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ABSTRACT

Through appropriate projections of an exact renormalization group equation, we study fixed points, critical exponents and nontrivial renormalization group flows in scalar field theories in $2 < d < 4$. The standard upper critical dimensions $d_k = \frac{2k}{k-1}$, $k = 2, 3, 4, \dots$ appear naturally encoded in our formalism, and for dimensions smaller but very close to d_k our results match the ϵ -expansion. Within the coupling constant subspace of mass and quartic couplings and for any d , we find a gradient flow with two fixed points determined by a positive-definite metric and a c -function which is monotonically decreasing along the flow.

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P00022533

UB-ECM-PF#93-20

October 1993

* This work is supported in part by funds provided by AEN 90-0033 Grant (Spain), and by M.E.C. (Spain).

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UB-ECM-PF 93-20
SW 9415

1. Introduction

Wilson's exact renormalization group[1] provides a functional differential equation which dictates the way short-distance physics gets integrated into a long-distance effective action. This equation and its analogues (Wegner-Houghton[2] and Polchinski[3]) are certainly powerful but too complex to be of practical use. Different approximations and projections have been devised to bring these equations to more workable, yet non-perturbative, settings.

In this Letter we elaborate on a projection of the Wegner-Houghton exact renormalization group equation due to Hasenfratz and Hasenfratz[4]. The basic idea is to focus on the evolution of the zero mode of the scalar field, with the highest momentum modes being integrated out into a self-interaction term of the constant mode. This setting retains non-linearities from the original equation and is amenable to both analytical and numerical studies. Extending previous work along the same lines[5], we analyze the fixed points of scalar theories for arbitrary dimensions. We then address the renormalization flows between fixed-points and, in particular, the critical exponents. This somewhat conventional piece of work is then completed with a new study on the irreversibility of the flows. It is an open question whether renormalization group flows are gradient. A theorem due to Zamolodchikov[6] ensures such a property in two dimensions but only inconclusive work has been done in higher dimensions (see, for instance, [7]). Here, with some guesswork and brute force we are able to show that at first non-trivial order in our approach the flow is indeed gradient and thus irreversible. It is determined by a positive-definite metric in coupling constant space and a c -function which is monotonically decreasing along the flow, and it connects a unique Gaussian fixed point to a unique Wilson-fixed point for any $2 < d < 4$.

2. Projection of the Wegner-Houghton Equation

The basic flow equation we will use is that of Wegner and Houghton[2], while our notation follows that of [4]. The basic and fairly intuitive procedure of Wegner and Houghton is to consider a generic scalar field theory action S , with an UV momentum cutoff Λ_0 . Starting from that, one then performs in the path integral an integration only over the outermost infinitesimal momentum shell, with momenta $e^{-t}\Lambda_0 \leq q \leq \Lambda_0$, and t small. An effective action will result for the unintegrated fields, now with a slightly smaller cutoff, $\Lambda = e^{-t}\Lambda_0$. Momenta in this effective action are then rescaled so their range again becomes $0 \leq q \leq \Lambda_0$, and with that the fields themselves and the measure in the action will scale with their appropriate scaling dimensions. When all this is put together, a differential equation results describing how the effective action changes as this second cutoff is lowered and more and more momentum degrees of freedom are integrated out. We do not give the derivation of this exact renormalization group equation, since it is presented in detail in

[2], but instead only state the final result:

$$\frac{\partial S}{\partial t} = \frac{1}{2t} \int_q' \left\{ \ln \frac{\partial^2 S}{\partial \phi(q) \partial \phi(-q)} - \frac{\partial S}{\partial \phi(q)} \frac{\partial S}{\partial \phi(-q)} \left(\frac{\partial^2 S}{\partial \phi(q) \partial \phi(-q)} \right)^{-1} \right\} - \int_q q_\mu \phi(q) \partial_{q_\mu}' \frac{\partial S}{\partial \phi(q)} + dS + \left(1 - \frac{d}{2} - \eta\right) \int_q \phi(q) \frac{\partial S}{\partial \phi(q)} + \text{const.}, \quad (2.1)$$

where the prime in the first integral above indicates integration only over the infinitesimal shell of momenta $\epsilon^{-t}\Lambda_0 \leq q \leq \Lambda_0$, and the prime in the derivative indicates that it does not act on the δ -functions in $\partial S/\partial \phi(q)$.

By projecting onto the constant mode $\phi(0)$ of $\phi(q)$, the above exact renormalization group equation is considerably simplified, and becomes the flow equation only for the effective potential of the theory (this, of course, also projects out some terms which could contribute to the flow of the effective potential, but the fact that we are still able to find a very rich structure in the ensuing flow motivates this projection). We furthermore use the approximation used in Ref.[4], constraining the effective action to have no other derivative pieces than the canonical kinetic term, that is, in coordinate space:

$$S = \int d^d x \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) \right\}. \quad (2.2)$$

This leads to the following flow equation for the effective potential:

$$\dot{V}(x, t) = \frac{A_d}{2} \ln(1 + V''(x, t)) + d \cdot V(x, t) + \left(1 - \frac{d}{2} - \eta\right) x V'(x, t) + \text{const.}, \quad (2.3)$$

where $A_d/2 = [(4\pi)^{d/2} \Gamma(d/2)]^{-1}$, the dot is a scale derivative $\partial/\partial t$, x is the constant mode $\phi(0)$, and we again refer the reader to the derivation in Ref.[4]. In the approximation we are using, Eq. (2.2), we actually leave no room for a wavefunction renormalization, and this turns out to imply that $\eta = 0$ above. For greater ease of calculations, we will actually study the equation for $f(x, t) = V'(x, t)$, trivially found from the above:

$$\dot{f}(x, t) = \frac{A_d}{2} \frac{f''(x, t)}{(1 + f'(x, t))} + \left(1 - \frac{d}{2}\right) x f'(x, t) + \left(1 + \frac{d}{2}\right) f(x, t), \quad (2.4)$$

with η already set to 0. We remark here that the constant A_d can be absorbed by a rescaling of x , thus disappearing from the equation above, a fact we will make use of later. This is a reflection of universality in Eq. (2.4), whereby the shape of f^* will depend on A_d but the critical exponents will not. This is the starting point of our calculations. From here one can proceed either by investigating numerical solutions[4] or by analytical means. We choose the latter, where we will use the following polynomial approximation for $f(x, t)$:

$$f(x, t) = \sum_{m=1}^M c_{2m-1}(t) x^{2m-1}, \quad (2.5)$$

with M an arbitrary integer (and, naturally, better approximations will have larger M), and where only odd powers are chosen because we want the potential V to be reflection-symmetric. This approximation has been widely used in the past (cf.[5],[8]).

3. Fixed Points

With Eqs.(2.4) and (2.5) as our starting point, our first objective is to determine the allowed fixed point solutions and their properties. This is easily done by substituting a polynomial fixed-point solution

$$f^*(x) = \sum_{m=1}^M c_{2m-1}^* x^{2m-1} \quad (3.1)$$

with finite but arbitrarily large M into the fixed-point equation, i.e., Eq.(2.4) with $\dot{f} = 0$. A Taylor expansion in x then leads to a set of M nonlinear algebraic equations, of the form

$$\begin{aligned} w_1(c_1^*, c_3^*) &= 2c_1^* + \frac{3A_d c_3^*}{1 + c_1^*} = 0 \\ w_2(c_1^*, c_3^*, c_5^*) &= (4 - d)c_3^* + \frac{10A_d c_5^*}{(1 + c_1^*)} - \frac{9A_d c_3^{*2}}{(1 + c_1^*)^2} = 0 \\ &\vdots \\ w_{M-1}(c_1^*, c_3^*, \dots, c_{2M-1}^*) &= 0 \\ w_M(c_1^*, c_3^*, \dots, c_{2M-1}^*) &= 0, \end{aligned} \quad (3.2)$$

which can *always* be solved exactly and recursively up to w_{M-1} , giving $c_3^*, c_5^*, \dots, c_{2M-1}^*$ as a function of c_1^* . That is substituted in w_M , which then becomes a polynomial of order M in c_1^* with the form:

$$w_M = k(d) c_1^* (\alpha_0(d) + \alpha_1(d) c_1^* + \dots + \alpha_{M-1}(d) c_1^{*M-1}) = 0, \quad (3.3)$$

with $\alpha_i(d)$ being polynomials of order $M - 1$ in d . (Note that $c_1^* = 0$ ($\Rightarrow c_{i>1}^* = 0$) is always a solution for any d . This is the Gaussian fixed point.) As an example, for $M = 6$, we find:

$$\begin{aligned} k(d) &= \frac{1}{155925A_d^5} \\ \alpha_0(d) &= (d - 4)(d - 3)(3d - 8)(2d - 5)(5d - 12) \\ \alpha_1(d) &= -699456 + 899960d - 436386d^2 + 95973d^3 - 8651d^4 + 150d^5 \\ \alpha_2(d) &= -8763072 + 9018200d - 3349144d^2 + 527126d^3 - 30234d^4 + 300d^5 \\ \alpha_3(d) &= -31764096 + 27691776d - 8408924d^2 + 1030750d^3 - 43166d^4 + 300d^5 \\ \alpha_4(d) &= -42872880 + 32815292d - 8479920d^2 + 851409d^3 - 28049d^4 + 150d^5 \\ \alpha_5(d) &= -19261320 + 13251980d - 2990962d^2 + 254333d^3 - 6903d^4 + 30d^5 \end{aligned} \quad (3.4)$$

For any given M , a complicated phase space of solutions (d, c_1^*) can be found and plotted numerically. We have done this up to $M = 7$, and the important aspects of these solutions can be summarized as follows:

i) The first important feature is that, rather unexpectedly, $\alpha_0(d)$ always factorizes into

$$\prod_{m=2}^M (d - d_m) = \prod_{m=2}^M \left(d - \frac{2m}{m-1}\right) = (d-4)(d-3)\left(d - \frac{8}{3}\right) \dots \quad (3.5)$$

This means that at the upper critical dimensions $d = d_k, k = 1, 2, \dots, c_1^* = 0$ is actually a double solution to $w_M = 0$, which indicates a branching of fixed-point solutions below these critical dimensions. This is in perfect agreement with the multicritical fixed-point solutions known to exist below these dimensions.

ii) To further corroborate the above, an ϵ -expansion of Eqs.(3.2) and (3.3) about any critical dimension (with $M = k$) also leads to the known ϵ -expansion solution given by Hermite polynomials, that is, for $d = d_k - \epsilon$,

$$f^*(x) = \kappa_k \epsilon H_{2k-1}(x/\lambda_k) + \mathcal{O}(\epsilon^2), \quad \lambda_k = \sqrt{\frac{2A_d k}{d_k - 2}}, \quad (3.6)$$

where κ_k is a constant depending on d_k (for instance, for $k = 2, d_2 = 4$, and $\kappa_2 = \sqrt{A_2/72}$). Note that a simple ϵ -expansion of Eq.(2.4) will lead to a linear equation and thus cannot furnish this constant κ_k . At higher orders in ϵ we expect our results not to agree with the standard ϵ -expansion since the present approximation does not allow for wavefunction renormalization.

iii) When M is increased by 1, the first $M - 1$ equations in (3.2) remain unchanged. The solution to the last one seems, as far as we have investigated, to lead to convergence of previous solutions as M gets larger (see also [5]).

iv) The lower the dimension, the less trustworthy is this approximation or, conversely, the larger is the M needed. Altogether, we find that, for any d , some solutions represent true fixed points while others are spurious. For lower dimensions, the number of true nontrivial fixed points increases, but so does the number of spurious solutions.

4. Flows and Critical Exponents

Having found particular fixed point solutions in some approximation M , we can now study how the renormalization flow approaches these solutions by determining the critical exponents. To find them, we study small t -dependent departures from some fixed-point profile $f^*(x)$:

$$f(x, t) = f^*(x) + g(x, t), \quad (4.1)$$

where again a polynomial *ansatz* is chosen for $g(x, t)$:

$$g(x, t) = \sum_{m=1}^M \delta_{2m-1}(t) x^{2m-1}. \quad (4.2)$$

When (4.1) is substituted in Eq.(2.4) and only linear terms in g are kept, the resulting equation is

$$\dot{g} = \frac{A_d}{2} \left[\frac{1}{(1+f^*)} g'' - \frac{f^{*''}}{(1+f^*)^2} g' \right] + \left(1 - \frac{d}{2}\right) x g' + \left(1 + \frac{d}{2}\right) g. \quad (4.3)$$

With the *ansatz* (4.2) and matching powers of x in (4.3), we then find:

$$\dot{\delta}_i = \sum_{j=1}^M \Omega_{ij}(c^*, d) \delta_j, \quad (4.4)$$

where Ω_{ij} is an $M \times M$ matrix which depends on the input values c_i^* and d . For higher M , the entries are rather long and unwieldy; here we present as an example Ω_{ij} for the $M = 3$ case:

$$\Omega_{ij} = \begin{pmatrix} 2 - \frac{6A_d c_3^*}{(1+c_1^*)^2} & \frac{6A_d}{1+c_1^*} & 0 \\ \frac{4A_d(9c_3^{*2} - 5c_1^*(1+c_1^*))}{(1+c_1^*)^3} & (4-d) - \frac{36A_d c_3^*}{(1+c_1^*)^2} & \frac{20A_d}{1+c_1^*} \\ \frac{-18A_d c_3^*(9c_3^{*2} - 10c_1^*(1+c_1^*))}{(1+c_1^*)^4} & \frac{18A_d(9c_3^{*2} - 5c_1^*(1+c_1^*))}{(1+c_1^*)^3} & (6-2d) - \frac{90A_d c_3^*}{(1+c_1^*)^2} \end{pmatrix} \quad (4.5)$$

The critical exponents will be given by the characteristic frequencies of the Eq.(4.4), i.e. the eigenvalues of Ω . For the fixed points continuously connected to the only nontrivial fixed point above $d = 3$, we have calculated the critical exponents numerically up to $M = 7$ for dimensions between 2 and 4 in steps of 0.1. For $2.9 \leq d \leq 4$ our results are plotted in Fig. 1. It is worthwhile noting that as $d \rightarrow 4$, the critical exponents merge with the tower of canonical dimensions $(2, 0, -2, -4, \dots)$, which are precisely the critical exponents of the trivial Gaussian theory at $d = 4$ (i.e., the canonical dimensions of the $(\phi^2, \phi^4, \phi^6, \dots)$ couplings in $d = 4$). This is an indication in our setting of the existence of a unique (Gaussian) fixed point at $d = 4$. We furthermore note that for $d = 3$ our two leading exponents

$$\nu = \frac{1}{\omega_1} = 0.656 \quad (4.6)$$

$$\omega_2 = -0.705$$

match fairly well results gotten by other methods (in [4], $\nu = 0.687$ and $\omega_2 = -0.595$; field theory calculations, high temperature expansions and Monte Carlo methods[10] all yield $\nu = 0.630 \pm 0.003$ and $-1.0 < \omega_2 < -0.5$).

It is also possible to perform an ϵ -expansion on Eq.(4.3) around a critical dimension $d = d_k$. We have done this for $d = d_2 = 4$, calculating the critical exponents analytically in ϵ to $\mathcal{O}(\epsilon^2)$ and for all operators, and the procedure is identical for all higher multicritical points. To simplify our calculations we make use of the universality of the critical exponents in A_d and set it to one. Then, we substitute Eq.(3.6) for f^* in Eq.(4.3) (and here it is important to have the correct coefficient κ_k in Eq.(3.6), as well as the $\mathcal{O}(\epsilon^2)$ terms), and make the following *ansatz* for $g(x, t)$:

$$g(x, t) = \exp\{(\omega_\ell^{(0)} + \epsilon \omega_\ell^{(1)} + \epsilon^2 \omega_\ell^{(2)})t\} (g_0(x) + \epsilon g_1(x) + \epsilon^2 g_2(x)), \quad (4.7)$$

where $\omega_\ell^{(0)} = 2(2-\ell)$, ℓ integer ≥ 1 , is any one of the critical exponents at $d = 4$. At zeroth order in ϵ , one finds $g_0(x) \sim H_{2\ell-1}(x)$. At first order in ϵ , using the previous solution for $g_0(x)$, one finds an equation of the form:

$$\begin{aligned} \mathcal{H}_\ell g_1(x) &\equiv \left(\frac{d^2}{dx^2} - 2x \frac{d}{dx} + 2(2\ell-1) \right) g_1(x) \\ &= \left(\omega_\ell^{(1)} + \left(\frac{1}{2} + \frac{(2\ell-1)(2\ell-3)}{6} \right) \right) g_0(x) + \dots, \end{aligned} \quad (4.8)$$

where \mathcal{H}_ℓ is the Hermite operator which annihilates $H_{2\ell-1}(x)$ and the dots are other Hermite polynomials. Since $g_0(x)$ on the r.h.s. is precisely this Hermite polynomial, its coefficient must vanish for the equation to be consistent. This determines $\omega_\ell^{(1)}$. At next order, $\omega_\ell^{(2)}$ is determined in essentially the same way, that is, by canceling on the r.h.s. a zero mode of the Hermite operator which appears on the l.h.s.. The final result is:

$$\omega_\ell = 2(2-\ell) - \frac{\epsilon}{2} \left(1 + \frac{(2\ell-1)(2\ell-3)}{3} \right) + \frac{2\epsilon^2}{9} \ell(2\ell-1)(2\ell-3) + \mathcal{O}(\epsilon^3), \quad (4.9)$$

where $\ell = 1, 2, 3, \dots$ labels the exponents for the different operators. At order ϵ this agrees exactly with standard field theory calculations in the ϵ -expansion[10]. At next order, as announced above, our result differs slightly from the standard one due to the absence of wave function renormalization and the truncation of the exact renormalization group equation itself. By a somewhat more cumbersome calculation it is also possible to find the $\mathcal{O}(\epsilon)$ correction to the critical exponents analytically in k for a generic multicritical point. The answer is:

$$\omega_{k,\ell} = 2 \frac{(2-\ell)}{(k-1)} - \epsilon \left(\ell - 1 - 2(k-1) \frac{(2\ell)! k!}{(2\ell-k)! (2k)!} \right) + \mathcal{O}(\epsilon^2). \quad (4.10)$$

Critical exponents only characterize the flow very close to a particular fixed point. Another option we have is to study the flow globally by substituting Eq.(2.5) directly into Eq.(2.4). Matching powers of x in a Taylor expansion leads to coupled nonlinear flow equations for $c_i(t)$ in the form:

$$\dot{c}_i = w_i(c), \quad i = 1 \text{ to } M, \quad (4.11)$$

where the $w_i(c)$ are given in Eq.(3.2). Arguably, a polynomial *ansatz* does introduce a perturbative element into the essentially nonperturbative nature of renormalization flows between distant fixed points, and our approximation very likely misses some features of the true flow. However, we believe that, again, the sensible and rich structure that emerges does justify the simplification.

We have solved the nonlinear flow (4.11) numerically with $M = 3$ in $d = 3$:

$$\begin{aligned} \dot{c}_1 &= 2c_1 + \frac{3}{2\pi^2} \frac{c_3}{1+c_1} \\ \dot{c}_3 &= c_3 - \frac{9}{2\pi^2} \frac{c_3^2}{(1+c_1)^2} + \frac{5}{\pi^2} \frac{c_5}{1+c_1} \\ \dot{c}_5 &= \frac{27}{2\pi^2} \frac{c_3^3}{(1+c_1)^3} - \frac{45}{2\pi^2} \frac{c_3 c_5}{(1+c_1)^2}. \end{aligned} \quad (4.12)$$

The $(c_1(t), c_3(t))$ subspace of that flow is shown in Fig. 2. We note there the presence of a Gaussian (at $(0,0)$) and a Wilson fixed point, and a unique trajectory leading from the former to the latter. To determine that this flow is gradient and permits a c -function description is the object of the next section.

5. c -Function

We now study some features of the geometry of the space of local interactions. If the beta functions of a theory can be written as a gradient in the space of coupling constants,

$$\beta^i(c) = -g^{ij} \frac{\partial \mathcal{C}}{\partial c_j} \quad (5.1)$$

where g^{ij} is a positive-definite metric, we know that the set of renormalization flows becomes irreversible[9]. In such a case, there exists a function \mathcal{C} of the couplings which is monotonically decreasing along the flows:

$$\frac{d\mathcal{C}}{dt} = \beta_i \frac{\partial \mathcal{C}}{\partial c^i} = -g^{ij} \frac{\partial \mathcal{C}}{\partial c^i} \frac{\partial \mathcal{C}}{\partial c^j} \leq 0, \quad (5.2)$$

making their irreversibility apparent, so that recurrent behaviors such as limit cycles are forbidden. In two dimensions it is possible to prove that the fixed points of the flow are the critical points of \mathcal{C} and that the linearized RG generator in a neighborhood of a fixed point is symmetric with real eigenvalues (the critical exponents).

The renormalization group flows found in the previous section are all well-behaved. Therefore it becomes natural to ask whether these flows are gradient, *i.e.*, whether there exists a globally defined Riemannian metric g_{ij} and a non-singular potential \mathcal{C} satisfying Eq. (5.1). The general solution for an arbitrary number of couplings M would be extremely difficult. However, we find that it is possible to treat the case $M = 2$, namely, the subspace of mass and quartic couplings. The beta functions corresponding to the two couplings c_1, c_3 are given in Eq. (4.12) (where we restrict to $c_5 = 0$). Because of the positivity of \dot{c}_3 (c_3 is the coefficient of ϕ^4 in V and is required to be positive for stability of the path integral) it is appropriate to make the following coupling constant reparametrization:

$$\begin{aligned} c_1 &\rightarrow m^2 = c_1 \\ c_3 &\rightarrow \lambda^2 = 6A_d c_3. \end{aligned} \quad (5.3)$$

In these new variables the beta functions take the form

$$\begin{aligned} \frac{d m^2}{dt} &= 2m^2 + \frac{1}{2} \frac{\lambda^2}{(1+m^2)} \\ \frac{d \lambda}{dt} &= \frac{(4-d)}{2} \lambda - \frac{3}{4} \frac{\lambda^3}{(1+m^2)^2} \end{aligned} \quad (5.4)$$

and the fixed points become

$$\begin{aligned} \text{Gaussian : } & (m_G^2, \lambda_G) = (0, 0) \\ \text{Wilson : } & (m_W^2, \lambda_W) = \left(-\frac{4-d}{10-d}, \frac{\sqrt{24(4-d)}}{10-d}\right). \end{aligned} \quad (5.5)$$

Note that the Wilson fixed point merges with the Gaussian one at $d = 4$, similarly to the situation in Sec. 4. Now, by trial and error and considerable guesswork, the following solution to Eq. (5.1) can be found:

$$\mathcal{C}(m^2, \lambda) = \frac{1}{2}(1+m^2)^4 - \frac{2}{3}(1+m^2)^3 + \frac{1}{4}\lambda^2(1+m^2)^2 - \frac{3}{16}\frac{\lambda^4}{(4-d)} \quad (5.6)$$

and

$$g^{ij} = \frac{1}{(1+m^2)} \begin{pmatrix} 1 & 0 \\ 0 & 4-d \end{pmatrix}. \quad (5.7)$$

$\mathcal{C}(m^2, \lambda)$ has the expected properties of a c -function: *i*) it has a maximum at the Gaussian fixed point, *ii*) it has a saddle at the Wilson fixed point, and *iii*) there is only one flow connecting both points (we have not normalized the c -function to one for the Gaussian fixed point as often done in the literature). Naturally, this description corresponds to our particular parametrization in terms of m and λ , which implicitly carries a choice of subtraction point. The variation of \mathcal{C} between fixed points is reparametrization invariant and its positivity amounts to physical irreversibility of the flow. A contour plot of \mathcal{C} for $d = 3$ is given in Fig. 3. which depicts the space of theories in the basis given by m and λ as a hilly landscape.

For the sake of completeness, let us comment that the first mention of irreversibility of the renormalization group flow was spelled out in the context of perturbation theory by Wallace and Zhia [9]. Later, Zamolodchikov[6] proved a theorem in two dimensions, the c -theorem, which relates the irreversibility of the flows to the basic assumption of unitarity in the Hilbert space of the theory. Several authors[7] have subsequently come to the conclusion that a similar theorem holds in any dimension in perturbation theory. More generally, any expansion where the space of theories is reduced to a manifold in a space of couplings will accommodate a c -theorem. Our setting in this Letter does not clearly fall into this category, due to the appearance of rational functions of the couplings in Eq. (5.4), and the explicit construction of the c -function, though to first non-trivial order, might be of relevance.

A systematic approach to the irreversibility of the renormalization group flow in the projected Wegner-Houghton equation should rely upon a computation of Zamolodchikov's metric (*i.e.* all two-point correlators between composite operators in the theory). This will require an exact renormalization group equation for the generating functional equipped with a source for composite scalar fields.

Acknowledgments

We thank A. Cappelli for useful comments and discussions.

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Figure Captions

Fig. 1. Critical exponents for $2.9 \leq d \leq 4$ corresponding to the relevant, marginal and the first two irrelevant operators in the $M = 7$ approximation.

Fig. 2. $d = 3$ Renormalization group flows projected on mass and quartic coupling subspace in the $M = 3$ approximation. c_1 is plotted on the x -axis and c_3 on the y -axis.

Fig. 3. c -function contour of Eq. (5.6). The Gaussian point is at the top of the hill $(0, 0)$, whereas the Wilson point lies on the saddle $(-1/7, \sqrt{24}/7)$.

