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Abstract

The crystal-spectrometer X-ray experiment on the π^--^{24}Mg system which provides the most precise determination of the pion mass allows for two possible configurations of the electronic K-shell population. This leads to two possible solutions for the mass of the negative pion: $m_\pi(A) = 139.56782 \pm 0.00037$ MeV/c^2 and $m_\pi(B) = 139.56995 \pm 0.00035$ MeV/c^2. If one assumes that solution B is realized and uses the published value of the muon momentum in the π^+ decay at rest, then the mass of the muon neutrino is $m_{\nu_\mu} < 0.27$ MeV/c^2 (90% confidence level).

1 The ambiguity of the mass experiment

The most precise determination of the charged pion mass derives from the crystal spectrometer experiment of Jeckelmann et al. [1] in which the 4f-3d X-ray transition in pionic ^{24}Mg was measured. The X-ray line consists, in general, of three components, depending on the number of K-electrons present during the X-ray transition (two, one or zero K-electrons, respectively). Since the electron screening differs for the three situations, the three components are shifted in energy. Two of the three components have clearly been observed in the high-resolution experiment of ref. [1]. The two components can be assigned in two different ways to the K-electron states: they correspond either to the (2, 1)-electron configuration or to the configuration (1, 0). Clearly, two different values for the pion mass result for the two assignments.

This ambiguity had been resolved in ref. [1] on the basis of a measurement of the X-ray intensity ratio I_x(4f-3d)/I_x(3d-2p). This ratio (along with the experimental information on the weak X-ray population from the non-circular orbits and the very weak Auger depopulation of the 3d state) yields the decay rate of the 4f state by Auger effect which, in turn, depends (sensitively) on the K-electron configuration. The decision of rejecting one of the two assignments was taken in ref. [1] on the basis of a 3.6 standard deviation difference for the corresponding average K-shell occupation numbers.

Very recently, pionic X-ray intensity measurements were published by Shinohara et al. [2] which appear to yield a smaller value for the above intensity ratio than the one used in ref. [1]. This would lead to an increase in the average K-shell occupation number, thus rendering the earlier decision between the two assignments somewhat marginal. In view of this new situation and the importance of the matter, we find it justified (from the point of view of our experiment), to accept **both** solutions on equal footing. We therefore give in the present work an evolution of the mass experiment of ref. [1] for **both** possibilities, the (1,0)-electron configuration (solution A) and the (2,1)-electron configuration (solution B). We use an improved wavelength calibration standard. The decision between the two mass values, based on other experiments, will be discussed in Sect. 3.

2 The wavelength (of the dominant component) of the 4f-3d transition

The 25.7 keV γ-ray wavelength standard [3] was used which was derived from the ^{170}Tm 84 keV wavelength standard of the National Institute for Standards and Technology (NIST, formerly NBS). The Si lattice spacing measurement of the NBS group was shown later to be incorrect [4]; the present recommended wavelength of the 84 keV γ-line is lower by 1.9 ppm: $\lambda_{84} = 14.715402(13)$ pm [5]. This correction leads to a new value for the wavelength of the 25.7 keV γ-line of

$$\lambda_\gamma = 48.334396(64) \text{ pm} \quad . \tag{1}$$

The same data analysis procedure used in ref. [1] for solution A was applied to solution B. We also reevaluate the wavelength λ^{1K}_{exp} for solution A (see Table 2) by taking into account the new wavelength standard (1). We recall that the data analysis involves fits to the 4f-3d X-ray (composite) line with three essential parameters. The most important one is of course the Bragg-angle position of the dominant component of the line; the second is the intensity ratio between the two most intense components of the line or, alternatively, the ratio between two electronic K-shell occupation numbers, and the third is the K-shell vacancy width (see below).

We define a_0, a_1 and a_2 to be the probabilities for having zero, one and two K-electrons at the moment of decay of the 4f state, where $a_0 + a_1 + a_2 = 1$. The X-ray transition probabilities for the three cases are

$$a_0 \;\; ; \quad a_1 \frac{\gamma_R}{\gamma_R + \frac{1}{2}\gamma_A} \;\; ; \quad a_2 \frac{\gamma_R}{\gamma_R + \gamma_A} \quad , \tag{2}$$

respectively, where γ_R denotes the pionic X-ray transition probability per sec of the 4f-level and γ_A is the Auger transition probability of the same level, with a complete electronic

K-shell (two K-electrons). The three X-ray components thus have intensity ratios

$$N_0 \; : \; N_1 \; : \; N_2 \;=\; a_0 \; : \; a_1 \, \frac{2}{2+\alpha_K} \; : \; a_2 \, \frac{1}{1+\alpha_K} \quad , \tag{3}$$

where we have introduced the K-Auger conversion coefficient [1]

$$\alpha_K \;\equiv\; \gamma_A/\gamma_R \;=\; 0.343 \quad . \tag{4}$$

Similarly, we find the Auger transition probabilities for the three channels:

$$0 \;\; ; \quad a_1 \, \frac{\alpha_K}{2+\alpha_K} \;\; ; \quad a_2 \, \frac{\alpha_K}{1+\alpha_K} \quad . \tag{5}$$

We can define an average (electronic) K-shell occupation number $\overline{a}$ through the relation

$$\gamma_A^{(tot)}/\gamma_R^{(tot)} \;=\; \overline{a}\,\alpha_K \quad , \tag{6}$$

where $\gamma_A^{(tot)}$ and $\gamma^{(tot)}$ are the total Auger and radiative decay probabilities for the 4f-3d transition. From (6), using (2), (4) and (5) we find

$$\overline{a} \;=\; \frac{a_1(1+\alpha_K) \;+\; a_2(2+\alpha_K)}{a_0(2+\alpha_K)(1+\alpha_K) \;+\; 2a_1(1+\alpha_K) \;+\; a_2(2+\alpha_K)} \quad . \tag{7}$$

In the limit $\alpha_K \rightarrow 0$ (7) reduces to [1]

$$\overline{a} \;\approx\; \frac{1}{2}a_1 \;+\; a_2 \quad . \tag{8}$$

The shape of the component lines is dominated by the measured instrumental response function of the spectrometer (FWHM: 0.73 eV). In addition, there is a contribution from the natural widths associated with the three different electronic K-states. These widths are the sum of three parts: the total width of the initial (pionic) 4f-state, the total width of the final 3d-state [2] and the width of the corresponding electronic K-shell vacancy state (the details are given in ref. [1]). The line profiles for the three

[1]Note that in ref. [1] the approximation $\alpha_K \rightarrow 0$ was used; non of the numbers are, however, significantly different from those of Table 1 which are based on the accurate expressions.

[2]The uncertainty in the strong absorption width of the 3d level can be fully neglected here; a (conservative) value of 0.01 ± 0.01 eV was assumed.

component peaks are then obtained by folding the natural widths with the instrumental response function. The widths of the K-shell vacancy states are described with the (phenomenological) width parameter γ_x which is defined as the total width of the one-K-electron-vacancy state, and the remaining part of the atom in the (unknown) state which existed during the 4f-3d transition. Thus, γ_x should vary within the interval $0 \leq \gamma_x \leq \gamma_x^{neut}$, where γ_x^{neut} (= 0.24 eV) is the value calculated for the neutral atom. The widths of the K-shell vacancy states are then 0, γ_x, and $2\gamma_x$, corresponding to two, one and zero K-electrons, respectively. From the analysis of run C (see below), an upper limit $\gamma_x < 0.17$ eV is obtained for solution B; the final data evaluation was performed with $\gamma_x = 0.10 \pm 0.08$ eV. The differences in the peak positions of the component lines was taken from the calculations by Fricke and Rashid which are based on a self-consistent Dirac-Fock method (see ref. [1]).

Fig. 1 shows the fit to the high-resolution data (run C of ref. [1]), for solution B. From this (excellent) fit the ratios of the K-shell occupation probabilities shown in Table 1 (along with those for solution A) are derived.

Table 1: Ratios of K-shell occupation probabilities and the average K-shell occupation number $\overline{a}$, eq. (7), are shown for solutions A and B. Note that $a_0 + a_1 + a_2 = 1$.

	solution A	solution B
a_0/a_1	0.33(11)	–
a_2/a_1	0.05(4)	–
a_1/a_2	–	0.56(20)
a_0/a_2	–	0.03(3)
$\overline{a}$	0.37(4)	0.79(4)

The line-shape parameters thus obtained were then used to deduce from the data of run A and B (ref. [1]) the wavelengths of the dominant components. The experimental values of Table 2 contain uncertainties from the source hight effect, non-linearity, background inclination, inhomogeneous π-stop distribution, temperature effect and the line profile (see ref. [1]).

Table 2: Wavelengths of the dominant component of the 4f-3d transition for solutions A and B (experimental values and the corrections to obtain the point-nucleus values are given) along with the deduced pion-electron mass ratios and the pion mass values.

	solution A	solution B
	$\lambda^{1K}_{exp} = 47.83813(11)$ pm	$\lambda^{2K}_{exp} = 47.83820(11)$ pm
radiative corrections and strong-interaction shift (from ref. [1])	50.487(10) eV	50.489(10) eV
electron screening [eV]	1K electron: − 0.501(5) L_1 electrons: − 0.030(30)	2K electrons: − 0.941(9) L_1 electrons: − 0.022(22)
total correction [eV]	49.956(32)	49.526(26)
$\Delta\lambda$ [pm]	0.09239(6)	0.09159(5)
λ_0 [pm]	47.93052(12)	47.92979(12)
m_π/m_e	273.12735±2.60 ppm	273.13152±2.51 ppm
m_π [MeV/c^2]	139.56782±2.62 ppm	139.56995± 2.53 ppm)

The next step in the analysis is to arrive at the point-nucleus wavelengths of the dominant components. We calculate the total corrections to the transition energy which includes all radiative corrections, the strong-interaction shift and the electron screening shifts (see Table 2). The shift due to the 2K-electrons (solution B) is $\Delta E_{2K} = -0.941$ eV, and for the shift due to one L_1-electron we take half of the difference between the shift of the neutral atom and ΔE_{2K} (see ref. [1]) which amounts to -0.022 eV. We then assume (conservatively) the presence of (1 ± 1) L_1 electrons. The total energy shift is converted into a wavelength difference $\Delta\lambda$ (Table 2) from which we obtain the point-nucleus wavelengths λ_0.

3 The value of the pion mass

From the point-nucleus wavelengths we find, as a final step, the mass values given in Table 2 (see ref. [1]: the constants are from ref. [6]). It should be stressed that, in arriving at these mass values, all the necessary information on the occupation of the electronic states is derived from the line shape experiment of ref. [1] itself.

The pion-electron mass ratios are displayed in Fig. 2, together with earlier pionic-atom mass measurements [7, 8, 9] and the lower limit for m_{π^+} from the measurement of the muon momentum in the π^+ decay at rest [10] [3]. It is evident that the earlier pionic-atom experiment of Lu et al. [7] and the measurement of the muon momentum by Daum et al. [10] are the only published results of sufficient precision which could be used to decide between solutions A and B. The result of Lu et al. is compatible with our solution A and appears to rule out solution B (by 3.7 standard deviations). On the other hand, solution B is compatible with the lower limit of Daum et al. and seems to rule out solution A (by 2.7 standard deviations (at least)). We conclude that the experiments of Lu et al. and Daum et al. are mutually conflicting.

If we combine our solution A with the result of the muon momentum measurement [10], using momentum and energy conservation in the π^+ decay and respecting the CPT-theorem, we find a negative value (by 2.8 standard deviations) for the square of the

[3]It should be stressed that this experiment can only be used to derive a **lower limit** for m_π since the mass of the muon neutrino is unknown. This is not stated explicitly in ref. [11].

muon neutrino mass: $m^2_{\nu_\mu} = -0.127 \pm 0.046$ MeV/c^2. If we use, on the other hand, our solution B, $m^2_{\nu_\mu}$ is compatible with zero and we obtain

$$m_{\nu_\mu} < 273 \text{ keV/c}^2 \quad (90\% \text{ confidence level}) \quad . \tag{9}$$

Figure Captions

Fig. 1: Bragg reflex of the 4f-3d transition in pionic ^{24}Mg; the data are from ref. [1] (R is the interferometer reading in optical units). The solid line is the fit corresponding to solution B (see text); the three components refer to the cases of having two, one and zero K-electrons present during the pionic transition. The line shape of the components is obtained by folding the instrumental response function with the natural line width of the transition. The relative peak positions of the components are derived from screening calculations.

Fig. 2: Recent experimental determinations of the pion-electron mass ratio, in chronological order: (a) present work; (b) ref. [7]; (c) ref. [8]; (d) ref. [9]; and (e) ref. [10]. The values (b), (c), (d) and (e) are those given in ref. [11] (for discussion see text).

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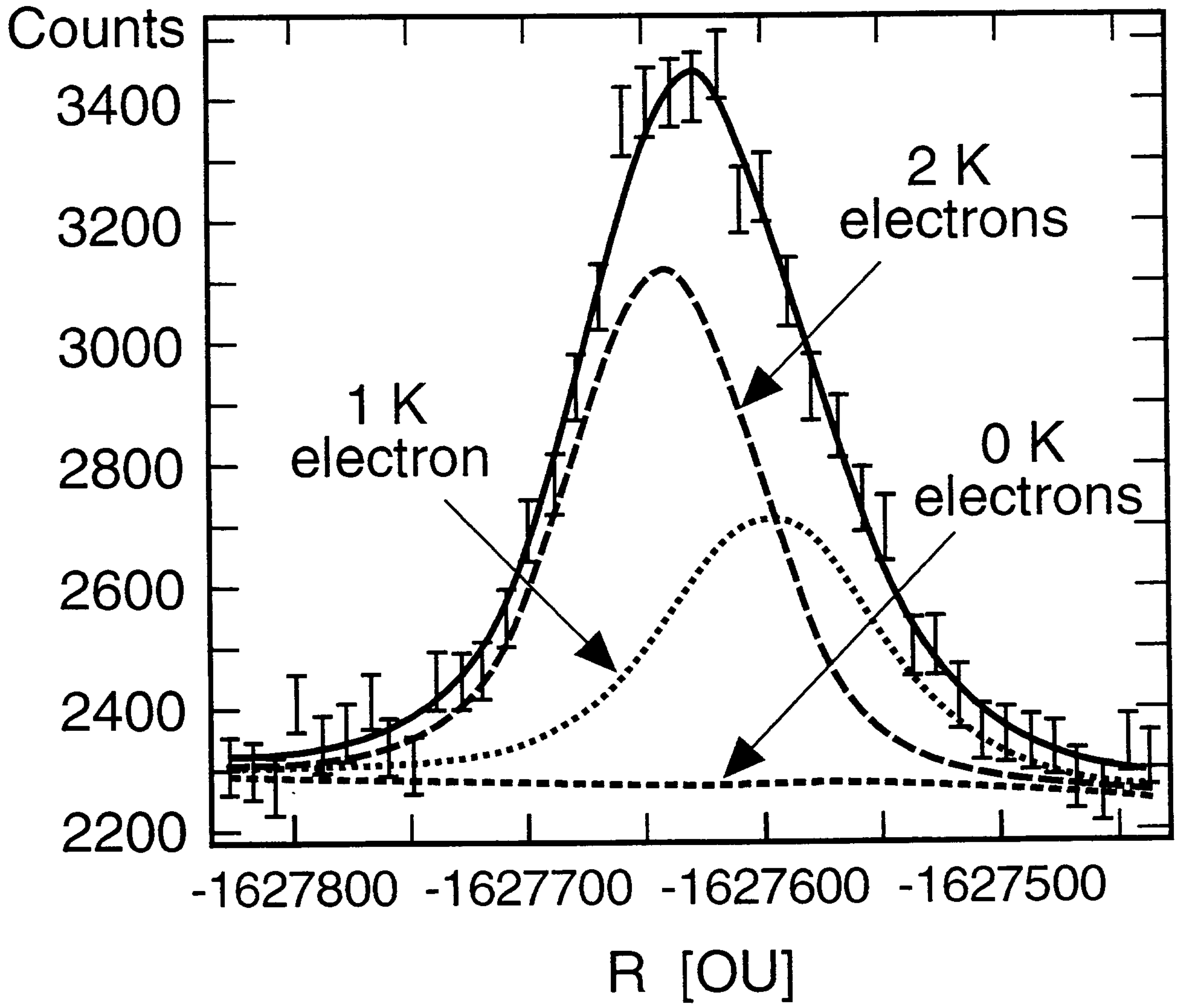

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Fig. 1

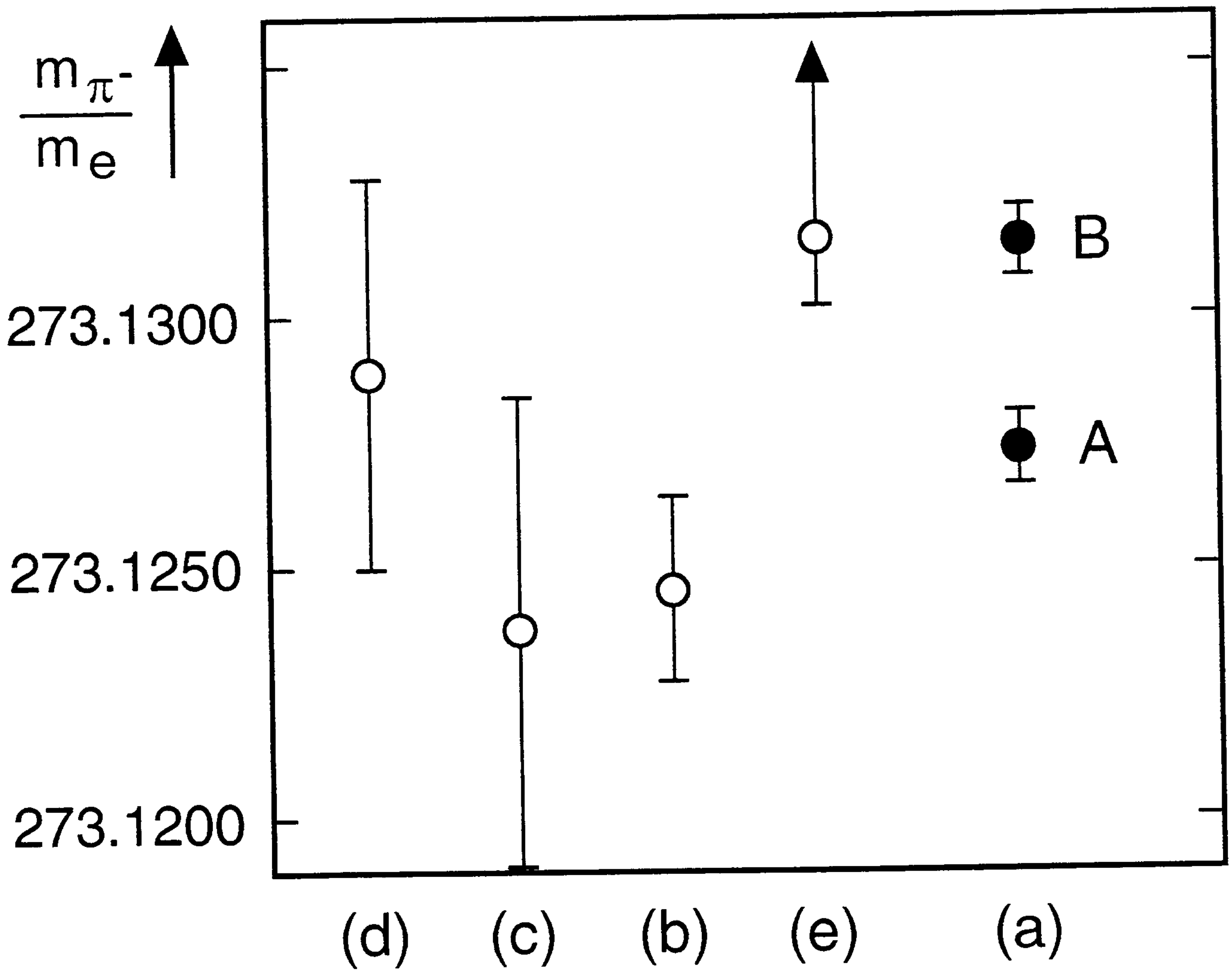

Fig. 2

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