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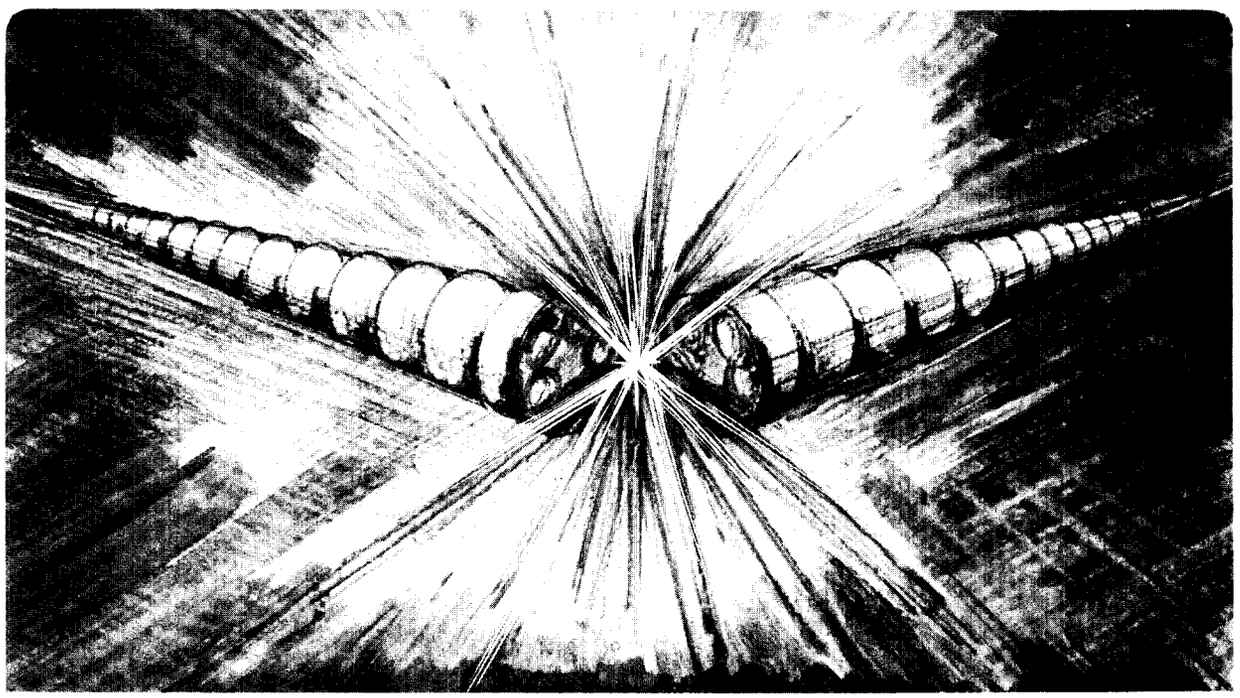

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Molecular dynamics is employed to study the low energy states of a beam of charged particles subject to circumferentially varying guiding and focusing forces and with Coulomb forces between the particles. In a constant gradient ring, the lowest energy state is never ordered, but in an alternating gradient structure, operating below the transition energy, the lowest state is ordered. The nature and characteristics of the ground state depends upon the beam density and the ring parameters. For zero temperature the crystal remains intact for a very long time, but at non-zero temperatures it gains energy from the lattice. A critical temperature exists above which the crystal melts rapidly.

PACS numbers: 52.60.+h, 29.20.Dh, 52.65.+z, 61.50.-f

The ground state of crystalline beams were first studied, in seminal work, by Schiffer and his colleagues. [1] Their work assumed a storage ring model in which charged particles are subject to time-independent harmonic forces in both transverse directions. Subsequently, they studied crystallization in a time-dependent focusing potential which replicated some of the features of alternating gradient (AG) focusing, and with time-dependent shearing which replicated some of the features of the alternate bending and straight sections of a storage ring.[2]

Nevertheless, questions remain about whether or not an ordered state can be created in a real storage ring. Furthermore, with laser cooling very low (longitudinal) temperatures of stored beams have recently been achieved.[3] Thus it is now prudent to develop the tools which will allow one to make calculations that incorporate the characteristics of actual storage rings. We have developed such a formalism, and in this Letter we describe the formalism, and then employ it to study the nature of the ordered state in actual storage rings. We find that in operation below the transition energy alternating gradient (AG) rings, as contrasted with constant gradient rings, can have a crystalline lowest energy--or ground--state. This state will change periodically in time, "breathing" as the particles go around the storage ring and are subject to periodic bending, straight sections, focusing lenses, and defocusing lenses. Under some conditions the changes are dramatic, the crystal periodically changing its shape and orientation, but the crystal remains for a very long time in the ground state; i.e., by this process very little heat is put into the crystal (possibly zero at zero temperature). In order to achieve the ordered state the beam must be very cold; we give results, in typical machine parameters, for just how cold (expressed in terms of energy spread and emittance) must be the beam. We show that there exists a critical temperature above which the crystal rapidly melts.

In order to use the molecular dynamics (MD) methods we must be in the frame of reference of the particles. That is, a rotating frame (x,y,z,τ) of a reference particle in which the orientation of the axis is also rotating so that the axis are constantly aligned to the radial (x) , vertical (y) and tangential (z) direction of motion. This is, of course, an accelerating frame of reference. We can derive the equations in the laboratory frame and then transform to the moving frame, but it is most convenient to derive the equations directly in the beam frame, employing the formalism of general relativity.[4] One may think of the result of this process as finding the relativistic generalization of centrifugal and Coriolis forces. In the reference particle frame of reference, the particle motion is non-relativistic. The rather lengthy derivation is given in a Laboratory Report and summarized in a Conference Proceeding Publication [5]; here we only present the results.

It is convenient to scale dimensions in terms of ξ, with $\xi^3 = r_0\rho^2/\beta^2\gamma^2$, where r_0 is the Classical particle radius (Z^2e^2/mc^2), the velocity of a reference particle is βc, its energy is γmc^2, and it moves on an orbit with bending radius ρ in magnetic field B_0. We measure time in units of $\rho/\beta\gamma c$ and energy in units of $\beta^2\gamma^2Z^2e^2/\xi$. In a bend region, with magnetic field B_0, we have the Hamiltonian:

$$H = (1/2)\{P_x^2 + P_y^2 + P_z^2\} - \gamma x P_z + (1/2)\, x^2 + V_c(x,y,z),$$

where the Coulomb potential

$$V_C = \sum \{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2\}^{-1/2},$$

and the summation, j, is over all the other particles.

In a straight section, where there is no bending of particles, there often are placed focusing magnets. If focusing is supplied by a quadrupole of field gradient B_1 so that $B_x = B_1 y$ and $B_y = B_1 x$, then in a straight region, where the focusing strength is characterized by $n = -B_1\rho/B_0$, we have the Hamiltonian

$$H = (1/2)\{P_x^2 + P_y^2 + P_z^2\} + (1/2)[-nx^2 + ny^2] + V_c(x,y,z).$$

As the particle goes about the storage ring the appropriate equations of motion must be employed. The change from those corresponding to a curved region to those corresponding to a straight section incorporate, in a quantitatively correct manner, the effect of shear (given by the term $\gamma x P_z$). Similarly the AG effect of alternate gradient is incorporated, quantitatively correctly, by changing the field gradient n (positive for vertical focusing, negative for vertical defocusing, and zero for an open straight section).

Consider a cyclotron magnet; i.e., a magnet that gives constant bending and constant focusing. If the gradient of the magnet is such that n lies between zero and one then, as is well-known, the magnet gives focusing in both the vertical and horizontal planes (including centrifugal force). Just the kind of storage ring, one would think, for the formation of a crystal and, yet, that is not true at all. The equations of motion, which follow from the Hamiltonian given above, are

$$x'' - \gamma z' + (-\gamma^2 + 1 - n)x = -\partial V_c/\partial x,$$

$$y'' + ny = -\partial V_c/\partial y,$$

$$z'' + \gamma x = -\partial V_c/\partial z.$$

Using these equations we have performed an eigenmode analysis [5] of small oscillations about an assumed crystalline state. The conditions that all eigenmodes be stable is that $n < 1 - \gamma^2$ and $n > 0$. Since $\gamma > 1$, we see that stability for all eogenmodes (both directions) can not be simultaneously satisfied. This result is because the centrifugal force is no longer focusing for a crystal.

Let us now, for pedagogical purposes, separate the effect of "shear" from that of time-dependent focusing. We shall study, as we did in our earlier work [5], an AG ring with a constant bending field. In fact, our analysis is made for a particular ring (FODO lattice), but the results are general. It can be shown, from the eigenmode analysis, that assuming there is horizontal and vertical focusing; i.e., $\nu_x > 0$ and $\nu_y > 0$, and provided that the ring is operated below the transition energy; i.e., $\gamma 2 < \nu_x 2$, where ν_x is the horizontal tune, then, a crystalline state exists.

For a given ring, the nature of the ground state (the periodic lowest energy state) depends upon the density of particles. When the density is low the ground state is a 1-D chain. If the density is larger then the ground state is a 2-D state that lies in the plane of weaker focusing, which is determined by whether or not $\nu_x^2 - \gamma^2$ is greater, or smaller, than ν_y^2. Notice that the focusing in a crystal is not determined simply by ν_x and ν_y, but by the factors $(\nu_x^2 - \gamma^2)^{1/2}$ and ν_y.

The density at which a 1-D structure changes into a 2-D structure can be determined analytically. It is given by

$$\nu_y^2, \nu_x^2 - \gamma^2 > 4.2 / \Delta_z^3,$$

where Δ_z is the nearest neighbor distance in z (given trivially in terms of the number of particles stored and the circumference of the storage ring). Notice that, in practice, one can change the focusing of a storage ring ("changing the operating point") and also change the storage energy. Thus the focusing can be readily changed and the effect of such change easily studied.

In an AG ring the 2-D crystal structure, as contrasted with the 1-D structure, will "breath" as the particles go about the storage ring. Despite this motion little, or no, energy is pumped into the crystal; it remains in its ground state for a very long time. Such behavior is not unexpected, for as particles go about AG storage rings the amplitude of their oscillation changes (β-function variation), and particles of different energy move

closer or further apart (η-function variation), yet particles can be stored forever.[6]

In general, when Coulomb interactions and AG focusing are present it is impossible to solve the equations analytically; we obtain numerical solutions using molecular dynamics. This method allows us to determine the lowest energy state in realistic storage rings (that is; the actual ring lattice can be inserted in the computation) and, also, allows us to study behavior as the crystal temperature is increased from absolute zero. We can also determine the temperature at which such a crystal melts; i.e., loses its long-range order as the particles go into a state in which they pass each other (as in a usual storage ring).

We find, numerically, that when the density is higher than that appropriate to a 2-D crystal, the particles arrange themselves into 3-D crystals. For even higher density the crystals become helices and then helices within helices. Two examples are shown in Figs 1 and 2. These structures are similar to, but differ in detail, from that given in Ref.1. It is seen that the inter-particle spacing in all these structures is approximately the same, and given (roughly) by the interparticle spacing when transition is made from a 1-D crystal to a 2-D crystal; i.e., can be characterized by ξ, except for dimensional numbers near unity. Thus the crystal forms cylinndrical shells within cylindrical shells, upon which the particles are deployed, in such a way that the interparticle spacing is about the same. This behavior is very similar to that which occurs in ordinary crystals. It appears to be the case that a perfect crystal (with many shells) forms no matter how high is the particle density.

Having determined that ground states can be formed in a storage ring with AG (time-dependent) focusing, we now study the effect of shear upon the ground state; that is, the effect of time varying bending. To do this in a systematic manner, we took an AG lattice of the FODO-type where with constant bending a ground state crystal can exist. The bending was then concentrted into a small region of the period. The result of this is shown in Fig 3. It can be seen that the crystal "takes up the difference" between constant angular velocity and constant linear velocity by adjusting the spacing between particles; i.e., by converting potential energy into kinetic energy.

A more extreme case is supplied by the Heidelberg lattice, which has a period of 2 and rather sharp bends in each of the corners.[7] We have taken the case N=20 and L=100ξ and find a 2D crystal which makes two rotations per turn, having a horizontal orientation in the straight sections and a separation of 1.7ξ and a vertical orientation with separation 7.8ξ in the bending region ! Because there is no shear for crystals extending in

the vertical direction, crystals can be very tall and very thin even if the shear is very large.

Having studied the ground state, we now investigated behavior at non-zero temperatures.[8] The temperature can readily be expressed in terms of the usual accelerator parameters of normalized emittance, ε, and relative energy spread, $\Delta E/E$, by: $\Delta\varepsilon_x = \xi^2\beta_x T_x/\rho^2$, $\Delta\varepsilon_y = \xi^2\beta_y T_y/\rho^2$, $\Delta E/E = \xi\gamma T_z^{1/2}/\rho$. The Hamiltonian for a crystalline beam is quite different from the conventional Hamiltonians encountered in condensed matter physics. It is time-dependent, and therefore energy is not conserved. Furthermore, the Hamiltonian is not bounded from below and the adiabatic approximation can not be used. It is precisely the dynamical coupling between the external focusing-defocusing force and the Coulomb interaction among the particles that gives us the well-defined periodic structures which we call the "ground state". However, the time dependence of the Hamiltonian can cause damage to the crystal, because energy can be pumped into the system by creating phonons. As a result, a crystalline beam, especially at non-zero temperatures, can not last forever unless energy is pumped out of the system by means of a refrigerator.

The frequency of the machine lattice is several times higher than the highest phonon frequency in the system, and the transfer of energy into the system can only be realized by multi-phonon emission. This process is expected to be strongly temperature-dependent. We have studied the relation between the heating rate and the temperature. Fig. 4 shows the survival time of a crystalline beam as a function of the initial temperature at which a molecular dynamics run starts. The survival time indeed strongly depends on the initial temperature, and rapidly tends to infinite as the initial temperature goes to a critical temperature. This indicates that at low temperature, the rate that energy transfers into the system is very low, and it is easy to maintain a crystalline beam for a long time.

In order to numerically study the correlation of particles we have evaluated the two-body correlation function $G_2(z)$. In Fig 5 we display this function for a variety of temperatures. One can see that there is a decided change as the temperature is increased.

Finally, it is well known from the work of Landau [9] and the work of Emery and Axe [10] that no sharp phase transition exists in one dimensional systems. The systems we study are all one-dimensional in the sense that in the two directions perpendicular to the beam, the systems are always finite, and therefore we might not expect sharp phase transitions between these phases. However, two qualitatively different states exist in these systems, one is a low temperature "condensed state" (crystalline beam), in which there is limited shearing motion in the z-

direction, but no passing of particles, and the other is a high temperature "running state" in which particles shear relative to each other. We believe that a sharp phase transition exists between these two states. Our numerical studies, to the accuracy of the computer, support that conjecture.

We want to acknowledge the assistance of R.W. Hasse in making our computer program more efficient. We had useful conversations with J.S. Hangst, D. Habs, J.P. Schiffer, V.J. Emery, and V.V. Avilov. This work was performed under the auspice of the U.S. Department of Energy, Office of Energy Research, Office of High Energy and Nuclear Physics, under Contract No. DE-AC03-76SF00098, as well as supported by NSF Grant DMR-91-15342.

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[6] With N=100 and L=50 and a FODO lattice the ground state is a string on axis plus one shell containing most of the particles. We ran the molecular dynamics calculation for 40,000 steps, or 2,000 structure periods, and saw no change. We then made a very long run for a lower density case (but still one shell) N= 40 and L=40. After 20 million steps, or 1 million structure periods, we could discern no change. We also studied a higher density case, where the ground state consists of five shells, for 20,000 steps, or 1,000 structure periods. Again we saw no change in time of the ground state.

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Figure Captions

Fig. 1 A 3D structure with particle positions projected (a) into the x-y plane and (b) into the ϕ-z plane, where ϕ is the polar angle. The lattice is a FODO lattice with $\gamma_T = 2.5$, $\nu_x = 2.7$ and $\nu_y = 2.3$ and the particle energy is $\gamma = 1.4$. The total number of particles is 40 and the MD period length is 10ξ. The particles move periodically in time, with the solid lines showing their trajectories and the circles indicating their position at the start and end of each lattice period.

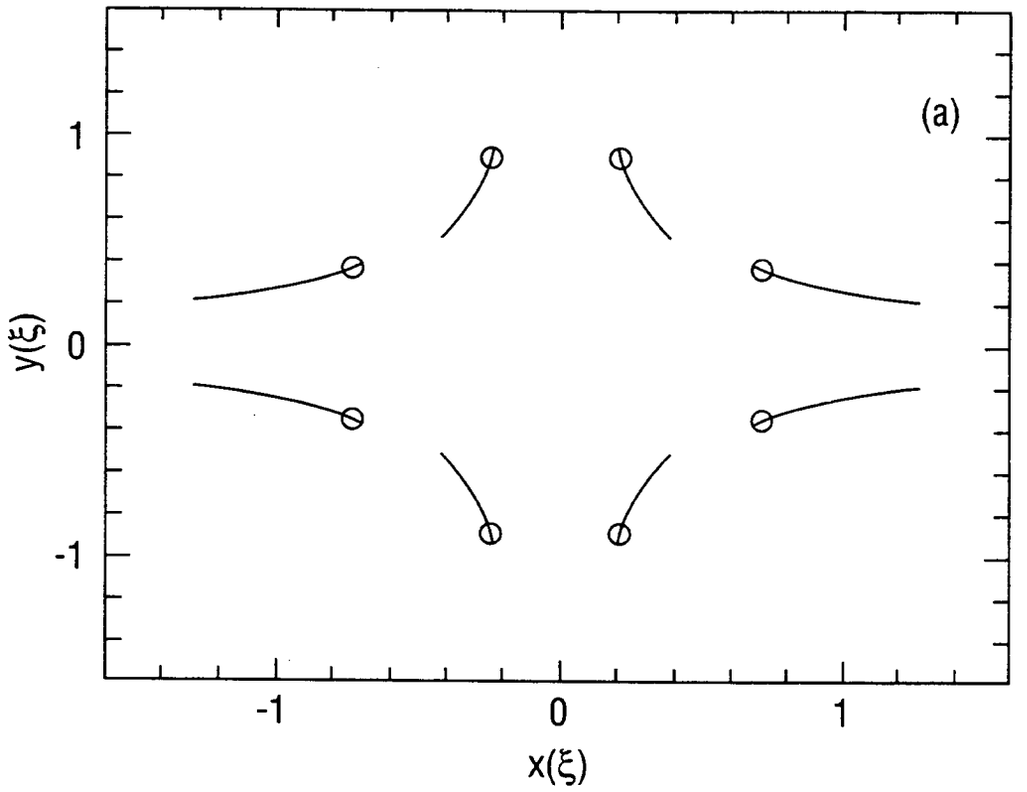
Fig. 2 A 3-D structure showing the effect of increasing density. The papameters are as in Fig 1, but now N = 60. A new ring is at the center, but Fig. 2b only shows the outer ring.

Fig. 3 The effect of shear. In this study N=40, L=40ξ. The FODO cell consists of 20 steps and bending occurs from steps 2 through 6. Motion occurs both (a) in the x-direction and (b) in the z-direction (shear).

Fig. 4 A study of survival time of a crystal as a function of the temperature of the crystal. For these studies a lattice of 10 periods was taken in a circumfrence of 25.1 m, with each period consisting of: O (2 step) B (10 steps) O (2 step) D (6 steps) O (14 steps) F (6 steps). The parameters were ρ=1m, $\gamma = 1.4$, $\gamma_T = 2.6$, $\nu_x = 2.8$ and $\nu_y = 2.1$. In all cases N=40 and L=40ξ. The fluctuation in survival time is due to different random seeds used to distribute initial particle position and velocity appropriate to the chosen temperature.

Fig. 5 Two-body correlation functions $G_2(z)$ corresponding to one of the cases shown in Fig 4. The ground state (a) has a very "sharp" function, but then as the temperature is increased the function "washes out" and, finally (d), becomes essentially uniform.

FIG. 1



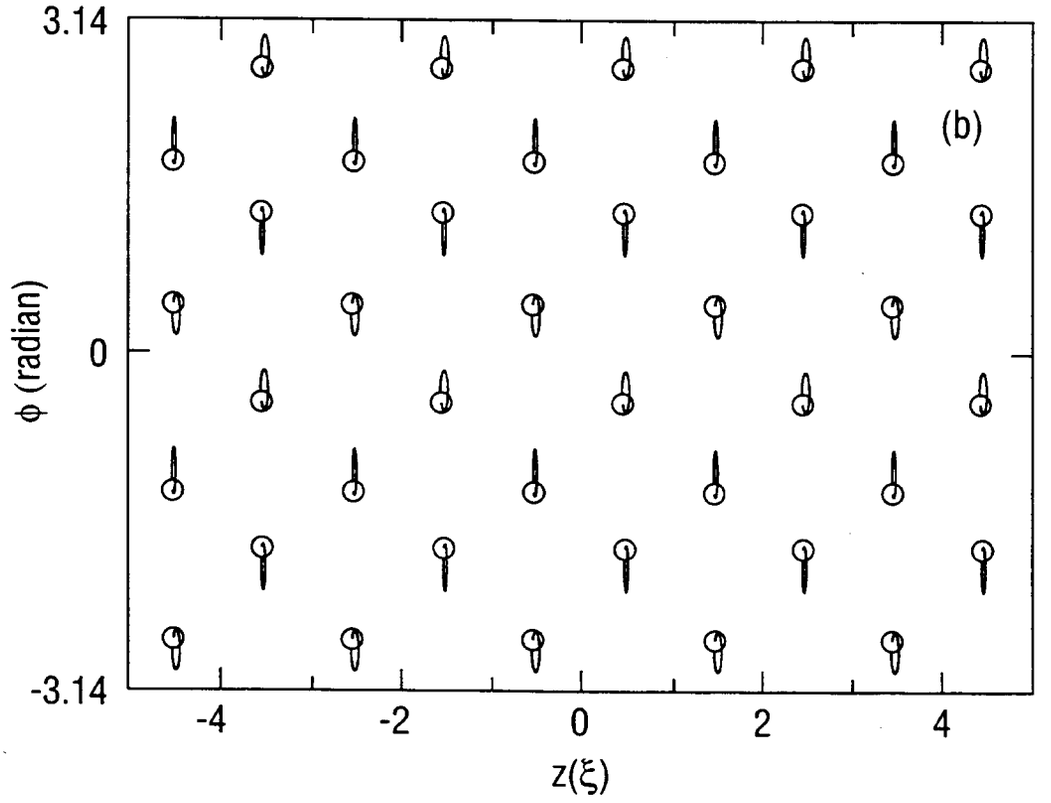
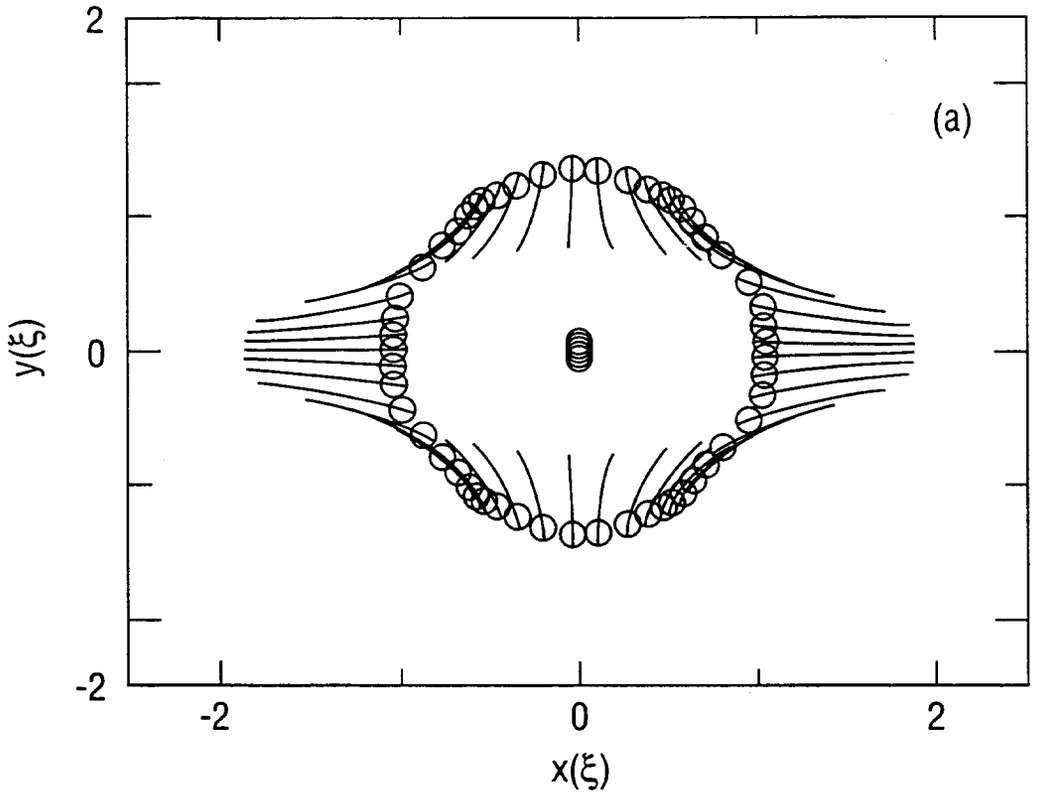


FIG. 2



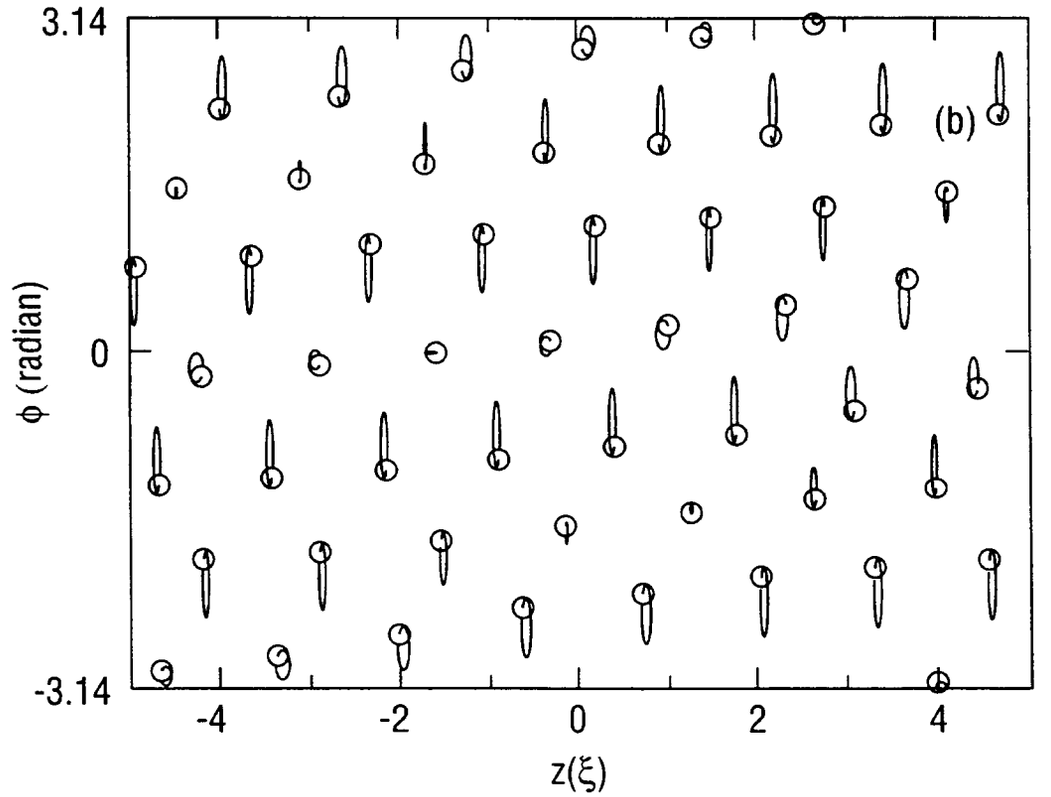


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FIG. 3a

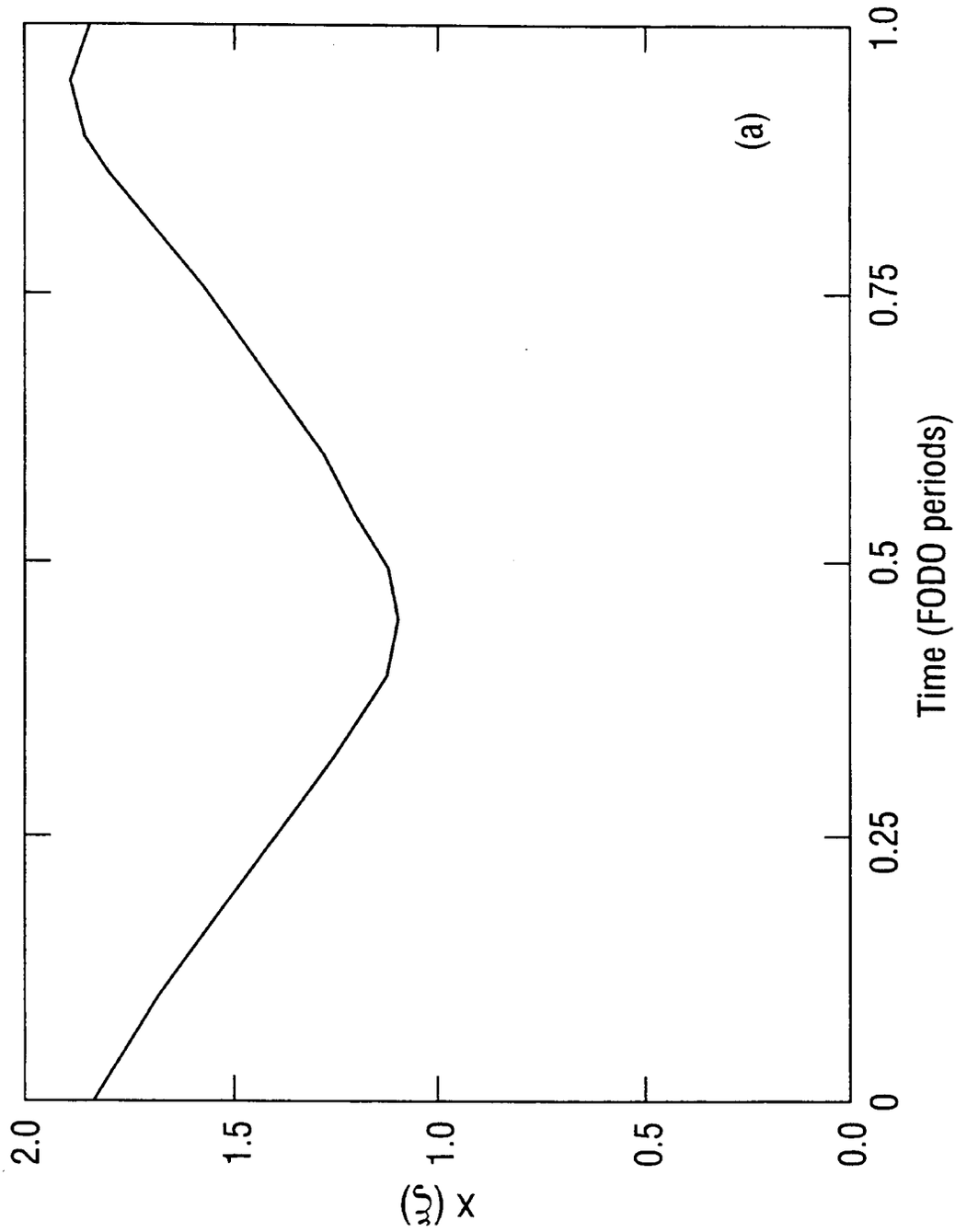

Fig. 3b

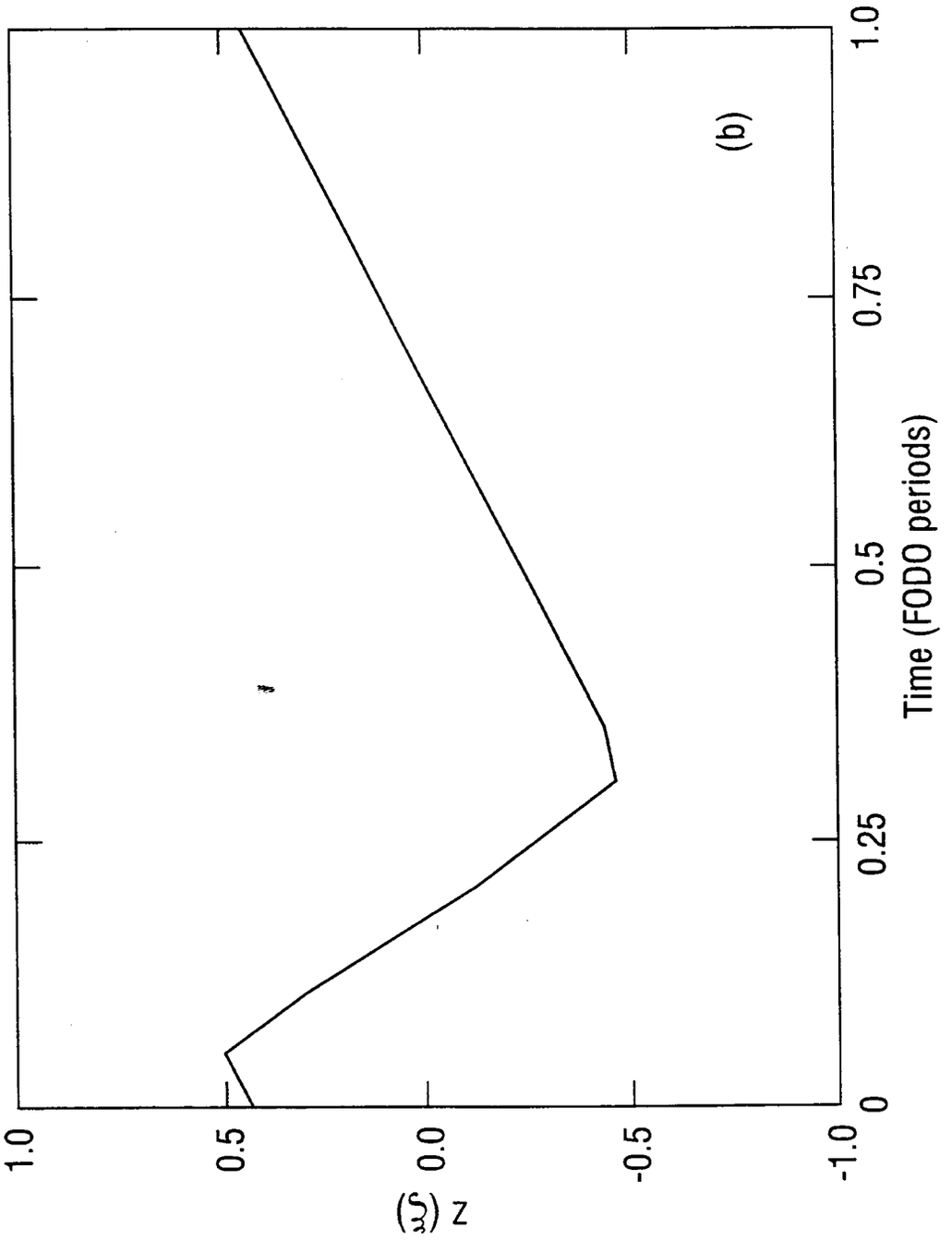

Fig. 4

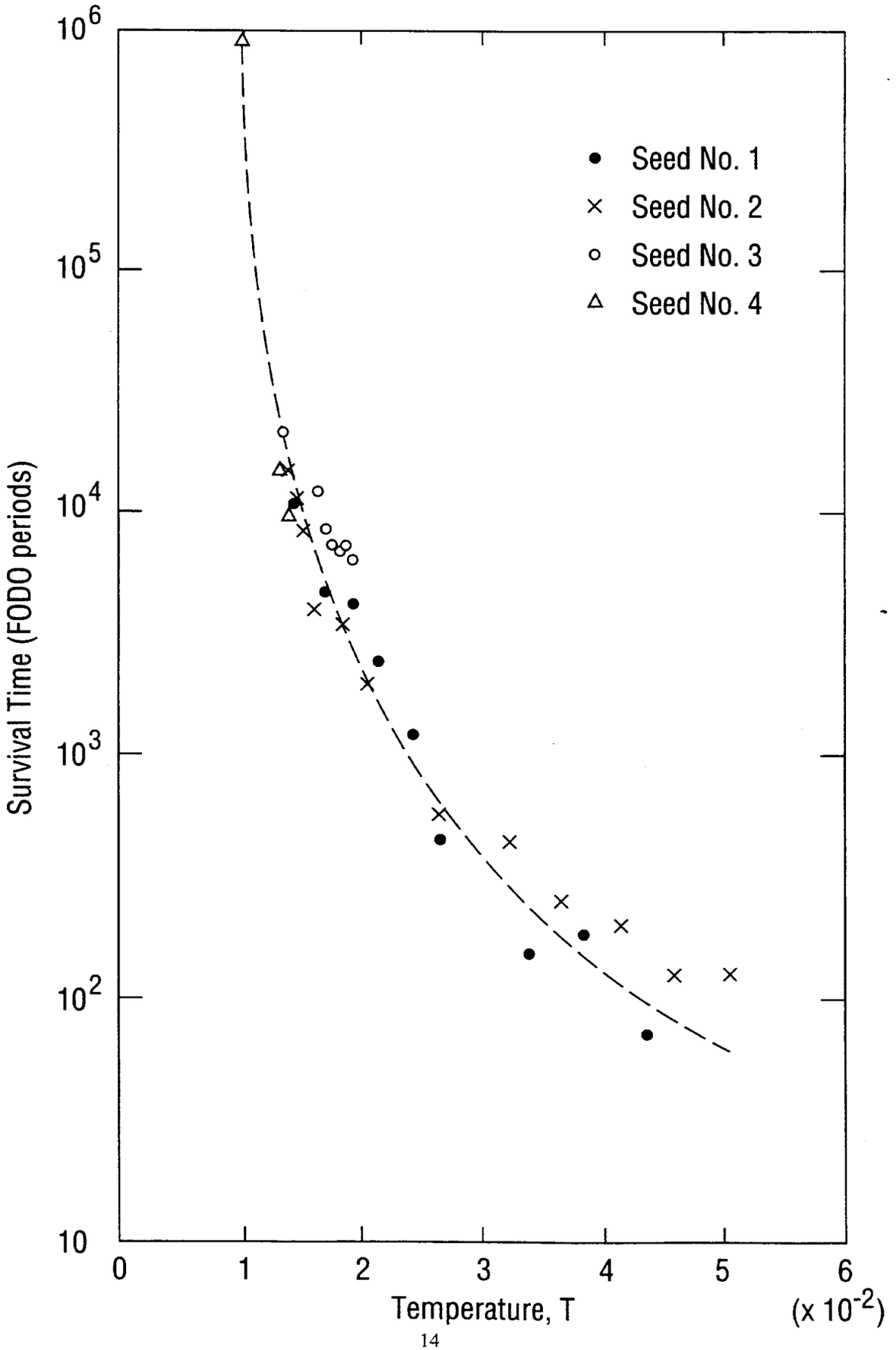
Survival Time (FODO periods)
10^6
10^5
10^4
10^3
10^2
10
0
1
2
3
4
5
6
Temperature, T
(x 10^{-2})
Seed No. 1
Seed No. 2
Seed No. 3
Seed No. 4

Fig. 5a

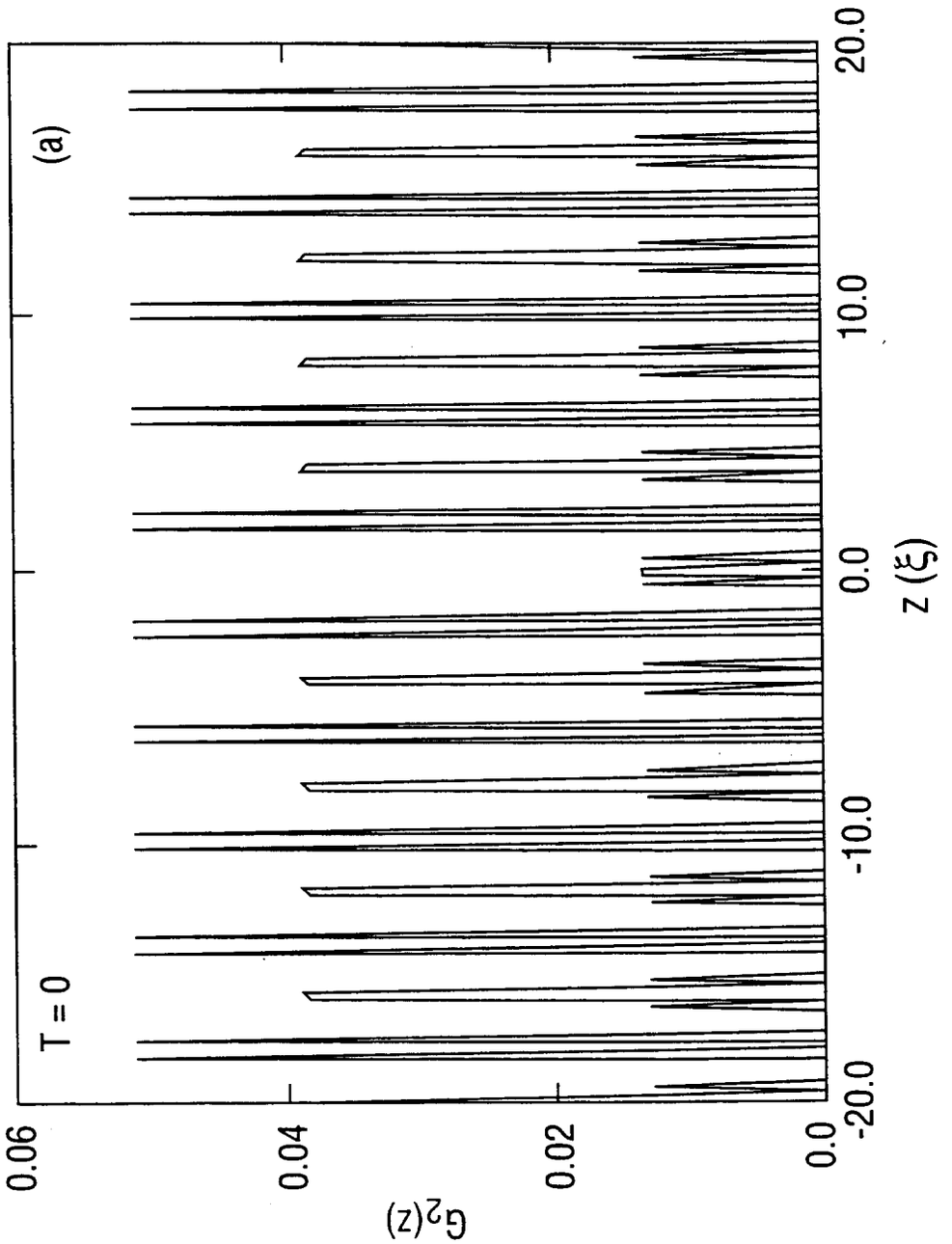

Fig. 5b

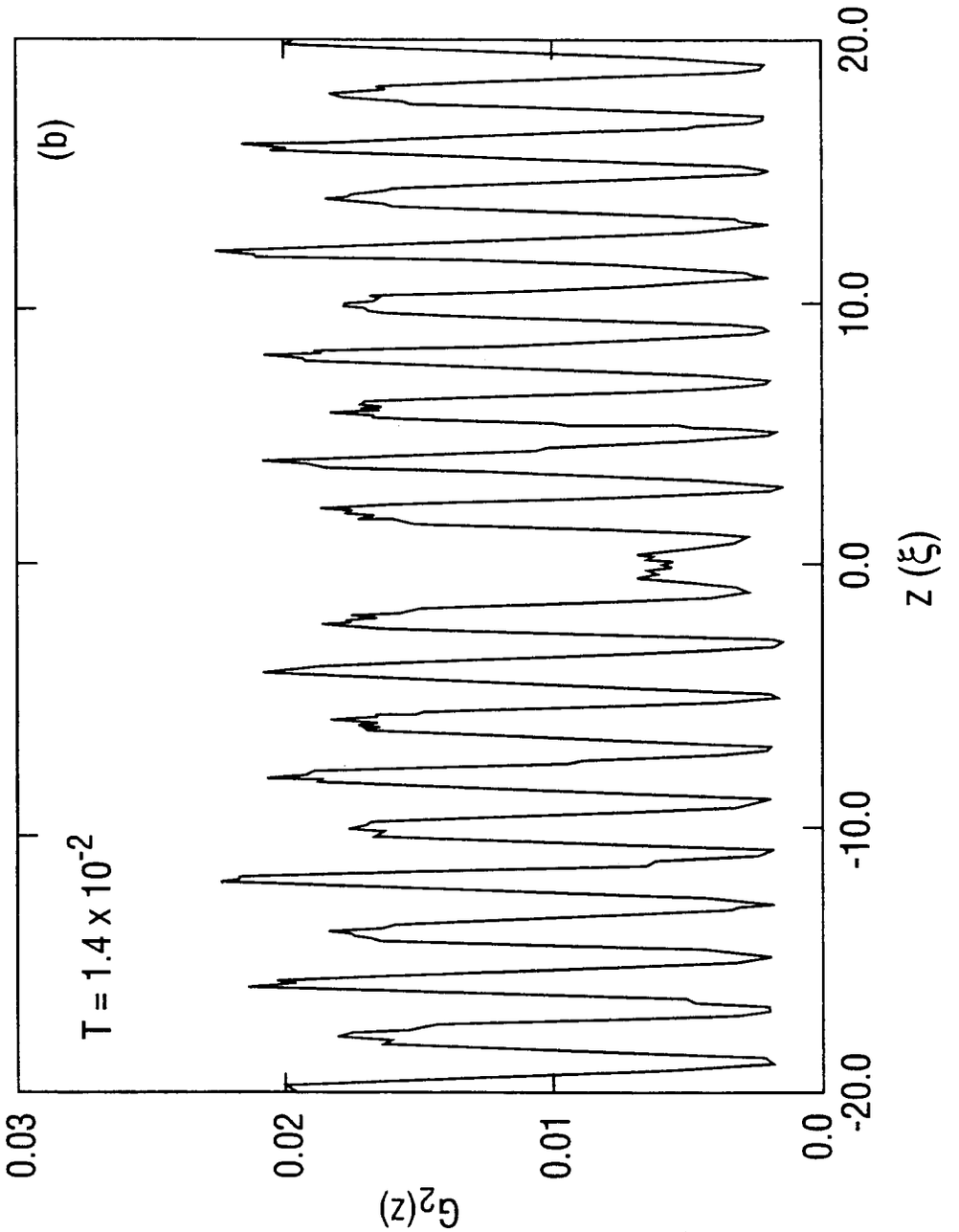

Fig. 5c

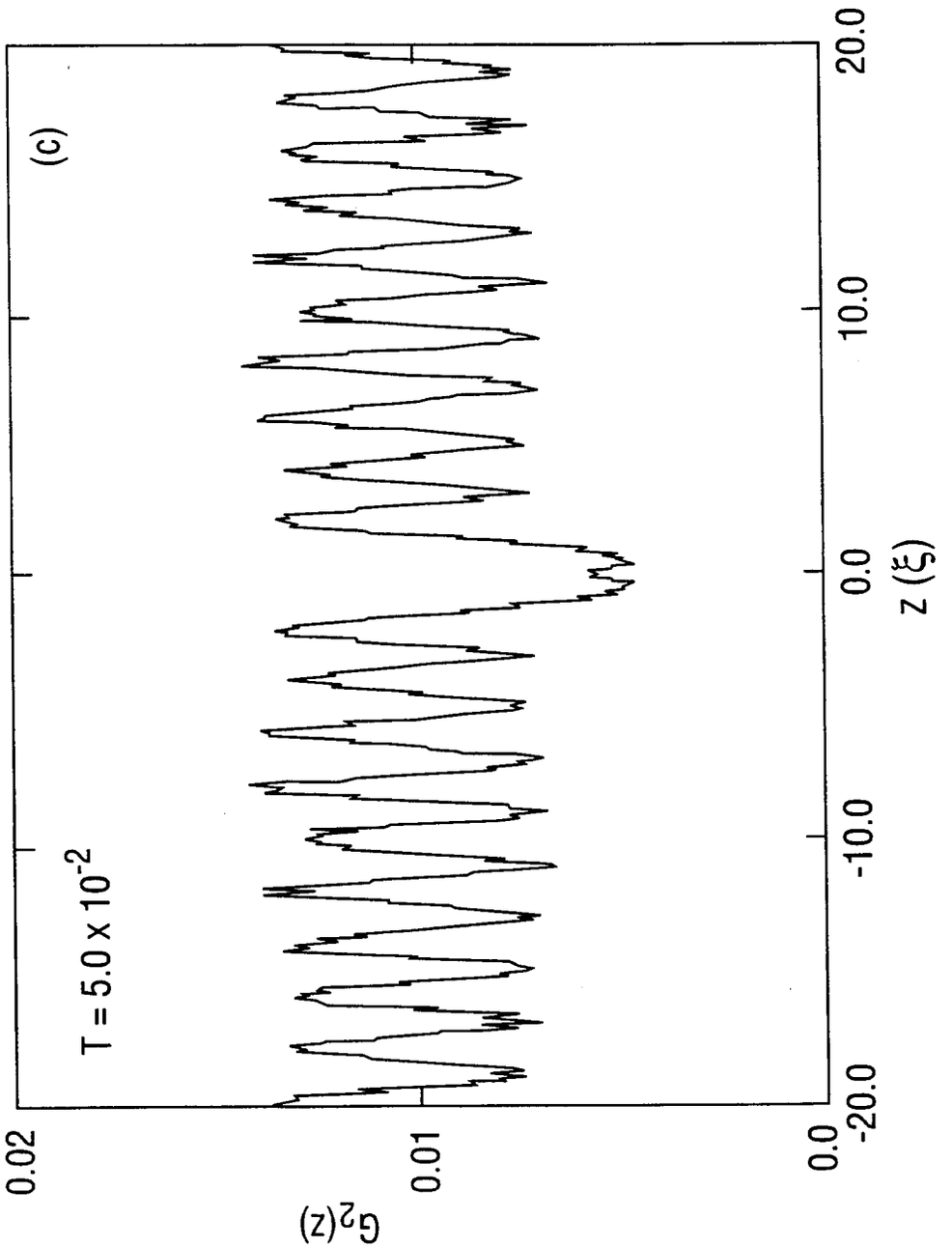

Fig. 5d