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A REVIEW ON MESONIC DECAY OF Λ HYPERNUCLEI

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Abstract

A review of the present situation of the mesonic decay of Λ hypernuclei is done. The link between the propagator method and the one with wave functions and nuclear matrix elements is established. The lack of links between the mesonic decay and the nucleon occupation number in nuclear matter is also discussed, as well as the effect of the ΛN short range repulsion in the mesonic decay of light hypernuclei. The relevance of the $2p2h$ induced Λ decay channel is also discussed. Finally an overview of the potential use of the process, when systematic measurements over the periodic table are done, is presented at the end.

Contents.

1 A brief historical introduction.

A paper devoted to the memory of our friend Hiroharu Bandō is the right place to recall some interesting events associated with him and the subject of this paper. In 1984 L.L. Salcedo was working in Valladolid for his Ph. D. on the decay of Λ hypernuclei and he surprised me (E.O.) with the results which indicated that the rate for pionic decay of heavy Λ hypernuclei was increased by about two orders of magnitude when a proper medium renormalization of the pion was done. At that time I was still not used to shake every one of the few times that Salcedo would open his mouth, so I told him to check again his program. He did so and came out with the same answer. This time I checked it myself and had to surrender to a reality that was difficult to swallow and worst, I had the feeling it would be difficult to sell. A paper was written and after a few useful corrections suggested by the referee the paper was published [1]. I should thank the generosity of the referee who, probably without believing the results, let it be published. I say that a posteriori because when my selling mission began I could not convince a single colleague of the soundness of the results. To make it short I recall the lapidary sentence of A. Gal: "I do not believe in any renormalization factor of 100".

With this predisposition from my colleagues I presented these results in the hypernuclear Conference at Brookhaven in 1985 [2]. I went there with clarifying and convincing arguments, with Feynman diagrams, with poles and cuts in the complex plane and Cutkosky rules to separate imaginary parts, and all this heavy artillery that should have crumbled the strongest walls. The result: a lost battle. The whole audience turned against me. Even my friends Gerry Brown and Torlif Ericson showed disbelief in their questions and comments at the end of the talk. But it was Torlif the one who opened my mind to what was happening: I was using an inappropriate language for that audience which was more used to the language of wave functions and matrix elements than to the one of propagators, selfenergies and cuts which I was using. And here came Bandō. I went to him and discussed with him. He was using this alternative language and he should be able to prove the same results using pionic wave functions in the nucleus and evaluating the proper matrix elements. He was the first person to take us seriously and he started to work with his colleagues Motoba and Itonaga on the issue. In ref. [2], after the experience with the audience and the discussions with Torlif, I wrote a section on "an alternative approach" sketching the way to follow using the language of wave functions and matrix elements, which was the one followed by Bandō and collaborators.

Their work has been very useful [3, 4, 5]. It not only confirmed the huge enhancement of the mesonic width found in [1], but produced detailed and quantitative results in many nuclei, taking into account shell effects, Q values and other details which go far beyond the nuclear matter and local density approach of ref. [1]. Since then the spectacular enhancement of the mesonic width has been universally accepted. Their predictions have been confirmed

by the new wave of experiments [6, 10].

I came to appreciate Bandō more with this incident. His quiet, flexible and gentle approach to the problems contrasted with my stern and temperamental one, and certainly proved to be much more efficient.

One of the interesting outputs of [3, 4, 5] is that the mesonic width is quite sensitive to the pion nucleus optical potential, for potentials which fit equally well the pion nucleus scattering data. This information is very useful and should serve as a check for different microscopic models of the pion nucleus interaction. The work done on the mesonic decay and its interconnection with the other Λ decay channels has also been essential to get a unified picture of the Λ decay in nuclei and has generated other interesting developments as we shall see below.

2 Formal derivation of the Λ width in nuclei. The propagator method.

The starting point is the $\Lambda \longrightarrow \pi N$ Lagrangian, accounting for this weak process, which is given by

$$L_{\pi\Lambda N} = G\mu^2 \bar{\psi}_N (A - B\gamma_5)\vec{\tau}\cdot\vec{\phi}_\pi \psi_\Lambda + h.c. \tag{1}$$

with

$$(G\mu^2)^2/8\pi = 1.94510^{-15}\,, \quad A = 1.06\,, \quad B = 7.10 \tag{2}$$

In eq. (1) the Λ is assumed to behave as the state $|1/2, -1/2\rangle$ of an isospin doublet with $T = 1/2$ and this imposes the $\Delta T = 1/2$ rule, which has as a consequence a strength double for the $\Lambda \longrightarrow \pi^- p$ channel than for the $\Lambda \longrightarrow \pi^0 n$ one. In eq. (1) the term A violates parity and the term B conserves it.

A practical way to evaluate the Λ width in nuclear matter and introduce the medium corrections is to start from the Λ selfenergy, Σ, associated to the diagram of fig. 1 and then use the relationship

$$\Gamma = -2\, Im\, \Sigma \tag{3}$$

The selfenergy is readily evaluated as

$$-i\Sigma(k) = 3(G\mu^2)^2 \int \frac{d^4q}{(2\pi)^4} G(k-q)D(q)[S^2 + (\frac{P}{\mu})^2 \vec{q}^{\,2}] \tag{4}$$

where G and D are the nucleon and pion propagators respectively and $P/\mu = B/2M$ with μ, M the pion and nucleon mass. By using the free nucleon and pion propagators, and making the typical nonrelativistic approximation $M/E = 1$, one obtains immediately the free Λ width [1, 11]

$$\Gamma_{free} \equiv \Gamma_\Lambda = 3(G\mu^2)^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega(\vec{q})} 2\pi\delta(E_\Lambda - \omega(\vec{q}) - E_N(\vec{k}-\vec{q}))$$

$$\times [S^2 + (\frac{P}{\mu})^2 \vec{q}^{\,2}] \tag{5}$$

In a Fermi sea of nucleons, both the nucleon and pion propagators are changed

$$G(p) = \frac{1 - n(\vec{p})}{p^0 - E(\vec{p}) - V_N + i\epsilon} + \frac{n(\vec{p})}{p^0 - E(\vec{p}) - V_N - i\epsilon} \tag{6}$$

$$D(q) = \frac{1}{q^{02} - \vec{q}^{\,2} - \mu^2 - \Pi(q^0, q)} \tag{7}$$

where V_N is the nucleon potential, $\Pi(q^0, q)$ is the pion selfenergy in the nuclear medium and $n(\vec{p})$ is the occupation number in the Fermi sea, $n(\vec{p}) = 1$ for $|\vec{p}| \leq k_F, n(\vec{p}) = 0$ for $|\vec{p}| > k_F$, with k_F the Fermi momentum. The practical way to perform the q^0 integral in eq.(4) is to perform a Wick rotation as shown in fig. 2, where the analytical structure of the integrand is shown. The shaded region accounts for the discontinuity of the pion propagator due to $Im\,\Pi$. The pole at $\tilde{\omega}(\vec{q})$ would correspond to a renormalized pion pole where

$$\tilde{\omega}(\vec{q})^2 - \vec{q}^{\,2} - \mu^2 - \Pi(\tilde{\omega}(\vec{q}), q) = 0 \tag{8}$$

Missing in the figure is the pole of $G(k-q)$ corresponding to the second term in (6). This pole lies in the lower halfplane of the figure and would contribute in the Wick rotation only when it happens to be in the third quadrant, i.e., $k^0 - E(\vec{k} - \vec{q}) - V_N < 0$. But this corresponds to $(\vec{k} - \vec{q})$ very large where $n(\vec{k} - \vec{q}) = 0$ and hence this term does not contribute. Thus we obtain for the width [1, 11]

$$\Gamma(k) = -6(G\mu^2)^2 \int \frac{d^3q}{(2\pi)^3} [1 - n(\vec{k} - \vec{q})] \theta(k^0 - E(\vec{k} - \vec{q}) - V_N)$$

$$\times [S^2 + (\frac{P}{\mu})^2 \vec{q}^{\,2}]\, Im \frac{1}{q^{02} - \vec{q}^{\,2} - \mu^2 - \Pi(q^0, q)} \Big|_{q^0 = k^0 - E(\vec{k}-\vec{q}) - V_N} \tag{9}$$

In the discussion here we neglect the role of correlations and form factors, which are obviously important and are treated in all detail in [1, 11]. The simplified formalism will allow us to concentrate on the qualitative aspects of the reaction.

In the first place we observe the Pauli blocking factor, $1 - n$, in eq. (9). Since a Λ with $k = 0$ decays into a nucleon and pion with $q \simeq 100\, MeV/c$, this momentum is smaller than the Fermi momentum for nuclear matter density,

$k_F = 270\, MeV/c$, and the decay is forbidden by Pauli blocking, i.e., $1 - n(\vec{k} - \vec{q}) = 0$. The overlap of the Λ wave function with the nuclear surface in finite nuclei still allows the Λ decay since at some radius the local Fermi momentum will be smaller than $100\ MeV/c$, and also because the momentum distribution of the Λ wave function helps a bit in allowing some nucleon momenta in the decay. Nevertheless the Λ mesonic width decreases drastically as a function of the mass number.

The language of propagators which we have used here is the most appropriate in order to provide a unified picture of the Λ nuclear decay. Indeed, eq. (9) contains not only the modified mesonic channel but also the nonmesonic one. This can be seen diagrammatically by expanding the pion propagator and taking a ph and Δh excitation to account for the pion selfenergy, Π. This is depicted in fig. 3. The imaginary part of a selfenergy diagram is obtained when the set of intermediates states cut by a horizontal line are placed simultaneously on shell in the intermediate integration. In fig. 3 we observe a source corresponding to placing on shell a nucleon and the ph of the pion selfenergy. This corresponds to a channel where there are no pions and only nucleons in the final state. The physical process which has occurred is $\Lambda N \rightarrow NN$ and this is the nonmesonic channel. Technically it would be obtained by substituting in eq. (9)

$$Im \frac{1}{q^{02} - \vec{q}^{\,2} - \mu^2 - \Pi} \longrightarrow \frac{Im\,\Pi_{ph}}{|q^{02} - \vec{q}^{\,2} - \mu^2 - \Pi|^2} \tag{10}$$

where Π_{ph} is the pion selfenergy due to the $1p1h$ excitation. There is no overlap between $Im\,\Pi_{ph}(q^0, q)$ and the pion pole in the propagator of eq. (9) and thus the separation is clear.

The mesonic channel would correspond to a different cut, the one where the N and the π are placed on shell. This is shown diagrammatically in fig. 4. The terms in fig. 4b, and further iterations contained in (9), lead to a renormalization of the mesonic width, and an appreciable one, as it was shown in ref [1].

Technically the mesonic width can be calculated from the total width, eq. (9), subtracting the nonmesonic width, or equivalently by obtaining the pion pole contribution in eq. (4) from the renormalized pion pole given in eq. (8).

The qualitative reason on why the mesonic width is so drastically changed is given in [1, 11]: The attractive character of the pion selfenergy leads to a larger pion momentum for the same pion energy and thus, to a larger nucleon momentum by momentum conservation. Thus, the nucleon has more chances to have a momentum bigger than the Fermi momentum, therefore increasing the mesonic width.

The width in finite nuclei is obtained in [1] via the local density approximation

$$\Gamma = \int d^3r |\phi_\Lambda(\vec{r})|^2 \Gamma(k, \rho(\vec{r})) \tag{11}$$

where ϕ_Λ is the Λ wave function. A further average over the momentum distribution of the Λ wave function is also done in [1].

3 Finite nuclei approach to the mesonic width. The wave function method.

This approach was sketched in [2] and carried out in detail in [3]. The mesonic width is given, in analogy to eq. (5) by

$$\Gamma^{(\alpha)} = \frac{1}{2} C^{(\alpha)} (G\mu^2)^2 \sum_{N \notin F} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega(\vec{q})} 2\pi\delta(E_\Lambda - \omega(\vec{q}) - E_N)$$
$$\times \{ S^2 \left| \int d^3x \phi_\Lambda(\vec{x}) \phi_\pi^{(-)}(\vec{q},\vec{x})^* \phi_N^*(\vec{x}) \right|^2$$
$$+ (\frac{P}{\mu})^2 \left| \int d^3x \phi_\Lambda(\vec{x}) \vec{\nabla} \phi_\pi^{(-)}(\vec{q},\vec{x})^* \phi_N^*(\vec{x}) \right|^2 \} \qquad (12)$$

where ϕ_N is the wave function of the nucleon states and $\phi_\pi^{(-)*}$ corresponds to an outgoing solution of the Klein Gordon equation normalized to a plane wave asymptotically ($e^{-i\vec{q}\vec{r}}$). The index α stands now for $\pi^- p$ or $\pi^0 n$ decay, with $C^{(p)} = 4, C^{(n)} = 2$, which one separates here since due to shell effects these channels can depart drastically from the elementary $\Delta T = 1/2$ rule.

The sum in eq. (12) runs only over non occupied nucleon states in the shell model. On the other hand the effects of using for $\phi_\pi^{(-)*}$ a solution of the Klein Gordon equation with a proper optical potential (or pion selfenergy, $\Pi = 2\omega V_{opt}$, V_c Coulomb potential), i.e.,

$$[-\vec{\nabla}^2 + \mu^2 + 2\omega V_{opt}(\vec{x})] \phi_\pi^{(-)}(\vec{q},\vec{x})^* = [\omega - V_c(\vec{x})]^2 \phi_\pi^{(-)}(\vec{q},\vec{x})^* \qquad (13)$$

[Binstead of a plane wave are rather drastic and increase the mesonic width in about two orders of magnitude in heavy nuclei [3, 5], in qualitative agreement with the nuclear matter results of ref. [1].

The arguments for the renormalization are expressed now in the alternative language as follows: the attraction caused by the pion selfenergy increases the pion momenta in the pion wave function. As a consequence the matrix element of the Λ wave function (in a $1s_{1/2}$ ground state of the Λ nucleus potential) and the nucleon wave function is considerably enhanced. Note that if the Λ and N potentials were the same, the Λ and $N \notin F$ states are orthogonal and the matrix elements of (12) would be zero for $q = 0$. The matrix elements thus necessarily increase with $\vec{q}$, for the moderately small values of $\vec{q}$ involved in the present process. In the two languages the physical consequences are the same: an increased probability of reaching the unoccupied states and thus an enhancement of the mesonic width.

4 Equivalence of the propagator and wave function methods.

The discussion above has shown that the physical and numerical results of the pion renormalization are the same. Yet, technically the two approaches look different. In this section we establish the equivalence of the two methods and the approximations implicit in them.

Let us start from the pion propagator in finite nuclei written in coordinate space

$$D(\vec{x},\vec{y},\omega) = \sum_n \frac{\varphi_n(\vec{x})\varphi_n^*(\vec{y})}{\omega^2 - \epsilon_n^2 + i\eta} \qquad (14)$$

where $\varphi_n(\vec{x})$ are the pion wave functions in the nucleus and ϵ_n their corresponding energies. Ignoring pionic bound states, which do not play a role in our problem, we can identify the pionic wave functions by the asymptotic momentum $\vec{q}$. Hence their energy is given by $\omega(\vec{q}) = (\vec{q}^2 + \mu^2)^{1/2}$. The sum over the index n is then replaced by an integral over $\vec{q}$ as given below

$$D_\pi(\vec{x}_1,\vec{x}_2,E_\pi) = \int \frac{d^3q}{(2\pi)^3} \frac{\phi_\pi(\vec{q},\vec{x}_1)\phi_\pi^*(\vec{q},\vec{x}_2)}{E_\pi^2 - \omega(\vec{q})^2 + i\eta} \qquad (15)$$

For simplicity in the derivation we shall take the s-wave part of the width (the one providing the largest contribution) and will not distinguish between π^0 or π^- decay. Hence, from eq. (12) we obtain

$$\Gamma_S = 3(G\mu^2)^2 S^2 \sum_{N \notin F} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega(\vec{q})} 2\pi\delta(E_\Lambda - E_N - \omega(\vec{q}))$$
$$\times \left| \int d^3x \phi_\Lambda(\vec{x}) \phi_\pi^*(\vec{q},\vec{x}) \phi_N^*(\vec{x}) \right|^2 \qquad (16)$$

which can be rewritten as

$$\Gamma_S = 3(G\mu^2)^2 S^2 \sum_{N \notin F} \int d^3x_1 \int d^3x_2 \phi_\Lambda^*(\vec{x}_1)\phi_\Lambda(\vec{x}_2)$$
$$\times \phi_N(\vec{x}_1)\phi_N^*(\vec{x}_2) \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega(\vec{q})} 2\pi\delta(E_\Lambda - E_N - \omega(\vec{q}))$$
$$\phi_\pi(\vec{q},\vec{x}_1)\phi_\pi^*(\vec{q},\vec{x}_2) \qquad (17)$$

or by virtue of eq. (15) as

$$\Gamma_S = 3(G\mu^2)^2 S^2 \sum_{N \notin F} \int d^3x_1 d^3x_2 \phi_\Lambda^*(\vec{x}_1)\phi_N(\vec{x}_1)(-2)$$
$$\times Im\, D_\pi(\vec{x}_1,\vec{x}_2; E_\pi = E_\Lambda - E_N)\theta(E_\Lambda - E_N)\phi_\Lambda(\vec{x}_2)\phi_N^*(\vec{x}_2) \qquad (18)$$

Now, in order to connect with eqs. (9) and (11) one makes a local density approximation. In the first step one evaluates Γ for a slab of infinite nuclear matter and in the second step one replaces the width in the infinite slab by an integral over the nuclear volume assuming slabs of matter in each d^3r of the nucleus with local density $\rho(\vec{r})$ and with a probability of finding the Λ particle given by $|\phi_\Lambda(\vec{r})|^2$. This last step is implemented by means of eq. (11). Hence we should see how we reproduce now eq. (9) when we assume in eq. (18) a slab of infinite nuclear matter. For this purpose we have to substitute for the nucleon sector

$$N \to \vec{p}$$

$$\sum_{N \notin F} \to V \int \frac{d^3p}{(2\pi)^3}(1 - n(\vec{p}))$$

$$\phi_N(\vec{x}) \to \frac{1}{\sqrt{V}} e^{i\vec{p}\vec{x}}$$

$$E_N \to E(\vec{p}) - V_N \tag{19}$$

and for the Λ wave function

$$\phi_\Lambda(\vec{x}) \to \frac{1}{\sqrt{V}} e^{i\vec{k}\vec{x}} \tag{20}$$

Now in the infinite slab of nuclear matter the pion propagator of eq. (15) is substituted by

$$D_\pi(\vec{x}_1, \vec{x}_2, E_\pi) \to \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}(\vec{x}_1 - \vec{x}_2)}}{E_\pi^2 - \omega(\vec{q})^2 - \Pi(E_\pi, \vec{q})} \tag{21}$$

where $\Pi(E_\pi, \vec{q})$ is the pion selfenergy, which is a function of ρ. Note that for values of $\vec{x}_1, \vec{x}_2$ far away from the nucleus eqs. (15) and (21) are equivalent since there $\rho = 0$ and Π (in the local density approximation) will be zero. At other densities, Π will be different of zero and the integral of eq. (21) gives rise to other momentum components, modulating the plane wave of the numerator and providing a kind of WKB approximation to the wave functions of the numerator of eq. (15). The local density approximation gives rise to a variable local momentum and hence a distorted pion wave.

By substituting eqs. (19), (20), (21) in eq. (18) we obtain:

$$\Gamma_S = -6(G\mu^2)^2 S^2 \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} (1 - n(\vec{p}))\, Im\, D_\pi(q) \theta(q^0)|_{q^0 = E_\Lambda - E(\vec{k} - \vec{q}) - V_N}$$

$$\times \int d^3x_1 d^3x_2 \frac{1}{V} e^{i\vec{p}(\vec{x}_1 - \vec{x}_2)} e^{i\vec{q}(\vec{x}_1 - \vec{x}_2)} e^{-i\vec{k}(\vec{x}_1 - \vec{x}_2)} \tag{22}$$

with $D_\pi(q)$ given by eq. (7). Finally, by means of the relationship $(2\pi)^3\delta^3(0) = \int d^3x = V$ we can cast eq. (22) as

$$\Gamma_S = -6(G\mu^2)^2 S^2 \int \frac{d^3q}{(2\pi)^3} (1 - n(\vec{k} - \vec{q}))\, Im\, D_\pi(q) \theta(q^0)|_{q^0 = E_\Lambda - E(\vec{k} - \vec{q}) - V_N} \tag{23}$$

which coincides with the s-wave contribution to Γ from eq. (9). This establishes the equivalence between the two methods within the local density approximation which we have done in the case of the propagator method.

5 The mesonic width and the occupation number.

We have seen that Pauli blocking is the major factor in the small mesonic width of heavy Λ hypernuclei. It was suggested that because real interacting nuclei have the "occupied" states partly unoccupied, the mesonic width should be enhanced with respect to a calculation with fully occupied Fermi levels [14]. In the nuclear matter approach of section 2 this is easily visualized by recalling a realistic picture of the occupation number of the Fermi sea [15], which is depicted in fig. 5. For the states below the Fermi energy the level of occupancy is of the order of 85% and by assuming that in the Λ decay the nucleons can occupy the 15% vacancy of these states we would guess that the mesonic width would stabilize at the level of about 10% of the free width for heavy nuclei (taking into account pion absorption in the way out of the pions). If this were the case the mesonic width could serve as a measure of the occupation number in the Fermi sea. The argument is very appealing and intuitive, however, it is incorrect and leads to an overestimate of the width in about three orders of magnitude in heavy nuclei.

The detailed discussion of this problem was done in ref. [16]. The fallacy in the argumentation lies in the fact that

$$\frac{1 - n_1(\vec{k})}{k^0 - E(\vec{k}) + i\epsilon} + \frac{n_1(\vec{k})}{k^0 - E(\vec{k}) - i\epsilon} \tag{24}$$

where $n_1(\vec{k})$ is the realistic occupation number in nuclear matter, is not an improvement over the propagator in eq. (6). The realistic N propagator for an interacting Fermi sea is given in terms of the spectral functions by

$$G(k^0, k) = \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, k)}{k^0 - \omega - i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, k)}{k^0 - \omega + i\epsilon} \tag{25}$$

with μ the chemical potential.

When performing the calculations of the mesonic width with this N propagator one obtains the factor

$$\int_\mu^\infty d\omega S_p(\omega, \vec{k}-\vec{q}) 2\pi\delta(k^0 - \omega - \omega(\vec{q})) \qquad (26)$$

replacing the factor

$$[1 - n(\vec{k}-\vec{q})] 2\pi\delta(k^0 - E(\vec{k}-\vec{q}) - \omega(\vec{q})) \qquad (27)$$

in eq. (5), when the Pauli blocking factor of eq. (9) is implemented. Eqs. (26), (27) bare some intuitive resemblance because

$$\int_\mu^\infty d\omega S_p(\omega, \vec{k}-\vec{q}) = 1 - \int_{-\infty}^{\mu} d\omega S_h(\omega, \vec{k}-\vec{q}) = 1 - n_1(\vec{k}-\vec{q}) \qquad (28)$$

However, in the presence of the δ function of eq. (26), the integral of eq. (28) cannot be factored out because the δ function in eq. (26) has ω in the argument. Furthermore because of restrictions of the phase space (energy and momentum conservation) the range of values of ω allowed are very small compared to the range (μ, ∞) needed in eq. (28) to obtain $1 - n_1(\vec{k}-\vec{q})$ of the interacting Fermi sea. In physical terms we can interpret it in the following way: the occupation number $n_1(\vec{k}-\vec{q})$ is an integral for all the energies of the nucleon, ω, of the probability of finding a nucleon with momentum $\vec{k}-\vec{q}$ and an energy ω, which is given by the spectral function $S_h(\omega, \vec{k}-\vec{q})$. However, in a physical decay process we have conservation of energy and momentum and hence there are severe restrictions to the values of the energies that the nucleon can have. This is why the occupation number $n_1(\vec{k}-\vec{q})$ cannot be factored out.

The actual calculations carried out in ref. [16] showed that for light and medium nuclei the use of the spectral representation for the nucleon propagator, eq. (25), instead of the one of the noninteracting Fermi sea, eq. (6), has negligible consequences in the mesonic width (of the order of 6% corrections in ^{16}O). The corrections can be of the order of 50% in heavy nuclei, but in all cases, when the pionic renormalization is taken into account, one can disregard these effects.

These findings have been of relevance in showing similar problems in the study of other physical processes, like in the contribution of the pion cloud to K^+ nucleus scattering where one can show [17] that one cannot relate the effect of the pion cloud to the pion excess number in the nucleus as assumed in refs. [18, 19].

6 Results for the mesonic width.

In refs. [3, 4, 5] one can find abundant results in different nuclei which are rather realistic. These results have been recently improved [13] by a more accurate description of the energy balance in the particular reactions, taking into account transitions to the bound and continuum N states and using a

pion nucleus optical potential which has been derived theoretically and leads to a good description of the data of pionic atoms and to elastic, reaction and absorption cross sections in the scattering processes [20]. The potential allows the separation of its imaginary part into two terms related to pion absorption and quasielastic scattering. In [13] the pion quasielastic events are not removed from the pion flux, as it corresponds to the actual experimental observation, while the use of a full distortion of the pion with the total optical potential, as done in [3, 4, 5], inevitably removes the pion quasielastic events, together with the pion absorption events. Though conceptually important, this refinement turns out to be of little practical relevance in the present problem given the small energy that the pions carry and the very small phase space for quasielastic collisions [13]. However, other considerations, particularly the energy balance in the reactions makes the widths in heavy nuclei for π^--decay about one order of magnitude smaller than those of ref. [5].

In fig. 6 we show the prediction of ref. [13] for different nuclei and for π^0 and π^- decay, with plane waves and the renormalized pion wave function. The drastic effects of the pion renormalization are seen there and are a bit smaller than in former works because the energy balance makes the pions come out with smaller energies than in the previous approaches and the attractive effects of the p-wave part of the optical potential are then diminished.

Of particular relevance are the results in $^{12}_{\Lambda}C$. One obtains the following

	$\Gamma_{\pi^0}/\Gamma_\Lambda$	$\Gamma_{\pi^-}/\Gamma_\Lambda$	$\Gamma_{\pi^0}/\Gamma_{\pi^-}$
[13]	0.159	0.086	1.86
[5]	0.13	0.098	1.32
exp	0.217 ± 0.084[10]	$0.052 \pm^{0.063}_{0.035}$	

Although with large errors the experimental results confirm these striking theoretical predictions which show a large violation of the $\Delta T = 1/2$ in nuclei ($\Gamma_{\pi^0}/\Gamma_{\pi^-}$ should be 0.5 under this rule) due mostly to nuclear shell effects.

Another interesting finding is seen in very light nuclei. The mesonic width of $^{5}_{\Lambda}He$ has attracted particular attention. There, in addition to the pion renormalization, the repulsive character of the ΛN interaction and the relatively weaker medium range attraction, compared to the NN interaction, has as an effect the pushing of the Λ to the surface of the nucleus, weakening the Pauli blocking effect and thus enhancing the mesonic decay [21, 22]. The experimental numbers clearly favour potentials with a repulsive ΛN core. One should note that such a repulsion automatically appears in quark based models of the ΛN interaction. A recent study of the $^{5}_{\Lambda}He$ decay using a quark model based hypernuclear wave function [23] leads to the following results

	$\Gamma_{\pi^-}/\Gamma_\Lambda$	$\Gamma_{\pi^0}/\Gamma_\Lambda$	$\Gamma_{tot}/\Gamma_\Lambda$
[23]	0.431	0.239	0.670
exp	0.44 ± 0.11 [9]	0.18 ± 0.20 [9]	$0.59 \pm^{0.44}_{0.31}$

These theoretical results are also in good agreement with those of ref. [24] when a Λ wave function from the modified YNG ΛN interaction of ref. [25], which has a strong repulsion at short distances, is used.

7 The $2p2h$ induced decay around the pion branch.

One of the interesting findings concerning the pionic decay was done in ref. [26]. The idea of this work, expressed in a different way, is the following: A real pion in a nuclear medium has a large width because of the coupling to $2p2h$ components which lead to pion absorption. This means that the strength of the pion is spread in a wide region, unlike a free pion which has all its strength accumulated in one point (one energy for a certain value of q). The decay leading to the emission of one pion is drastically reduced in nuclei because of Pauli blocking. However, one extreme of the pion distribution in the nucleus could be saved from Pauli blocking, because we can have a smaller energy for the pion and correspondingly more energy for the nucleon, and thus this part of the nuclear pion spectrum could participate in the Λ decay. Technically we could say that the strength of a free pion, which is accumulated in a δ function, becomes now a Breit Wigner distribution and part of the tail will correspond to a Pauli unblocked situation. Since the width of the Breit Wigner distribution at low pion energies is mostly due to pion absorption through $2p2h$ emission, the new mode would be observed as three particle emission from $\Lambda NN \longrightarrow NNN$. This is depicted in fig. 7.

In order to see this analytically we go back to eq. (9). $\Gamma(k)$ is related to $Im\, D(q)$. Assume we have

$$\Pi(q^0, q) = \Pi_{ph} + \Pi_{\Delta h} \tag{29}$$

as done in [1]. Then

$$Im\, D(q) = \frac{Im\, \Pi_{ph} + Im\, \Pi_{\Delta h}}{|q^{02} - \vec{q}^{\,2} - \mu^2 - \Pi|^2} \tag{30}$$

Around the pion pole, when the denominator in (30) vanishes, $Im\, \Pi_{\Delta h}$ is extremely small because there is little phase space for pion quasielastic collisions and the Δ is far off shell. In addition, there $Im\, \Pi_{ph} = 0$ because a real pion cannot be absorbed by just one nucleon in nuclear matter. As a consequence we have a δ like distribution which corresponds to a pion in the medium, renormalized by a real pion selfenergy $Re\, \Pi \simeq \Pi_{ph} + \Pi_{\Delta h}$. Now, if in addition we consider the $2p2h$ part of the pion selfenergy leading to pion absorption we would have

$$\Pi(q^0, q) = \Pi_{ph} + \Pi_{\Delta h} + \Pi_{2p2h} \tag{31}$$

and now $Im\, \Pi_{2p2h} \neq 0$ for (q^0, q) close to on shell pions. As a consequence we will have around the pion pole the following strength of the pion propagator

$$Im\, D(q) \simeq \frac{Im\, \Pi_{2p2h}}{(q^{02} - \vec{q}^{\,2} - \mu^2 - \Pi_{ph} - \Pi_{\Delta h} - Re\, \Pi_{2p2h})^2 + (Im\, \Pi_{2p2h})^2} \tag{32}$$

This is like a Breit-Wigner distribution in q^0, except for the fact that Π_{2p2h} depends explicitly on the variable q^0 (and $\vec{q}$).

Since now there is overlap between $Im\, \Pi_{2p2h}$ and the pion pole one has to be cautious in the separation of the pionic width and the one associated to $2p2h$ emission. In ref [26] the calculations were done in infinite nuclear matter at normal nuclear matter density where the mesonic decay channel is forbidden. Hence, all the strength from eq. (9) with $Im\, D(q)$ from eq. (32) was attributed to the $2p2h$ channel. In finite nuclei, where there is some mesonic decay allowed, eq. (9) with the distribution of eq. (32) accounts simultaneously for the mesonic and $2p2h$ excitation channel. The separation of the two channels can be done by calculating the contribution of the pion pole and associating it to pion emission, and then associating to the $2p2h$ excitation channel, the difference between the width calculated with eqs. (9), (32) and the width from the pion pole contribution. The pion pole contribution is calculated by means of eq. (9) substituting $Im\, D(q)$ by $-\pi\delta(q^{02} - \vec{q}^{\,2} - \mu^2 - Re\, \Pi(q^0, q))$.This is the way followed in ref. [27]. In addition a more realistic input for Π_{2p2h} is used in [27] taking care properly of the phase space available for the $2p2h$ excitation.

The results of [27] indicate that $\Gamma_{2p2h}/\Gamma_\Lambda \simeq 0.30$ for different nuclei from ^{12}C up to ^{208}Pb. In ref. [26] this ratio had a value around 0.60 for values of the g'_Λ parameter compatible with those used in [27].

Even with smaller values for $\Gamma_{2p2h}/\Gamma_\Lambda$ than those of ref. [26], the existence of this channel has important repercussions in the number of neutrons and protons emitted in the Λ decay process, a piece of information which is used to determine the ratio of proton to neutron induced Λ decay in the nonmesonic channel. It is clear that in view of the new results one cannot associate all n or p emerging from the experiment to the primary $\Lambda n \to nn$ or $\Lambda p \to np$ reactions and hence a reanalysis of the experimental data is needed. This analysis requires the consideration of the Λ nonmesonic decay channel, which we have not addressed here, hence the reader is addressed to this paper [27] for further details.

Up to now the experiments for Λ decay have focused on two channels, the mesonic and the non mesonic. In view of the former results and the fact that the $2p2h$ channel has a bigger strength than the mesonic one from nuclei like ^{16}O up, it would be very interesting to conduct experimental searches for this channel too.

8 Conclusions.

We have made a review of the present situation concerning the mesonic decay of Λ hypernuclei. We have established the formal link between the propagator method, where the huge enhancement of the pionic decay width was first reported, and the finite nuclei approach with wave functions and matrix elements. Shell effects and precise values of the nuclear binding energies are also important in the mesonic width and they are best taken into account in the finite nuclei approach. The intuitive and appealing, but fallacious, link between the nucleon occupation number and the mesonic width has also been discussed, which has served to unveil rough approximations used in other processes to link the pion excess number with contributions of the nuclear meson cloud to some physical observables, like K^+ nucleus scattering. We have also discussed the relevance of the short range ΛN repulsion in the mesonic width of light hypernuclei and showed how the repulsion provided by quark models of the ΛN interaction can naturally account for the present experimental widths. Finally we have discussed the Λ decay induced by pairs of nucleons through the tail of the pion distribution in the nucleus, which "cheats" the Pauli blocking and leads to a three nucleon decay channel, $\Lambda NN \to NNN$.

With the limited amount of experimental data available on the mesonic channel, the amount of physical information obtained is remarkable. There is support for strong ΛN repulsion at short distances providing indirect support for quark models of the ΛN interaction; the process provides us with the most striking renormalization effect due to the pion nucleus interaction. Furthermore, the "cheating" of Pauli blocking by the $2p2h$ induced decay can provide good information on the coupling of the pion to these nuclear components, a very valuable complement to real pion absorption, etc. The sensitivity of the Λ decay to the pion nucleus optical potential can also serve as a tool to choose between different theoretical descriptions of the complex mechanisms of pion nucleus interaction. The decay channel into π^0 can be an excellent instrument to learn about π^0 nucleus interaction, and so on.

It is clear that a systematic experimental search in many nuclei of the mesonic decay channel and its related $2p2h$ induced decay mode will provide us with very valuable information to unravel the intricacies of the pion nucleus interaction or the elementary properties of the ΛN interaction, as well as proper nuclear structure details of the Λ hypernuclei themselves.

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Figure captions.

Fig. 1 Feynman graph for the free Λ self-energy of eqs. (4), (5). The $\Lambda \to \pi N$ "cut" is shown (dotted line).

Fig. 2. Analytical structure of the integrand of eq. (4) in the complex q^0 plane with the nucleon and pion propagators of eqs. (6), (7). The renormalized pion propagator pole $\tilde{\omega}(q)$ is shown. The dashed lines close to the real axis indicate the analytical cut from $Im\,\Pi(q^0, q)$ related to the nonmesonic Λ decay channel.

Fig. 3 Λ self-energy diagrams included in eq. (4) with the nucleon and pion propagators of eqs. (6), (7). (a) Free self-energy graph. (b), (c) Insertion of p-wave pion self-energy at lowest order. (d) Generic RPA graph from the expansion of the pion propagator in powers of the pion selfenergy. (e) s-wave pion self-energy at lowest order. The cuts represent the nonmesonic decay channel.

Fig. 4. Free and lowest order Λ self-energy graph. The dotted cuts represent the mesonic decay channel.

Fig. 5. Schematic representation of the nucleon occupation number for an interacting Fermi sea.

Fig. 6. Pionic decay rate for π^0 and π^- as a function of the mass number (of the host nucleus, ^{16}O, ^{40}Ca, ^{90}Zr, ^{138}Ba, and ^{208}Pb). The two lower lines show the calculation with plane waves for the pion and the two upper lines the results with pion distorted waves.

Fig. 7. Schematic representation of the Λ decay coupling to $2p2h$ components through virtual (close to real) pion absorption.

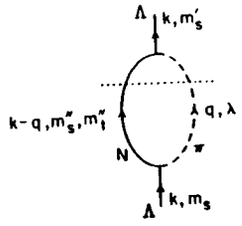

fig. 1

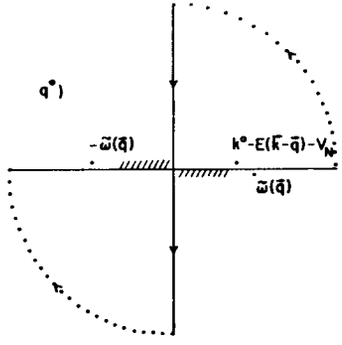

fig. 2

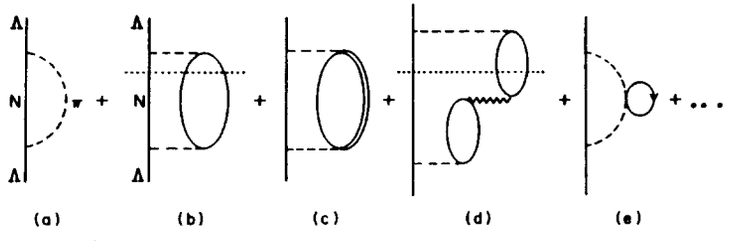

fig. 3

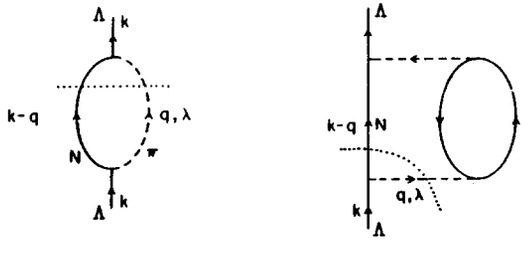

fig. 4

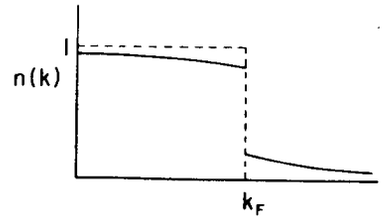

fig. 5

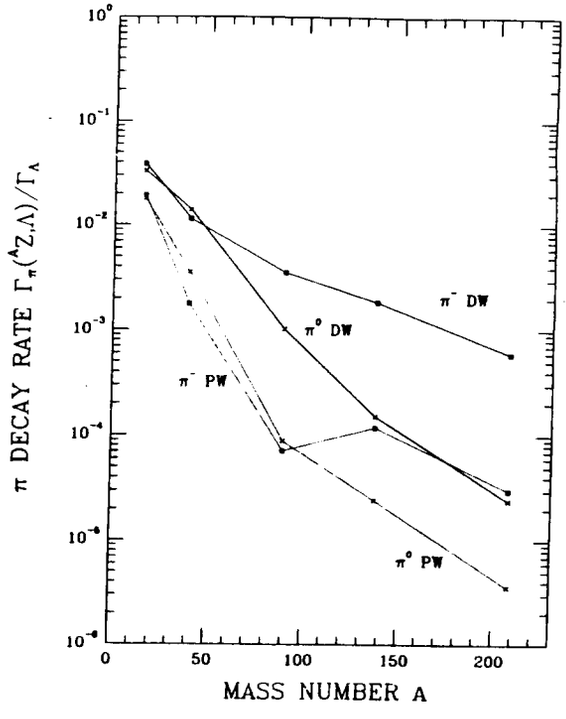

fig. 6

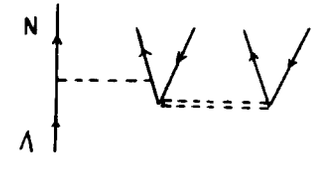

fig. 7