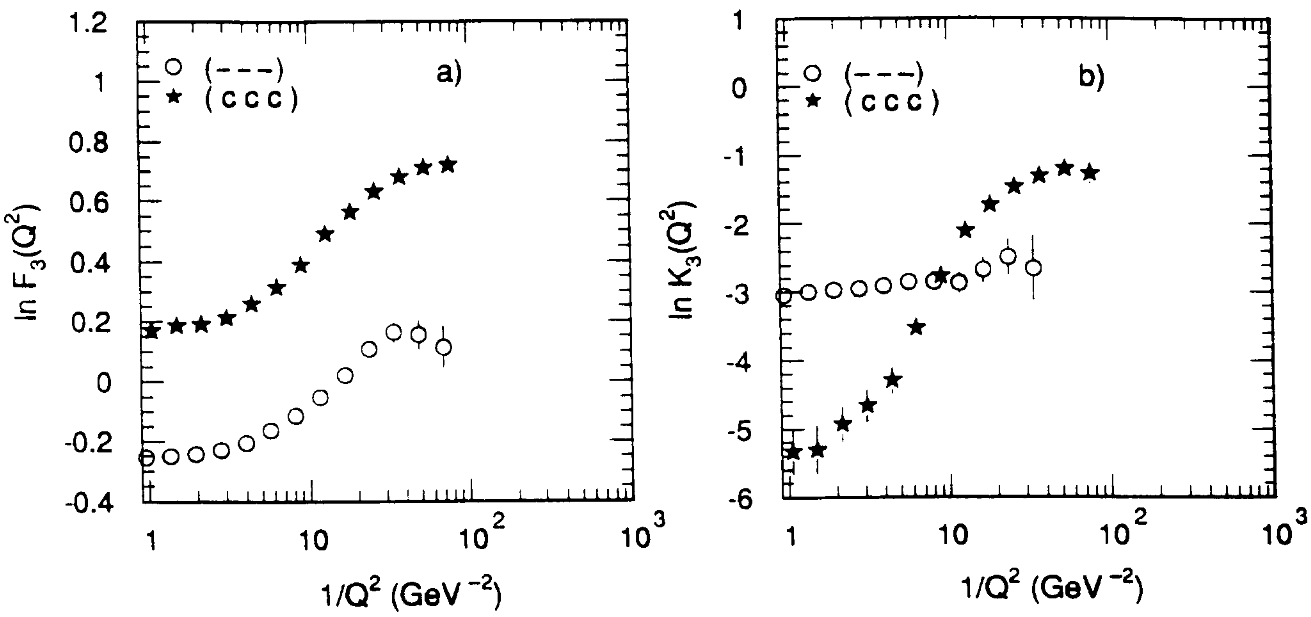

Fig. 4: Log-log plot of a) $F_3(Q^2)$ and b) $K_3(Q^2)$ for $(- - -)$ and (ccc), vs. $1/Q^2$, from the Lund Monte Carlo program (including resonances, but without Bose-Einstein correlations).