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GAUGE THEORY ON DISCRETE GROUPS AND THE STANDARD MODEL[1], [2]

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ABSTRACT

We introduce the gauge theory on discrete groups with Higgs fields being such gauge fields and the Yukawa coupling between Higgs and fermions is automatically given by the minimum coupling principle. We concentrate on the Z_2-symmetry taken as $\{e, r = (CPT)^2\}$, a subsymmetry of the CPT transformations. A $G_L \times G_R \times Z_2$-gauge invariant model is presented. The standard model for the electroweak-strong interaction is reformulated in detail. The method for the model building is very different from and much simpler than that of Connes and others by means of non-commutative geometry.

1. Introduction

It is well known that Higgs fields play a very important role in modern QFT, especially, in the standard model of electroweak-strong interaction. However, unlike Yang-Mills gauge fields, not only the introduction of Higgs fields but also the Yukawa couplings between Higgs and fermions seem to be artificial and have no profound meaning from ordinary differential geometrical point of view.

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Recently, Alain Connes [1] applied his non-commutative differential geometry to particle physics model building. An interesting result is that his formalism applied to a two-sheeted spacetime automatically presents the Higgs field as a gauge field associated to discreteness. Later, Connes and Lott [2, 3] and Kastler [4] made further study on Connes' approach. Since then, lots of efforts have been made along this direction [5-8]. In this approach the non-commutative Yang-Mills action together with fermions reproduces some constraints among free parameters at the tree level and at least some of the constraints, say, the Weinberg angle, seem to be unreasonable from the point of view of QFT and they do not survive standard quantum corrections [9]. On the other hand, however, this approach indicates that the Higgs fields have also deep geometrical meaning and there exists some nontrivial geometry behind the Weinberg-Salam model in the sense of the non-commutative differential geometry.

Among others Coquereaux et al [5] emphasized that it could be possible to formulate Higgs fields as gauge potentials with respect to discrete symmetry without any knowledge of non-commutative geometry. Very recently, Sitarz [10] proposed an approach towards the construction of a pure gauge theory on arbitrary discrete groups in which Higgs fields appear as gauge potentials on discrete groups. This approach also does not require entire knowledge of non-commutative geometry and for the two-point space (with Z_2 symmetry), it is more or less equivalent to the ones in previous works. Unfortunately, Sitarz [10] could not achieve the goal towards the realistic model building. In fact, the formalism in [10] has not been completed yet since fermion fields were not included into the formalism.

In [11-14], we have completed the construction of the gauge theory on discrete groups coupled to the fermions Namely, the ordinary Yang-Mills gauge theory is generalized in order to take not only Lie groups but also discrete groups as gauge groups. It has been shown that a simple complex Higgs field is such a gauge field with respect to Z_2-gauge symmetry over 4-dimensional spacetime M^4 and the Yukawa coupling between Higgs and fermions is automatically introduced via the covariant derivative or the minimum coupling principle with respect to Z_2-gauge potential. Therefore, together with the Yang-Mills fields, all fundamental bosonic fields are gauge potentials and their couplings with fermions are all given by the corresponding covariant derivatives or the minimum coupling principles. It has also been shown that the Weinberg-Salam model and the standard model for the electroweak-strong interaction can be reformulated. In [12, 13], for all these models, the Z_2-symmetry is taken to be a subsymmetry of the CPT transformations, i.e. $Z_2 = \{e, r = (CPT)^2\}$, and the Higgs appears as discrete gauge fields of this Z_2-gauge symmetry over spacetime M^4. And in [14], we further take a Z_4-symmetry with elements being CPT-transformation as a whole to be the gauge symmetry. With a simple ansatz, the Weinberg-Salam model has also been reformulated.

It is important to stress that in our approach there is no constraint on the Weinberg angle at the tree level and all other constraints the mass parameters are not direct consequences of the gauged discrete symmetry but totally depend on certain working hypotheses. Although we may get some constraints on m_t and M_H at the tree level under certain natural hypotheses. However, all of them could be completely released at all in principle.

In this talk, we briefly introduce our approach to the gauge theory on discrete groups We concentrate on the Z_2-symmetry taken as $\{e, r = (CPT)^2\}$, a subsymmetry of the CPT transformations [11, 12]. We first review how to construct the Higgs field as a gauge field on discrete gauge symmetry Z_2 over the spacetime M^4. Then we present a $G_L \times G_R \times Z_2$-gauge invariant model with Z_2-symmetry being $\{e, r = (CPT)^2\}$ and we reformulate the standard model for the electroweak-strong interaction in detail. The method for the model building is very different from and much simpler than that of Connes and others by means of non-commutative geometry. Finally, we end with some discussions and remarks. Especially, we emphasize the differences between our approach and the ones by means of the non-commutative geometry and explore the implication of the $Z_2 = \{e, r = (CPT)^2\}$-gauge symmetry.

2. Higgs As Gauge Field On Discrete Group Z_2

In this section, we outline the notion of a pure gauge field theory on discrete group Z_2 and construct the Lagrangian of a simple complex Higgs field being as the gauge field with respect to Z_2-gauge symmetry as a Yang-Mills like Lagrangian on spacetime M^4. For details, it is referred to [10-12].

Let us consider the discrete group $Z_2 = \{e, r\}$ and $\mathcal{A}$ the algebra of complex valued functions on $M^4 \times Z_2$. We extend the exterior derivative operator d_M on M^4 as follows:

$$d = d_M + d_{Z^2}, \tag{1}$$

where d_{Z^2} is the exterior derivative on Z_2. If $f(x,h)$, $h \in Z_2$ is a function, then

$$df = \partial_\mu f dx^\mu + \partial_r f \chi^r, \tag{2}$$

where ∂_r is the partial derivative defined by

$$(\partial_r) f(x,h) = f(x,h) - f(x, h\cdot r),$$

χ^r (sometime denoted as χ) is a one-form on Z_2 dual to ∂_r and a metric on Z_2 can be defined as

$$< \chi^r, \chi^r >= \eta^{rr},$$

sometime it is denoted as η for simplicity.

The nilpotency of d requires the nilpotency of both d_M and d_{Z^2} as well as

$$d_M d_{Z^2} = -d_{Z^2} d_M.$$

Acting with $d_M d_{Z^2}$ and $d_{Z^2} d_M$ on $f(x,h), h \in Z_2$ separately, it follows

$$dx^\mu \otimes \chi = -\chi \otimes dx^\mu. \tag{3}$$

On the other hand, the nilpotency of d_{Z^2} requires

$$\chi f(x,h) = f(x, h\cdot r)\chi, \quad d\chi = d_{Z^2}\chi = 2\chi \otimes \chi. \tag{4}$$

Let us now construct the generalized gauge theory on finite group Z_2. We take the gauge transformations to be unitary elements of $\mathcal{A}$ dependent on the both $x \in M^4$ and the Z_2 elements,

$$\mathcal{H} = \mathcal{U}(\mathcal{A}) = \{a \in \mathcal{A} : \quad aa^\dagger = a^\dagger a = 1\}.$$

It is easy to see that the exterior derivative d is not covariant so that we should introduce the covariant derivative $d + \Phi$, where $\Phi = \Phi(x,h)\chi$ is a generalized connection one-form. The Hermiticity requires that

$$\chi^* = -\chi, \quad \Phi(x,h) = \Phi^\dagger(x, h\cdot r). \tag{5}$$

The generalized curvature two-form should be defined as follows

$$\begin{aligned} F &= d\Phi + \Phi \otimes \Phi \\ &= F_{\mu r} dx^\mu \otimes \chi + F_{rr}\chi \otimes \chi \\ &= \partial_\mu \Phi dx^\mu \otimes \chi + (\partial_r \Phi + \Phi\Phi^\dagger - 2\Phi)\chi \otimes \chi. \end{aligned} \tag{6}$$

To get the Lagrangian of the pure generalized gauge field with respect to Z_2 on M^4, we define metrics as

$$< dx^\mu, dx^\nu >= g^{\mu\nu}, \quad < dx^\mu \otimes \chi, \chi \otimes dx^\nu >= g^{\mu\nu}\eta.$$

Then the Lagrangian of the pure generalized gauge field on M^4 reads

$$\begin{aligned} \mathcal{L}_{YM} &= -\tfrac{1}{N}\textstyle\int_{Z^2} < F, \overline{F} > \\ &= \tfrac{1}{N}\{\eta \partial_\mu \phi \partial^\mu \phi^\dagger - \eta^2(\phi\phi^\dagger - 1)^2\}, \end{aligned} \tag{7}$$

where $\phi = 1 - \Phi$, N is a normalization constant and $\int_{Z^2}$ is the Haar integral over Z_2 which is introduced as a complex valued linear functional on $\mathcal{A}$ that remains invariant under the action of Z_2. In general, for a discrete G, it is defined as:

$$\int_G f = \frac{1}{N_G}\sum_{g\in G} f(g),$$

which is normalized such that $\int_G 1 = 1$.

Now we get a full Lagrangian of Yang-Mills type for a simple complex Higgs field up to some coupling constants which will be introduced later. Note that there is a tiny but important difference between this Lagrangian and the conventional one. Namely, unlike in the conventional one, the ratio of coefficients of the kinetic term and the potential here is fixed by the metric we have chosen. It turns out that this ratio indicates a constraint. However, it could be released at all. We will mention this issue at the end of the talk. On the other hand, without loss of Z_2-gauge invariance we may also take the conventional Lagrangian as the one for the Higgs being Z_2-gauge field.

We may also introduce the coupling of the Higgs field with fermions. It turns out that the Yukawa couplings are the gauge couplings with respect to the gauge potential, the Higgs field, on the Z_2-gauge symmetry. Due to the limitation of the space, we do not illustrate it here. For this issue, it is referred to [11,12].

3. A Model with $G_L \times G_R \times Z_2$-Gauge Symmetry

Let us construct a model of the $G_L \times G_R \times Z_2$-gauge symmetry with $Z_2 \subset CPT$ symmetry with leptons $\psi(x,h),\ \ h \in Z_2$, Yang-Mills gauge potentials $A_\mu(x,h)$ and Higgs $\Phi(x,h)$. We assign them into two sectors according to two elements of the group Z_2 as follows:

$$\begin{aligned} \psi(x,e) = -\psi(x,r) &= \begin{pmatrix} L \\ R \end{pmatrix}; \\ A_\mu(x,e) = A_\mu(x,r) &= \begin{pmatrix} L_\mu & 0 \\ 0 & R_\mu \end{pmatrix}; \\ \Phi(x,e) = \Phi(x,r) &= \begin{pmatrix} \frac{\mu}{\lambda} & -\phi \\ -\phi^\dagger & \frac{\mu}{\lambda} \end{pmatrix}. \end{aligned} \tag{8}$$

where L (R) is the left (right) handed fermion, L_μ (R_μ) the gauge potential valued on the Lie algebra of the gauge group G_L (G_R) and coupled to the fermion L (R), μ and λ two constants. Note that the minus sign in $-\psi(x,r)$, $r = (CPT)^2$, is due to the transformation property of the fermion under $(CPT)^2$ modulo the gauge transformations of $G_L \times G_R$. ¿From the assignments, it is easy to see that the field contents of the model are of Z_2 symmetry and the Higgs in such a model may be regarded as the gauge field with respect to a gauged Z_2 subsymmetry of the group CPT. However, it should be mentioned that the assignments (8) not only assigns the fields to the elements of Z_2 but also implies that we arrange all fields into certain matrices. In fact, this arrangement is nothing to do with discrete gauge symmetry but for convenience in the forthcoming calculation. Of course, we must keep in mind

that this arrangement is a working hypothesis and sometimes one should avoid some extra constraints coming from this working hypothesis.

¿From the general framework we have developed in [11, 12], it follows the generalized connection one-form

$$A(x,h) = A_\mu(x,h)dx^\mu + \frac{\lambda}{\mu}\Phi(x,h)\chi, \quad h \in Z_2, \tag{9}$$

where χ denotes χ^r, and the generalized curvature two-form

$$\begin{aligned} F(h) &= dA(h) + A(h) \otimes A(h) \\ &= \tfrac{1}{2}F_{\mu\nu}(h)dx^\mu \wedge dx^\nu + \tfrac{\lambda}{\mu}F_{\mu r}(h)dx^\mu \otimes \chi + \tfrac{\lambda^2}{\mu^2}F_{rr}(h)\chi \otimes \chi. \end{aligned} \tag{10}$$

Using the above assignments, we get

$$\begin{aligned} F(x,e) &= F(x,r) \\ &= \tfrac{1}{2}\begin{pmatrix} L_{\mu\nu} & 0 \\ 0 & R_{\mu\nu} \end{pmatrix} dx^\mu \wedge dx^\nu + \tfrac{\lambda}{\mu}\begin{pmatrix} 0 & -D_\mu\phi \\ -D_\mu\phi^\dagger & 0 \end{pmatrix} dx^\mu \otimes \chi \\ &+ \tfrac{\lambda^2}{\mu^2}\begin{pmatrix} \phi\phi^\dagger - \frac{\mu^2}{\lambda^2} & 0 \\ 0 & \phi^\dagger\phi - \frac{\mu^2}{\lambda^2} \end{pmatrix} \chi \otimes \chi; \end{aligned} \tag{11}$$

where

$$D_\mu\phi = \partial_\mu\phi + L_\mu\phi - \phi R_\mu. \tag{12}$$

Having these building blocks, we may get the generalized gauge invariant Lagrangian including both the bosonic part and the fermionic one as well as their interactions via generalized minimum coupling principle in the conventional way. From the field contents (8), it follows Lagrangian of the ordinary type in gauge invariant models. On the other hand, we may also introduce the generalized gauge invariant Lagrangian with respect to each element of Z_2 first, then take the Haar integral of them over Z_2.

For the Lagrangian of the bosonic sector with respect to each element of Z_2, we have

$$\begin{aligned} \mathcal{L}_{YM-H}(x,e) &= \mathcal{L}_{YM-H}(x,r) \\ &= -\tfrac{1}{4N_L}Tr_L(L_{\mu\nu}L^{\mu\nu}) - \tfrac{1}{4N_R}Tr_R(R_{\mu\nu}R^{\mu\nu}) \\ &+2\eta\tfrac{\lambda^2}{\mu^2}Tr(D_\mu\phi(x))(D^\mu\phi(x))^\dagger \\ &-2\eta^2\tfrac{\lambda^4}{\mu^4}Tr(\phi(x)\phi(x)^\dagger - \tfrac{\mu^2}{\lambda^2})^2 + const; \end{aligned} \tag{13}$$

where N_L and N_R are normalization constants, η is a metric parameter defined by $\eta = <\chi, \chi>$, $Dim(\eta) = \mu^2$. Here we suppose that both G_L and G_R are semi-simple.

For the fermionic sector, the Lagrangian with respect to each element of Z_2 may also be given as follows:

$$\begin{aligned}\mathcal{L}_F(x,e) &= \mathcal{L}_F(x,r) \\ &= i\overline{L}\gamma^\mu(\partial_\mu + L_\mu)L + i\overline{R}\gamma^\mu(\partial_\mu + R_\mu)R - \lambda(\overline{L}\phi R + \overline{R}\phi^\dagger L).\end{aligned} \tag{14}$$

It is easy to get the entire Lagrangian for the model:

$$\mathcal{L}(x) = \mathcal{L}_F(x,e) + \mathcal{L}_{YM-H}(x,e). \tag{15}$$

Obviously, this is the one of SSB type and there may exist some constraints among the coupling constants and mass parameters, which will be explained for the concrete model.

4. The Standard Model

We now turn to the standard model for electroweak-strong interactions. Namely, we take into account the colour degree of freedom together with the weak isospin and the weak hypercharge degrees of freedom for both leptons and quarks in three families with the gauge group $SU(2)_L \times U(1)_Y \times SU(3)_c$. We make use of the field assignments in (8) with the field contents in the standard model. Then we present the generalized gauge-invariant Lagrangian including both the bosonic part and the fermionic one as well as their interactions via generalized minimum coupling principle. As in the previous section, we introduce the Lagrangian with respect to each element of Z_2 first, then take the Haar integral of them over Z_2.

Let us take the assignment for the fermions with respect to Z_2 symmetry as follows:

$$\psi(x,e) = -\psi(x,r) = \begin{pmatrix} L \\ R \end{pmatrix}, \tag{16}$$

with

$$L = \begin{pmatrix} u^c \\ \vdots \\ b^c \\ \nu_e \\ \vdots \\ \tau \end{pmatrix}_L^{GFC}, \quad R = \begin{pmatrix} u^c \\ \vdots \\ b^c \\ e \\ \mu \\ \tau \end{pmatrix}_R. \tag{17}$$

Here superscript c stands for the colour degree of freedom. Taking into account all strong and electroweak interactions among leptons and quarks, we assign the gauge fields as follows:

$$A_\mu(x,e) = A_\mu(x,r) = \begin{pmatrix} L_\mu & 0 \\ 0 & R_\mu \end{pmatrix},$$

with

$$\begin{aligned} L_\mu = & -\begin{pmatrix} \frac{ig}{2}\tau_i^l W_\mu^i \otimes I_3^G \otimes I_3^C & \\ & \frac{ig}{2}\tau_i^l W_\mu^i \otimes I_3^G \end{pmatrix} \\ & -ig'B_\mu \begin{pmatrix} \frac{1}{6}I_2 \otimes I_3^G \otimes I_3^C & \\ & -\frac{1}{2}I_2 \otimes I_3^G \end{pmatrix} - \begin{pmatrix} I_2 \otimes I_3^G \otimes \frac{ig_c}{2}G_\mu^i\lambda_i^C & \\ & 0 \end{pmatrix}, \end{aligned}$$

$$R_\mu = -ig'B_\mu \begin{pmatrix} \begin{pmatrix} \frac{2}{3} & \\ & -\frac{1}{3} \end{pmatrix} \otimes I_3^G \otimes I_3^C & \\ & -I_3^G \end{pmatrix} - \begin{pmatrix} I_2 \otimes I_3^G \otimes \frac{ig_c}{2}G_\mu^i\lambda_i^C & \\ & 0 \end{pmatrix}, \tag{18}$$

where $G_\mu^i, i = 1, \cdots, 8$, are gluons, λ_i 3×3 Gell-Mann matrices, and I_n $n \times n$ unit matrices.

For the Higgs field, we take

$$\Phi(x,e) = \Phi(x,r) = \begin{pmatrix} \frac{\mu}{\lambda} & -\phi(x) \\ -\phi(x)^\dagger & \frac{\mu}{\lambda} \end{pmatrix}. \tag{19}$$

as before. But, the field content of $\phi(x)$ being gauge field with respect to Z_2-symmetry is more complicated:

$$\phi(x) = \begin{pmatrix} \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \otimes I_3^G \otimes I_3^C & \\ & \begin{pmatrix} \phi^\dagger \\ \phi^0 \end{pmatrix} \otimes I_3^G \end{pmatrix} \cdot \begin{pmatrix} M_1 \otimes I_3^C & & \\ & M_2 \otimes I_3^C & \\ & & M_3 \end{pmatrix}.$$

Now we may write down the generalized connection one-form including both ordinary Yang-Mills potentials and the Higgs field and the generalized curvature two-form. Especially, the components $F_{\mu r}$ of the generalized field strength are the ordinary covariant derivatives of the Higgs field as before:

$$D_\mu\phi = \partial_\mu\phi + L_\mu\phi - \phi R_\mu.$$

Introducing the original Higgs doublet π,

$$\pi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{20}$$

a straightforward calculation shows that

$$Tr D_\mu\phi(D^\mu\phi)^\dagger = \Sigma_1(D_\mu\pi)^\dagger D^\mu\pi, \tag{21}$$

where

$$D_\mu \pi = (\partial_\mu - \frac{ig}{2}\tau_i W^i_\mu - \frac{ig'}{2}B_\mu)\pi,$$

$$\Sigma_1 = Tr\begin{pmatrix} M_1M_1^\dagger \otimes I_3 & & \\ & M_2M_2^\dagger \otimes I_3 & \\ & & M_3M_3^\dagger \end{pmatrix}. \quad (22)$$

It is worthy to see that this is just the ordinary covariant derivative for the Higgs field in the standard model with both correct weak isospin charge and weak hypercharge assignments. We have in the Lagrangian:

$$Tr(\phi^\dagger\phi - \mu^2)^2 = \Sigma_2(\pi^\dagger\pi - \frac{\Sigma_1}{\Sigma_2}\mu^2) + const, \quad (23)$$

where

$$\Sigma_2 = Tr\begin{pmatrix} (M_1M_1^\dagger)^2 \otimes I_3 & & \\ & (M_2M_2^\dagger)^2 \otimes I_3 & \\ & & (M_3M_3^\dagger)^2 \end{pmatrix}. \quad (24)$$

The bosonic part of the entire gauge invariant Lagrangian, by some straightforward calculation, is

$$\begin{aligned} \mathcal{L}_{YM-H} &= -< F, \overline{F} > \\ &= -\tfrac{1}{4N_L}6g^2 W^i_{\mu\nu}W^{i\mu\nu} - \tfrac{1}{4N_Y}10g'^2 B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4N_c}6g_c^2 G^i_{\mu\nu}G^{i\mu\nu} \\ &\quad +2\eta\tfrac{\Sigma_1}{\mu^2}(D_\mu\pi)^\dagger D^\mu\pi - 2\eta^2\tfrac{\Sigma_2}{\mu^2}(\pi^\dagger\pi - \tfrac{\Sigma_1}{\Sigma_2}\mu^2)^2, \end{aligned} \quad (25)$$

where N_L, N_Y, N_c are normalization constants with respect to gauge fields W, B, and G respectively. The normalization of the coefficients of the terms in the Lagrangian leads to

$$N_L = 6g^2, \quad N_Y = 10g'^2, \quad N_c = 6g_c^2, \quad 2\frac{\Sigma_1}{\mu^2}\eta = 1. \quad (26)$$

This gives rise to the following form for the Yang-Mills-Higgs Lagrangian

$$\begin{aligned} \mathcal{L}_{YM-H} = &-\tfrac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} \\ &+(D_\mu\pi)^\dagger D^\mu\pi - \tfrac{1}{2}\tfrac{\Sigma_2}{\Sigma_1^2}(\pi^\dagger\pi - \tfrac{\Sigma_1}{\Sigma_2}\mu^2)^2. \end{aligned} \quad (27)$$

It is easy to see that together with the Lagrangian of the usual gauge fields the kinetic energy of Higgs field and the interaction between Higgs field and the usual gauge fields are all included here.

It is well known that the mass of top quark is much heavier than other fermions. If we set $m_t \gg m_i$, where m_i is the mass for the i-th fermion except t, we have

$$\frac{\Sigma_2}{\Sigma_1^2} = \frac{1}{3}, \quad \frac{\Sigma_1}{\Sigma_2} = \frac{1}{m_{tt}^2}, \quad \epsilon = \sqrt{\frac{\eta}{\mu^2}} = \frac{g}{m_{tt}} \quad (28)$$

where m_{ii} are the eigenvalues of the fermion mass matrices, and m_{tt} is the one corresponding to the top quark. Then the Lagrangian for the generalized gauge fields can be rewritten as

$$\begin{aligned} \mathcal{L}_{YM-H} = &-\tfrac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} \\ &+(D_\mu\pi)^\dagger D^\mu\pi - \tfrac{1}{6}(\pi^\dagger\pi - \tfrac{\mu^2}{m_{tt}^2})^2. \end{aligned} \quad (29)$$

Consequently, when π field takes value $|\pi| = \frac{\mu}{m_{tt}}$, the Higgs potential is at its minimum. If we set

$$<\pi> = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad v = \frac{\sqrt{2}}{m_{tt}}\mu$$

the symmetry $SU(2)_L \times U(1)_Y$ will spontaneously be broken down.

Introducing new field η to replace the field $|\pi|$ in eq.(29) and adding the fermionic part through covariant derivative, we get the final expression of the entire Lagrangian as follows

$$\begin{aligned} \mathcal{L}(x) = &\textstyle\sum_i \overline{q_i}(i\gamma^\mu D_\mu - m_{q_i})q_i + \sum_i \overline{l_i}(i\gamma^\mu D_\mu - m_{l_i})l_i \\ &-\tfrac{1}{M_{tt}^q}\eta\textstyle\sum_i M^q_{ii}\overline{q_i}q_i - \tfrac{1}{M_{tt}^q}\eta\sum_i M^l_{ii}\overline{l_i}l_i \\ &-\tfrac{1}{2}W^\dagger_{\mu\nu}W^{\mu\nu} - \tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \tfrac{1}{4}A_{\mu\nu}A^{\mu\nu} - \tfrac{1}{4}G^i_{\mu\nu}G^{i\mu\nu} \\ &+\tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta + \tfrac{g^2}{4}(v+\eta)^2W^-W^+ + \tfrac{g^2}{8\cos\theta_w^2}(v+\eta)^2 Z_\mu Z^\mu \\ &-\tfrac{1}{6}(v^2\eta^2 + v\eta^3 + \tfrac{\eta^4}{4}), \end{aligned} \quad (30)$$

where the photon and Z boson are given as usual:

$$\begin{aligned} A_\mu &= B_\mu cos\theta_W + W^3_\mu sin\theta_W \\ Z_\mu &= B_\mu sin\theta_W - W^3_\mu cos\theta_W \\ g\sin\theta_W &= g'\cos\theta_W = \tfrac{gg'}{\sqrt{g^2+g'^2}} = e. \end{aligned} \quad (31)$$

Using (26), we get

$$sin^2\theta_W = \frac{g'^2}{g^2+g'^2} \quad \{= \frac{3N_Y}{5N_L + 3N_Y}\}. \quad (32)$$

It is easy to see that the neutrinos, photon and gluons remain massless while other particles become massive. And we can also get the following mass relations,

$$M_W = \frac{1}{2}gv, \quad M_Z = \frac{M_W}{\cos\theta_w}M_{Higgs} = \frac{v}{\sqrt{3}}, \quad m_t \approx \mu. \tag{33}$$

Then we may have

$$\frac{M_{Higgs}}{M_W} \approx \frac{2}{\sqrt{3}g}, \quad \frac{m_t}{M_W} = \frac{\sqrt{2}}{\epsilon}, \tag{34}$$

where ϵ is a free parameter coming from the metric $< \chi^r, \chi^r >$ as introduced before. If we take $\epsilon = 1$, we may get a mass ratio m_t/M_W at the tree level as well. Of course, this also means, on the other hand, the mass ratio constraint can be released by keeping ϵ to be an arbitrary constant.

5. Concluding Remarks

Now we summarize what we have done as follows:

First, we have completed the construction of the generalized gauge theory in which Higgs fields are introduced as gauge fields with respect to the discrete gauge groups and their Yukawa couplings with fermions are automatically given by the generalized minimum coupling principle. It is obvious that our approach is influenced by the approach due to A. Connes [1-4] by means of the non-commutative geometry. On the other hand, however, we have not made use of any concrete knowledge of beautiful but very abstract content in the non-commutative geometry. In terms of the formalism we have completed, we have presented a model with $G_L \times G_R \times Z_2$-gauge symmetry and reformulated the standard model for electroweak-strong interaction among all leptons and quarks in three families. Obviously, the model building introduced here is much simpler than, in the sense of without using the non-commutative geometry, and very different from, in some important aspects, that of Connes and others by means of non-commutative geometry. It turns out, in certain sense, that our formalism is easier to be handled and more reasonable from physical point of view.

One of these very important points is about the constraints among the coupling constants and the mass parameters. We stress that all these constraints are not direct consequences of the gauged discrete symmetry but totally dependent on certain working hypotheses. Therefore, all of them could be completely released at all. In all these models, we have no constraint with the Weinberg angle. We have also shown that the mass ratio m_t/M_W depends on the metric parameter ϵ on the group $Z_2 = \{e, r = (CPT)^2\}$. As for the mass ratio M_H/M_W, it can also be released, say, by introducing a relative metric parameter in the metric on $M^4 \times Z_2$ or by other means [15]. Of course, it should be interesting to see whether these two mass ratios could survive quantum correlations. This is one of the open questions to be understood in

our approach. Since our approach is very different from the ones by means of non-commutative geometry, the answer to this question may also different. Especially, it is important to see what the role played by the symmetry $Z_2 = \{e, r = (CPT)^2\}$ is in the quantum version of the present formalism. Of course, to understand these problems needs further investigations.

Another important point different from the ones by Connes and others is about the fundamental fields. In our approach the gauge fields and Higgs have been directly dealt with as fundamental dynamical variables rather than the ones defined by a huge number of auxiliary scalar fields introduced by Connes and others in their non-commutative geometry approach. Eventually, it is somehow difficult to explain what the physical meaning is for that huge number of auxiliary scalar fields and how to quantize them.

The third important point different from the ones by Connes and others is as follows. We first arrange the fermions, gauge bosons and Higgs into two sectors and assign them to two elements of the discrete group Z_2. Then we gauge the Z_2 symmetry and regard Higgs as a generalized gauge field with respect to the Z_2-gauge symmetry. In other words, we have dealt with two sectors of fields on a 4-dimensional spacetime M^4 rather than two sheeted spacetime [1-4] or two parallel universes [5]. Those fields transform to each other under $Z_2 = \{e, r = (CPT)^2\}$. Eventually, the symmetry $Z_2 = \{e, r = (CPT)^2\}$ is *an intrinsic symmetry* of these models in their conventional version. What we have done is to gauge it, in the sense that the gauge transformations depend on the both spacetime points and the Z_2 elements, and to show its gauge potential is just the Higgs field in these models. Especially, the role played by Higgs is something just like a bridge linking the fermions and gauge bosons in the two sectors.

It is worthy to note that one of the important points in our approach is the link between the Z_2-gauge symmetry in CPT and the standard model. It implies that the CPT symmetry as a whole probably should be gauged. In fact, we have gauged what we call in [14] the restricted CPT symmetry and reformulated the Weinberg-Salam model with a simple Ansatz for Higgs. It also implies that the CPT symmetry as a whole probably should be gauged. Why the CPT symmetry should be gauged is in fact a simple but profound question similar to the questions why the Yang-Mills gauge fields should be introduced and why the Lorentz group should be gauged. Of course, by gauging a discrete group we always mean that the the gauge transformations are dependent on the elements of these discrete groups rather than the discrete group elements are of the functions of the spacetime points. As is well-known, the content and implication of the CPT symmetry is very rich. Therefore, to gauge the entire CPT symmetry may shed light on some fundamental problems, such as the CP violation, the generation of mass and so on. Recently, Hall and Weinberg [16] have suggested that in order to explain the CP nonconservation systematically, the flavour change scalar interactions should be introduced in some extended vertion of the standard model. In fact, what they introduced is something like the Yukawa couplings among the three generations via Higgs fields in addition to the Higgs in the ordinary version of the standard model. But, in their approach, the full Higgs potentials are to complicated to write down and there are too many free parameters involved. ¿From the point of view of the present approach, the whole structure of the Yukawa couplings in their approach indicates that we should take $Z_3 \times Z_2$ as a discrete gauge group where Z_3 is of the generations. The full potential for the Higgs could also be written explicitly and the total number of the free parameters is much less than that of conventional version. This also indicates that there might be more prediction power in the present approach. Needless to say, These topics also need further investigation.

Finally, it should be mentioned that there is also another candidate for the discrete gauge group Z_2 in the standard model. Namely, the fourth homotopy group of the gauge group in the standard model $\pi^4(SU(3) \times SU(2) \times U(1)) = \pi^4(SU(2)) = Z_2$. The role played by this group might be quite remarkable. This also needs further investigation.

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