AD

UTHEP 272

su 9416

DOE/ER/40427-24-N93
UTHEP-272

Transverse Quark Distribution in Mesons - QCD Sum Rule Approach -

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Abstract

QCD sum rules are used to compute the first few moments of the mesonic quark momentum. Transverse, longitudinal and mixed transverse-longitudinal components are examined. The transverse size of the pion is shown to be dictated by the gluon condensate, even though the mass and the longitudinal distribution are dominated by the quark condensate. The implications of our results for color transparency physics and finite temperature QCD are discussed.

PACS numbers:13.40.-f,13.40.Fn,12.38.Lg
to appear in Physical Review Letters

Mesonic wave functions have been the focus of much recent theoretical activity regarding exclusive processes [1, 2, 3] and color transparency [4]. In the former case, the asymptotic formula of exclusive process can be expressed, via factorization, in terms of a hard scattering amplitude T_H and the quark distribution function $\phi(\xi, Q^2)$ [5] (ξ is the light-cone momentum fraction of quarks and Q^2 is the typical external momentum). However, there have been many suggestions that such formulae are not valid at presently accessible energy ranges [6, 7]. Furthermore, computations of form factors often employ a wave function with a spread of $\vec{b}$ (the transverse separation between quarks) [1, 8]. Sterman and collaborators have computed the Sudakov effects [9] for the pion and proton form factors[1, 10]. This involves "wave function" of the form $e^{S(b,\xi,Q^2)}$. The Sudakov effect should dominate at high Q^2, but it would be surprising if the b-dependence resides only in the function S.

The transverse size dependence plays also a key role in color transparency physics. A Gaussian dependence on the transverse momentum $k_\perp^2$ ($\vec{k}_\perp$ is canonically conjugate to $\vec{b}$) implies that the effects of color transparency would never be observed, but a power fall off does allow such observations [4]. Furthermore, QCD lattice calculations [11] now provide hadronic wave functions, and understanding these with analytic techniques would be useful.

The use of QCD sum rules (QSR) has been a particularly useful example of such an analytic technique. QSR have long been known to provide reasonable estimates of hadronic properties [12] and distribution functions needed as inputs to perturbative QCD (PQCD) calculations. In particular, Chernyak and Zhitnitsky [13, 14] have determined ξ^{2n} moments of the distribution function $\phi(\xi)$.

The present paper, concerns a new application of QCD sum rules(QSR): the determination of the $\xi^{2l}k_\perp^{2n}$ moments of mesonic wave functions. We derive new Borel sum rules for these moments, which makes us possible to extract physical picture of the quark momentum distributions. (For an application of a finite energy sum rule, see ref. [15].) The starting point is the Bethe-Salpeter amplitude for the pion

$$\begin{aligned} if_\pi p_\alpha \phi_\pi(z\cdot p, z^2)_\mu + z_\alpha \chi_\pi(z\cdot p, z^2)_\mu &= \langle 0|\bar{d}(z)\gamma_\alpha\gamma^5 e^{ig\int_{-z}^{z} d\sigma\cdot A} u(-z)|\pi^+(p)\rangle_\mu \\ &= \sum_n \frac{(i)^n}{n!}\langle 0|\bar{d}(0)\gamma_\alpha(iz\cdot \overleftrightarrow{D})^n u(0)|\pi^+(p)\rangle_\mu, \end{aligned} \tag{1}$$

where ϕ_π and χ_π represent two different Bethe-Salpeter amplitudes, $i\overleftrightarrow{D} = i\vec{D} + (i\vec{D})^\dagger$, $\vec{D} = \vec{\partial} - ig\vec{B}^a\frac{\lambda_a}{2}$, μ is the renormalization scale and f_π is the pion decay constant. First let us introduce a light-like four vector y and a transverse vector $t = (0, \vec{b}, 0)$ which is perpendicular to the hadron momentum $p = (p^0, \vec{0}, p_z)$. Then we make a decomposition $z = y + t$ so that $z^2 = -b^2$, and $z\cdot p = y\cdot p$. Since we have chosen z to have both longitudinal and transverse components, the matrix element in eq.(1) can be further expanded in terms of y and $-b^2$. The result is that

$$\langle 0|\bar{d}(0)y^\alpha\gamma_\alpha(iz\cdot \overleftrightarrow{D})^n u(0)|\pi^+(p)\rangle_\mu = if_\pi\sum_{l=0}^{n}(y\cdot p)^{n-l+1}(-b^2)^{\frac{l}{2}}M_{n-l,l}, \tag{2}$$

after contracting with y^α, and where the normalization is $M_{0,0} = 1$. Since $y\cdot z = 0$, the contraction of y^α with eq. (1) enables us to extract ϕ_π on which we focus our attention in this paper. $M_{n-l,l}$ are related to the matrix elements with $n-l$ covariant derivatives in the longitudinal direction and l in the transverse direction. Both n and l are even numbers. The moments

$M_{n-l,l}$ can be related to spin $n+1$ and twist $l+2$ matrix elements by identifying independent symmetric traceless matrices. For example, $-if_\pi M_{0,2}p_\mu = \frac{5}{9}i\langle 0|\bar{d}\gamma_\mu\gamma_5\sigma_{\alpha\beta}gG^{\alpha\beta}u|\pi(p)\rangle$.

The function $\phi_\pi(z\cdot p, z^2)_\mu$ depends on coordinates; the relation to the momentum space wave function is given by

$$\phi_\pi(y\cdot p, -b^2)_\mu \equiv \int_{-1}^{1} d\xi \int^{\mu^2} d^2k_\perp e^{i\xi y\cdot p} e^{-i\vec{k_\perp}\cdot\vec{b}} \phi_\pi(\xi, k_\perp^2)_\mu, \tag{3}$$

while the light-cone quark distribution function is given by

$$\phi_\pi(\xi)_\mu = \int d^2k_\perp \theta(\mu^2 - k_\perp^2)\phi_\pi(\xi, k_\perp^2)_\mu. \tag{4}$$

Expanding the integrand of eq.(3) as a power series in $y\cdot p$ and $(b^2)^{\frac{1}{2}}$ leads to

$$\phi_\pi(y\cdot p, -b^2)_\mu = \sum_n \frac{(i)^n}{n!}\sum_{l=0}^{n}(y\cdot p)^{n-l}(b^2)^{\frac{l}{2}} \;\; {}_nC_l\Gamma_l\langle\xi^{n-l}k_\perp^l\rangle_\mu \tag{5}$$

where

$$\langle\xi^{n-l}k_\perp^l\rangle_\mu = \int_{-1}^{1}\int d^2k_\perp \xi^{n-l}k_\perp^l \phi_\pi(\xi, k_\perp^2)_\mu, \tag{6}$$

$\Gamma_l \equiv \Gamma(1/2 + l/2)/(\Gamma(1/2)\Gamma(1+l/2))$ and ${}_nC_l = \frac{n!}{(n-l)!l!}$. The desired moments are now identified by using eqs. (5) and eq.(2) in the contraction of eq. (1) with y^α and equating the coefficients of the terms in the expansion. The result is

$$< \xi^{n-l}k_\perp^l >= \frac{1}{{}_nC_l\Gamma_l} M_{n-l,l}(-1)^{l/2}. \tag{7}$$

We proceed by using QCD sum rules to extract $M_{n-l,l}$ from eq.(2) and thereby obtain the moments from eq.(7). Consider the correlation function $T_{n,0}$,

$$T_{n,0}(y\cdot q, -b^2, q^2) = i\int d^4x e^{iqx}\langle 0|T[\bar{d}(x)y^\alpha\gamma_\alpha\gamma_5(z\cdot \overleftrightarrow{D})^n u(x), \bar{u}(0)y^\alpha\gamma_\alpha\gamma_5 d(0)]|0\rangle. \tag{8}$$

The complete sum over intermediate states is approximated by the lowest mass pion pole term and a continuum contribution. Then the spectral density can be expressed as

$$\frac{1}{\pi}ImT_{n,0}(y\cdot q, -b^2, q^2) = \sum_{l=0}^{n} f_\pi^2(y\cdot q)^{n-l+2}(-b^2)^{\frac{l}{2}}\left(M_{n-l,l}\delta(q^2 - m_\pi^2) + c(n,l)\theta(q^2 - S_0)\right), \tag{9}$$

where $c(n,l)$ are coefficients chosen to match the perturbative part in the Operator Product Expansion (OPE) at $Q^2 \to \infty$. We have carried out the OPE for $T_{n,0}$ for operators of dimension less than or equal to six:

$$\begin{aligned} T_{n,0}(y\cdot q, -b^2, q^2) &= \sum_{l=0}^{n} {}_nC_l\Gamma_l(y\cdot p)^{n-l+2}(b^2)^{l/2}(q^2)^{l/2}[-C_{n,l}^{pert}\ln(-q^2) \\ &\quad -(C_{n,l}^1\ln(-q^2) + C_{n,l}^2)\langle\frac{\alpha}{\pi}G^2\rangle\frac{1}{q^4} - C_{n,l}^4(-1)^{l/2}\langle\sqrt{\alpha}\bar{q}q\rangle^2\frac{1}{q^6}], \end{aligned} \tag{10}$$

where

$$C_{n,l}^{pert} = \frac{3}{16\pi^2}(-1)^n B(\frac{n-l}{2}+\frac{1}{2},\frac{l}{2}+2),$$

$$C_{n,l}^{1} = \frac{1}{24}B(\frac{n-l}{2}+\frac{1}{2},\frac{l}{2}+1)(\frac{l}{2})(\frac{l-2}{2})$$
$$[1+\theta(n-2-l)(n-l)\frac{l+2}{2}+\theta(l-2)l\frac{l+3}{2}],$$

$$C_{n,l}^{2} = \frac{1}{24}B(\frac{n-l}{2}+\frac{1}{2},\frac{l}{2}+1)(l-1)$$
$$[1+\theta(n-2-l)(n-l)\frac{l+2}{2}+\theta(l-2)l\frac{l}{2}\frac{l+1/2}{l-1}],$$

$$C_{n,l}^{4} = +\delta_{l,0}\frac{32\pi}{81}(4n+11)+\delta_{l,2}\frac{2\pi}{9_n C_l \Gamma_l}n(n-1), \tag{11}$$

with $B(n,l) = \Gamma(n)\Gamma(l)/\Gamma(n+l)$ and $\theta(n) = 1$ for $n \geq 0$. The OPE for the ρ meson is as above except for the four quark condensate term $C_{n,l}^4$ for which $(4n+11) \to (4n-7)$.

We may immediately examine the physical consequences of these expressions. The gluon condensates arising from the covariant derivative in eq.(8) give the terms proportional to θ's in $C_{n,l}^1$ and $C_{n,l}^2$. These terms together with the other gluon condensate become increasingly important as l increases. Conversely, one of the four quark condensates with hard gluon line (Fig. 3 of [13]), which gives the dominant power correction for the $l = 0$ sum rules, does not contribute at all for $l \neq 0$. This is because the internal gluon line in the relevant tree graph carries no transverse momentum. Thus the power correction is dominated by the gluon condensate and the quark condensate plays only a minor role. This is opposite to the usual sum rules for longitudinal moments ($l = 0$) and for the meson masses where the quark condensate term is essential. In the constituent quark picture of hadrons, the mass of the constituent quark is obtained through the dynamical breaking of chiral symmetry and the transverse wave function is determined by the confining force between the constituent quarks. This picture is consistent with our observations about the OPE results.

To go beyond these qualitative aspects we follow the QSR standard procedure and, equate the phenomenological side eq.(9) to the OPE side eq.(10) using the the standard Borel transformed dispersion relation on q^2, $\frac{1}{\pi}\int ds e^{-s/M^2}\mathrm{Im}T_{n,0}(s) =$ Borel trans[Re$T_{n,0}$], and identify the coefficients of the double expansion. The result is

$$f_\pi^2\langle\xi^n k_\perp^l\rangle = C_{n+l,l}^{pert}(-\frac{d}{d(1/M^2)})^{l/2}M^2(1-e^{-s/M^2})$$
$$+\theta(2-l)C_{n+l,l}^2(-1)^{l/2+1}\frac{1}{(\frac{2-l}{2})!}(M^2)^{l/2-1}\langle\frac{\alpha}{\pi}G^2\rangle$$
$$+\theta(l-4)C_{n+l,l}^1(\frac{l-4}{2})!(M^2)^{l/2-1}\langle\frac{\alpha}{\pi}G^2\rangle$$

$$+\theta(2-l)C^4_{n+l,l}(-1)^l\frac{1}{(\frac{4-l}{2})!}(M^2)^{l/2-2}\langle\sqrt{\alpha}\bar{q}q\rangle^2 \quad . \tag{12}$$

The above sum rule reduces to the sum rule for f_π [12] for $n = l = 0$ and to the longitudinal sum rule [13, 14] for $l = 0$. We will use the physical value $f_\pi = 133$ MeV and derive each moment for different value of S_0. For the vacuum condensates, we use the standard values [12]:

$$\langle\sqrt{\alpha}\bar{u}u\rangle^2 \simeq 1.83\cdot 10^{-4}\ \ \mathrm{GeV}^6, \qquad \langle\frac{\alpha}{\pi}G^2\rangle \simeq 1.2\cdot 10^{-2}\ \ \mathrm{GeV}^4. \tag{13}$$

The outline of the method used to analyze the sum rule is the following:

1. Find the Borel window, i.e., find M^2_{min} such that the total power correction is less than 30 % of the perturbative part. This will insure that the neglected higher dimensional operators cannot be large for $M > M_{min}$. We then choose M^2_{max} arbitrarily say $M^2_{max} = M^2_{min} + 0.3$ or $0.4\mathrm{GeV}^2$.

2. Find the value of S_0 which minimizes the dependence on M^2. Then the extracted moments will not depend too much on the exact choice of M^2_{max}. Since we do not take into account the a_1-meson explicitly in eq.(9), our S_0 should be regarded as an *effective* threshold in the pseudo-vector channel. This is the reason why we obtain a relatively small value S_0 in Table 1.

3. The physical quantity is then obtained from the average over the Borel window M^2.

Chernyak and Zhitnitsky used a similar procedure for the case of $l = 0$.

The form of the Wilson coefficients dictates the characteristic behavior of the sum rule as a function of the variables n, l. **Increasing n for fixed l** generally enhances the coefficient of both the gluon and 4-quark condensate in the OPE. Then the Borel window (range of M^2) appears at higher values of M^2 and the value of S_0 increases. Note also that the power correction becomes large as n increases, so that the sum rule is not reliable for $n > 6$. **Increasing l for fixed n** generally enhances the coefficient of the gluon condensate but will reduce that of the 4-quark condensate. In particular, there is no 4-quark condensate for $l > 2$. This competing effect tends to keep the Borel window fixed or moves it to slightly higher values of M^2. For $l > 6$, the sum rule is again sensitive to S_0 and one cannot make reliable estimates. The value of S_0 must be reduced compared to other moments to reduce the M^2 dependence.

Table 1 shows the result of our analysis. The moments for $l = 0$ correspond to the values obtained in ref.[13, 14]. The main sources of errors in Table 1 are the contribution from higher dimensional operators and the uncertainty in the exact value of S_0. The first issue is related to Mikhailov and Radyushkin's (MR) [16] criticism of ref.[13, 14]. MR stress the importance of including non-local condensates [17]. Although, there are some ambiguities associated with modeling different types of non-local condensates, we take the MR results as a guide to estimate the uncertainties of our calculation. The Gaussian model of MR leads to longitudinal moments about 50 % smaller than that of ref.[13, 14]. We expect similar behavior to be true for sum rules with $l \neq 0$ and estimate the uncertainty associated with unknown higher dimensional operators to be -50 % for all moments. A second source of errors is the lack of knowledge of S_0 which strongly influences the knowledge of moments with $l \neq 0$. For example, increasing the value of S_0 to its value for $l = 0$, causes a 50% increase in the moments. This gives an upper limit for the $l \neq 0$ moments. So overall, the estimated errors for all the moments are ± 50 %.

In the conventional notation, where the separation between quarks is b instead of $2b$, our $n = 0, l = 2$ moment implies an average transverse momentum of $(300\text{MeV})^2$. This is a characteristic hadronic scale and is consistent with a rough estimate $(323\text{MeV})^2$ in ref.[14].

We can use our results to study three features of the wave function; approximate factorization of the longitudinal and transverse directions; power law behavior of the transverse wave function; and the low temperature behavior of the transverse wave function. Let us discuss each one separately.

Here the term "factorization" refers to the property that $\langle \xi^n k_\perp^l \rangle \approx \langle \xi^n \rangle \langle k_\perp^l \rangle$. The results shown in Table 1 are consistent with the factorization property at least for low moments $(n, l \leq 4)$. Although the numerical origin of factorization is natural and can be traced from the expressions provided here, further works should be done to study the validity of the approximate factorization for the wave function $\phi(\xi, k_\perp) = \phi(\xi)\psi(k_\perp)$. [18] A recent calculation [19] of color transparency effects in high energy pion-nucleus scattering assumed this factorization.

The second point is the $k_\perp$ dependence of the transverse wave function. Table 1 shows a large fluctuation of the transverse momentum

$$\Delta \equiv [\langle k_\perp^4 \rangle - \langle k_\perp^2 \rangle^2] / \langle k_\perp^2 \rangle^2 \simeq 8 \quad . \tag{14}$$

We can then determine which of two popular forms in the factorized wave function, a Gaussian $\psi_G(k_\perp) = A\text{exp}(-k_\perp^2/m^2)$ and a power law $\psi_{PL}(k_\perp) = A\frac{1}{(k_\perp^2+m^2)^2}$, has a value of Δ consistent with eq.(14). One may use the ψ's to calculate Δ; it is first necessary to introduce a value of μ^2 (see eqs. (3) and (4)) which is expected to be of the order of 1 GeV^2. For the Gaussian wave function $\psi_G(k_\perp)$, $\Delta_\text{G} \sim 1 - O(exp(-\mu^2/m^2))$. The exponential term can be estimated by reproducing the value of $\langle k_\perp^2 \rangle$ shown in Table 1. For μ^2 larger than 1 GeV^2, the $O(exp(-\mu^2/m^2))$ turns out to be always less than 0.1. For the power law, we can again determine μ^2 by reproducing the value of $\langle k_\perp^2 \rangle$. Then $\Delta_\text{PL} = 28$ for $\mu^2 = 1$ GeV^2 and $\Delta_\text{PL} = 6$ for $\mu^2 = 2$ GeV^2 and always larger than 5 for larger values of μ^2. So the sum rule values for Δ are more consistent with a power law type of transverse wave function even if we take into account the estimated errors. More generally, the concept of a large value of Δ is consistent with the notion of significant fluctuations of the momentum, a property that favors the possibility that color transparency would occur [4].

Another interesting question is the low temperature generalization of our moment sum rules. This generalization is achieved by applying the low temperature pion gas approximation to the thermal expectation value of the hadronic correlation functions.[20, 21] The use of soft pion theorems lead to the result that to lowest order in $\epsilon = T^2/6f_\pi^2$, the axial vector and vector correlation functions at finite temperature can be expressed in terms of a linear combination of vector and axial vector vacuum correlation functions with temperature dependent residues [20], such that the only effect at finite temperature is a renormalization of $f_\pi^T = f_\pi(1 - \epsilon/2)$. We find that the same is true for the transverse moments, i.e.,

$$M_{0,l}^A(T) = (1 - \epsilon)M_{0,l}^A(T = 0) + \epsilon M_{0,l}^V(T = 0)$$

$$M^V_{0,l}(T) = (1-\epsilon)M^V_{0,l}(T=0) + \epsilon M^A_{0,l}(T=0), \tag{15}$$

where the superscripts V and A represent moments for the vector and axial vector currents. So the change at finite temperature can all be accommodated by a change in f_π with no change in the transverse moments. This suggests that to lowest order in T, the wave functions do not change when the temperature is increased from zero to a small value.

In summary, we have derived sum rules for the transverse and longitudinal moments of the pion (and the ρ meson) wave function. These results can be used to understand the role of the quark wave functions in exclusive processes and at finite temperature.

We thank M. Chu, T. DeGrand, R. Gupta, and J. Negele for useful discussions at the Institute for Nuclear Theory, Univ. of Washington. This work was supported in part by the US Department of Energy under grant DE-FG06-88ER40427.

$\langle \xi^n k_\perp^l \rangle$	$l=0$			$l=2$			$l=4$		
	moments	M^2_{min} (GeV^2)	S_0 (GeV^2)	moments (GeV^2)	M^2_{min} (GeV^2)	S_0 (GeV^2)	moments (GeV^4)	M^2_{min} (GeV^2)	S_0 (GeV^2)
$n=0$	1	0.6	0.9	0.36	0.8	0.5	1.17	1.2	0.4
$n=2$	0.4	1.3	1.9	0.14	1.3	0.9	0.30	1.5	0.5
$n=4$	0.24	1.9	2.8	0.09	1.9	1.4	0.20	2.0	0.5

Table 1: Mixed moments of the quark transverse/longitudinal momentum distribution.

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