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SCALING BEHAVIOR IN FIRST-ORDER QUARK-HADRON PHASE TRANSITION

RUDOLPH C. HWA

Institute of Theoretical Science and Department of Physics
University of Oregon, Eugene, OR 97403

Abstract

It is shown that in the Ginzburg-Landau description of first-order quark-hadron phase transition the normalized factorial moments exhibit scaling behavior. The scaling exponent ν depends on only one effective parameter g, which characterizes the strength of the transition. For a strong first-order transition, we find $\nu = 1.45$. For weak transition it is 1.30 in agreement with the earlier result on second-order transition.

1 Introduction

It has been suggested after extensive studies in the lattice gauge theory [1, 2, 3] that the QCD phase transition may be second order or a weak first order, depending upon the number of quarks in the problem. If there are only two massless quarks, indications are that the transition is second order, whereas if there are three massless quarks, then it is first order. For a realistic strange quark mass, the situation is somewhere in between, and may be near the tricritical point [4].

While nonperturbative calculations can shed some light on the order of the phase transition (PT) they cannot at present make any predictions that can be tested in heavy-ion collisions where the quark to hadron transition is to take place. It is therefore important to find phenomenological consequences of PT that can directly be measured in the experiment so as to check the nature of the transition. It would also indirectly verify the formation of quark-gluon plasma (QGP), since there would not be any PT without the prior existence of a QGP. The fluctuations of hadron multiplicities at varying resolution scale can readily be measured in heavy-ion collisions. The nature of the fluctuations should reveal some aspects of the dynamical processes in which the hadrons are produced. In the case of second-order PT we have found a scaling behavior in the framework of the Ginzburg-Landau theory [5]. Furthermore, we determined a numerical scaling exponent ν that is independent of the details characterizing the PT [6]. Current experiments on pp and AA collisions all give values of ν larger than the critical value 1.304, signifying the absence of PT [7]. However, in quantum optics the production of photons at the threshold of lasing has long been known to be a process describable by second-order PT [8], and a recent experiment has verified $\nu = 1.304$ to a high degree of accuracy [9].

The purpose of this paper is to extend the consideration to first-order PT. We shall continue to use the Ginzburg-landau (GL) formalism and investigate the question of scaling behavior. The dependence of ν on the details of the PT will be discussed.

It should be mentioned that the investigation reported in this paper complements the work done in Ref. [10], which also studies the first-order PT. There, no thermal equilibrium is assumed, and the focus is on hadronic cluster formation as explored by a cellular automaton. Here, we use the Ginzburg-Landau formalism which is both a mean-field theory (when expressed in terms of fields) and a phenomenological theory (when expressed in terms of phenomenological order parameter); it does not matter whether the hadrons produced form clusters or not.

2 First-Order Phase Transition

If the number of flavors of massless quarks is 2, there are cogent reasons to believe that the QCD transition is second order [1]-[4] The Ginzburg-Landau free energy density may be written in the form

$$\mathcal{F} = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\left(\Phi^2\right)^2 + \frac{1}{2}\left(\nabla\Phi\right)^2 \tag{1}$$

where Φ is a 4-vector $(\sigma, \vec{\pi})$, consisting of the σ and $\vec{\pi}$ fields. When the strange quark mass is brought down from infinity, the system goes through a tricritical point [4, 11], since it is known that for three massless quarks the chiral phase transition is first order [12]. When the renormalization effect on the coupling λ due to the introduction of the strange quark makes λ small or negative, then it becomes necessary to include the sixth-order term in Φ, so (1) should be replaced by

$$\mathcal{F} = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\left(\Phi^2\right)^2 + \frac{1}{6}\kappa\left(\Phi^2\right)^3 + \frac{1}{2}\left(\nabla\Phi\right)^2 \quad . \tag{2}$$

When λ is sufficiently negative, the value of Φ^2 where $\mathcal{F}$ is at the minimum jumps discontinuously and the system undergoes a first-order PT.

Expressed in terms of the fields Φ, the free energy $\mathcal{F}$ does not enable us to determine the multiplicity distribution of the pions produced in a heavy-ion collision. For that what is more pertinent is the phenomenological GL free energy $\mathcal{F}[\phi]$, which on the one hand has the structure of (1) or (2) that contains the mechanism of second- or first-order PT, but on the other hand is expressed in terms of some phenomenological order parameter ϕ that is closely related to the hadron multiplicity. In [6] that was done for second-order PT, where we considered

$$\mathcal{F}[\phi] = a\,|\phi|^2 + b\,|\phi|^4 + c\,|\nabla\phi|^2 \quad . \tag{3}$$

ϕ is a complex number that is related to the hadron density

$$\rho_0 = |\phi|^2 \tag{4}$$

for a pure coherent state, characterized by

$$\hat{a}|\phi> = \phi|\phi> \tag{5}$$

where $\hat{a}$ is the hadron annihilation operator. The quark-gluon system undergoing PT need not be in a pure coherent state. The multiplicity distribution P_n is therefore to be determined from the Poisson distribution $P_n^0[\phi]$ (for a pure coherent state) by a functional integration over ϕ with a Boltzmann factor specified by $\mathcal{F}[\phi]$, i.e.,

$$P_n = Z^{-1} \int \mathcal{D}\phi\, P_n^0[\phi] e^{-F[\phi]}, \tag{6}$$

where

$$F\,[\phi] = \int_V d^d r \mathcal{F}\,[\phi] \tag{7}$$

$$Z = \int \mathcal{D}\phi\, e^{-F[\phi]}, \tag{8}$$

$$P_n^0[\phi] = \frac{1}{n!}\,(\bar{n}_V)^n \exp\left(-\bar{n}_V\right) \quad , \tag{9}$$

$$\bar{n}_V \;=\; \int_V d^d r < \phi|\hat{a}^\dagger(r) a(r)|\phi > = \int_V d^d r |\phi|^2 \quad . \tag{10}$$

Knowing the GL parameters a, b and c, one can calculate P_n and learn about the fluctuations from the average near the critical point.

The phenomenological consequences of this approach to second-order PT and the scaling behavior that can be deduced and experimentally tested will be summarized in the following section. We proceed now to the modification of the formalism appropriate for first-order PT.

To consider first-order PT, in a step parallel to the change of $\mathcal{F}$ from (1) to (2), we now also add a sixth-order term to (3) and write

$$\mathcal{F}[\phi] = a\,|\phi|^2 + b\,|\phi|^4 + k\,|\phi|^6 + c\,|\nabla\phi|^2 \quad . \tag{11}$$

with the recognition that the $k\,|\phi|^6$ term becomes important if b is negative. The average hadron multiplicity is determined by the location of the minimum of the first three terms of the RHS of (11), which we rewrite as

$$a\,|\phi|^2 + b\,|\phi|^4 + k\,|\phi|^6 = \left(a^3/k\right)^{1/2} t f(t) \quad , \tag{12}$$

where

$$f(t) = 1 - 2(1+g)t + t^2 \quad , \tag{13}$$

$$t = \sqrt{\frac{k}{a}}\,|\phi|^2 \quad , \tag{14}$$

$$g = -\left(1 + \frac{b}{2\sqrt{ak}}\right) \quad . \tag{15}$$

Since $f(t)$ has two real roots when g is positive, the minimum jumps from $t = 0$ to a value between the two roots when

$$b < -2\sqrt{ak} \tag{16}$$

for a and k both positive. This is a manifestation of the first order PT. In Fig. 1 we show $tf(t)$ for several values of g.

Although the GL parameters a, b and k are not known functions of T, we know that at the transition temperature T_c (16) would be an equality and $g = 0$. However, depending on how the latent heat released is dissipated, the system may have to supercool, and g would be positive for hadrons to form. In the absence of detailed knowledge about the values of those parameters, let alone their dependences on the hydrodynamical variables of the problem, we shall regard g as a parameter that characterizes the nature of the first-order PT.

The exponential factor in (6) involves a spatial integral of $\mathcal{F}[\phi]$ over a volume V. In heavy-ion collisions the hydrodynamical expansion of the quark-gluon system relates spatial positions in the system to specific cells in the momentum space in which the detected hadrons populate. Thus the cell size that is under experimental control affects the multiplicity distribution P_n of hadrons detected. The theoretical and experimental investigations can be done in any dimension of that space: y, $\ln p_T$ and ϕ in $d = 3$, or any projections of it in lower d.

The factor $e^{-F[\phi]}$ suppresses large fluctuations of $|\phi|$ from the most favored $|\phi_0|$ at the minimum of the well in Fig. 1, depending on the cell size $V = \delta^d$. The larger V is, the smaller the fluctuations will be allowed. Hence, if we focus on the multiplicity fluctuation as a function of V, we would be studying the properties of $\mathcal{F}[\phi]$ near the well minimum. The characteristics of $\mathcal{F}[\phi]$ in the neighborhood of the well minimum does not differ drastically from that of (3) in the case of the second-order PT. Therein lies the clue to the search for observable consequences of the first-order PT that may share the same scaling behavior already found for the second-order case [4].

3 Scaling Behavior

Let us first summarize for the case of second-order PT what possesses the scaling property and what the universal exponent ν is. Since the GL parameters are not known for the quark-gluon system, it is important that measurable predictions should be as free of sensitive dependence on those parameters as possible. It turns out that the measure of fluctuations that can convey such features is the normalized factorial moments

$$F_q = \frac{\langle n(n-1)\dots(n-q+1)\rangle}{\langle n\rangle^q}. \tag{17}$$

where $\langle\dots\rangle$ is an average over P_n. F_q was originally suggested by Białas and Peschanski for the study intermittency [13]. Although there is no intermittency in the strict sense for the GL problem, i.e., power-law dependence on the bin width δ, F_q does possess the scaling behavior

$$F_q \propto F_2^{\beta_q} \tag{18}$$

with β_q satisfying the remarkably simple formula

$$\beta_q = (1-1)^\nu \quad . \tag{19}$$

In the case of a homogeneous medium (for which c may be set to 0) we obtained [6]

$$\nu = 1.304 \quad . \tag{20}$$

This value is independent of a, b and the dimension d. Even when the gradient term in (3) is taken into account the value of ν is not changed significantly; it may be summarized as being in the range [14].

$$\nu = 1.316 \pm 0.012 \quad . \tag{21}$$

Thus we have a scaling exponent ν that is essentially independent of all parameters in the theory, and can be regarded as a numerical signature of second-order PT. For photon production at the threshold of lasing in a homogeneous laser $c = 0$, the value in (20) has been verified to an accuracy within 1% error [9].

Our task now is to examine whether the scaling behavior described above persists to be valid when the PT is first order. If so, what is the corresponding scaling exponent ν?

Defining

$$f_q = \langle n(n-1)\dots(n-q+1)\rangle = \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n \tag{22}$$

we have from (6), (9), and (10)

$$f_q = Z^{-1} \int \mathcal{D}\phi \left(\int_V d^d r \, |\phi(r)|^2 \right)^q e^{-F[\phi]} \tag{23}$$

It then follows from (17) that

$$F_q = f_q / f_1^q \quad . \tag{24}$$

Note that if $e^{-F[\phi]}$ were replaced by $\delta(\phi - \phi_0)$, corresponding to a pure state, then $f_q = \left(\delta^d |\phi_0|^2\right)^q$ and $F_q = 1$. Thus any nontrivial property of F_q is a measure of the dynamical fluctuations due to PT as prescribed by the GL $\mathcal{F}[\phi]$.

We proceed by considering, as in [6], the case of a homogeneous system ($c = 0$), relying on the result of [14] to regard the correction due to the $c \neq 0$ term in (11) to be also small. In that case ϕ is independent of r and we have

$$f_q = \delta^{dq} I_q / I_0, \tag{25}$$

where

$$\begin{aligned} I_q &= \int_0^\infty d|\phi|^2 \, |\phi|^{2q} e^{-\delta^d \left(a|\phi|^2 + b|\phi|^4 + k|\phi|^6 \right)} \\ &= (a/k)^{(q+1)/2} \, J_q \quad , \end{aligned} \tag{26}$$

$$J_q = \int_0^\infty dt \, t^q e^{-xtf(t)} \quad , \tag{27}$$

$$x = \delta^d \left(a^3 / k \right)^{1/2} \quad . \tag{28}$$

We note that the coefficients in (25) and (26) all get cancelled in the normalized moments, i.e.,

$$F_q = I_q \, I_1^{-q} \, I_0^{q-1} = J_q \, J_1^{-q} \, J_0^{q-1}. \tag{29}$$

Thus F_q is a function of x and g only. The parameter g is not subject to experimental control, but x is proportional to δ^d, which is the cell volume that can be varied to effect measurable variation in F_q. The dependence of F_q on x cannot be directly tested because of the unknown parameters a and k. However, the dependence of F_q on F_2 as δ is varied is accessible to experimental verification, and can be theoretically calculated by varying x for every fixed g.

It is straightforward to determine F_q by use of (27) and (29). In Fig. 2 we show the results plotted in $\ln F_q$ vs $\ln F_2$ for $q = 3, \ldots, 6$ and for several values of g. Evidently, they all exhibit linear dependences, describable by the power law (18). Thus we have found the same scaling behavior of F_q as we did for second-order PT, except that now the slope β_q depends on g, and only g. For every fixed g, it is found remarkably that β_q again satisfies extremely well the simple formula (19), as shown in Fig. 3. The dependence on the scaling exponent ν on g is shown by the dots in Fig. 4. The error bar on the $g = 0.6$ point indicates the extent of the deviation of the corresponding plots in Fig. 2 from exactly straight lines. The solid line in Fig. 4 is a smooth fit of the dots, and the dashed line is for $\nu = 1.304$, the result for second-order PT.

We see that, at large g, ν is approximately equal to 1.3. From Fig. 1 it is clear that as g is increased, the well of $\mathcal{F}[\phi]$ becomes deeper and resembles more the familiar dip associated with $a|\phi|^2 + b|\phi|^4$ for the second-order case with $a < 0$ and $b > 0$. For that reason the value of ν for such a large value of g is about the same for either order of PT. However, if an extensive and prolonged mixed phase is to exist in the system, the relevant value of g would be smaller, since for g close to zero the free energies for the quark and hadron phases are more nearly equal. If that is the case, then our result indicates that ν should be larger, but does not exceed 1.45. Since at present our theoretical understanding of nonperturbative QCD cannot yet lead to a determination of the value of g, we cannot predict a unique value of ν. What we can state is that if in a future experiment in heavy-ion collisions ν is found to be less than 1.45, then we have a strong reason to suggest the possibility of the occurrence of a PT worthy of closer examination. So far all experimental values of ν for nuclear collisions are greater than 1.45 [6], implying no PT. Barring other complications that have not been taken into account in this consideration, an accurate determination of the experimental value of ν, if it is < 1.45, may even reveal which order of PT that the quark-gluon system has undergone.

4 Conclusion

In our search for observable consequences of quark-hadron phase transition, we have found some unifying features of the first- and second-order cases. The Ginzburg-Landau phenomenological theory is well suited to describe the problem of our concern, since it is coarse-grained enough compared to the underlying field theory to be able to address the hadronic observables in an experiment, while it is at the same time fine-grained enough compared to the hydrodynamical studies to be able to account for the fluctuations in phase space. Furthermore, the GL theory is capable of describing both the first- and second-order PT.

Although the GL free energy density $\mathcal{F}[\phi]$ has many terms, each with a parameter that is not known from first principles, only one effective parameter g turns out to affect our result. That is due to our choice of the normalized factorial moments F_q to measure the multiplicity fluctuations. Their dependences on F_2 with the cell size δ treated as a parametric variable, exhibit scaling behavior just as in the second-order case. For reasons that are not self-evident, the slopes β_q satisfy the $(q-1)^\nu$ description extremely well, rendering the scaling exponent ν a simple and effective characterization of the nature of dynamical fluctuation due to the PT. Depending on

g, the value of ν varies from 1.45 to 1.30.

One can see from Fig. 1 that when g is as large as 0.6, the well is so deep that it hardly describes a two-phase system that is necessary for a strong first-order PT. Thus one may regard a weak first-order PT to be one describable by a GL $\mathcal{F}[\phi]$, in which the parameters vary with temperature in such a way that g is effectively large as the system goes through phase transition. On the other hand, for a strong first-order PT the system would remain in a state describable by a small g during the phase transistion. What our calculation has shown is that the two cases have distinguishable values of ν: 1.30 to 1.45 from weak to strong. This simple criterion is made highly relevant to heavy-ion experiments by the fact that the ν is measurable in the laboratory.

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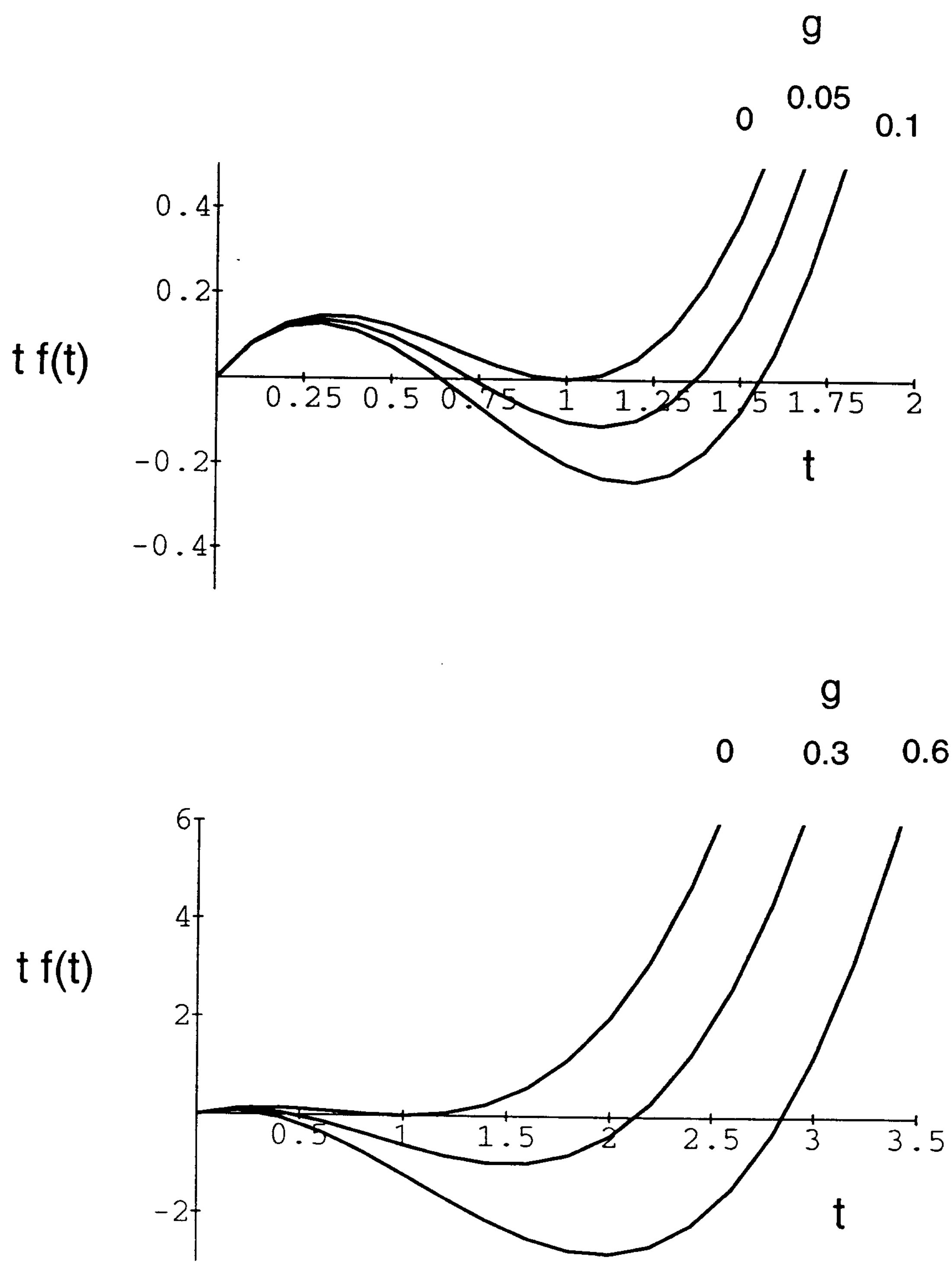
Figure Captions

Fig. 1 The function $tf(t)$, defined in Eq. (13), for several values of g.

Fig. 2 Scaling behavior of F_q for several values of g.

Fig. 3 β_q vs q. Dots are the slopes determined from Fig. 2, and the curves are determined from the formula $\beta_q = (q-1)^\nu$.

Fig. 4 The values of $\nu(g)$ are determined from Fig. 3 (dots). Solid line is a smooth-curve fit and the dashed line is for $\nu = 1.304$ [6].


g
0
0.05
0.1
0.4
0.2
t f(t)
0.25
0.5
0.75
1
1.25
1.5
1.75
2
-0.2
t
-0.4
g
0
0.3
0.6
6
4
t f(t)
2
0.5
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2
2.5
3
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-2
t

Fig. 1

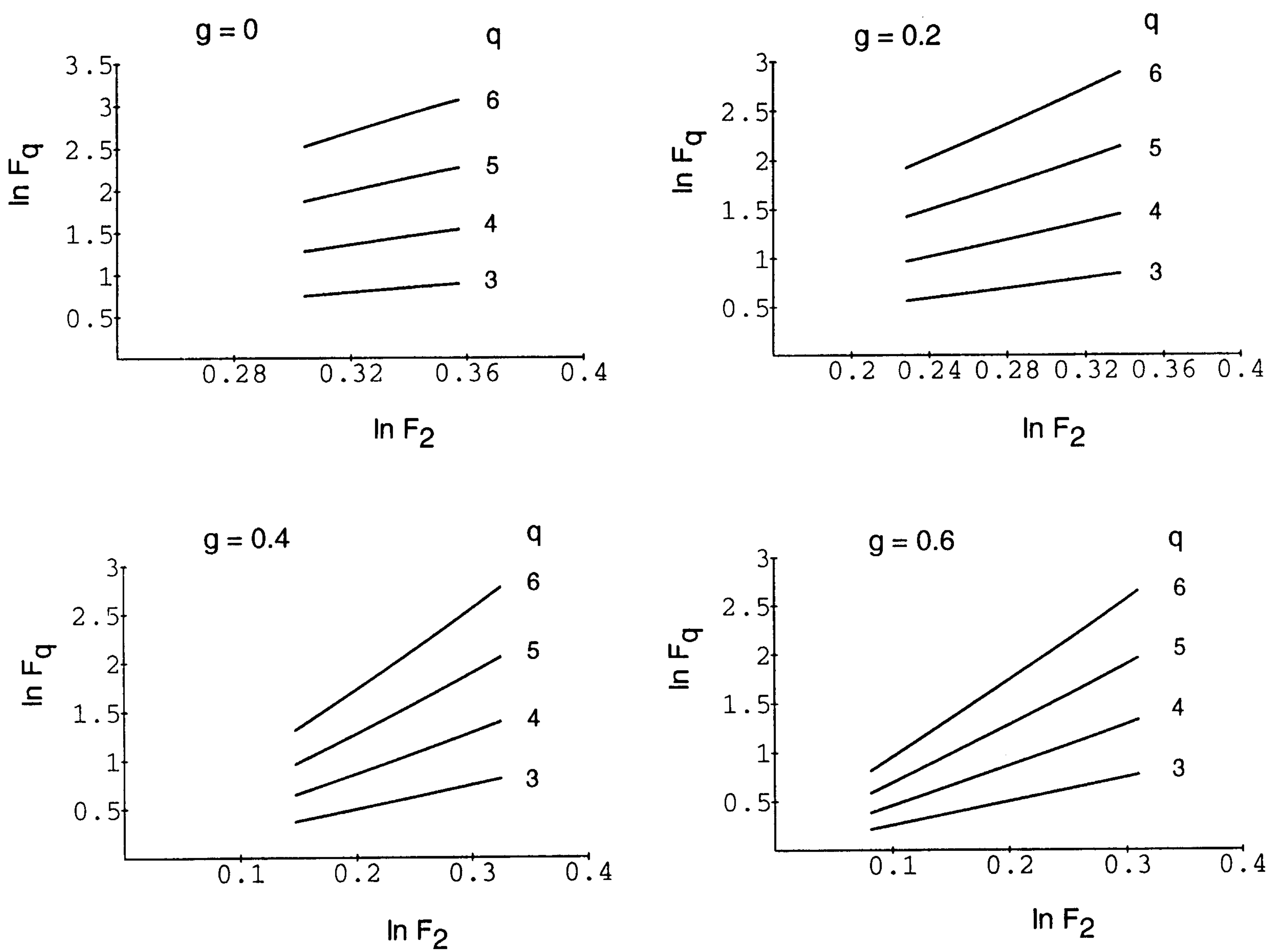

Fig. 2

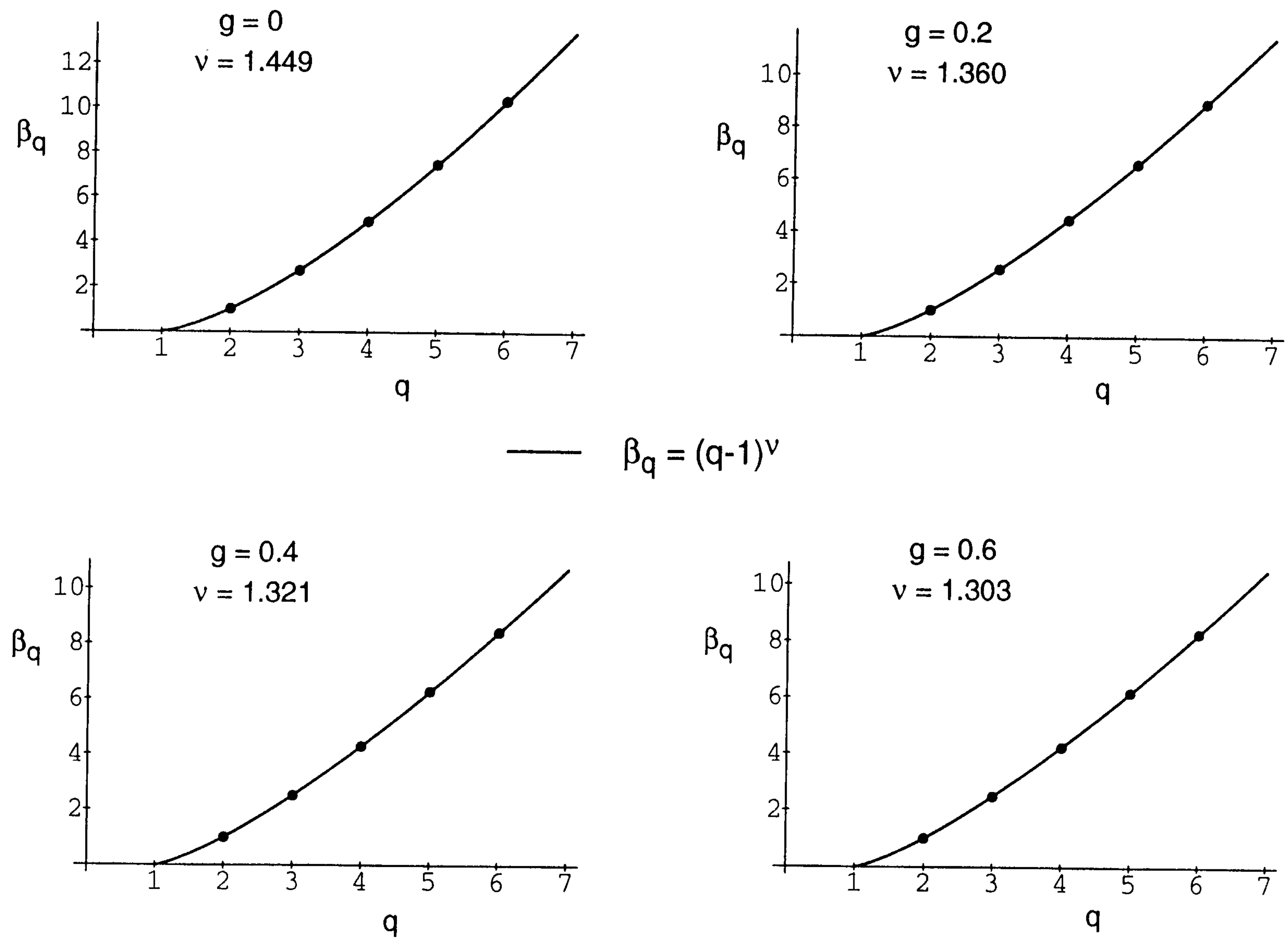
g = 0
ν = 1.449
g = 0.2
ν = 1.360
g = 0.4
ν = 1.321
g = 0.6
ν = 1.303
β_q
q
1 2 3 4 5 6 7
2 4 6 8 10 12
β_q = (q-1)^ν

Fig. 3

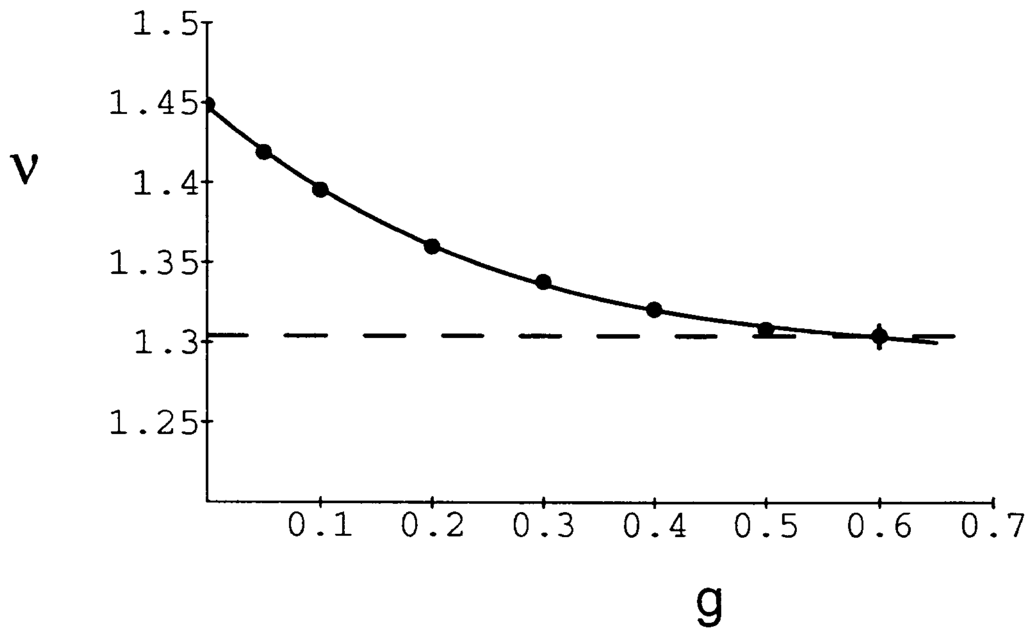

Fig. 4