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CP Violations in $D^{\pm}$ and $B^{\pm}$ Boson Decays*

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Abstract

We will study some possible tests of CP-violations in $D^{\pm}$ and $B^{\pm}$ boson decays. Especially, possible inequivalence of Dalitz plot for 3 or 4 body decay mode of D^{+} or B^{+} with the corresponding one of its anti-particle D^{-} or B^{-} may be used to test the CP. Also, 4-body decay modes such as $D^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}K^{0}(\text{or } \overline{K}^{0})$ can be used to separately detect possibly large P and C violations, even though the CP may be conserved.

*Paper dedicated to Memorial Volume for Professor Robert Marshak.

1. Introduction

Weak interaction physics had been one of the most favorite subjects of the late Professor R. E. Marshak during his long illustrious career. The discovery of the weak V-A theory[1] in collaboration with E. C. G. Sudarshan in 1957 was the starting point of the now canonical unified electro-weak theory of Glashow, Salam, and Weinberg, where the Lagrangian responsible for non-leptonic decays is given[2] by

$$L_W = g \begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix} \gamma_\mu (1+\gamma_5) V \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\mu + \text{ hermitian conjugate} \quad . \tag{1.1}$$

Here, W^μ stands for the intermediate vector boson, and the 3×3 matrix V is the Cabibbo-Kobayashi-Maskawa mixing matrix:

$$\begin{aligned} V &= \begin{pmatrix} V_{ud}, & V_{us}, & V_{ub} \\ V_{cd}, & V_{cs}, & V_{cd} \\ V_{td}, & V_{ts}, & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13}, & s_{12}c_{13}, & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{12}s_{13}e^{i\delta_{13}}, & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}}, & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}}, & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}}, & c_{23}c_{13} \end{pmatrix} \end{aligned} \tag{1.2}$$

where we adopted the parametrization of the C-K-S matrix given by the Particle-Data Group[3]. In Eq. (1.2), s_{ij} and c_{ij} stand for

$$s_{ij} = \sin\theta_{ij} \quad , \quad c_{ij} = \cos\theta_{ij} \tag{1.3}$$

with experimental values[3] of

$$\begin{aligned} s_{12} &= 0.218 \sim 0.224 \\ s_{23} &= 0.032 \sim 0.054 \\ s_{13} &= 0.002 \sim 0.007 \end{aligned} \tag{1.4}$$

while the value of the CP-violating phase factor δ_{13} is not accurately known.

The purpose of the present paper which is dedicated to the memory of Prof. Marshak is to study some aspects of the CP-violation processes in the weak decays of $B^\pm$ and $D^\pm$ boson decays.

First, we note the following known facts:

(i) The TCP invariance alone guarantees the equivalence[4)–7)] of the total decay rates of a particle and its anti-particle. For example, we will have

$$\Gamma\left(K^{+} \rightarrow \text{ all}\right) = \Gamma\left(K^{-} \rightarrow \text{ all}\right) \quad . \tag{1.5}$$

(ii) Individual partial decay rate of a particle will differ[4)–7)] in general from that of the corresponding anti-particle, if both C and CP are violated in the process. An example will be

$$\Gamma\left(K^{+} \rightarrow \pi^{+}\pi^{+}\pi^{-}\right) \neq \Gamma\left(K^{-} \rightarrow \pi^{-}\pi^{-}\pi^{+}\right) \quad . \tag{1.6}$$

(iii) The Dalitz plot of any three body (or more generally multi-body) decay mode (such as $K^{+} \rightarrow \pi^{+}\pi^{+}\pi^{-}$) will be different[8)] from that of its anti-particle (such as $K^{-} \rightarrow \pi^{-}\pi^{-}\pi^{+}$), unless C or CP is conserved in the process.

(iv) The decay asymmetry parameters of a particle with non-zero spin (such as Σ^{+}) will be different[5),6)] from that of its anti-particle (such as $\overline{\Sigma}^{+}$) under the same condition of C and CP non-invariances.

All these facts are more or less well-known perhaps except for the non-equivalence of the Dalitz plots as in the third statement. To show it explicitly, it is convenient to consider an effective weak Hamiltonian H_W and write

$$M(i \rightarrow f) = < f(\text{out})|H_W|i(\text{in}) > \tag{1.7a}$$

$$M(\bar{i} \rightarrow \bar{f}) = < \bar{f}(\text{out})|H_W|\bar{i}(\text{in}) > \tag{1.7b}$$

for the decay matrix elements of a particle i and its anti-particle $\bar{i}$ into states f and $\bar{f}$, respectively. Here, the symbol "out" and "in" refer to outgoing and incoming wave conditions, respectively. However, we will delete them for simplicity in what follows. Suppose now that H_W has a form of

$$H_W = \sum_{j=1}^{N} \left(g_j H_j + g_j^* H_j^{\dagger}\right) \tag{1.8}$$

where g_j's are some complex coupling constants, and H_j's transform covariantly under CP according to

$$\text{CP} \ : \ H_j \leftrightarrow H_j^\dagger \qquad (j = 1, 2, \ldots, N) \quad . \tag{1.9}$$

Moreover, if H_j is responsible to a decay $i \to f$, then its anti-particle decay $\bar{i} \to \bar{f}$ will proceed via $H_j^\dagger$. For example, consider decays $D^\pm \to \pi^\pm\pi^\pm\pi^\mp$ where H_1 and g_1 may be given as in Eqs. (2.4) and (2.5) in the V-A form of

$$H_1 =: \ \big(\bar{u}\gamma_\mu(1+\gamma_5)d\big)\big(\bar{d}\gamma^\mu(1+\gamma_5)c\big) \ : \tag{1.10a}$$

$$g_1 = \frac{1}{\sqrt{2}} \ G \ V_{ud}V_{cd}^* \tag{1.10b}$$

with

$$G = \sqrt{2} \ g^2/(m_W)^2 \quad . \tag{1.11}$$

We can rewrite Eqs. (1.7) then as

$$M(i \to f) = \sum_{j=1}^{N} g_j < f|H_j|i > \tag{1.12a}$$

$$M(\bar{i} \to \bar{f}) = \sum_{j=1}^{N} g_j^* < \bar{f}|H_j^+|\bar{i} > \quad . \tag{1.12b}$$

Choosing $\bar{f}$ and $\bar{i}$ be CP-conjugates of f and i, respectively, i.e.

$$|\bar{f}> = CP|f> \quad , \quad |\bar{i}> = CP|i> \quad , \tag{1.13}$$

Eqs. (1.12b) and (1.9) lead to

$$M(\bar{i} \to \bar{f}) = \sum_{j=1}^{N} g_j^* < f|H_j|i > \quad , \tag{1.14}$$

so that we calculate

$$|M(\bar{i} \to \bar{f})|^2 + |M(i \to f)|^2 = 2 \sum_{j,k=1}^{N} \text{Re}(g_i g_k^*)\text{Re}(< f|H_j|i >< f|H_k|i >^*) \tag{1.15a}$$

$$|M(\bar{i} \to \bar{f})|^2 - |M(i \to f)|^2 = 2 \sum_{j,k=1}^{N} \text{Im}(g_i g_k^*)\text{Im}(< f|H_j|i >< f|H_k|i >^*) \tag{1.15b}$$

from Eqs. (1.12a) and (1.14), where Re A and Im A designate real and imaginary parts, respectively, of A. Therefore, we find the formula

$$\Delta \equiv \frac{\Gamma(\bar{i} \to \bar{f}) - \Gamma(i \to f)}{\Gamma(\bar{i} \to \bar{f}) + \Gamma(i \to f)} = \frac{\sum_{j,k=1}^{N} \mathrm{Im}(g_j g_k^*) \cdot \mathrm{Im}(< f|H_j|i >< f|H_k|i >^*)}{\sum_{j,k=1}^{N} \mathrm{Re}(g_j g_k^*) \cdot \mathrm{Re}(< f|H_j|i >< f|H_k|i >^*)} \quad (1.16)$$

so that we will have in general

$$\Gamma(\bar{i} \to \bar{f}) \neq \Gamma(i \to f) \quad , \quad (1.17)$$

proving the assertion made in the statements (ii) and (iii) given in the beginning. Note that Δ can be non-zero only if $N \geq 2$. Contrarily, we will have

$$\Gamma(\bar{i} \to \bar{f}) = \Gamma(i \to f) \quad (1.18)$$

for $N = 1$, corresponding to the case that H_W is effectively CP-conserving. From Eq. (1.16), it is also clear that the CP-violation will be found only if we have $\mathrm{Im}(g_j g_k^*) \neq 0$ and $\mathrm{Im}(< f|H_j|i >< f|H_k|i >^*) \neq 0$. In general, the decay matrix element $< f|H_j|i >$ can acquire its complex phase via strong final-state interactions. Indeed, the time-reversal invariance of H_j's which is a consequence of Eq. (1.9) and of the TCP theorem will lead to the validity of

$$< f|H_j|i > = \sum_n S(\hat{f} \to n) < n|H_j|\hat{i} >^* \quad . \quad (1.19)$$

Here, $|\hat{f} > = T|f >$ and $|\hat{i} > = T|i >$ refer to the time-reversed states of f and i, respectively, and $S(\hat{f} \to n)$ stands for the S-matrix element for the strong final-state transition $\hat{f} \to n$. Hence, if more decay channels exist, then the larger will be complex phases of the decay matrix elements.

In order to estimate the magnitude of the CP-violation, suppose for simplicity that $g_1 H_1$ is dominant, i.e.

$$|g_1 < f|H_1|i > | \gg |g_j < f|H_j|i > | \qquad (j = 2, 3, \ldots, N) \quad . \quad (1.20)$$

Eq. (1.16) will then reduce to a simple formula

$$\Delta \simeq 2\sum_{j=2}^{N} \text{Im}\left(\frac{g_j}{g_1}\right) \cdot \text{Im}\left(\frac{< f|H_j|i >}{< f|H_1|i >}\right) \quad . \tag{1.21}$$

However, in reality, it is very difficult to compute the imaginary part of the ratios of the decay matrix elements.

We will now demonstrate the equivalence of total decay rates of a particle and its anti-particle. We first note

$$\Gamma(i \to \text{ all}) = N_0 \sum_f |< f|H_W|i >|^2 \delta(E_i - E_f) \tag{1.22a}$$

$$\Gamma(\bar{i} \to \text{ all}) = N_0 \sum_f |< \bar{f}|H_W|\bar{i} >|^2 \delta(E_i - E_f) \tag{1.22b}$$

for a normalization constant N_0. Then, Eq. (1.15b) leads to

$$\Gamma(\bar{i} \to \text{ all}) - \Gamma(i \to \text{ all}) = 2N_0 \sum_{j,k=1}^{N} \text{Im}(g_j g_k^*) \; \text{Im}\left(< i|H_k^\dagger \delta(E_i - H_0) H_j|i >\right) \tag{1.23}$$

where H_0 stands for the strong QCD Hamiltonian. Now, the time-reversal invariance of $H_j (j = 1, 2 \dots, N)$ and H_0 implies the validity of

$$\begin{aligned} &< i|H_k^\dagger \delta(E_i - H_0) H_j|i > \\ &= < \hat{i}|H_j^\dagger \delta(E_i - H_0) H_k|\hat{i} > \\ &= < i|H_j^\dagger \delta(E_i - H_0) H_k|i > \end{aligned} \tag{1.24}$$

where we utilized the rotation-invariance of theory for the 2nd equation to replace the time-reversed state $|\hat{i}>$ by $|i>$. Then, from Eqs. (1.23) and (1.24), we find

$$\Gamma(\bar{i} \to \text{ all}) = \Gamma(i \to \text{ all}) \quad . \tag{1.25}$$

Although we have derived this result on the basis of Eqs. (1.8) and (1.9), we can actually dispense with the assumption. Its validity depends[5),6)] only upon the CPT invariance of H_W. However, we will not go into detail.

In sections 2 and 3, we will study some consequences of CP-violations for $D^{\pm}$ and $B^{\pm}$ decays. We note that some two-body CP-violations for B-decays have been studied[9),10)] by some authors.

2. CP-Violations in $D^{\pm}$ Decays

Let S be the strangeness quantum number of the s-quark, and write for simplicity

$$(\bar{q}_1 q_2)(\bar{q}_3 q_4) = :\ (\bar{q}_1 \gamma_\mu (1+\gamma_5) q_2)(\bar{q}_3 \gamma^\mu (1+\gamma_5) q_4) : \tag{2.1}$$

for the effective V-A 4-fermi interaction with the maximal C and P violations satisfying the condition Eq. (1.9). Then, H_W is given by

$$H_W = H_W(\Delta S = 0) + H_W(\Delta S = 1) + H_W(\Delta S = -1) \tag{2.2}$$

where we have

$$H_W(\Delta S = 1) = \frac{1}{\sqrt{2}}\ G\ V_{us} V_{cd}^{*} (\bar{u} s)(\bar{d} c) + h.c. \tag{2.3a}$$

$$H_W(\Delta S = -1) = \frac{1}{\sqrt{2}}\ G\ V_{ud} V_{cs}^{*} (\bar{u} d)(\bar{s} c) + h.c. \tag{2.3b}$$

for $\Delta S = \pm 1$ with $N = 1$. Especially, these processes are essentially CP-conserving. The effective decay Hamiltonian for $\Delta S = 0$ is written as

$$H_W(\Delta S = 0) = \sum_{j=1}^{3} \left(g_j H_j + g_j^{*} H_j^{\dagger} \right) + \left(H_W^{(S)} + H_W^{(P)} + h.c. \right) \tag{2.4}$$

where we have set

$$\sum_{j=1}^{3} g_j H_j = \frac{1}{\sqrt{2}}\ G \left\{ V_{ud} V_{cd}^{*} (\bar{u} d)(\bar{d} c) + V_{us} V_{cs}^{*} (\bar{u} s)(\bar{s} c) + V_{ub} V_{cb}^{*} (\bar{u} b)(\bar{b} c) \right\} \tag{2.5}$$

while $H_W^{(S)}$ and $H_W^{(P)}$ are self-energy and Penguinn diagram[10] terms, respectively, which correspond to Fig. 1 and 2. Contrarily, Eq. (2.5) is the result of the familiar W-exchange diagram Fig. 3.

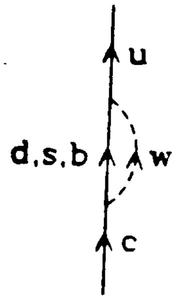

Fig. 1

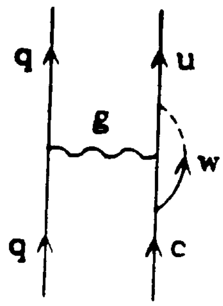

Fig. 2

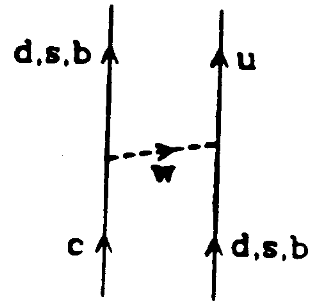

Fig. 3

In Fig. 2, the wavy line g stands for the gluon propagator and q can be any of u, d, and s-quarks. Then, $H_W^{(P)}$ is non-local with quark structure of $(\overline{q}\gamma_\mu q)\cdot\{\overline{u}(a+b\gamma_5)\gamma^\mu c\}$.

With respect to $H_W^{(S)}$, we must first subtract all divergences inherent in the Fig. 1. The remaining finite term will lead then to a form of

$$\begin{aligned} H_W^{(S)} &= \overline{u}(g_4+g_4'\gamma_5)c+\overline{u}(g_5+g_5'\gamma_5)i\gamma^\mu\partial_\mu c \\ &+\partial_\mu\overline{u}(g_6+ig_6'\gamma_5)\partial^\mu c \\ &+\ldots\ldots \quad . \end{aligned} \tag{2.6}$$

Actually, there exist some ambiguities for exact values of these coupling parameters, depending upon how much we should subtract finite terms. We could for instance subtract all kinematical and mass terms, so that we may set $g_4=g_4'=g_5=g_5'=0$, if we wish.

Now, the Hamiltonian H_3 proportional to $(\overline{u}b)(\overline{b}c)$ in Eq. (2.5) will give negligible contribution for decays of D-mesons, because of the quark-line rule[11)] since it will involve the annihilation diagram of $b\overline{b}\to d\overline{d}$, $u\overline{u}$, or $s\overline{s}$. Similarly, contributions from $H_W^{(S)}$ and $H_W^{(P)}$ for D-decays appear to be small[10] for the decay of D mesons. Then, we estimate

$$\Delta \simeq 2\ s_{13}\sin\delta_{13}\ R \quad , \tag{2.7a}$$

$$R\simeq\frac{\mathrm{Im}(<f|H_1|i><f|H_2|i>^*)}{|<f|H_1|i>-<f|H_2|i>|^2} \tag{2.7b}$$

for CP-violating $\Delta S = 0$ reactions such as $D^+ \to K^+\overline{K}^0$ and $D^- \to K^-K^0$ with $H_1 = (\overline{u}d)(\overline{d}c)$ and $H_2 = (\overline{u}s)(\overline{s}c)$. Noting $s_{13} \simeq (2 \sim 7) \times 10^{-3}$, and assuming the value of $R \sin\delta_{13} \simeq 0.2$, this would give $\Delta \simeq 10^{-3}$ at most.

In spite of this likely small value for CP-violation, we will proceed now to some explicit examples.

(i) Two Body Decays

We expect to have

$$\Gamma(D^+ \to \pi^+\pi^0) \neq \Gamma(D^- \to \pi^-\pi^0) \tag{2.8a}$$

$$\Gamma(D^+ \to K^+\overline{K}^0) \neq \Gamma(D^- \to K^-K^0) \quad . \tag{2.8b}$$

However, we will have

$$\Gamma(D^+ \to K^0\pi^+) = \Gamma(D^- \to K^0\pi^-) \tag{2.9a}$$

$$\Gamma(D^+ \to \overline{K}^0\pi^+) = \Gamma(D^- \to K^0\pi^-) \tag{2.9b}$$

since decay modes for $\Delta S = \pm 1$ are essentially CP-conserving. Note that the decay $D^+ \to K^0\pi^+$ is one of the dominant Cabibbo allowed mode.

(ii) Three-Body Decays

We will have

$$\Gamma\left(D^+ \to \pi^+\pi^+\pi^-\right) \neq \Gamma\left(D^- \to \pi^-\pi^-\pi^+\right) \quad . \tag{2.10}$$

Moreover, the Dalitz plot for $D^+ \to \pi^+\pi^+\pi^-$ will differ from that of $D^- \to \pi^-\pi^-\pi^+$. However, we must have

$$\Gamma\left(D^+ \to K^+\pi^+\pi^-\right) = \Gamma\left(D^- \to K^-\pi^-\pi^+\right) \tag{2.11}$$

for the dominant Cabibbo-allowed decay with the identical Dalitz plots for decay modes of both sides. We note that if the equalities in Eqs. (2.9) and (2.11) are experimentally found to be violated, then it would imply that the present minimum standard model must be somehow modified.

(iii) 4-Body Decay

Again, we expect

$$\Gamma\left(D^+ \to \pi^+\pi^+\pi^-\pi^0\right) \neq \Gamma\left(D^- \to \pi^-\pi^-\pi^+\pi^0\right) \tag{2.12}$$

but

$$\Gamma\left(D^+ \to K^0\pi^+\pi^+\pi^-\right) = \Gamma\left(D^- \to \overline{K}^0\pi^-\pi^-\pi^+\right) \quad . \tag{2.13}$$

One interesting fact about 4-body decays is that we can experimentally test separate C and P invariances. To see it, let

$$M\left(D^+ \to \pi^+\pi^+\pi^-\pi^0\right) = A + B(\underline{k}_1 \times \underline{k}_2)\cdot\underline{k}_3 \quad , \tag{2.14a}$$

$$M\left(D^- \to \pi^-\pi^-\pi^+\pi^0\right) = \overline{A} + \overline{B}(\underline{k}_1 \times \underline{k}_2)\cdot\underline{k}_3 \quad , \tag{2.14b}$$

where $\underline{k}_1$, $\underline{k}_2$, and $\underline{k}_3$ are momenta of 3 charged pions $\pi^\pm$, $\pi^\pm$, and $\pi^\mp$, respectively in the rest frames of $D^\mp$. Also, A, $\overline{A}$, B and $\overline{B}$ are some scalar functions of the products $\underline{k}_i \cdot \underline{k}_j$ $(i,j = 1,2,3)$. Note that the Bose statistics requires that A and $\overline{A}$ are symmetric for the exchange of $\underline{k}_1 \leftrightarrow \underline{k}_2$, while B and $\overline{B}$ must change their signs under the exchange. Now, if the CP-invariance holds, then we must have

$$\overline{A} = -A \quad , \quad \overline{B} = B \tag{2.15}$$

while the C-invariance requires the validity of

$$\overline{A} = A \quad , \quad \overline{B} = B \quad . \tag{2.16}$$

Further, the P-invariance demands

$$\overline{A} = A = 0 \quad . \tag{2.17}$$

Note that $(\underline{k}_1 \times \underline{k}_2) \cdot \underline{k}_3$ plays the role of the spin-dependent term $\underline{\sigma} \cdot \underline{k}$ in the $\Sigma^+ \to p\pi^0$ decay[5]. Similarly, comparison of decays $D^+ \to \pi^+\pi^+\pi^-K^0$ and $D^- \to \pi^-\pi^-\pi^+\overline{K}^0$ will reveal possibly large P and C violations, even if CP is conserved.

3. CP-Violations in $B^\pm$ Decays

Let C and S be the charm and strangeness quantum numbers, respectively. Then, the effective Hamiltonian for B-decays will be represented by any of $H_W^{(j)} (j = 1, 2, \ldots, 6)$ below:

(i) $\Delta S = \Delta C = 0$

$$H_W^{(1)} = \frac{1}{\sqrt{2}} G\{V_{ud}^* V_{ub}(\bar{d}u)(\bar{u}b) + V_{cd}^* V_{cb}(\bar{d}c)(\bar{c}b) + V_{td}^* V_{tb}(\bar{d}t)(\bar{t}b)\} + H_W^{(S)} + H_W^{(P)} + h.c. \tag{3.1}$$

where $H_W^{(S)}$ and $H_W^{(P)}$ are again self-energy and Penguinn-diagram terms specified by Figs. 4 and 5, respectively, and q in Fig. 5 may assume any of u, d, s, and c.

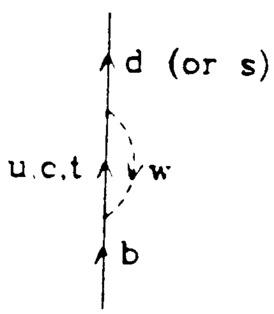

Fig. 4

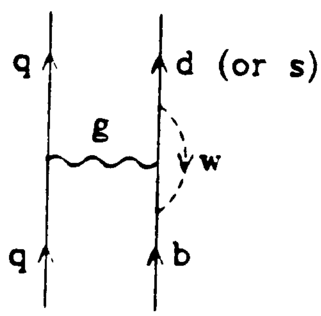

Fig. 5

Especially, $H_W^{(S)}$ will have the same form as Eq. (2.6) by replacing $c \to b$ and $u \to d$.

(ii) $\Delta S = 1,\ \Delta C = 0$

$$H_W^{(2)} = \frac{1}{\sqrt{2}} G\{V_{us}^* V_{ub}(\bar{s}u)(\bar{u}b) + V_{cs}^* V_{cb}(\bar{s}c)(\bar{c}b) + V_{ts}^* V_{tb}(\bar{s}t)(\bar{t}b)\} + H_W^{\prime(S)} + H_W^{\prime(P)} + h.c. \tag{3.2}$$

which has the same form as $H_W^{(1)}$ if we replace the symbol d there by s.

(iii) $\Delta S = 0,\ \Delta C = 1$

$$H_W^{(3)} = \frac{1}{\sqrt{2}}\, G\ V_{ud}^* V_{cb} (\overline{d}u)(\overline{c}b) + h.c. \tag{3.3}$$

(iv) $\Delta S = 0,\ \Delta C = -1$

$$H_W^{(4)} = \frac{1}{\sqrt{2}}\, G\ V_{cd}^* V_{ub} (\overline{d}c)(\overline{u}b) + h.c. \tag{3.4}$$

(v) $\Delta S = -\Delta C = 1$

$$H_W^{(5)} = \frac{1}{\sqrt{2}}\, G\ V_{cs}^* V_{ub} (\overline{s}c)(\overline{u}b) + h.c. \tag{3.5}$$

(vi) $\Delta S = \Delta C = 1$

$$H_W^{(6)} = \frac{1}{\sqrt{2}}\, G\ V_{us}^* V_{cb} (\overline{s}u)(\overline{c}b) + h.c. \quad . \tag{3.6}$$

We note that $H_W^{(1)}$ and $H_W^{(2)}$ are CP-violating while all other terms $H_W^{(3)}$, $H_W^{(4)}$, $H_W^{(5)}$ and $H_W^{(6)}$ are essentially CP-conserving. Moreover, $H_W^{(3)}$ is the dominant Cabibbo-allowed term while all others are Cabibbo-suppressed.

In contrast to the case of the D-decay, the Penguinn term $H_W^{(P)}$ in both $H_W^{(1)}$ and $H_W^{(2)}$ can be large,[10] because of the large t-quark mass. However, since its matrix element is difficult to estimate, we will consider only the first two terms in Eqs. (3.1) and (3.2). Then, the analysis of the degree of CP-violations for the present case proceeds as in the previous D-case. Consider decay modes such as $B^+ \to D^+\overline{D}^0$, $B^+ \to D^+D^-\pi^+$ and so on. Then, the ratio

$$R = \frac{< f|H_1|i >}{< f|H_2|i >} = \frac{< f|(\overline{d}u)(\overline{u}b)|i >}{< f|(\overline{d}c)(\overline{c}b)|i >} \tag{3.7}$$

is expected to be small in view of the fact that the quark-diagram for $< f|H_1|i >$ requires to pick up $c\overline{c}$ pair from the vacuum sea. Therefore, interchanging the role of H_1 and H_2, the formula Eq. (1.21) will lead now to

$$\Delta \simeq 2 \text{ Im } \left(\frac{g_1}{g_2}\right) \cdot \text{ Im } R \simeq \frac{2s_{13}}{s_{12}} \sin\delta_{13} \text{ Im } R \simeq (2 \sim 7) \times 10^{-2} \sin\delta_{13} \cdot \text{ Im } R \qquad (3.8)$$

Assuming a tentative value of $\sin\delta_{13} \cdot \text{ Im } R \simeq 10^{-2}$, this will give

$$\Delta \simeq (0.2 \sim 0.7) \times 10^{-3} \quad . \qquad (3.9)$$

Thus, a more promising prospect would be to include the contribution from the interference of Penguinn diagram with H_1 and H_2, which may be considerably more large,[10] for example, for $B^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decays.

At any rate, possible CP-violations due to $H_W^{(1)}$ will be found in

$$\Gamma(B^+ \to K^+\overline{K}^0) \neq \Gamma(B^- \to K^- K^0) \qquad (3.10a)$$

$$\Gamma(B^+ \to \pi^+\pi^+\pi^-) \neq \Gamma(B^- \to \pi^-\pi^-\pi^+) \qquad (3.10b)$$

$$\Gamma(B^+ \to D^+\overline{D}^0) \neq \Gamma(B^- \to D^- D^0) \qquad (3.10c)$$

etc. with inequivalent Dalitz plots for three body decays, while $H_W^{(2)}$ predicts

$$\Gamma(B^+ \to K^0\pi^+) \neq \Gamma(B^- \to \overline{K}^0\pi^-) \qquad (3.11a)$$

$$\Gamma(B^+ \to K^+\psi/J) \neq \Gamma(B^- \to K^-\psi/J) \qquad (3.11b)$$

$$\Gamma(B^+ \to K^0\pi^+\pi^-) \neq \Gamma(B^- \to \overline{K}^0\pi^-\pi^+) \quad . \qquad (3.11c)$$

All these decays are Cabibbo-suppressed. However, for the dominant Cabibbo-allowed mode $H_W^{(3)}$, we will have

$$\Gamma\left(B^+ \to D^+\pi^+\pi^-\right) = \Gamma\left(B^- \to D^-\pi^-\pi^+\right) \quad . \qquad (3.12)$$

with identical Dalitz plots.

We can also study separate C and P violations for 4-body decays $B^\pm \to D^\pm\pi^+\pi^-\pi^0$ just as in $D^\pm$-decays.

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