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Letter of Intent to measure Vacuum Magnetic Birefringence: the VMB@CERN experiment

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Abstract

Non linear electrodynamic effects have been predicted since the formulation of the Euler effective Lagrangian in 1935. These include processes such as light-by-light scattering, Delbrück scattering, g-2 and vacuum magnetic birefringence. This last effect deriving from quantum fluctuations appears at a macroscopic level. Although experimental efforts have been active for about 40 years (having begun at CERN in 1978) a direct laboratory observation of vacuum magnetic birefringence is still lacking: the predicted magnetic birefringence of vacuum is $\Delta n = 4.0 \times 10^{-24}$ @ 1 T.

Key ingredients of a polarimeter for detecting such a small birefringence are a long optical path within an intense magnetic field and a time dependent effect. To lengthen the optical path a Fabry-Perot interferometer is generally used. Interestingly, there is a difficulty in reaching the predicted shot noise limit of such polarimeters. The cavity mirrors generate a birefringence-dominated noise whose ellipticity is amplified by the cavity itself limiting the maximum finesse which can be used.

This Letter of Intent proposes an experiment which overcomes this difficulty by using a LHC superconducting magnet together with a novel polarisation modulation scheme for the polarimeter. The proposing authors all come from previous experimental efforts to measure vacuum magnetic birefringence and represent the maximum expertise in the field. Using the proposed setup, vacuum magnetic birefringence should be detected with an SNR = 1 in less than 1 day.

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The first detection of VMB would result in a direct observation of the fluctuations of the electron-positron field, and it would pave the way both to an accurate test of QED and to the observation of higher order effects.

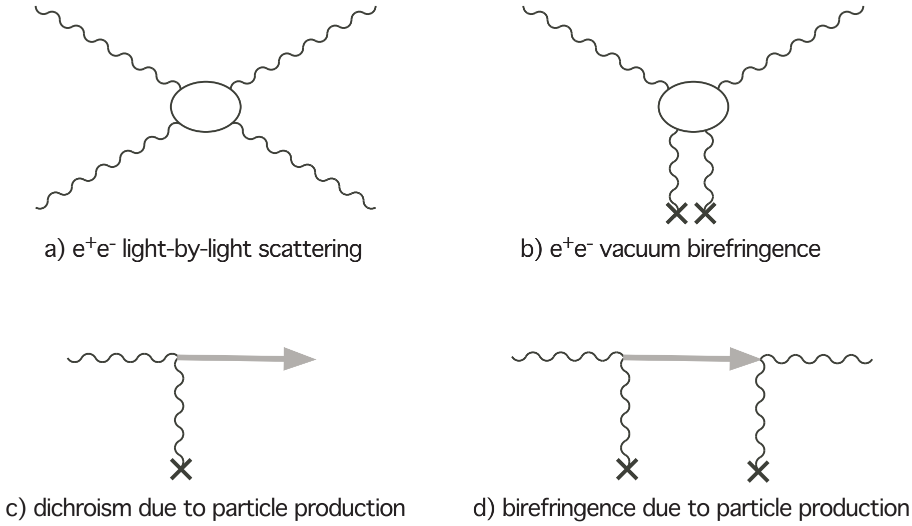

Figure 1: Lowest order non-linear elementary processes in vacuum: a) light-by-light elastic scattering, b) vacuum magnetic birefringence, c) Primakoff process leading to particle production and dichroism, d) second-order Primakoff process leading to birefringence.

1 Introduction

1.1 Vacuum Magnetic Birefringence

Non linear electrodynamics in vacuum leading to light-by-light elastic scattering was first imagined in 1933 [1] by O. Halpern after the postulation of Heisenberg's Uncertainty Principle and of Dirac's equation of the electron which predicted the anti-electron (positron). O. Halpern describes vacuum as having scattering properties on light. An effective Lagrangian density describing such a quantum effect in vacuum was first written by H. Euler and B. Kockel in 1935 and shortly after generalised by H. Euler, W. Heisenberg and V. S. Weisskopf [2, 3, 4, 5] in 1936. This Lagrangian leads to Light-by-Light (LbL) scattering [6, 7, 8], to a reduction of the velocity of light c in in the presence of an external field and to vacuum anisotropy through the Vacuum Magnetic Birefringence (VMB) [9, 10, 11, 12, 13, 14, 15], namely a difference in the indices of refraction for light polarised parallel and perpendicular to an external magnetic field $\vec{B}_{ext}$. VMB due to QED is a macroscopic purely quantum effect due to fermion fluctuations. The two closely related processes LbL and VMB are shown in Figure 1, top row, using today's Feynman diagrams.

For electric and magnetic fields well below their critical values ($E \ll E_{\rm crit} = \frac{m_e^2 c^3}{e\hbar} = 1.38 \times 10^{18}$ V/m and $B \ll B_{\rm crit} = \frac{m_e^2 c^2}{e\hbar} = 4.4 \times 10^9$ T) the free field electromagnetic Lagrangian density which takes into account e^+e^- quantum vacuum fluctuations and describes non linear electrodynamic effects in vacuum can be written as

$$\mathscr{L}_{\rm EHW} = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} - B^2\right) + \frac{A_e}{\mu_0}\left[\left(\frac{E^2}{c^2} - B^2\right)^2 + 7\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] + \ldots, \tag{1}$$

where

$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!^{-}{}_e^3}{m_e c^2} = 1.32 \times 10^{-24}\ {\rm T}^{-2}. \tag{2}$$

The parameter A_e describes the entity of the non linear correction to the Classical Lagrangian. Here $\lambda\!\!\!^{-}{}_e = \hbar/m_e c$ is the Compton wavelength of the electron and $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ is the fine structure constant. As of today, $\mathscr{L}_{\rm EHW}$ still needs direct experimental confirmation. Evidence of LbL scattering of high energy virtual photons has been recently published by the ATLAS collaboration [16] and an indirect evidence of VMB has been published by Mignani *et al.* [17] (with some criticism [18]) as a result of observations of polarised light from a neutron star.

QED is an extremely well tested theory but always in the presence of charged particles either in the initial and/or final states. The observation of VMB would be the first verification of QED with only low energy photons in the initial and final states. Furthermore, it would also put forth a purely quantum effect involving electron fluctuations at a macroscopic level.

The macroscopic properties of the quantum vacuum in the presence of an external field can be studied through the complex index of refraction $\tilde{n} = n - i\kappa$ where n is the index of refraction and κ is the extinction coefficient describing absorption. The index of refraction n can be determined from $\mathscr{L}_{\rm EHW}$ by applying the constitutive relations which define $\vec{D}$ and $\vec{H}$ as a function of $\vec{E}$ and $\vec{B}$:

$$\vec{D} = \frac{\partial \mathscr{L}_{\rm EHW}}{\partial \vec{E}} \quad \text{and} \quad \vec{H} = -\frac{\partial \mathscr{L}_{\rm EHW}}{\partial \vec{B}}. \tag{3}$$

Considering the case in which a linearly polarised beam of light is traversing perpendicularly an intense external magnetic field $\vec{B}_{\rm ext}$ it can be shown that the index of refraction depends on the polarisation direction with respect to the external field:

$$\begin{aligned} n_{\|} &= 1 + 7A_e B_{\rm ext}^2 \\ n_{\perp} &= 1 + 4A_e B_{\rm ext}^2. \end{aligned} \tag{4}$$

Here $\|$ and $\perp$ indicate polarisation directions parallel and perpendicular to the external field. Two facts are apparent: the velocity of light in an external field is no longer c and vacuum becomes anisotropic with a birefringence Δn given by equations (4). The resulting birefringence predicted by $\mathscr{L}_{\rm EHW}$ in the presence of an external magnetic field is [9, 10, 11, 12, 13, 14, 15]

$$n_{\|} - n_{\perp} = \Delta n = 3A_e B_{\rm ext}^2 = 3.96 \times 10^{-24} \left(\frac{B_{\rm ext}}{1\ \rm T}\right)^2. \tag{5}$$

Even with a magnetic field intensity of 9.5 T, as could be reached with an LHC magnet, the birefringence would still be $\Delta n^{\rm LHC} = 3.6 \times 10^{-22}$, an extremely small value. An electric field could also be used to generate a birefringence, in which case

$$n_{\|} - n_{\perp} = \Delta n = -3A_e \left(\frac{E_{\rm ext}}{c}\right)^2. \tag{6}$$

Experimentally, higher values of $B_{\rm ext}^2$ can be obtained with respect to $\left(\frac{E_{\rm ext}}{c}\right)^2$.

Higher order corrections have been calculated resulting in a correction $\delta(\Delta n)/\Delta n = \frac{25\alpha}{4\pi}$ to the birefringence. This accounts for a 1.45% increase in Δn [19].

It must also be noted that the imaginary part of $\tilde{n}$ is substantially zero for QED. Indeed the leading non zero term to photon splitting in an external field is with six photons connected to a loop [14, 15, 20] and is unmeasurably small. The diagram comprises an incident real photon, three external field lines and two exiting real photons. This implies the absence of a Vacuum Magnetic Dichroism VMD (polarisation dependent absorption) in QED.

At present VMB has not been directly detected yet. The best experimental limit obtained by the PVLAS collaboration is [21]

$$\frac{\Delta n^{\rm PVLAS}}{B^2} = (1.9 \pm 2.7) \times 10^{-23}\ \rm T^{-2}. \tag{7}$$

The 1-σ uncertainty is about a factor 7 larger than the predicted value from QED and was obtained with a total integrating time $T = 5 \times 10^6$ s.

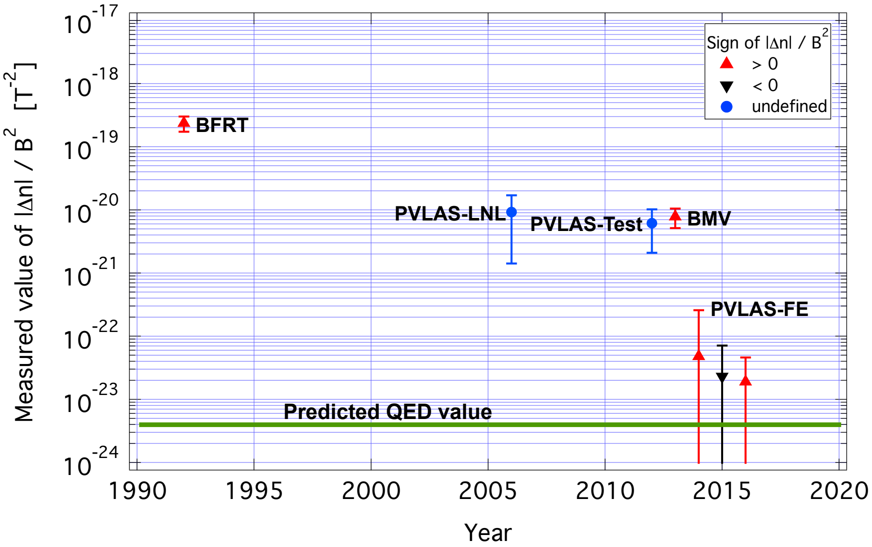

Figure 2: VMB measurements from different experimental efforts during the last 25 years. 1-σ statistical uncertainties are reported. Systematic errors are present in several efforts (incompatibility with zero). BFRT [22], PVLAS-LNL [24, 25], PVLAS-TEST [26], BMV [27], PVLAS-FE [28, 29, 21].

In Figure 2 we report the evolution of the results of experimental efforts to measure $\Delta n/B_{\rm ext}^2$ due to VMB performed during the last 25 years.

1.2 VMB beyond first order QED

The detection of VMB would be an extremely important verification of QED and its underlying bases. It would also demonstrate the validity of using low energy photons for particle physics in the eV−sub-eV domain. Several other effects, either predicted or hypothetical, could be studied. Below are listed a few of these:

- **Higher order corrections.** Radiative corrections to (5) due to QED have also been calculated resulting in a relative increase of $\delta(\Delta n)/\Delta n = \frac{25\alpha}{4\pi} = 1.45\%$ and could be hunted for. Other more difficult corrections to observe could involve hadron fluctuations, but it is expected that these are out of reach [30].

- **Post-Maxwellian non-linear electrodynamics.** In a more generalised formulation of the non-linear electrodynamics [31] the Lagrangian density will depend on the three parameters ξ, η_1 and η_2:

$$L_{\rm pM} = \frac{\xi}{2\mu_0}\left[\eta_1\left(\frac{E^2}{c^2}-B^2\right)^2+4\eta_2\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] \quad (8)$$

where $\xi = 1/B_{\rm crit}^2$, and η_1 and η_2 are dimensionless parameters depending on the chosen model. The density (8) reduces to (1) with $\eta_2^{\rm (QED)} = \frac{7}{4}\eta_1^{\rm (QED)} = \alpha/(45\pi)$. In this generalisation, one finds that the birefringence induced by a transverse magnetic field is

$$\Delta n^{\rm (pM)} = 2\xi(\eta_2-\eta_1)B^2 \quad (9)$$

to be compared with equation (5). It is also interesting to note that, considering the forward LbL scattering amplitudes, $n_\parallel$ depends only on η_1 whereas $n_\perp$ depends only on η_2. Therefore birefringence is only sensitive to the difference $\eta_2-\eta_1$. For example in the Born-Infeld electrodynamics

[32, 33] where $\eta_1 = \eta_2$, magnetically induced birefringence is not expected even though $n > 1$ in vacuum and LbL scattering is permitted.

– **Search for Axion-Like Particles (ALPs) and Milli-Charged Particles (MCPs).** Other effects may generate both VMB and VMD. These include hypothetical axion-like particles (Figure 1 bottom row) [34, 35, 36, 37] and milli-charged particles [38, 39, 40, 41]. In the case of ALPs, a detection of both VMB and VMD would allow to disentangle their mass and coupling constant to photons. Furthermore, a recent arXiv preprint [42] has proposed the possibility to detect ALPs via their vacuum fluctuations. In the case of MCPs and depending on their mass, these could generate both a birefringence in the same way as e^+e^- pairs and a dichroism when the virtual pairs become real at the expense of the incoming light.

2 Polarimetry

To determine a birefringence one can measure the ellipticity ψ induced on a linearly polarised beam of light with wavelength λ passing through the medium with uniform birefringence Δn and effective length L. The expression for ψ ($\psi \ll 1$) is

$$\psi = \pi \frac{L}{\lambda} \Delta n \sin 2\vartheta = \psi_0 \sin 2\vartheta \tag{10}$$

where ϑ is the angle between the magnetic field and the polarisation direction.

The first proposal to detect VMB using an optical polarimeter, published by E. Zavattini and E. Iacopini from CERN [43], dates back to 1979 and some very preliminary tests were then performed in the early ’80s at CERN [44]. Optical techniques have greatly improved since, and today several experiments are underway to attempt to detect either LbL scattering or VMB. Of these, the most sensitive at present are based on optical polarimeters with very high finesse Fabry-Perot cavities and variable magnetic fields [27, 28, 29, 45, 46].

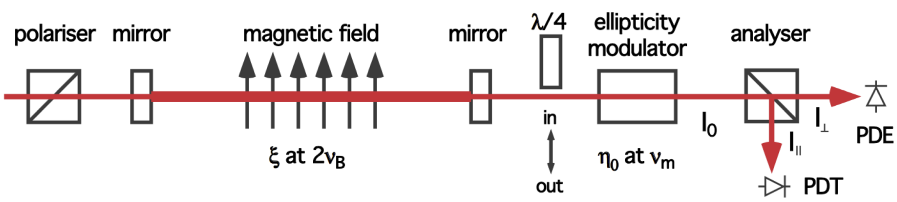

Figure 3: A polarimeter based on a Fabry-Perot cavity with a time-dependent signal ξ and heterodyne detection using an ellipticity modulator of amplitude η_0. With the quarter-wave plate extracted, the polarimeter is sensitive to induced ellipticities. With the quarter-wave plate inserted, instead, rotations are measured. PDE: Extinction Photodiode; PDT: Transmission Photodiode.

A typical scheme of such a polarimeter is shown in Figure 3. The Fabry-Perot cavity is necessary to increase the optical path within the magnetic field region by a factor $N = \frac{2\mathcal{F}}{\pi}$, where $\mathcal{F}$ is the finesse of the cavity, whereas the variable magnetic field is necessary to induce a time dependent effect. The ellipticity $\eta(t)$ generated by the modulator allows heterodyne detection linearising the induced ellipticity $\psi(t)$. The presence of the extractable quarter-wave plate is to permit the measurement of rotations instead of ellipticities.

The variability of the external field may be either in its intensity, thereby modulating directly $\Delta n(t)$, or in its direction thereby modulating the angle $\vartheta(t)$. Both the Fabry-Perot and the time dependent signal significantly increase the sensitivity of such polarimeters. In general, the induced ellipticity due to a magnetic birefringence is therefore

$$\psi(t) = \frac{2\mathscr{F}}{\lambda} \int_{\text{field}} \Delta n \, dl \sin 2\vartheta(t) \Big|_{\text{variable } \vartheta} \tag{11}$$

if the direction between the polarisation and the magnetic field is varied (rotating magnets as in the PVLAS and Q&A [46] experiments) or

$$\psi(t) = \frac{2\mathscr{F}}{\lambda} \int_{\text{field}} \Delta n(t) \, dl \sin 2\vartheta \Big|_{\text{variable } B_{\text{ext}}} \tag{12}$$

if the field intensity is modulated (pulsed magnets as in BMV and OVAL or ramped fields as in BFRT).

At the output of the polarimeter of Figure 3 (with the quarter-wave plate extracted) the intensity will be

$$I_{\text{out}}(t) = I_0 \left\{ \sigma^2 + [\psi(t) + \eta(t) + \gamma(t)]^2 \right\} \simeq I_0 \left[\sigma^2 + \eta(t)^2 + 2\psi(t)\eta(t) + 2\eta(t)\gamma(t) + ... \right] \tag{13}$$

where σ^2 is the extinction ratio of the polarisers and $\gamma(t)$ are slowly varying spurious ellipticities. The term of interest is the product $\eta(t)\psi(t)$. If $\eta(t)$ and $\psi(t)$ are sinusoidal functions of time, well defined Fourier components will appear in I_{out}. If the angular frequency of the ellipticity modulator and of the VMB induced ellipticity are respectively Ω and ω_{VMB} with respective amplitudes η_0 and ψ_0, and assuming I_{out} is demodulated at Ω and 2Ω, the amplitude ψ_0 can be determined from

$$\psi_0(\omega_{\text{VMB}}) = \frac{A_\Omega(\omega_{\text{VMB}})}{A_{2\Omega}(\text{dc})} \frac{\eta_0}{4}. \tag{14}$$

Here $A_\Omega(\omega_{\text{VMB}})$ is the Fourier amplitude at ω_{VMB} of I_{out} demodulated at Ω and $A_{2\Omega}(\text{dc})$ is the dc amplitude of I_{out} demodulated at 2Ω. As will be discussed below, the spurious ellipticities $\gamma(t)$ will generate a continuous noise spectrum $S_{A_{\omega_{\text{VMB}}}}(\nu)$ which limits the sensitivity for very high finesse cavities.

Modulating intense magnetic fields at frequencies $\sim 10 - 100$ Hz presents serious difficulties. For this reasons the use of high field superconducting magnets, such as those used in the LHC and HERA accelerators, to increase the signal has not been until today a viable solution.

Another possibility of modulating $\vartheta(t)$ is to rotate the polarisation of the incident light. This was attempted by the OSQAR collaboration [47, 48, 49] at CERN using one LHC magnet but the presence of the Fabry-Perot cavity, whose mirrors always present an intrinsic birefringence and whose induced ellipticity is orders of magnitude larger than the ellipticity due to VMB, have made this idea unfeasible.

None of the methods adopted until now have allowed the detection of VMB.

Several other ideas have been put forth but never implemented. These include using gravitational wave antennas with magnets along one or both of the arms [50, 51], using X-ray polarimetry [52] or using crossed high intensity lasers [53, 54].

In this Letter, a novel polarisation modulation scheme is presented that might profitably be employed with large superconducting magnets present a CERN for a first direct detection of VMB.

3 State of the art

Shot-noise limits the ultimate ellipticity sensitivity of a polarimeter. Indeed the maximum ellipticity ψ_0 ($\sin 2\vartheta = 1$) is related to the phase difference φ acquired by two orthogonal polarisation components of light when crossing a birefringent medium: $\psi = \varphi/2$. Therefore the ultimate ellipticity sensitivity is governed by the relative phase noise between the two perpendicular components of the polarisation of the light which in turn depends only on the power of the light. The rms shot-noise ellipticity sensitivity, $S_{\psi_{\rm shot}}$, is

$$S_{\psi_{\rm shot}} = \sqrt{\frac{2e}{I_0 q}\left(\frac{\sigma^2+\eta_0^2/2}{\eta_0^2}\right)} \approx \sqrt{\frac{e}{I_0 q}} \tag{15}$$

for $\eta_0^2/2 \gg \sigma^2$. Here q is the detector efficiency in ampere/watt. A better parameter to evaluate the sensitivity of an ellipsometer, rather than the ellipticity, is the optical path difference $\mathscr{D} = \int \Delta n\, dl$. It allows the comparison of different polarimeters independently of their finesse and/or wavelength. The optical path difference noise $S_{\mathscr{D}}$ results to be

$$S_{\mathscr{D}} = \int S_{\Delta n}\, dl = S_{\psi}\frac{\lambda}{2\mathscr{F}}. \tag{16}$$

Typical experimental parameters of today's polarimeters are $I_0 \simeq 1-10$ mW, $q = 0.7$ A/W, $\mathscr{F} \simeq 500000$ and $\lambda = 1064$ nm. These values would predict a shot-noise sensitivity in optical path difference of

$$S_{\mathscr{D}_{\rm shot}} = \sqrt{\frac{e}{I_0 q}}\frac{\lambda}{2\mathscr{F}} \simeq (5-16)\times 10^{-21}\ \frac{\rm m}{\sqrt{\rm Hz}}. \tag{17}$$

Given the magnets used in the different efforts and given the optical path difference $\mathscr{D}_{VMB}$ due to VMB to be measured of

$$\mathscr{D}_{VMB} = \int 3A_e B^2\, dl \simeq 4-40\times 10^{-23}\ \rm m, \tag{18}$$

a continuous integration time of $T = 10^2 - 10^5$ s should have allowed its detection.

Excess noise has always afflicted high finesse polarimeters. By plotting the noise in optical path difference of the different experiments as a function of their working frequency one finds that all seem to lie on a power law as can be seen in Figure 4. Although the measured optical path difference noise of

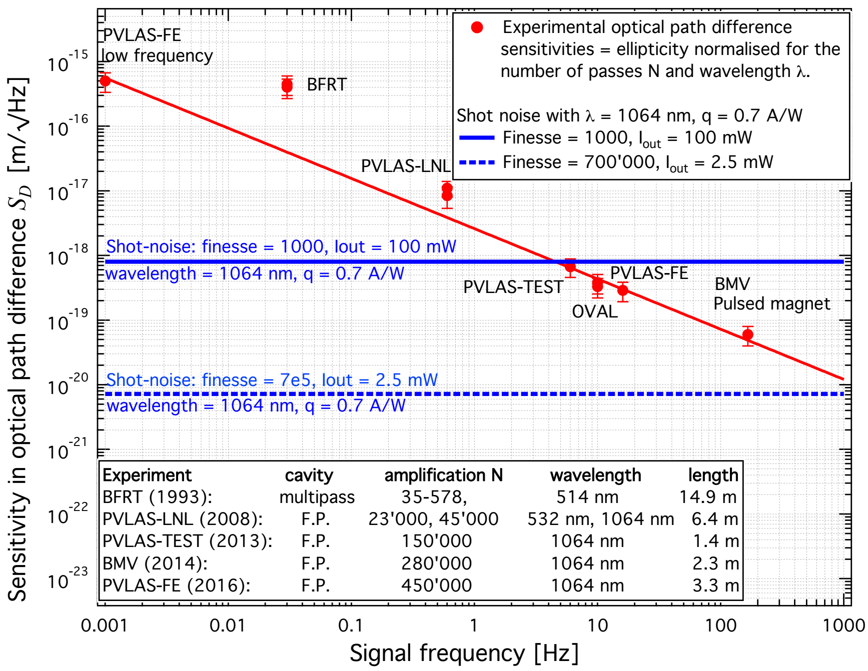

Figure 4: Noise in optical path difference versus working frequency for different polarimeters using cavities (multi-pass or Fabry-Perot) for improved sensitivity. All seem to lie on a common curve indicating a common noise source. Shot-noise sensitivities for two different configurations are superimposed.

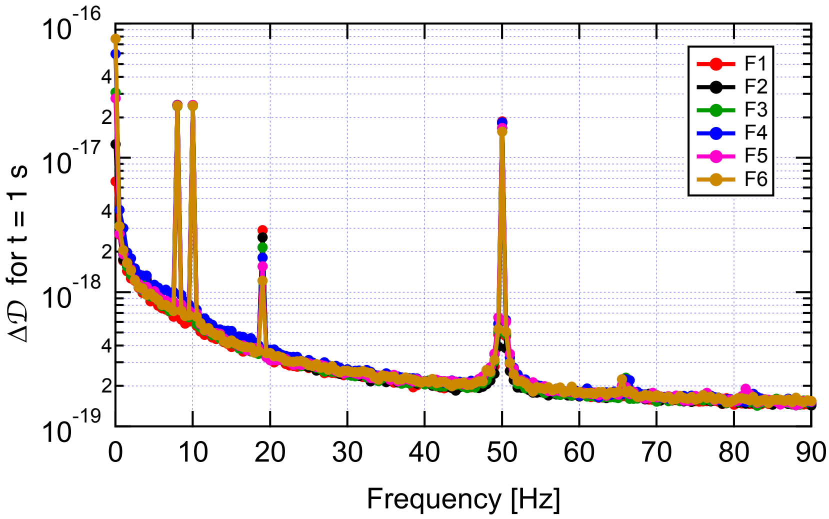

Figure 5: Optical path difference sensitivities measured for six different finesse values of the PVLAS cavity (data from ref. [58]). All spectra overlap indicating a common noise source originating from the mirrors. This noise is amplified in the same way as a birefringence signal resulting in a SNR independent of the finesse.

$S_{\mathscr{D}} \approx 10^{-18} - 10^{-19}$ m/$\sqrt{\mathrm{Hz}}$ @ $10 - 100$ Hz is quite impressive it is significantly above shot-noise. The working frequency of each experiment was defined by the modulating technique of the magnetic field: ramping the magnetic field strength of a superconducting magnet (BFRT [22]), rotating a superconducting magnet[1] (PVLAS-LNL [23, 24, 25]), rotating permanent magnets (Q&A [46], PVLAS-Test [26] and PVLAS-FE [21, 28, 29]) and pulsing magnets (BMV [27] and OVAL [45]).

Other than the ellipticity induced by the magnetic birefringence, a polarimeter is sensitive to any spurious birefringence between the polariser and the analyser: this includes the mirrors of the Fabry-Perot cavity. As discussed below, recent measurements [58] have shown that there is indeed an excess noise originating from the reflective coatings of the cavity mirrors and dominating the sensitivity for high finesse values. Increasing the finesse above a certain value increases both the induced ellipticity signal ψ_{VMB} to be measured and the ellipticity noise $S_\psi(t)$ generated by the mirrors. The SNR becomes *independent* of the finesse.

Comparing this intrinsic mirror noise to the expected rms shot-noise of a polarimeter shows that there is a limiting finesse which can be used to increase the SNR given by

$$\mathscr{F}_{\mathrm{max}} = \sqrt{\frac{e}{I_0 q}} \frac{\lambda}{2 S_{\mathscr{D}}(\nu)}. \tag{19}$$

Given a noise $S_{\mathscr{D}} \approx 5 \times 10^{-19}$ m/$\sqrt{\mathrm{Hz}}$ and an output power $I_0 \approx 1$ mW the highest usable finesse (above which the SNR does not improve) is about $\mathscr{F}_{\mathrm{max}} = 16000$, well below the typically used values.

The use of relatively high field (2.5 T) permanent rotating magnets at present has given the best results. The long integration time achieved by the PVLAS experiment was possible thanks to these magnets and to the detailed study of systematic noise sources: two identical separate magnets rotating at different frequencies were used to study the delicate systematic effects and keep them under control.

In Figure 5 one can see the superposition of the noise in optical path difference $S_{\mathscr{D}}$ measured with the PVLAS apparatus at 6 different finesse values ranging from $\mathrm{F}_1 = 688000$ to $\mathrm{F}_6 = 256000$ [58]. The peaks at 8 and 10 Hz are due to the Cotton-Mouton effect used for calibration. These too all have the

[1] This 1 m long magnet from the ’80s, generating a record field of 7.6 T [55], was designed and tested by Mario Morpurgo as a prototype for a 6 m long version required for the original proposal to measure VMB at CERN [56, 57].

same value for $\mathscr{D}$ independently of the finesse, as it should be for a true signal. As mentioned previously, the SNR does not depend on the finesse so long as $\mathscr{F} > \mathscr{F}_{\rm max}$ in equation (19).

To improve the SNR by a factor of at least 10 so as to measure VMB at SNR = 1 in about 10^5 s continuous integration, there are a few possibilities:

- Further increase the frequency of the signal modulation. At present, a factor 10 would mean reaching frequencies of hundreds of hertz with a $\int B^2\, dl \simeq 10$ T^{2}m. This would require also reducing the finesse by at least a factor 10 to avoid cavity filtering. Since the SNR does not depend on the finesse, this would not alter the sensitivity $S_{\mathscr{D}}$. High frequencies are reached by pulsed magnets with fields lasting a few millisecond. Furthermore these are expected to reach $\int B^2\, dl \approx 600$ T^{2}m [27] in the near future (BMV experiment). This would lead to a VMB optical path difference $\mathscr{D}_{\rm BMV} \approx 2.4 \times 10^{-21}$ m. With a noise $S_{\mathscr{D}_{\rm BMV}} \approx 6 \times 10^{-20}$ m/$\sqrt{\rm Hz}$ @ 200 Hz this results in a *total* integration time with the field on of $T \sim 1000$ s meaning $\sim 10^6$ pulses. A pulse repetition time interval of a few minutes makes this solution extremely difficult too.
- Assuming the noise $S_{\mathscr{D}}$ reported in Figures 4 and 5 has a thermo-elastic origin in the coatings of the mirrors which decreases proportionally with temperature T, one could imagine to reduce the temperature of the cavity mirrors to about $T \lesssim 30$ K. Furthermore Brownian noise contributions have not been evaluated and if these were also to be significant, they decrease as $T^{1/2}$ making everything very difficult.
- Increase the parameter $\int B^2\, dl$ of the dipole magnets is a third option. This could be viable if one could use LHC magnets (with a static field) and obtain the necessary modulation of the signal by rotating the polarisation at the entrance of the cavity. As mentioned above this was proposed in the past by the OSQAR collaboration, but the intrinsic mirror birefringence of the cavity mirrors was a show stopper. Recently a new optical design has been proposed which could work around this problem: the polarisation is rotated only inside the magnetic field but not on the cavity mirrors.

4 New optical scheme

In 2016 a new optical scheme has been proposed [59] which could work around the problem of the intrinsic birefringence of the Fabry-Perot mirrors when trying to modulate the signal by rotating the polarisation. As will be introduced below this novel scheme limits the maximum finesse to about $\mathscr{F} \approx 1000 - 5000$. The new scheme is based on inserting two co-rotating half-wave plates *inside* of the Fabry-Perot, near each mirror and therefore outside of the magnetic field. The linearly polarised light traveling from the first mirror of the Fabry-Perot encounters the first rotating half-wave plate. This causes the angle between the polarisation and the magnetic field, described by ϑ, to rotate at twice the rotation frequency of the wave plate. Indicating with $\phi(t)$ the angular position of the first wave-plate then $d\vartheta(t)/dt = 2d\phi(t)/dt = 2\omega_{\rm HWP}$. The light with rotating polarisation then traverses the *static* magnetic field. Given that the induced VMB ellipticity is proportional to $\sin 2\vartheta(t)$ [see equation (10)] the ellipticity signal will have a frequency component at the fourth harmonic of the rotation frequency of the half-wave plates. Finally, the light encounters the second half-wave plate co-rotating with the first one. This second wave plate stops the rotation of the polarisation. Hence, the polarisation direction on the mirrors remains fixed while it is rotating inside the static magnetic field generating the desired modulation in the induced ellipticity. In Figure 6 one can see the scheme of the setup. Besides the two half-wave plates, the scheme is identical to the scheme shown in Figure 3.

The presence of the two wave plates will limit the maximum finesse due principally to the reflection on their surfaces. At present we are performing some preliminary test with two wave plates with an antireflective coating with $R \approx 0.1\%$ resulting in a finesse $\mathscr{F} \simeq 1000$. Contacts with companies seem to indicate that a reflectivity of 0.01% should be possible resulting in a finesse of $\mathscr{F} \simeq 10000$.

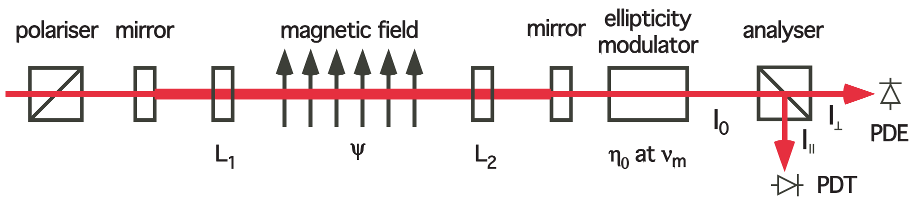

Figure 6: Proposed modulation scheme. $L_{1,2}$: rotating half-wave-plates. PDE: Extinction Photodiode; PDT: Transmission Photodiode.

In the presence of perfect half-wave plates and perfect polarisers the output signal in the new scheme would be

$$I_{\rm out} = I_0 \left\{ \eta(t)^2 + 2\eta(t)\psi_0 \sin[4\phi(t)] \right\} \tag{20}$$

where $\phi(t)$ is the azimuthal angle of the first half-wave plate such that at $t = 0$ the polarisation entering the magnetic field is parallel to the field direction. Interestingly the VMB signal in $I_{\rm out}$ appears, in the demodulated signal $A_\Omega(\nu)$, at the fourth harmonic of the half-wave plates' rotation frequency.

Small imperfections $\alpha_{1,2}$ with respect to a phase π in the two half-wave plates will certainly be present. By including these in the calculation of $I_{\rm out}$ and by introducing the relative (fixed) angle, $\Delta\phi$, between the two half-wave plates, the extinction ratio σ^2 of the polariser and analyser and the spurious ellipticities $\gamma(t)$, one finds

$$I_{\rm out} = I_0 \Big\{ \eta(t)^2 + \sigma^2 + 2\eta(t) \Big[\psi_0 \sin[4\phi(t)] + C \sin[2\phi(t) + 2\beta] + \gamma(t) \Big] \Big\} \tag{21}$$

where

$$C = \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \cos 2\Delta\phi} \tag{22}$$

and

$$\tan 2\beta = \frac{\alpha_1 + \alpha_2 \cos 2\Delta\phi}{\alpha_2 \sin 2\Delta\phi}. \tag{23}$$

Furthermore the wave-plate imperfections may themselves depend on their intrinsic azimuthal angle ϕ'. Expanding $\alpha_{1,2}$ in powers of $\cos n\phi'$ one can write

$$\alpha_{1,2}(t) = \alpha_{1,2}^{(0)} + \alpha_{1,2}^{(1)} \cos\phi'(t) + \alpha_{1,2}^{(2)} \cos 2\phi'(t) + \ldots \tag{24}$$

The term proportional to $\cos\phi'(t)$ can be generated by a non parallelism of the two faces of the wave plates whereas the term proportional to $\cos 2\phi'(t)$ represents a 'saddle' shape defect. Therefore the term in $\alpha_{1,2}^{(0)}$ will generate a Fourier component in $A_\Omega(\nu)$ at $2\omega_{\rm HWP}$ and the term in $\alpha_{1,2}^{(1)}$ will generate Fourier components at $\omega_{\rm HWP}$ and $3\omega_{\rm HWP}$. These will not disturb the VMB signal which appears at $\omega_{\rm VMB} = 4\omega_{\rm HWP}$. The term in $\alpha_{1,2}^{(2)}$, on the other hand, will generate a component at $4\omega_{\rm HWP}$. This is the really critical value to be understood: we must have $\alpha_{1,2}^{(2)} \ll \psi_0$.

Half-wave plates can be rated with $\alpha_{1,2}^{(0)} \lesssim \frac{\pi}{1000}$. A parallelism of the two wave-plate faces better than $\varepsilon \approx 10^{-6}$ rad is also typical. With a beam shifted from the center of rotation by $d \lesssim 0.1$ mm this implies ($\Delta n_{\rm quartz} = 9 \times 10^{-3}$) $\alpha_{1,2}^{(1)} < \frac{2\pi}{\lambda} d\varepsilon\Delta n_{\rm quartz} = 6 \times 10^{-6}$. As for $\alpha_{1,2}^{(2)}$, it is very difficult to estimate it.

5 The VMB@CERN initiative

The technique described above allows the use of superconducting *static* fields as those from an LHC magnet generating fields up to 9.5 T over a length of 14.3 m resulting in $\int_{LHC} B^2\, dl = 1290$ T^{2}m. If the intrinsic noise in the mirrors is given by the curve in Figure 4 and considering a finesse of $\mathscr{F} = 1000$

then with 100 mW exiting the cavity the shot-noise will dominate $S_{\mathscr{D}}$ for signal frequencies above about 5 Hz resulting in $S_{\mathscr{D}} = 10^{-18}$ m/$\sqrt{\text{Hz}}$ (blue horizontal line in Figure 4). This implies rotating the wave plates faster than 1.5 Hz. Therefore assuming $S_{\mathscr{D}}$ to be shot-noise limited the necessary integration time to reach a SNR = 1 of VMB is

$$T = \left(\frac{\sqrt{\frac{e}{I_0 q}} \frac{\lambda}{2\mathscr{F}}}{3A_e \int_{LHC} B^2 \, dl} \right)^2 \approx 7\,\text{h}. \tag{25}$$

Preliminary tests with a cavity of finesse $\mathscr{F} = 850$ and with two wave plates inserted in the cavity, but not rotating, show a flat noise spectrum $S_{\mathscr{D}} \simeq 10^{-17}$ m/$\sqrt{\text{Hz}}$ above a few hertz in agreement with the expected shot-noise given by the experimental conditions ($I_{\text{out}} = 1$ mW, $q = 0.7$ A/W). With the same cavity but with the wave plates extracted, the finesse increases to $\mathscr{F} \simeq 2500$ with an increased output power of 6 mW with a noise of $S_{\mathscr{D}} \simeq 6 \times 10^{-18}$ m/$\sqrt{\text{Hz}}$. The improved sensitivity in $S_{\mathscr{D}}$ without the wave plates is due to the increased finesse and output power. We believe that at the level of $S_{\mathscr{D}} \simeq 10^{-17}$ m/$\sqrt{\text{Hz}}$ the wave plates do not introduce extra optical path difference noise. More testing is underway.

At the moment the new technique seems very promising already with standard commercial half-wave plates.

During the past years, several groups have attempted measuring VMB leading to a strong expertise in polarimetry. Furthermore, some optics experts from the gravitational wave interferometer community have also become interested in VMB. Given the existence of the LHC dipole magnets at CERN, which at present generate the highest $\int B^2\, dl$ value, these groups are moving towards a joint effort at CERN. These include the PVLAS collaboration, the OSQAR collaboration, the Q&A collaboration and a group from LIGO-Cardiff. These add up to about 16 members from 12 Institutes from Czech Republic, France, Italy, Poland, Republic of China and Wales.

We believe this is a unique opportunity in which researchers with the right expertise have united to a joint effort for measuring VMB at CERN where magnet technology developments can provide the required magnetic field integral with dedicated infrastructures. All these points are being discussed within the Physics Beyond Collider (PBC) working group to inscribe this initiative in the next update of the European Strategy for Particle Physics.

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