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Semileptonic B_c Decays from QCD Sum Rules

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Abstract

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The semileptonic decays of B_c-mesons in the framework of QCD are considered with use of the spectra of the excited state masses of participating mesons. The obtained results are in a good agreement to other calculations carried out within QCD SR. The reasons for the discrepancy observed between the QCD SR data and the quark models are discussed. It is shown how to eliminate these discrepancies using higher QCD corrections.

Аннотация

В.В.Киселев, А.В.Ткабладзе. Полулептонные распады B_c из правил сумм КХД: Препринт ИФВЭ 93-46. – Протвино, 1993. – 14 с., библиогр.: 25.

В рамках ПС КХД рассмотрены полулептонные распады B_c-мезонов с использованием спектра масс возбужденных состояний участвующих в процессе мезонов. Полученые результаты согласуются с другими вычислениями в ПС КХД. Рассмотрены причины расхождения данных ПС КХД и кварковых моделей. Показано, что с учетом высших КХД поправок эти расхождения можно устранить.

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1. Introduction

The B_c meson, composed of two heavy quarks, occupies a prominent place in the B meson physics, which has been intensively discussed recently. Being a nonrelativistic system, like ψ and Υ, the B_c is intermediary between charmonium and bottonium (the mass spectrum and radiative transitions in the family of B_c, see [1], [2]), and so provides a specific possibility for investigating the heavy meson production and decay mechanisms. The available estimates of the B_c production cross section achieved at TeV colliders $\sigma(B_c)/\sigma(b\bar{b}) \approx (2-4)\ 10^{-3}$ [3], [4], [5] point to the possibility to observe B_c in practice [6].

Because of the absence of annihilation decay channels due to the strong and electromagnetic interactions (like $J/\psi \to ggg \to hadrons$, $J/\psi \to \gamma^* \to \mu^+\mu^-$ for J/ψ), the lightest state $(b\bar{c})$ of the system, the pseudoscalar meson $B_c(0^-)$, is long-lived, similarly with ordinary B-mesons with a light quark, since it decays due to the weak interaction. The preliminary estimates of the widths of some decay channels of B_c in the potential model were made in [4], [7] and completed in [8]. A detailed consideration of $B_c$decays in the WSB quark model of the relativistic oscillator [9] was first carried out in [10] and then in [11], where the ISGW quark model of the meson [12] was exploited. Recently, the widths of semileptonic decays of B_c were calculated in [13,14]. The aspects of CP violation in B_c decays were discussed in [15]. The application of the heavy quark spin symmetry to the formfactor calculation in the semileptonic B_c decays was studied in [16]

Alongside with small differences in the partial decay widths in quark models, the current estimates indicate a significant discrepancy between the predictions made on the basis of the QCD sum rules and quark models. For instance, the difference in the estimates of the decay width of $B_c^+ \to \psi e^+ \nu_e$ is of the order of 10 times.

In our earlier consideratoions of B_c-meson we applied the method, allowing one within the QCD sum rules to determine the axial constant f_{B_C} using the data on the mass of radial excited states of B_c [17]. In [14] this very method was applied to determine the formfactors in the transitions $B_c^+ \to \psi(\eta_c)e^+\nu_e$. In [13] the same formfactors are calculated in the framework of Borel sum rules. Although consistent with each other, the estimated widths in [13] and 14 therein differ considerably from the results obtained in other approaches.

In the present paper, our goal is to try to explain the reasons of the observed discrepancy between the predictions of the quark models and QCD SR concerning the widths of semileptonic decays of B_c mesons. In Sect.2 we will describe the method that was applied in [14]. In Sect. 3 the higher corrections to the spectral densities are calculated. Sect. 4 presents numerical results comparing them with other data available.

2. Description of the Method

The hadronic matrix elements of the transitions $B_c^+ \to \psi(\eta_c)e^+\nu_e$ are described as follows

$$< \eta_c(p_2)|V_\mu|B_c(p_1)> \; = \; f_+(p_1+p_2)_\mu + f_- q_\mu \,, \tag{1}$$

$$\frac{1}{i} < J/\psi(p_2)|V_\mu|B_c(p_1)> \; = \; iF_v \epsilon^{\mu\nu\alpha\beta}\epsilon^{*\nu}(p_1+p_2)_\nu q_\mu \,, \tag{2}$$

$$\frac{1}{i} < J/\psi(p_2)|A_\mu|B_c(p_1)> \; = \; F_0^A \epsilon_\mu + F_+^A(\epsilon^* p_1)_\mu + F_-^A(\epsilon^* p_1) q_\mu \,, \tag{3}$$

where $q = p_1 - p_2$ and ϵ_μ is the polarization vector for J/ψ. V_μ and A_μ are the flavor changing vector and pseudoscalar currents. In virtue of the transversality of the lepton current $l_\mu = e\gamma_\mu(1+\gamma_5)\nu$ (in the limit $m_t \to 0$), the probabilities of the decays are independent of f_- and F_-^A. To calculate the formfactors, consider the three-point functions:

$$\Pi_\mu(p_1,p_2,q^2) =$$
$$i^2 \int dxdy e^{i(p_2x-p_1y)} < 0|T\{\bar{c}(x)\gamma_5 c(x), V_\mu(0), \bar{b}(y)\gamma_5 c(y)\}|0> \,, \tag{4}$$
$$\Pi_{\mu\nu}^{V,A}(p_1,p_2,q^2) =$$
$$i^2 \int dxdy e^{i(p_2x-p_1y)} < 0|T\{\bar{c}(x)\gamma_\nu c(x), J_\mu^{V,A}(0), \bar{b}(y)\gamma_5 c(y)\}|0> \,. \tag{5}$$

Isolate the Lorentz structures in the correlators:

$$\Pi_\mu \; = \; \Pi_+(p_1+p_2)_\mu + \Pi_- q_\mu, \tag{6}$$

$$\Pi_{\mu\nu}^V \; = \; i\Pi_V \epsilon_{\mu\nu\alpha\beta} p_2^\alpha p_1^\beta, \tag{7}$$

$$\Pi_{\mu\nu}^A \; = \; i\Pi_V g_{\mu\nu} + \Pi_1^A p_2^\mu p_1^\nu + \Pi_2^A p_1^\mu p_1^n u + \Pi_3^A p_2^\mu p_2^\nu + \Pi_4^A p_1^\mu p_2^\nu. \tag{8}$$

The formfactors f_+, f_V, F_0^A and $F_=A$ will be determined, respectively, from the amplitudes Π_+, Π_V, Π_0^A and $\Pi_+^A = \frac{1}{2}(\Pi_1 + \Pi_2)$. For these one can write the double dispersion representation with an accuracy up to the required subtractions:

$$\Pi_i(p_1^2, p_2^2, q^2) = -\frac{1}{(2\pi)^2} \int \frac{\rho_i(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} ds_1 ds_2, \tag{9}$$

where $Q^2 = -q^2 \geq 0$. The integration region in (9) is determined by the condition:

$$-1 < \frac{2s_1 s_2 + (s_1 + s_2 - q^2)(m_b^2 - m_c^2 - s_1)}{\lambda^{1/2}(s_1, s_2, q^2)\lambda^{1/2}(m_c^2, s_1, m_b^2)} < 1, \tag{10}$$

where $\lambda(x_1, x_2, x_3) = (x_1 + x_2 - x_3)^2 - 4x_1 x_2$.

Let us consider the physical part of the SR. One of the sources of the ambiguity in the SR is the modelling of the hadron spectral density. In the Borel SR, it is generally assumed that this density includes the contribution of the low-lying meson and the hadronic continuum. The latter is approximated by the perturbative part of the spectral function starting from some threshold value s_0. In our approach, on the contrary, we assume $\rho_i(s_1, s_2, Q^2)$ to be saturated by the contribution of the infinite number of narrow resonances. As a result, we can get rid of the parameters s_{01} and s_{02}:

$$\begin{aligned}\rho_+(s_1, s_2, Q^2) = {} & (2\pi)^2 \sum_{i,j=1}^{\infty} f_{B_C}^i \frac{{M_{B_C}^i}^2}{m_b + m_c} f_{\eta_C}^j \frac{{M_{\eta_c}^j}^2}{2m_c} f_+^{ij}(Q^2) \\ & \delta(s_1 - {M_{B_C}^i}^2)\delta(s_2 - {M_{\eta_C}^j}^2)\end{aligned} \tag{11}$$

$$\begin{aligned}\rho_V(s_1, s_2, Q^2) = {} & 2(2\pi)^2 \sum_{i,j=1}^{\infty} f_{B_C}^i \frac{{M_{B_C}^i}^2}{m_b + m_c} \frac{{M_\psi^j}^2}{g_\psi} F_V^{ij}(Q^2) \\ & \delta(s_1 - {M_{B_C}^i}^2)\delta(s_2 - {M_\psi^j}^2)\end{aligned} \tag{12}$$

$$\begin{aligned}\rho_{0,+}^A(s_1, s_2, Q^2) = {} & (2\pi)^2 \sum_{i,j=1}^{\infty} f_{B_C}^i \frac{{M_{B_C}^i}^2}{m_b + m_c} \frac{{M_\psi^j}^2}{g_\psi} F_{0,+}^{ij}(Q^2) \\ & \delta(s_1 - {M_{B_C}^i}^2)\delta(s_2 - {M_{J/\psi}^j}^2).\end{aligned} \tag{13}$$

In the following, we are going to derive the formula for only $f_+(Q^2)$. The obtained result can easily be generalized for other formfactors.

The physical part of $\Pi_\mu(p_1^2, p_2^2, Q^2)$ has the form

$$\Pi_+(p_1^2, p_2^2, Q^2) = -\sum_{i,j=1}^{\infty} f^i_{B_C} \frac{{M^i_{B_C}}^2}{m_b+m_c} f^j_{\eta_C} \frac{{M^j_{\eta_c}}^2}{2m_c} f^{ij}_+(Q^2) \frac{1}{(s_1-p_1^2)(s_2-p_2^2)}. \quad (14)$$

Applying to (9) and (14) two Borel operators, $\hat{L}_{\tau_1}(-p_1^2)$ and $\hat{L}_{\tau_2}(-p_2^2)$, we come to the following SR:

$$\sum_{i,j=1}^{\infty} f^i_{B_C} {M^i_{B_C}}^2 f^j_{\eta_c} {M^j_{\eta_c}}^2 f^{ij}_+(Q^2) e^{-{M^i_{B_C}}^2\tau_1 - {M^j_{\eta_c}}^2\tau_2} =$$

$$= \frac{2(m_b+m_c)m_c}{(2\pi)^2} \int ds_1 ds_2 \rho_+(s_1, s_2, Q^2) e^{-s_1\tau_1 - s_2\tau_2}, \quad (15)$$

where

$$\hat{L}_\tau(x) = \lim_{\substack{n,x\to\infty \\ n/x=\tau}} \frac{x^{n+1}}{n!}\Big(-\frac{d}{dx}\Big)^n_{\cdot},$$

Introduce now the notations:

$$S_i = \sum_{j=1}^{\infty} f^j_{\eta_c} {M^j_{\eta_c}}^2 f^{ij}_+(Q^2) e^{-{M^j_{\eta_c}}^2\tau_2} \quad (16)$$

and transform the left-hand side of (15) by means of the Euler-McLohren formula as[18]

$$\sum_{i=1}^{\infty} f^i_{B_C} {M^i_{B_C}}^2 S_i e^{-{M^i_{B_C}}^2\tau_1} = \int_{M^k_{B_C}}^{\infty} dM^n_{B_C} \frac{dn}{dM^n_{B_C}} f^n_{B_C} {M^n_{B_C}}^2 S_n e^{-{M^n_{B_C}}^2\tau_1}$$

$$+ \sum_{n=0}^{n=k-1} f^n_{B_C} {M^n_{B_C}}^2 S_n e^{-{M^n_{B_C}}^2\tau_1} + \cdots \quad (17)$$

Acting on (15) by the operator $\hat{L}_{\tau_1}((M^k_{B_C})^2)$ and taking into account (17), we get

$$\sum_{j=1}^{\infty} f^j_{\eta_c} {M^j_{\eta_c}}^2 f^{kj}_+(Q^2) e^{-{M^j_{\eta_c}}^2\tau_2} =$$

$$= \frac{2m_c(m_b+m_c)}{(2\pi)^2} \frac{dM^k_{B_C}}{dk} \frac{2}{M^k_{B_C} f^k_{B_C}} \int \rho({M^k_{B_C}}^2, s_2, Q^2) e^{-s_2\tau_2}. \quad (18)$$

Here the following property of the Borel operator has been used:

$$\hat{L}_\tau(x)(x^n e^{-bx}) \to \delta^{(n)}_+(\tau - b)$$

After repeating an analogous procedure for the sum of η_c^l resonances one obtains that

$$f_+^{kl}(Q^2) = \frac{8m_c(m_b + m_c)}{M_{B_C}^k M_{\eta_c}^l f_{B_C}^k f_{\eta_c}^l} \frac{dM_{B_C}^k}{dk} \frac{dM_{\eta_c}^l}{dl} \frac{1}{(2\pi)^2} \rho_+(M_{B_C}^{k\,2}, M_{\eta_c}^{l\,2}, Q^2). \tag{19}$$

The proper choice of k and l-discriminates the transitions between the prescribed resonances. The sought-for $f_+(Q^2)$ is obtained at $k = l = 1$.

Now write down the expressions for the other formfactors:

$$F_V^{kl}(Q^2) = \frac{2(m_b + m_c) g_\psi^l}{M_{B_C}^k M_\psi^l f_{B_C}^k} \frac{dM_{B_C}^k}{dk} \frac{dM_\psi^l}{dl} \frac{1}{(2\pi)^2} \rho_V(M_{B_C}^{k\,2}, M_\psi^{l\,2}, Q^2), \tag{20}$$

$$F_{0,+}^{kl}(Q^2) = \frac{4(m_b + m_c) g_\psi^l}{M_{B_C}^k M_\psi^l f_{B_C}^k} \frac{dM_{B_C}^k}{dk} \frac{dM_\psi^l}{dl} \frac{1}{(2\pi)^2} \rho_{0,+}(M_{B_C}^{k\,2}, M_\psi^{l\,2}, Q^2). \tag{21}$$

Therefore, in the place of the additional parameters s_0, i.e. the continuum thresholds, we use the phenomenological parameters dM_k/dk that are, in fact, the inverse value to the density of the quarkonium states with the given quantum numbers. These parameters may be defined with a rather good precision. The masses of the radial excitations of ψ are known from experiment [19], and for the B_c and η_c systems, composed of heavy quarks, one might use the predictions of the potential models [1], [2].

3. Calculating Spectral Densities

In line with the general ideology of the QCD SR [20], the right-hand (theoretical) side of (59) may be calculated at large Euclidean p_1^2 and p_2^2 using the operator expansion (OPE). The perturbative part (the unit operator in the OPE) was obtained in the one-loop approximation in [13]. As we consider the systems containing the heavy quarks only, one may neglect the power corrections [13], so that

$$\begin{aligned} \rho_+(s_1, s_2, Q^2) = \; & \frac{3}{2k^{3/2}}\{\frac{k}{2}(\Delta_1 + \Delta_2) - k[m_3(m_3 - m_1) + \\ & m_3(m_3 - m_2)] - [2(s_1\Delta_2 + s_2\Delta_1) - u(\Delta_1 + \Delta_2)] \\ & [m_3^2 - \frac{u}{2} + m_1 m_2 - m_2 m_3 - m_1 m_3]\} \end{aligned} \tag{22}$$

$$\begin{aligned} \rho_V(s_1, s_2, Q^2) = \; & \frac{3}{k^{3/2}}\{(2s_1\Delta_2 - u\Delta_1)(m_3 - m_2) + (2s_2\Delta_1 - u\Delta_2) \\ & (m_3 - m_1) + m_3 k\} \end{aligned} \tag{23}$$

$$\rho_0^A(s_1,s_2,Q^2) = \frac{3}{k^{1/2}}\{9m_1 - m_2)[m_3^2 + \frac{1}{k}(s_1\Delta_2^2 + s_2\Delta_1^2 - u\Delta_1\Delta_2)] - m_2(m_3^2 - \frac{\Delta_1}{2}) - m_1(m_3^2 - \frac{\Delta_2}{2}) + m_3[m_3^2 - \frac{1}{2}(\Delta_1 + \Delta_2 - u) + m_1 m_2]\} \quad (24)$$

$$\rho_+^A(s_1,s_2,Q^2) = \frac{3}{k^{3/2}}\{m_1[2s_2\Delta_1 - u\Delta_2 + 4\Delta_1\Delta_2 + 2\Delta_2^2] + m_1 m_3^2[4s_2 - 2u] + m_2[2s_1\Delta_2 - u\Delta_1] - m_3[2(3s_2\Delta_1 + s_1\Delta_2) - u(3\Delta_2 + \Delta_1) + k + 4\Delta_2\Delta_1 + 2\Delta_2^2 + m_3^2(4s_2 - 2u)] + \frac{6}{k}(m_1 - m_3)[4s_1 s_2\Delta_1\Delta_2 - u(2s_2\Delta_1\Delta_2 + s_1\Delta_2^2 + s_2\Delta_1^2) + 2s_2(s_1\Delta_2^2 + s_2\Delta_1^2)]\}. \quad (25)$$

Here $k = (s_1 + s_2 + Q^2)^2 - 4s_1 s_2$; $u = s_1 + s_2 + Q^2$; $\Delta_1 = s_1 - m_1^2 + m_3^2$; $\Delta_2 = s_2 - m_2^2 + m_3^2$. In the decays $B_c \to \eta_c(J/\psi)e\nu$ one has $m_1 = m_b$ and $m_2 = m_3 = m_c$. The procedure of calculating the α_s-corrections to the triangle is rather complicated which is the reason why they are still unknown.

The formfactors of the transitions are determined in (19)-(21) by the spectral densities $\rho_i(s_1,s_2,q^2)$ at $s_1 = M_{B_C}^2$ and $s_2 = M_{\eta_c}^2, M_{J/\psi}^2$. In fact, the $\rho_i(s_1,s_2,q^2)$ are calculated in the near-threshold region, where instead of α_s, the expansion should be done in the parameters $(\alpha_s/v_{1,2})$, with $v_{1,2}$ meaning the relative velocities of quarks in $(b\bar{c})$- and $(c\bar{c})$-systems. For the heavy quarkonia, where the quark motion velocity is small, these corrections take essential significance (as in the case of two-point functions [21,22]). The α_s/v - corrections, that are responsible for the Coulomb-like interaction of quarks, are related with the ladder diagrams in Fig. 2. In the calculation of the electromagnetic widths of Υ and J/ψ mesons [23] and the axial constant of the B_c meson [17], summation of (α_s/v)-terms resulted in a double-triple multiplication of the Born value of the imaginary part of the correlator.

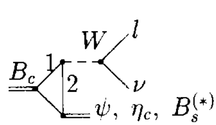

Fig.1. Semileptonic B_c decay due to the transition of quark 1 into quark 2.

Let us estimate the contribution from analogous corrections to the spectral densities $\rho_i(s_1,s_2,q^2)$. Consider the three-point function (4) $\Pi_\mu(P_1,P_2,q)$ at

$q^2 = q^2_{max}$, where q^2_{max} is the maximum invariant mass of the lepton pair in the decay $B_c \to \eta_c e\nu$. Introduce the notations: $P_1 \equiv (m_b + m_c + E_1, 0)$ and $P_2 \equiv (2m_c + E_2, 0)$. At $s_1 = M^2_{B_C}$ and $s_2 = M^2_{\eta_c}$ $E_1 \ll (m_b + m_c)$ и $E_2 \ll 2m_c$. In this kinematics, the quark velocities are small, and so the diagram in Fig. 2 may be considered in the nonrelativistic approximation. We will apply the Coulomb gauge, in which the ladder diagrams with the Coulomb-like gluon exchange are dominant. The gluon propagators have the form:

$$D^{\mu\nu}(k) = i\delta^{\mu 0}\delta^{\nu 0}/k^2.$$

The consideration of the nonrelativistic quarks in the static approximation brings us to the interaction potential, which in the momentum representation has the form [24]:

$$V(\vec{k}) = -\frac{4}{3}\alpha_s(\vec{k}^2)\frac{4\pi}{k^2}, \qquad \alpha_s(\vec{k}^2) = 4\pi/b_0 ln(\vec{k}^2/\Lambda^2),$$

$$b_0 = 11 - \frac{2}{3}n_f, \qquad \Lambda = \Lambda_{\bar{\mu s}} exp[\frac{1}{b_0}(\frac{31}{6} - \frac{5}{9}n_f)].$$

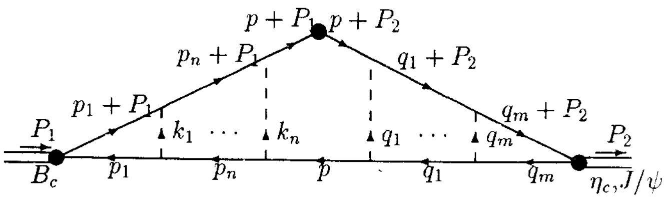

Fig.2. The ladder diagrams of the coulomb-like quark interaction.

The notations concerning Fig. 2 are:

$$k_i = p_{i+1} - p_i, \qquad l_i = q_i - q_{i-1}, \qquad p_{n+1} \equiv q_0 \equiv p.$$

In the nonrelativistic approximation, the fermionic propagators have only one pole, corresponding to either a particle or an antiparticle:

$$S_F(p_i + P_1) = \frac{i(1 + \gamma_0)/2}{E_1 + p_i^0 - |\vec{p}_i|^2/2\mu_1 + i0}$$

$$S_F(p_i) = \frac{-i(1 - \gamma_0)/2}{-p_i^0 - |\vec{p}_i|^2/2\mu_1 + i0}.$$

In this approximation it is not difficult to integrate over loops (Fig. 2). The gluonic propagators are independent of the time components of the momenta, and the integrals over p_i^0, p^0 and q_i^0 can easily be obtained by means of the residues. As a result, for the sum of the ladder diagrams we obtain the expression:

$$\Pi_\mu(E_1,E_2,q) =$$
$$2g_{\mu 0}\sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{d\vec{p}_i}{|\vec{p}_i|^2/2\mu_1 - E_1 - i0}\tilde{V}((\vec{p}_{i+1}-\vec{p}_i)^2)\frac{1}{|\vec{p}|^2/2\mu_1 - E_1 - i0}$$
$$\frac{1}{|\vec{p}|^2/2\mu_2 - E_2 - i0}\sum_{k=1}^{\infty}\prod_{j=1}^{k}\frac{d\vec{q}_j}{|\vec{q}_j|^2/2\mu_2 - E_2 - i0}$$
$$\tilde{V}((\vec{q}_j-\vec{q}_{j-1})^2)\frac{d\vec{p}}{(2\pi)^3}, \qquad (26)$$

$$\tilde{V}((\vec{p}_{n+1}-\vec{p}_n)^2) \equiv \tilde{V}((\vec{p}-\vec{p}_n)^2), \qquad \tilde{V}((\vec{q}_1-\vec{q}_0)^2) \equiv \tilde{V}((\vec{q}_0-\vec{p})^2).$$

μ_1 and μ_2 are the reduced masses of the $(b\bar{c})$- and $(c\bar{c})$-systems, respectively. The three-point function may be expressed in terms of the Green's functions of the relative motion of $(b\bar{c})$- and $(c\bar{c})$-systems in the Coulomb field $G_E^{(i)}(\vec{x},\vec{y})$:

$$G_E^{(i)}(\vec{x},\vec{y}) = \sum_{n=1}^{\infty}\Big(\prod_{k=1}^{n}\int\frac{d\vec{p}_k}{(2\pi)^3(\vec{p}_k^2/2\mu_i - E - i0)}\Big)$$
$$\prod_{k=1}^{n-1}\tilde{V}((p_k-p_{k+1})^2)e^{i\vec{p}_1\vec{x}-i\vec{p}_n\vec{y}}. \qquad (27)$$

The comparison of expressions (26) and (27) yields:

$$\Pi_\mu(E_1,E_2,q^2_{max}) = 2g_{\mu 0}\int G^1_{E_1}(\vec{x}=0,\vec{p})G^2_{E_2}(\vec{p},\vec{y}=0)\frac{d\vec{p}}{(2\pi)^3} =$$
$$2g_{\mu 0}\int G^1_{E_1}(0,z)G^2_{E_2}(z,0)\frac{d^3z}{(2\pi)^3}. \qquad (28)$$

For the Green's function we use the representation:

$$G_E(\vec{x},\vec{y}) = \sum_{l,m}\Big(\sum_{n=l+1}^{\infty}\frac{\Psi_{nlm}(\vec{x})\Psi^*_{nlm}(\vec{y})}{E_{nl} - E - i0} + \int_0^{\infty}\frac{dk}{2\pi}\frac{\Psi_{klm}(\vec{x})\Psi^*_{klm}(\vec{y})}{k/2\mu - E - i0}\Big). \qquad (29)$$

Provided $\vec{x}=0$, only the terms with $l=0$ are retained in the sum. Then for the spectral density one has (to within the inessential variables):

$$\rho_\mu(E_1,E_2,q^2_{max}) \simeq g_{\mu 0}\Psi_1^C(0)\Psi_2^C(0)\int\Psi^C_{1E_1}(\vec{z})\Psi^C_{2E_2}(\vec{z})d\vec{z}, \qquad (30)$$

where Ψ_i^C are the Coulomb wave functions, $(b\bar{c})$ or $(c\bar{c})$. An analogous expression may also be derived in the Born approximation:

$$\rho_\mu^B(E_1, E_2, q_{max}^2) \simeq g_{\mu 0}\Psi_1^f(0)\Psi_2^f(0)\int \Psi_{1E_1}^f(\vec{z})\Psi_{2E_2}^{f*}(\vec{z})d\vec{z}. \qquad (31)$$

Here Ψ_i^f mean the functions of the free motion of particles. Since the continuous spectrum functions should have one and the same normalization for the free and Coulomb solutions, we obtain that

$$\rho_\mu(E_1, E_2, q_{max}^2) = \rho_\mu^B(E_1, E_2, q_{max}^2)\frac{\Psi_1^C(0)\Psi_2^C(0)}{\Psi_1^f(0)\Psi_2^f(0)} \equiv \rho_\mu^B(E_1, E_2, q_{max}^2)\mathbf{C}, \qquad (32)$$

$$\mathbf{C} = \Big(\frac{4\pi\alpha_s}{3v_1}(1 - exp(-\frac{4\pi\alpha_s}{3v_1}))\frac{4\pi\alpha_s}{3v_2}(1 - exp(-\frac{4\pi\alpha_s}{3v_2}))\Big)^{\frac{1}{2}},$$

where $v_{1,2}$ are relative velocities in the $(b\bar{c})$- and $(c\bar{c})$-systems, respectively. In accordance with (6), isolate the Lorentz structures in $\rho_\mu(p_1, p_2, q)$:

$$\rho_\mu(p_1, p_2, q^2) = (p_1 + p_2)_\mu\rho_+(q^2) + q_\mu\rho_-(q^2). \qquad (33)$$

The nonrelativistic expression of $\rho(E_1, E_2, q_{max}^2)$ is proportional to the vector $(g_{\mu 0})$, whose most general form is $(A_+(p_1 + p_2) + A_-q)_\mu$. $A_- \sim (M_{B_C} - M_{\eta_c})$ due to the vector current conservation in the limit $M_{B_C} \to M_{\eta_c}$. Since in the same limit $\rho(E_1, E_2, q^2) \neq 0$, then $A_+ \neq 0$. Hence, $\rho_+(q_{max}^2)$ is also described by the relation:

$$\rho_+(q_{max}^2) = \rho_+^B(q_{max}^2)\mathbf{C}, \qquad (34)$$

where the factor $\mathbf{C}$ was specified in (32).

In the case of the $B_c \to J/\psi e\nu$ transition, the one can more easily obtain analogous relation for $\rho_0^A(q^2)$ (note, that the formfactor $F_0^A \sim \rho_0^A$ determines, in the main, the width of this decay [16]). In the nonrelativistic approximation one has:

$$\Pi_{\mu\nu}^A \simeq (g_{\mu\nu} - g_{\mu 0}g_{\nu 0})\int G_{E_1}^{(1)}(0, z)G_{E_2}^{(2)}(z, 0)\frac{d^3z}{(2\pi)^3} \qquad (35)$$

In the given kinematics, the tensor components $\Pi_{\mu\nu}^A(q^2)$ at $\mu, \nu \neq 0$ are determined only in terms of $\Pi_0^A(q^2)$:

$$\Pi_{\mu\nu}^A(p_1, p_2, q) = g_{\mu\nu}\Pi_0^A(q^2).$$

Then, remembering expression (35) for $\rho_0^A(q_{max}^2)$, we come to

$$\rho_0^A(q_{max}^2) = \rho_0^{A^B}(q_{max}^2)\mathbf{C}, \qquad (36)$$

To conclude this section, note that the derivation of formulas (34),(36) is purely formal, since the spectral densities are not specified at q^2_{max} ($\rho_i^B(q^2)$ can easily be shown to be singular at this point). Therefore, the resultant relations are true for only q^2 approximating q^2_{max}.

Unfortunately, the q^2-dependence of the factor **C** has not been established. But we suppose that the **C** does not affect the pole behavior of the formfactors. Therefore, the resultant widths of the transitions may be treated as the upper bounds in QCD sum rules.

4. Numerical Results

The formulas that we have obtained for formfactors (19)-(21) include the phenomenological parameters dM_k/dk. Proceeding from the experimental data concerning the family ψ [16] and the predictions of the potential models for the B_c and η_c systems [1], [2], the above derivative will be found with three points, using the simple rules of numerical differentiation [18]. For the systems, composed of heavy quarks,the dependence of M_k on the principal quantum number is smooth, and so the three points determine the derivative with a good accuracy. For instance, the difference between the values of $dM_k/dk\ |_{k=1}$ for ψ-system, obtained by three and four points, will be less than 1%. The use of three points is also justified in view of the fact that in the potential model a rise of the principal quantum number degrades the precision of the estimate of the excited state mass. The values of dM_k/dk at $k = 1$ for the systems of interest are presented in Table 1.

Table 1. The derivatives dM_k/dk for low-lying mesons.

M_1	B_c	J/ψ	η_c
$dM_k/dk\ \|_{k=1}$	0.75	0.75	0.76

Choose the following values for the parameters: $f_{B_C} = 360$ MeV, $f_{\eta_C} = 330$ MeV, [13],[17], $m_b = 4.6 \pm 0.1$ GeV, $m_c = 1.4 \pm 0.05$ GeV, $g_{J/\psi} = 8.1$ (from the data on $\Gamma(J/\psi \to e^+e^-)$). The B_c-meson mass will range from 6.245 to 6.284 GeV (the data come from different potential models). Note, this choice of the parameter values will not take us beyond the integration region (10). The values of different formfactors at $Q^2 = 0$ may be found in Table 2. The results of [13] are also included.

Table 2. The formfactors of the transitions $B_c \to J/\psi(\eta_c)e\nu$ at $Q^2 = 0$.

$f_+(0)$	$F_V(0)$ GeV^-1	$F_+^A(0)$ GeV^-1	$F_0^A(0)$ GeV	Ref.
0.23 ± 0.01	0.035 ± 0.03	-0.024 ± 0.002	2 ± 0.2	Present
0.2 ± 0.02	0.04 ± 0.01	-0.03 ± 0.01	2.5 ± 0.3	[13]

The deviation from the mean values in Table 2 corresponds to the quark and B_c-meson mass variations within the above range.

Assume that the formfactors have the following pole behavior:

$$F_i(Q^2) = \frac{F_i(0)}{1+Q^2/m_{pole}^2}\phi_i(Q^2) , \tag{37}$$

where $m_{pole} = 6.3 \div 6.4$ GeV, and $\phi_i(Q^2) = 1 + a_i Q^2$. Fitting with (73)-(75), we get the following values: -0.025, -0.007, -0.012, -0.02 for the formfactors f_+, F_v, F_0^A and F_+^A respectively. For the widths of the transitions $B_c \to \eta_c e\nu$ and $B_c \to J/\psi e\nu$ there are:

$$\Gamma(B_c \to \eta_c e\nu) = 1.4\ 10^{-6}\ eV , \tag{38}$$

$$\Gamma(B_c \to J/\psi e\nu) = 4.6\ 10^{-6}\ eV . \tag{39}$$

These estimates are quite close to those obtained within the Borel sum rules [13].

Let us now consider the contribution from the (α_s/v)-corrections. In Ref. [17], the formula for specifying the axial decay constant of a meson composed of heavy quarks has been derived:

$$f_k^2 = \frac{2(m_Q+m_q)^2}{M_k^3}\frac{dM_k}{dk}\frac{1}{\pi}Im\Pi(M_k^2). \tag{40}$$

In that case, the imaginary part was calculated near the threshold. Clearly, there are also the (α_s/v)-corrections which have turned out to be most important. The summation of these corrections yielded [22]:

$$Im\Pi(q^2) = Im\Pi^B(q^2)\frac{|\Psi^C(0)|^2}{|\Psi^f(0)|^2},$$

where $Im\Pi^B(q^2)$ is the value of the imaginary part of the correlator calculated in the Born approximation.

To calculate the formfactors, let us make use of f_{B_C} and f_{η_c} found from formula (40). As can easily be seen, the factors allowing for the (α_s/v)-corrections in ρ_+, f_{B_C} and f_{η_c}, are cancelled. In [25], where B-meson semileptonic decays were considered, to reduce the ambiguities of the higher QCD corrections, the spectral densities and the axial constants of the relevant mesons were calculated in one and the same order (the lower one). In our approach, we will act in the same way. For the formfactor F_0^A we will use the experimental value $g_{J/\psi}$. In this case only the wave functions relevant to the B_c-system cancel, eliminating thereby the strong dependences of the B_c-meson and b-quark masses. The formfactors will no more be proportional to dM_k/dk, which is another source of ambiguity (especially in the case of the B_c- and η_c-families, whose masses have not yet been established experimentally), but they are in proportion to the root of this factor.

As a result, the following values for the formfactors are obtained at $Q^2 = 0$:

$$f_+(0) = 0.85 \pm 0.15 \quad \text{and} \quad F_0^A = 6.5 \pm 1 GeV.$$

The widths of the corresponding decays are given in Table 3 together with the data of different quark models:

Table 3. Partial widths of the semileptonic decays of B_c in units 10^{-6} GeV.

mode	ISGW [11]	WSB [11]	[14]	present
$B_c^+ \to \psi e^+ \nu_e$	38.5	21.8	37.3	44
$B_c^+ \to \eta_c e^+ \nu_e$	10.6	16.5	20.4	15

Note, we have ignored the contributions from other formfactors participating in the decay $B_c \to J/\psi e\nu$, which may result in an overestimation of the width by 10-20%. The obtained widths values does not contradict the results of the quark models (Table 3) in the limits of the theoretical ambiguities of the methods applied.

Conclusion

In the present paper, the semileptonic decays of the B_c-meson with the J/ψ- and η_c-mesons in the final state have been considered. The formfactors of the decays have been obtained in the framework of QCD SR using the excited states mass spectra of participating mesons. The obtained values of the formfactors and decay widths fit well the results of the Borel sum rules [13].

To elucidate the observed discrepancy between the data of QCD SR and quark models, the higher QCD-corrections to the spectral densities in the theoretical part of the SR have been considered. In particular, we have summed the (α_s/v)-corrections, following from the ladder diagrams with the Coulomb-like gluon exchange at q^2 close to q^2_{max} (q^2 is the invariant mass of the lepton pair). The account of these corrections reduces to multiplying the Born value of the absorptive part of the three-point function by the nonrelativistic factor **C**:

$$\rho_i(q^2_{max}) = \rho_i^B(q^2_{max}) \frac{\Psi_1^C(0)\Psi_2^C(0)}{\Psi_1^f(0)\Psi_2^f(0)} \equiv \rho_i^B(q^2_{max})\mathbf{C}.$$

On the assumption that there should be a weak dependence of **C** on q^2, the obtained decay widths may be understood as the upper bounds in QCD SR. Our estimates do not come into contradiction with the data of the quark models (see Table 3).

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