ETH ITP
Institut für Teilchenphysik

ETHZ-ITP PR-94-4
March 1994

The Relativistic Mean-Field Model for the pion-nucleus interaction and the s-wave repulsion

P.F.A. Goudsmit, H.J. Leisi and E. Matsinos

Institute for Particle Physics (ITP), ETH Zurich,
CH-5232 Villigen PSI, Switzerland

(Submitted to Physics Letters B)

Institute for Particle Physics
ETH Zurich
CH-5232 Villigen PSI
Switzerland

The Relativistic Mean-Field Model for the pion-nucleus interaction and the s-wave repulsion

P.F.A. Goudsmit, H.J. Leisi and E. Matsinos

Institute for Particle Physics, ETH Zurich, CH-5232 Villigen PSI, Switzerland

Abstract

The Relativistic Mean-Field Model for the pion-nucleus interaction is modified in such a way as to implement the properties of the free pion-nucleon scattering amplitude. The experimental data of pionic atoms of isoscalar nuclei are very well described with this model. The phenomenon of the s-wave repulsion is shown to be due to the nucleon and Δ-isobar effective masses inside the nuclear environment. The value of the effective-mass parameter for the Δ-isobar has been determined to be close to the one corresponding to the nucleon.

Rather recently [1], pionic-atom data of isoscalar nuclei were analyzed within a modified version of the Relativistic Mean-Field (RMF) Theory [2] which is an alternative to the Multiple Scattering Approach ([3]-[4]) for the treatment of the low-energy pion-nucleus interaction; therein, it was shown that the local real part of the interaction could be described with the two parameters of the RMF Theory, namely the strength of the nuclear scalar-meson field for pions (S_π) and the pseudoscalar mixing parameter in the pion-nucleon ($\pi - N$) vertex (x). Our RMF Model provided an explanation to the long-standing problem of the s-wave repulsion [1].

In order to investigate the compatibility of our RMF Model with the pion-nucleon sector, a $\pi - N$ interaction model (consisting, at the tree level, of the lowest-order elementary contributions of the RMF Theory) was introduced [1]. Very recently, the general validity of this model was thoroughly investigated ([5]-[8]); our predictions were compared with $\pi - N$ scattering data (from threshold up to the Δ_{33}-resonance) and excellent description of the s- and p-wave parts of the interaction was achieved, including the $\pi - N$ Σ-term (which was calculated in the tree-level approximation). However, one important modification of the original form of the $\pi - N$ model was **essential** in order to achieve these results: **the Δ-isobar had to be treated in the most general manner**; i.e. the Rarita-Schwinger condition (projection of the Δ-isobar amplitudes onto pure spin-$\frac{3}{2}$ states), which is also imposed in the original form of the RMF Theory [2], had to be retracted. It was found that a considerable admixture of spin-$\frac{1}{2}$ in the Δ-isobar intermediate state was necessary in order to account for the energy dependence of the s-wave part of the $\pi - N$ scattering amplitude. A plausible question to be asked now is whether our RMF Model can still reproduce the pionic-atom data after the implementation of the properties of the free pion-nucleon interaction. This subject is investigated in the present work.

The RMF Theory is an extension of the Relativistic Mean-Field Approach in Nuclear Physics (the $\sigma - \omega$ model [9]) in such a way as to include the interaction with the π-meson. The pion interacts in two ways with the nucleus (we restrict ourselves to **isoscalar** nuclei): it couples to the mean static scalar-meson field $S_\pi(r)$ (which is due to the nucleons in the nuclear ground state) and it polarizes the nucleus through absorption and reëmission on a bound nucleon (with a nucleon or a Δ-isobar in the intermediate state).

The $\pi-$nucleus interaction is described with an optical potential which contains local (s-wave) and non-local (p-wave) parts and is complex in order to account for the pion absorption [3]. The non-local part and the pion absorption will be treated phenomenologically. Within the context of our model, there are then three contributions to the local real part of the optical potential (all these contributions refer to threshold, since we will be dealing with pionic atoms):

- The contribution of the scalar-meson field has the form

$$2\,\mu\,U^{(\sigma)}(r) \;=\; 2\,m_\pi\;S_\pi(r) \quad , \tag{1}$$

where m_π is the charged-pion mass and μ is the reduced mass of the pion-nucleus system [1].

[1]Note that the quadratic term in $S_\pi(r)$ of the original form of the RMF Theory [2] (also adopted in ref. [1]) has been excluded in this work; instead, we follow the prescription of ref. [10], page 531. The reason is that the elementary scalar interaction in our (very successful) $\pi - N$ model corresponds to the (simpler) linear form (in $S_\pi(r)$) in the $\pi-$nucleus optical potential.

The scalar-meson field is assumed to follow the nuclear density $\rho(r)$:

$$S_\pi(r) \;=\; S_\pi\,\frac{\rho(r)}{\rho_0} \quad ; \quad \rho_0 \;=\; 0.17\,fm^{-3} \quad . \tag{2}$$

S_π is the strength parameter related to this field.

• The nucleon contribution reads as

$$2\mu U^{(N)}(r) \;=\; \frac{g^2_{\pi NN}}{(1+x)^2}\left[\frac{m^2_\pi}{4\mathcal{M}(r)}\left(\frac{x}{\mathcal{M}(r)}+\frac{1}{m}\right)^2+\frac{x^2}{\mathcal{M}(r)}\right]\rho(r) \quad , \tag{3}$$

where $g_{\pi NN}$ stands for the coupling constant corresponding to the $\pi - N$ vertex, x for the pseudoscalar admixture in the $\pi - N$ vertex and m for the nucleon mass (average of the proton and the neutron masses) [1].

$\mathcal{M}(r)$ denotes the nucleon effective mass and is defined according to the formula

$$\mathcal{M}(r) \;=\; m\,\left(1 \;-\; c_N\,\frac{A-1}{A}\,\rho(r)\,\right); \;\; c_N \;=\; \frac{V_N - S_N}{2m\rho_0} \quad , \tag{4}$$

where V_N and S_N are respectively the strength parameters of the vector(ω)- and scalar-meson fields for the nucleon. The parameter c_N has been calculated in three independent ways: from nuclear matter properties [9], from shell-model parameters [11] and from the parameters of the $N - N$ One-Boson-Exchange model [12]: $c_N = 2.06 fm^3$ with an estimated relative uncertainty of 15%. A denotes the mass number of the corresponding nucleus. The term $(A-1)/A$ is due to the fact that the struck nucleon interacts with the fields created by the remaining $A - 1$ nucleons.

• The contribution of the Δ-isobar graphs to the local real part of the $\pi-$nucleus optical potential has been calculated to be [2]

$$2\mu U^{(\Delta)}(r) \;=\; -\frac{2g^2_{\pi N\Delta}m^2_\pi Y}{9m^2\mathcal{M}^2_\Delta(r)}\left[3\mathcal{M}_\Delta(r)+(2\mathcal{M}_\Delta(r)-\mathcal{M}(r))\,Y\right]\rho(r) \quad , \tag{5}$$

[2] The calculation was performed in the framework of the RMF Theory [2] with the Δ-isobar propagator and the $\pi N\Delta$ interaction vertex of ref. [13], page 562.

where $g_{\pi N\Delta}$ denotes the $\pi N\Delta$ coupling constant, $Y = Z - \frac{1}{2}$ (the parameter Z describes the spin-$\frac{1}{2}$ admixture in the Δ-isobar field [3]) and $\mathcal{M}_\Delta(r)$ denotes the Δ-isobar effective mass, which can be parametrized (similarly to the nucleon case) as

$$\mathcal{M}_\Delta(r) = M \left(1 - c_\Delta \frac{A-1}{A} \rho(r)\right) \quad , \tag{6}$$

where M stands for the 'free' Δ-isobar mass. Since no information is available for c_Δ, it will be treated as a free parameter.

The local real part $2\mu U^{(lr)}(r)$ of the optical potential equals to the sum of the three individual contributions;

$$2\mu U^{(lr)}(r) = 2\mu U^{(\sigma)}(r) + 2\mu U^{(N)}(r) + 2\mu U^{(\Delta)}(r) \quad . \tag{7}$$

The complete strong-interaction equivalent potential $2\mu U(r)$ should include the non-local (gradient) term and imaginary terms (both in the local and non-local parts). Thus,

$$2\mu U(r) = 2\mu U^{lr}(r) - 4\pi \left[i\frac{1}{2} Im\bar{B}_0 \rho^2(r) - \vec{\nabla}\left(\frac{\bar{c}_0}{p_1}\rho(r) + \frac{1}{2}\bar{C}_0\rho^2(r)\right)\cdot\vec{\nabla}\right] \quad , \tag{8}$$

where $p_1 = 1 + m_\pi/m$. The parameter $\bar{C}_0$ is complex. The optical potential of equation (8) contains five parameters: c_Δ in the local real part, two parameters in the non-local real part ($\bar{c}_0$ and $Re\bar{C}_0$) and two parameters for the pion absorption ($Im\bar{B}_0$ for the s-wave absorption and $Im\bar{C}_0$ for the p-wave absorption). The remaining parameters (in the local real part) have been taken from the $\pi - N$ interaction model. For details, the reader is referred to refs. [6] and [7], where the parameters of the $\pi - N$ model are defined and their values determined. Equally good fits to

[3]Note that the imposition of the Rarita-Schwinger condition, which at threshold is equivalent to having $Z = \frac{1}{2}$, would lead to vanishing $U^{(\Delta)}(r)$.

the $\pi - N$ phase-shift data were obtained within a whole range of values of the (independent) parameter $G_\rho^{(V)}$. In the present work, the strength of the scalar-meson field S_π has been chosen as an independent parameter; S_π is related with the scalar coupling constant G_σ of the $\pi - N$ model through the formula [1]

$$G_\sigma \;\cong\; -S_\pi \cdot \left(\frac{\rho(0)}{\rho_s(0)}\right)\frac{1}{\rho_0} \quad , \tag{9}$$

where $\rho(0)/\rho_s(0) \;\cong\; 1.08$, see ref. [14].

In our analysis, we restrict ourselves to **isoscalar** nuclei. This is because we intend (as a first step) to study the dominant feature of the interaction (i.e. the isoscalar part) with a clear separation from the (smaller) isovector part; furthermore, the problem of the poorly known neutron distributions is eliminated in the case of isoscalar nuclei (the nuclear density $\rho(r)$ is double the measured nuclear charge density).

The analysis is based on the same experimental input as ref. [1]; the data set consists of the energy shifts and widths of ^{6}Li, ^{10}B, ^{12}C, ^{14}N, ^{16}O and ^{20}Ne for the $1s$ states, and of ^{12}C, ^{16}O, ^{24}Mg, ^{28}Si, ^{32}S and ^{40}Ca for the $2p$ states (24 entries in total). In order to emphasize the role of the $1s$ level shifts in determining the local real part of the potential, the errors of all other experimental input data (i.e. of the widths and of the $2p$ energy shifts) were doubled. With our optical potential of equation (8), we will fit to these data. The numerical work, i.e. solving the Klein-Gordon equation with the strong interaction potential and calculating the radiative corrections, was done with the computer code BIPA [15].

For different values of S_π (our fits to the $\pi - N$ phase shifts ([6]-[7]) indicate a range of S_π-values between -50 and -30 MeV), one may then fit for the parameters of our optical potential of equation (8). In all cases, **very good** fits were realized. This can be clearly seen in figs. 1(a) ($1s$ energy shifts), 1(b) ($2p$ energy shifts) and 1(c) ($1s$ and $2p$ widths). From fig. 1(a), it is evident that the local real part of our optical potential (equation (7)) can account for the $1s$ energy shifts of pionic-atoms of isoscalar nuclei at a **few-percent level.**

It is interesting to notice that these results have been obtained with only one parameter in the local real part of the π−nucleus optical potential (c_Δ); its dependence on S_π is shown in fig. 2. It is remarkable that the values of this parameter, obtained by the fits, are very reasonable, i.e. close to the values for the nucleon effective-mass parameter c_N.

Let us finally note that higher-order effects have been dealt with as in ref. [1]; they originate from rescattering processes [16] and from pion absorption (dispersive effect) [17]. Both effects make contributions to the quadratic term in the nuclear density (thus, they affect $Re\bar{B}_0$ in the notation of ref. [3]). In ref. [1], we showed that these two contributions almost cancel one another; $Re\bar{B}_0^{ho} \sim 0 \pm 0.06\ m_\pi^{-4}$. The dashed lines of fig. 2 correspond to the uncertainty in $Re\bar{B}_0^{ho}$.

Taking into account the variation of S_π and the uncertainty in c_N, we have obtained: $c_\Delta = 2.4 \pm 0.5\ fm^3$. The (small) statistical error is also included in the uncertainty. The higher-order effects (previously mentioned) induce additional (average) uncertainties of $^{+0.5}_{-0.7}\ fm^3$ in c_Δ.

The values of the parameters $\bar{c}_0$, $Re\bar{C}_0$, $Im\bar{B}_0$ and $Im\bar{C}_0$ of our optical potential (8), obtained with our fits, are $0.192 \pm 0.005\ m_\pi^{-3}$, $0.05 \pm 0.03\ m_\pi^{-6}$, $0.112\ m_\pi^{-4}$ and $0.09\ m_\pi^{-6}$, respectively. These values correspond to averages over the whole domain of variation of S_π. The errors exclusively correspond to the variation of S_π and to the uncertainty in c_N; for the parameters $Im\bar{B}_0$ and $Im\bar{C}_0$, these uncertainties are negligibly small. The parameters $\bar{c}_0$ and $Re\bar{C}_0$ are strongly anti-correlated.

The individual contributions to the local real part of the $\pi-$nucleus potential are shown in figs. 3 as a function of S_π for two cases:

- $c_N \neq 0$, $c_\Delta \neq 0$ and
- $c_N = c_\Delta = 0$.

Evidently, the phenomenon of the s-wave repulsion can not be accounted for in the latter case.

To summarize, our Relativistic Mean-Field (RMF) Model for the $\pi-$nucleus interaction is put on a firm basis by implementing the properties of the free $\pi - N$ interaction. This is achieved with the help of our $\pi-$nucleon interaction model ([5]-[8]): the structure and the parameter values of this model are introduced into the RMF Approach. The main results are:

- The phenomenon of the s-wave repulsion is shown to be due to the nucleon and Δ-isobar effective masses inside the nuclear environment.
- The effective-mass parameter of the Δ-isobar is determined to be $c_\Delta \sim 2.4\ fm^3$ (i.e. close to the corresponding value for the nucleon).

Interesting further developments of the RMF Model might now be attempted in the following directions: with the inclusion of the isovector part (by introducing the ρ-meson vector field), with the description of the non-local part of the interaction and, finally, with the extension of the model above threshold in order to describe the $\pi-$nucleus elastic scattering and single-charge exchange reactions.

Clearly, the present work establishes a promising new link between the Pion and Nuclear Physics, and could stimulate additional work in the domain of the Relativistic Hadron Physics.

We would like to thank B.L. Birbrair for his contribution in the implementation of the treatment of the Δ-isobar in our model. We acknowledge helpful discussions with A.B. Gridnev.

References

[1] P.F.A. Goudsmit, H.J. Leisi and E. Matsinos, Phys. Lett. **B271** (1991) 290

[2] B.L. Birbrair, V.N. Fomenko, A.B. Gridnev and Yu.A. Kalashnikov, J. Phys. **G9** (1983) 1473; J. Phys. **G11** (1985) 471

[3] M. Ericson and T.E.O. Ericson, Ann. Phys. **36** (1966) 323

[4] B.L. Birbrair and A.B. Gridnev, Nucl. Phys. **A528** (1991) 647

[5] P.F.A. Goudsmit, H.J. Leisi and E. Matsinos, Helv. Phys. Acta **65** (1992) 878

[6] P.F.A. Goudsmit, H.J. Leisi and E. Matsinos, Phys. Lett. **B299** (1993) 6

[7] P.F.A. Goudsmit et al., to be published in Nuclear Physics A

[8] P.F.A. Goudsmit, H.J. Leisi and E. Matsinos, ETHZ-ITP Preprint 94-1; submitted to Nuclear Physics A

[9] B.D. Serot and J.D. Walecka, Advances in Nuclear Physics 1986 (Vol. 16, 1), eds. J.W. Negele and E. Vogt

[10] G. Baym, Lectures on Quantum Mechanics (The Benjamin/Cummings Publishing Company, Reading, Massachusetts, 1969)

[11] J.V. Noble, Nucl. Phys. **A329** (1979) 354

[12] F. Gross, J.W. van Orden and K. Holinde, Phys. Rev. **C41** (1990) R1909

[13] G. Höhler, Pion-Nucleon Scattering (Landolt-Börnstein, Vol. **I/9b2**, ed. H. Schopper, Springer Verlag, 1980)

[14] C.J. Horowitz and B.D. Serot, Nucl. Phys. **A368** (1981) 503

[15] B. Jeckelmann, BIPA, Ein Program zur Berechnung von Bindungsenergien in pionischen Atomen, ETHZ-IMP Internal Report LB-85-03

[16] R. Seki, K. Masutani and K. Yazaki, Phys. Rev. **C27** (1983) 2817

[17] C. Garcia-Recio, E. Oset and L.L. Salcedo, Phys. Rev. **C37** (1988) 194

Figure caption

Figures 1:

The relative differences $(\varepsilon_0(th) - \varepsilon_0(exp))/\varepsilon_0(th)$ for the $1s$ (fig. 1(a)) and $2p$ states (fig. 1(b)) of pionic atoms of isoscalar nuclei and the ratio $(\Gamma(th) - \Gamma(exp))/\Gamma(th)$ for the widths of these states (fig. 1(c)), as functions of the mass number A. $\varepsilon_0(th)$ denotes the difference $E_{th} - E_{el}$; E_{th} is the relevant transition energy, predicted with the strong potential of equation (8), and E_{el} is pure electromagnetic energy difference of the corresponding levels. $\varepsilon_0(exp)$ stands for the difference $E_{exp} - E_{el}$; E_{exp} is the measured energy of the transition in question. $\Gamma(exp)$ denotes the measured width of a level and $\Gamma(th)$ is the fitted value obtained with the optical potential of equation (8). Average values of the energy shifts and widths (over the whole domain of variation of S_π) have been considered. The errors represent the experimental uncertainties and the (much smaller) uncertainties in our prediction (which are due to the variation of S_π and to the uncertainty in c_N).

Figure 2:

The S_π-dependence of the parameter c_Δ. The solid line corresponds to c_N fixed at 2.06 fm^3. The errors due to the 15% uncertainty in c_N are designated by the dotted lines. The dashed lines correspond to the limits of the c_Δ-values when the corrections due to higher-order effects (see text) are taken into account. The hatched region indicates the (15%) uncertainty intervals in the nucleon effective-mass parameter c_N (around the value of 2.06 fm^3).

Figures 3:

The individual contributions to the local real part of the π−nucleus potential as a function of S_π for two cases:

- $c_N = 2.06\ fm^3$, c_Δ taken from fig. 2 (solid line) and
- $c_N = c_\Delta = 0$.

The figure corresponds to ^{12}C; in the calculation, the central density of this nucleus ($\rho(0) \cong 0.16\ fm^{-3}$) has been used.

Figs. 1

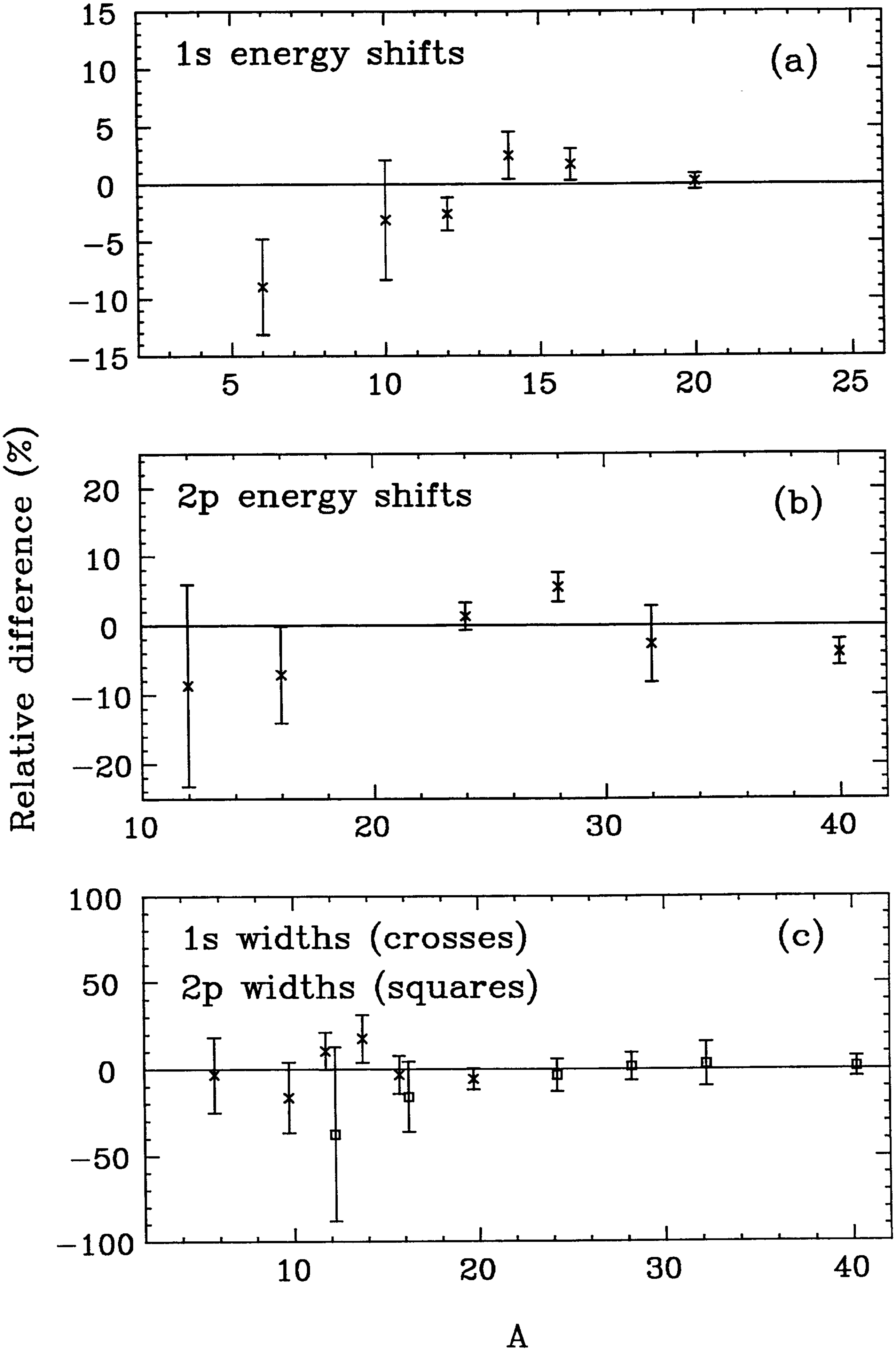
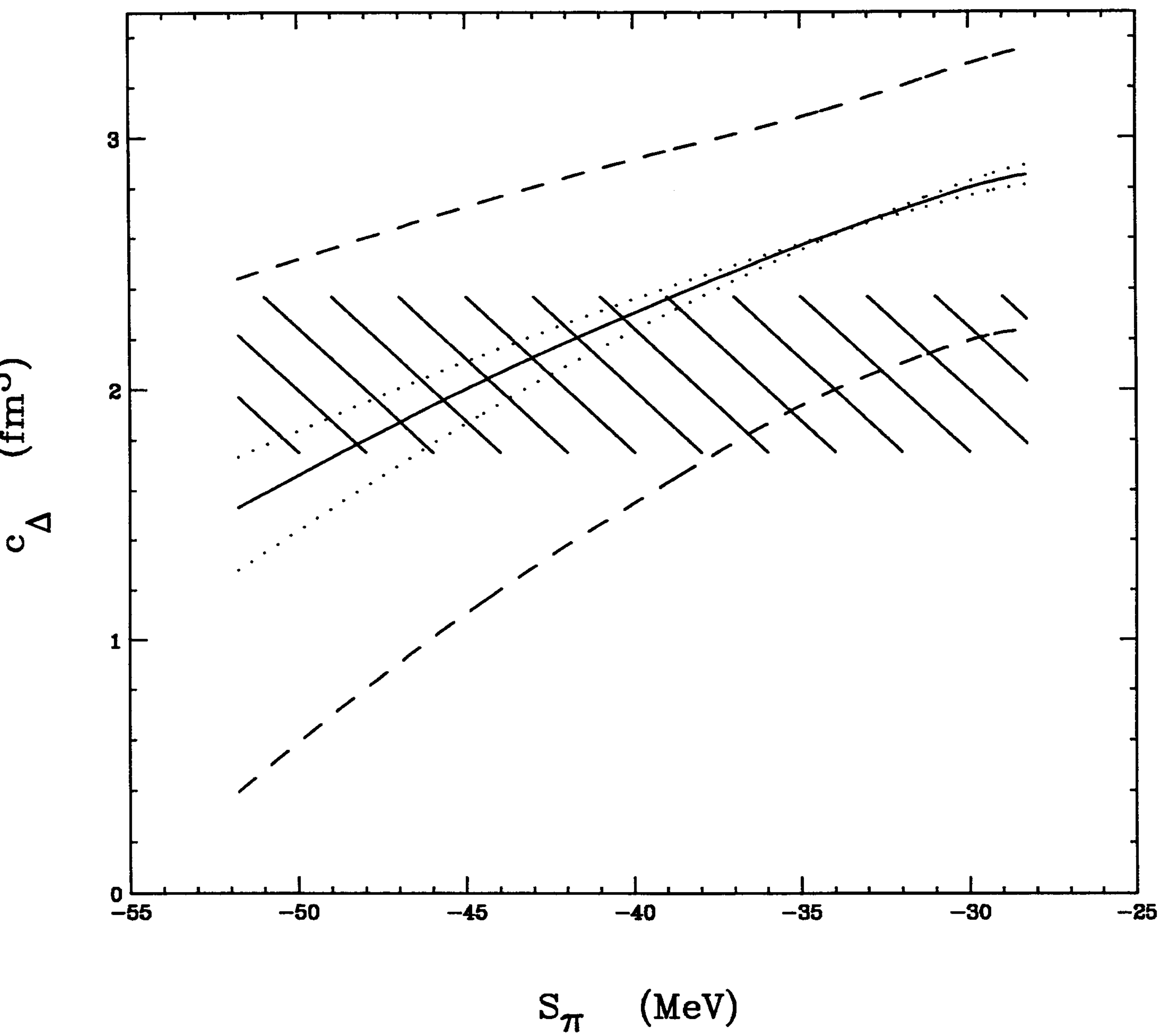
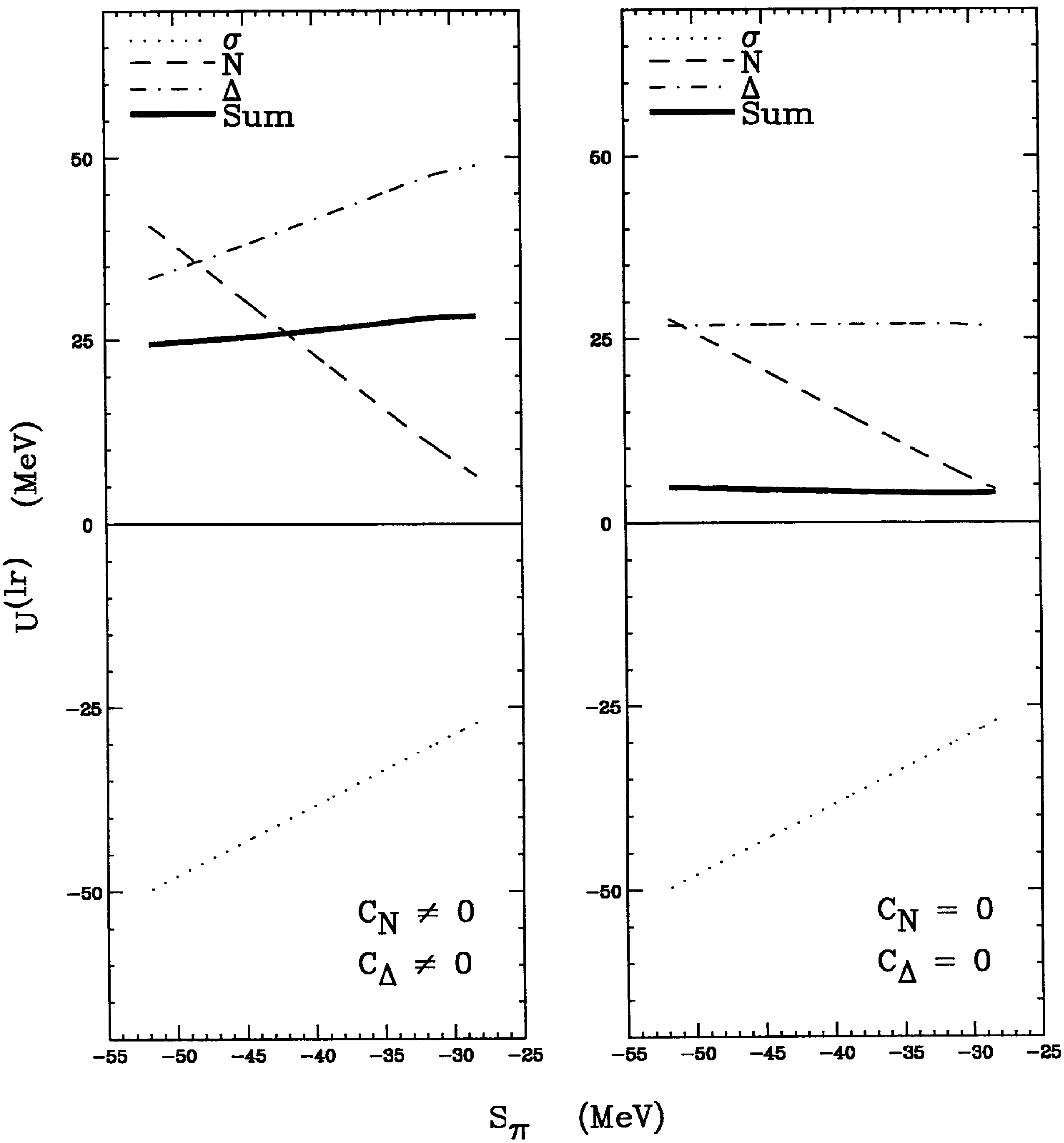

1s energy shifts
(a)
2p energy shifts
(b)
1s widths (crosses)
2p widths (squares)
(c)
Relative difference (%)
A
15
10
5
0
−5
−10
−15
5
10
15
20
25
20
10
0
−10
−20
10
20
30
40
100
50
0
−50
−100
10
20
30
40

Fig. 2



Figs. 3


σ
N
Δ
Sum
50
25
0
−25
−50
U(lr) (MeV)
$C_N \neq 0$
$C_\Delta \neq 0$
$C_N = 0$
$C_\Delta = 0$
−55 −50 −45 −40 −35 −30 −25
S_π (MeV)