CERN-ACC-NOTE-2018-0059

11 December 2017
Olav.Ejner.Berrig@cern.ch

Review of path-length calculations for rectangular bending magnets with arbitrary pole-face angles

O. Berrig
CERN, CH-1211 Geneva, Switzerland

Keywords: Bending Magnet, path length, pole face angle, beam trajectory, survey, GEODE

Summary

This document shows how to calculate the path-length of rectangular bending magnets in a beam line. The path-length depends on the pole-face angles and the body geometry, i.e. how the magnet is positioned in the line. The majority of bending magnets are installed with identical pole-face angles at the start and the end, but in certain cases the pole-face angles are different e.g. in the CERN PS BOOSTER BTP and BTY extraction lines, the BHZ10 magnet have a special positioning in order not to perturb the optics of any of the lines unfavorably. The path-length corresponds to the s-parameter in MADX, and must be calculated precisely, in order to get a correct survey, which needs to be correct to the 10 micron level.

Contents

1 Introduction

This note will describe how to calculate the path length of a bending magnet. Throughout this note the bending magnet is always rectangular, but is defined according to the MADX sector magnet definition SBEND (See ref. [6]). The RBEND definition , i.e. rectangular magnet definition, is not used. The reason is that a sector magnet definition can also model a rectangular magnet, so it is easier just to use the sector bending magnet definition, SBEND, always. The sector magnet definition, SBEND, is characterized by its arc length "L_{arc}", its bending angle "ϕ" and its pole-face angles "$E1$ and $E2$", (see Fig. 1).

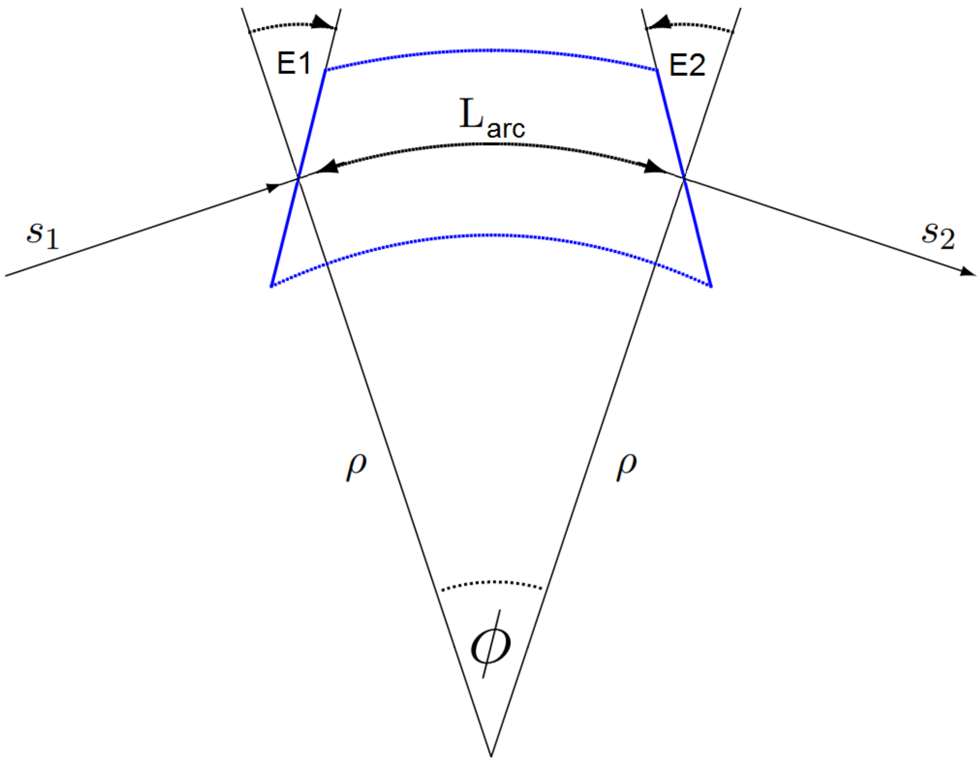

Figure 1: Standard magnet layout for a sector bending magnet

The definition of a sector magnet in MADX is the following:

$$\begin{aligned}
label :& SBEND, L = real, ANGLE = real, TILT = real,\\
& K0 = real, K1 = real, K2 = real, K1S = real,\\
& E1 = real, E2 = real,\\
& FINT = real, FINTX = real, HGAP = real, H1 = real, H2 = real, THICK = logical;
\end{aligned}$$

In the above definition of an SBEND magnet "L" is the arc length "L_{arc}", "$ANGLE$" is the bending angle "ϕ" and "E1" & "E2" are the pole-face angles.

NB! Please note that for Fig.1, the definition of a positive bending angle as well as the pole-face angles depends on the charge of the particle that moves through the bending magnet. If a positively charged particle is bent to the right, then the bending angle is positive. If a negatively charged particle is bent to the right, then the bending angle is negative.

CERN use four different definitions of magnet length. The reason is that several groups are interested in different aspects of magnet length (see Fig. 2):

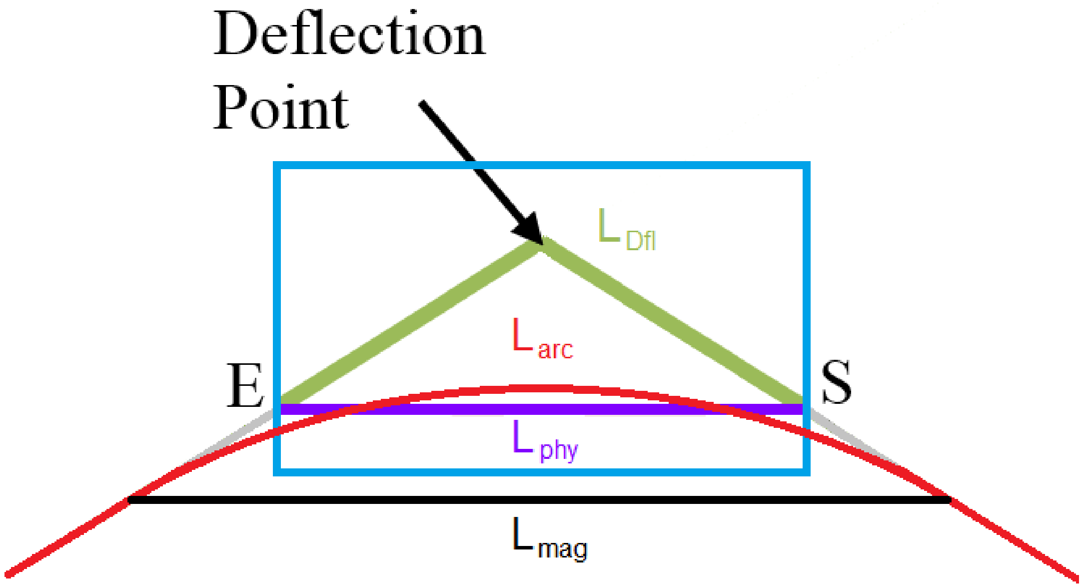

Figure 2: Definition of four different length for a bending magnet. The rectangular blue box is the physical bending magnet itself. The straight purple line from point E to point S is the physical length L_{phy} (It is used by the magnet group, the drawings group, the Machines & Experimental Facilities group and the survey group). The Red circular line is the arc length L_{arc} (It is used by the beam physics group and the Machines & Experimental Facilities group). The straight black line below is the magnetic length L_{mag} (It is used by the magnet group and the beam physics group). The two angled green straight lines from point E to point S is the length via the deflection point L_{Dfl} (It is used by the drawings group).

1. **The physical length L_{phy}.** This is the length given in drawings. It is only used for survey calculations, but never for optics calculations. The survey group defines a set of "ENTRÉE" and "SORTIE" points for each equipment to indicate the survey coordinates where the equipment starts and ends. "ENTRÉE" and "SORTIE" are the names defined in the survey database (See Ref. [2]). The "ENTRÉE" and "SORTIE" points are indicated in Fig. 2 as E and S. Because the survey group also accepts the magnetic length as a basis for survey calculations, then the physical length is basically never used. The physical length is defined as the length of the iron core.

2. **The magnetic length L_{mag}.** This is used by the beam physics group as a basis to calculate the arc length, which in turn is used by the MADX program for optics

calculations. The magnetic length is calculated from magnet measurements. For a rectangular magnet the magnetic length is based on the integral of the B-field along a straight line going through the middle of the magnet and extending over the two ends See Ref. [3]. The formula for the magnetic length is: $L_{Mag} = \frac{\int B \cdot dl}{B_{Max}}$, where B_{Max} is the maximum B field in the center of the magnet. The magnetic length is longer than the physical length because the magnetic field exceeds the iron core of the magnet.

3. **The arc length $\mathbf{L_{arc}}$**. This is used by the beam physics group for input to the MADX program. It is the length of the beam trajectory and is the length given in the SBEND command. The arc length is also called the path length as it describes the path that the particles move through the magnet. The arc length L_{arc} is extremely important for optics calculations and is equal to the increase in the s variable from the entry to the exit of the magnet.

4. **The deflection length $\mathbf{L_{Dfl}}$**. This is used by the drawings group. The drawings group define a beam line as passing through the deflection point. This makes the beam line independent of the characteristics of the bending magnet. A change of the length of a bending magnet will not change the length of the beam line. Unfortunately if there is a bending magnet on the beam line, then downstream of the bending magnet, the position coordinates of equipment on a beam line - as defined by the drawings group - are different from the position coordinates - as defined by the beam physics group. The reason is that the length of the magnet on the beam line is defined as $\mathrm{L_{Dfl}}$ by the drawings group and as $\mathrm{L_{arc}}$ by the beam physics group.

NB! Please be very careful to check whether a physical length or a magnetic length is used. Check e.g. with the NORMA magnet database. See Ref.[1]

2 How to position a straight vacuum chamber to maximize the aperture for the beam

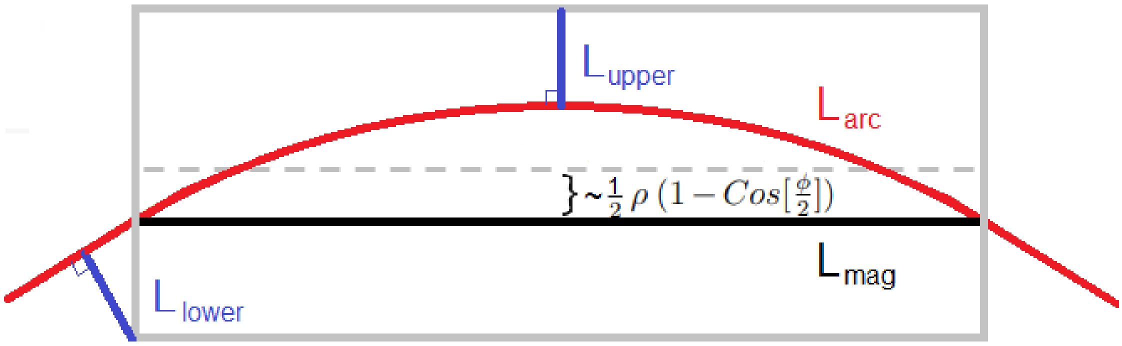

Figure 3: Calculation of vacuum chamber position. The vacuum chamber is the gray rectangle. The vacuum chamber is here positioned in the best position, so that the two blue lines L_{upper} and L_{lower} have equal lengths. The dashed grey line is the middle of the vacuum chamber.

In Fig. 3, the red line represents the beam trajectory. We will assume that the vacuum chamber is straight i.e. it is not bend around the beam. Furthermore we will also assume that the beta-function is constant in the whole region inside & around the magnet and that the vacuum chamber starts and ends at the same points as the magnetic length (i.e. the black line in Fig. 3). The two purple lines (L_{upper} and L_{lower} in Fig. 3) represent the aperture i.e. the space available for particles to oscillate around the beam trajectory. The two blue lines are both perpendicular to the beam trajectory. They both start on the beam trajectory and ends on the wall of the vacuum chamber; moving the vacuum chamber up or down will thus change the length of each of the purple lines. E.g. moving the vacuum chamber up will shorten L_{upper} and make L_{lower} longer. The two purple lines should have the same length because the oscillation amplitude of the beam is the same on both sides of the beam.

Our aim is therefore to place the vacuum chamber so that the lengths of the two purple lines are the same. We must therefore solve the equations:

$$\begin{aligned} &\text{Width of vacuum chamber:} && W = L_{upper} + \rho\,(1 - Cos[\frac{\phi}{2}]) + \frac{L_{lower}}{Cos[\frac{\phi}{2}]} \\ &\text{and:} && L_{upper} = L_{lower} \\ &\text{which gives:} && L_{upper} = L_{lower} = \frac{W + \rho\,(Cos[\frac{\phi}{2}] - 1)}{1 + \frac{1}{Cos[\frac{\phi}{2}]}} \end{aligned} \quad (1)$$

The maximum aperture is thus obtained if the vacuum chamber is moved a bit to the outside; i.e. "The middle of the vacuum chamber should be the distance $\frac{W}{2} - \frac{L_{lower}}{Cos[\frac{\phi}{2}]}$ above the L_{mag} line in Fig. 3." *If the bending angle ϕ is small, then the middle of the vacuum chamber should be approximately the distance $\frac{1}{2}\,\rho\,(1 - Cos[\frac{\phi}{2}])$ above the L_{mag} line.*

3 Three layouts for a rectangular magnet

3.1 The standard magnet layout

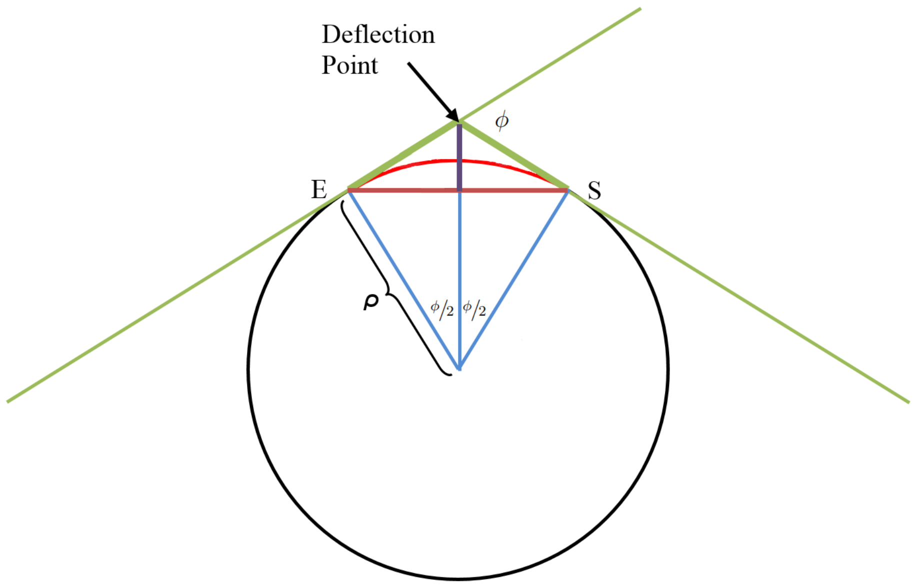

Figure 4: Calculation of path length ($= L_{arc}$) for the standard magnet layout. The red circular line from E to S is the arc length.

$$L_{arc} = \rho \cdot \phi$$

$$L_{ES} = 2 \cdot \rho \cdot Sin(\frac{\phi}{2})$$

$$L_{mag} = L_{ES}$$

$$L_{Dfl} = \frac{L_{ES}}{Cos(\frac{\phi}{2})} = 2 \cdot \rho \cdot Tan(\frac{\phi}{2}) = 2 \cdot L_{arc} \cdot \frac{Tan(\frac{\phi}{2})}{\phi}$$

where

$L_{arc} =$ *arc length (also called path length) i.e the length of the beam trajectory*

$L_{ES} = L_{mag}$. *The survey points E and S are in this drawing based on the magnetic length.*

$L_{mag} =$ *magnetic length of the rectangular bending magnet.*

$L_{Dfl} =$ *Length via the deflection point*

$\rho =$ *bending radius of the rectangular bending magnet*

$\phi =$ *bending angle of the rectangular bending magnet*

3.2 Rectangular bending magnet with zero pole-face angle at ENTRÉE

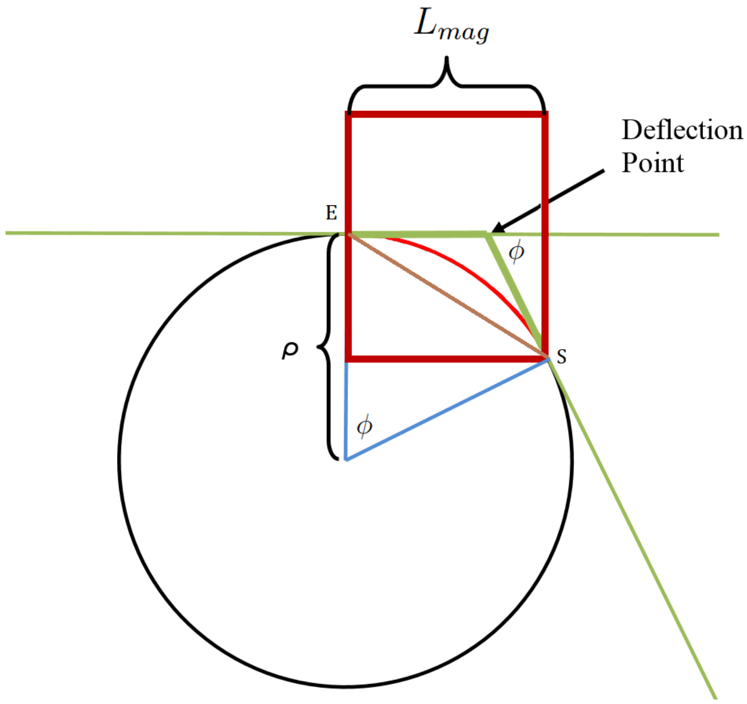

Figure 5: Calculation of path length ($= L_{arc}$) for magnet aligned with ENTRÉE. The red circular line from E to S is the arc length.

$$L_{arc} = \rho \cdot \phi$$

$$L_{ES} = \rho \cdot 2 \cdot Sin(\frac{\phi}{2})$$

$$L_{mag} = \rho \cdot Sin(\phi)$$

$$L_{Dfl} = \frac{L_{ES}}{Cos(\frac{\phi}{2})} = 2 \cdot \rho \cdot Tan(\frac{\phi}{2}) = 2 \cdot L_{arc} \cdot \frac{Tan(\frac{\phi}{2})}{\phi} = \frac{L_{mag}}{Cos(\frac{\phi}{2})^2}$$

where

L_{arc} = *arc length (also called path length) i.e the length of the beam trajectory*

L_{ES} = *straight length between ENTRÉE "E" and SORTIE "S" points.*

L_{mag} = *magnetic length of the rectangular bending magnet.*

L_{Dfl} = *length via deflection point.*

ρ = *bending radius of the rectangular bending magnet.*

ϕ = *bending angle of the rectangular bending magnet.*

3.3 Rectangular bending magnet arbitrary pole-face angles

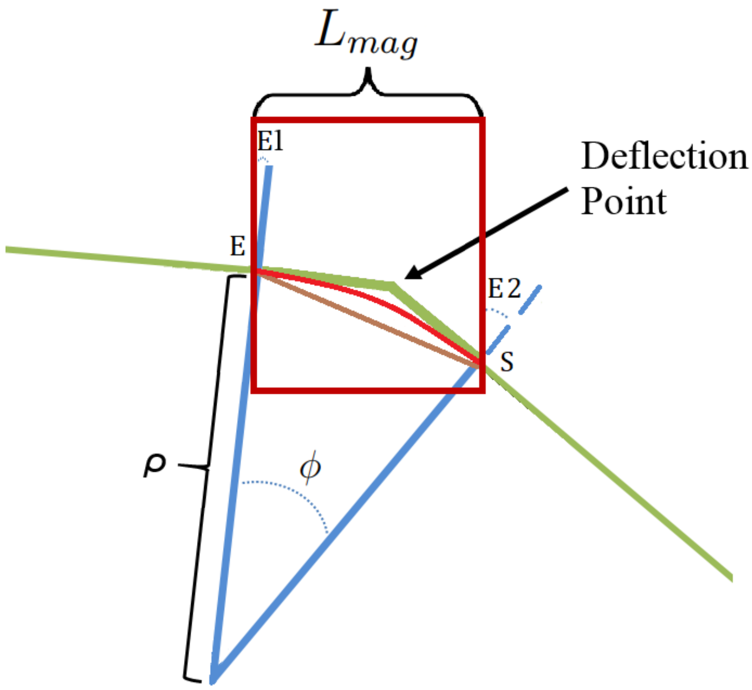

Figure 6: Calculation of path length (= L_{arc}) for magnet with arbitrary pole-face angles. The red circular line from E to S is the arc length. In the above Fig. 6 the pole-face angle at the ENTRÉE $E1$ is negative because is is on the inside of the magnet. The pole-face angle at SORTIE $E2$ is positive. From geometric considerations it can be seen that the sum of the pole-face angles is equal to the bending angle: $E1 + E2 = \phi$

$$\begin{aligned} L_{arc} &= \rho \cdot \phi \\ L_{ES} &= \rho \cdot 2 \cdot Sin(\frac{\phi}{2}) \\ L_{mag} &= \rho \cdot (Sin(E1) + Sin(E2)) \\ L_{Dfl} &= \frac{L_{ES}}{Cos(\frac{\phi}{2})} = 2 \cdot \rho \cdot Tan(\frac{\phi}{2}) = 2 \cdot L_{arc} \cdot \frac{Tan(\frac{\phi}{2})}{\phi} \end{aligned}$$

where

$$\begin{aligned} L_{arc} &= \textit{arc length (also called path length) i.e the length of the beam trajectory} \\ L_{ES} &= \textit{straight length between ENTRÉE “E” and SORTIE “S” points.} \\ L_{mag} &= \textit{magnetic length of the rectangular bending magnet.} \\ L_{Dfl} &= \textit{length via deflection point} \\ \rho &= \textit{bending radius of the rectangular bending magnet} \\ \phi &= \textit{bending angle of the rectangular bending magnet} \end{aligned}$$

3.4 Some concluding comments about survey

In order to compare positions on a drawing with the corresponding positions in the MADX sequence file, if there is a bending magnet, one must be careful to check how the drawing positions downstream of this bending magnet are calculated.

Assuming that the length of a bending magnet, on a drawing, is calculated as the length via the deflection point; then to get the MADX positions of the elements downstream the bending magnet for this drawing, one has to subtract the difference between the length via the deflection point and the arc length from all these downstream elements.

As an example, take the bending magnet LT.BHZ20 in the LT transferline between LINAC2 and the PSB (PS BOOSTER). LT.BHZ20 has a magnetic length of 1.045 m and a bending angle of 0.279226 rad.

The length via the deflection point is 1.055268 m (=1.045/Cos[0.279226/2])

while the length via the arc is 1.048403 m (=1.045 · (0.279226/2)/Sin[0.279226/2]).
The difference is 6.865 mm, which is a large distance in terms of survey. One will therefore not find identical values of the positions in the drawing (Ref. [4]) and the MADX file (Ref. [5]), even though the the two sources describe the exact same information.

References

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[6] Laurent Deniau; Hans Grote; Ghislain Roy; Frank Schmidt; Iselin F.C.; Herr W. and many other contributors. MADX. http://mad.web.cern.ch/mad/.