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CONSTRUCTION OF HIGGS POTENTIAL WITH SPONTANEOUS CP VIOLATION IN TERMS OF THE GAUGE THEORY ON $z_2 \times z_3$ GROUP

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Construction of Higgs Potential with Spontaneous CP violation in Terms of the Gauge Theory on $z_2 \times z_3$ Group [1]

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Abstract

The gauge theory on $z_2 \times z_3$ group is presented according to the general formalism of the gauge theory on discrete group. The Yang-Mills action, i.e, the Higgs potential is constructed with the assumption of the gauge fields consisted of three Higgs doublets and the assumption of the metric on the differential algebra. There are two types of solutions to the constrained equations imposed on the vacuum expected values of the three Higgs doublets, one is conserving the CP symmetry and the other violating it. The violating solution is corresponding to the true vacuum.

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1. Introduction

Although various models for the spontaneous CP violation[5-8] have been advocated in the past three decades, the concrete form of the Higgs potential, the most important issue for these models, were generally put in them by hand.

Thanks to the non-commntative geometry[9-13] and the gauge theory on discrete group[1], we are given the ability to deduce the Higgs potential from the gauge theory on a discrete group. Futhermore, this gauge field will be connected to the Yukawa coupling between the fermion and the scalar. If one knows the symmetry of the lagrangian among the fermions, for example the (approximate) z_3 generation symmetry or the z_2 left-right chiral symmetry, he can construct a gauge theory on the group which reflects that symmetry, and get the Higgs potential from the Yang-Mills theory on those discrete groups. In fact, the symmetry manifests itself in the Yukawa coupling and it is in this aspect that the Hall-Weinberg model[4] gives us some hints to the concrete discrete group and the gauge field on it.

In this paper we will construct the Higgs potential on the $z_2 \times z_3$ group from the general formalism of the gauge theory on discrete group, and discuss the issue of the spontaneous CP violation.

To implement the spontaneous CP violation, there must be several Higgs doublets and in order to make the Higgs's acquiring a complex vacuum expectation values (VEV's), the Higgs potential must have some special properties:

(1) the 2-nd order term is negative definite;

(2) it has non-self-conjugate term (4-th order and/or 2-nd order) and the coefficients of these terms have to meet some special conditions. (ex. Branco's condition in the NFC condition[7].)

This strongly limits the choice of the model——the discrete group, the concrete form of the construction of the Yang-Mills action and the metric on the space of the differential algebra.

To show the main spirit and main results of the spontaneous CP violation model from the gauge theory on discrete group, in this paper we will consider a toy model in which the Higgs potential is contributed by the gauge field which couples to fermions in a special manner. The steps to build our model are the following:

(1) the choice of the discrete group——$z_2 \times z_3$;

(2) the assignment of the quarks and the leptons——their chiralities and generations are corresponding to the elements of group z_2 and z_3 respectively;

(3) the assignment of the Higgs——to fit to its Yukawa coupling with the quarks and the leptons, we will assume it has no-null elements connected to the elements g_3, g_4, g_5 of the group only;

(4) the metric on the space of the differential algebra——include the special term only for commutative group, this term include the main contribution to the CP violation (assume that the terms connected to the elements e, g_1, g_2 may be neglected.)

We will construct the gauge theory on $z_2 \times z_3$ group in the section 2 first, then in terms of it to build a toy model of the spontaneous CP violation in particle physics in section 3, and finally make some remarks on the results of this paper in discussions.

2. The gauge theory on $z_2 \times z_3$ group

In this section we will show the main results of the gauge theory on $z_2 \times z_3$ group. The elements of $z_2 \times z_3$ group are $\{e, g_1, g_2, g_3, g_4, g_5\}$ and the multiplication rule of these elements is in the following table:

	e	g_1	g_2	g_3	g_4	g_5
e	e	g_1	g_2	g_3	g_4	g_5
g_1	g_1	g_2	e	g_4	g_5	g_3
g_2	g_2	e	g_1	g_5	g_3	g_4
g_3	g_3	g_4	g_5	e	g_1	g_2
g_4	g_4	g_5	g_3	g_1	g_2	e
g_5	g_5	g_3	g_4	g_2	e	g_1

(1)

The general formalism of the differential geometry on discrete group has been introduced in detail in the reference[1], [2]. For the group $G = z_2 \times z_3$, concretely, there are five independent one-forms in the one form space of $\Omega^{(1)}$, the index of those one-forms are denoted by the subscripts $\{g_1, g_2, g_3, g_4, g_5\} \in G' = G/\{e\}$. The components of the gauge field on the discrete group G, correspondingly the components $\Phi_g(h)$ of the Higgs, are denoted by two entries, one for the entry of one forms in the space of $\Omega^{(1)}$, i.e. $g \in G'$, and the other for the entry of the space of the group G, i.e. $h \in G$. For the assignment of the Higgs, as we said before, we assume that $\Phi_{g_1}(h) = 0$, $\Phi_{g_2}(h) = 0$ and its no-null elements are $\Phi_{g_3}(h)$, $\Phi_{g_4}(h)$, $\Phi_{g_5}(h)$ in order to fit its Yukawa coupling with the fermions. So we must consider the fermions at the first. In this paper, coresponding to the elements $e, g_1, g_2, g_3, g_4, g_5$ of group G, the assignment of the quarks and leptons is a 6×1 matrix, each element in this matrix is the $SU(2)$ doublet, as follows:

$$Q \equiv \begin{bmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_L \\ \begin{pmatrix} c \\ s \end{pmatrix}_L \\ \begin{pmatrix} t \\ b \end{pmatrix}_L \\ \begin{pmatrix} u \\ d \end{pmatrix}_R \\ \begin{pmatrix} c \\ s \end{pmatrix}_R \\ \begin{pmatrix} t \\ b \end{pmatrix}_R \end{bmatrix}, \quad L \equiv \begin{bmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L \\ \begin{pmatrix} \nu \\ \mu \end{pmatrix}_L \\ \begin{pmatrix} \nu \\ \tau \end{pmatrix}_L \\ \begin{pmatrix} 0 \\ e \end{pmatrix}_R \\ \begin{pmatrix} 0 \\ \mu \end{pmatrix}_R \\ \begin{pmatrix} 0 \\ \tau \end{pmatrix}_R \end{bmatrix} \tag{2}$$

To fit its Yukawa coupling with the quarks and the leptons, then, the assignment of the Higgs fields is as the following:

$$\Phi^Q \equiv \begin{bmatrix} & & & \Phi_3(e) & \Phi_4(e) & \Phi_5(e) \\ & & & \Phi_5(g_1) & \Phi_3(g_1) & \Phi_4(g_1) \\ & & & \Phi_4(g_2) & \Phi_3(g_2) & \Phi_3(g_2) \\ \Phi_3(g_3) & \Phi_4(g_3) & \Phi_5(g_3) & & & \\ \Phi_5(g_4) & \Phi_3(g_4) & \Phi_4(g_4) & & & \\ \Phi_4(g_5) & \Phi_5(g_5) & \Phi_3(g_5) & & & \end{bmatrix} = \begin{bmatrix} 0 & \Phi^Q_{ij} \\ \Phi^{Q\dagger}_{ij} & 0 \end{bmatrix} \tag{3}$$

Similarly, the lepton part is:

$$\Phi^L \equiv \begin{bmatrix} & & & \Phi'_3(e) & 0 & 0 \\ & & & 0 & \Phi'_3(g_1) & 0 \\ & & & 0 & 0 & \Phi'_3(g_2) \\ \Phi'_3(g_3) & 0 & 0 & & & \\ 0 & \Phi'_3(g_4) & 0 & & & \\ 0 & 0 & \Phi'_3(g_5) & & & \end{bmatrix} = \begin{bmatrix} 0 & \Phi^L_{ij} \\ \Phi^{L\dagger}_{ij} & 0 \end{bmatrix} \tag{4}$$

where, following Hall and Weinberg[4], we assume Φ^Q_{ij}, Φ^L_{ij} have facterized forms as follows,

$$\Phi^Q_{ij} \equiv Q_i \left[\begin{matrix} U_j \tilde{\phi}_n & D_j \phi_n \end{matrix} \right] \tag{5}$$

and

$$\Phi^L_{ij} \equiv L_i \delta_{ij} \left[\begin{matrix} 0 & \phi_n \end{matrix} \right] \tag{6}$$

where the coefficents Q_i, U_j, D_j and L_i will be explained latter and $\tilde{\phi}_n$ and ϕ_n are usual Higgs doublets (see eq. (16), (17) in the following).

According to the differential calculus on discrete groups[1], we get the curvature tensor in terms of the Higgs field introduced above,

$$F_{gh}(k) = \Phi_g(k) R_g \Phi_h(k) - \Phi_{h\otimes g}(k) \tag{7}$$

where

$$F_{gh}(k) = \Phi_g(k)\Phi_h(kg) - I, \qquad (g, h \in \{g_3, g_4, g_5\}) \tag{8}$$

$$F_{gh}(k) = \Phi_g(k) - \Phi_{h\otimes g}(k), \qquad (g \in \{g_3, g_4, g_5\}, h \in \{g_1, g_2\}) \tag{9}$$

And from

$$\Phi^\dagger_g(h) = \Phi_{g^{-1}}(hg) \tag{10}$$

we find the following conjugate properties of the curvature tensor for a commutative discrete group,

$$F^\dagger_{gh}(k) = F_{h^{-1}g^{-1}}(kgh) \tag{11}$$

To construct the action of the gauge field on discrete group we choose the metric on the space of one forms in the differential algebra as:

$$< \chi^g, \chi^h > = E(g)\delta_{gh} \tag{12}$$

and the metric on the space of two forms,

$$< \chi^g \otimes \chi^h, \chi^{g'} \otimes \chi^{h'} > = E(g)E(h)(\alpha\delta_{gg'}\delta_{hh'} + \beta\delta_{gh'}\delta_{hg'}) \tag{13}$$

where $E(g), \alpha, \beta$ are free parameters, in this paper we will set $E(g) = 1$, for simplisity.

Then the Yang-Mills action is expressed as:

$$S_{YM} = \alpha \int F_{gh}(k) F^\dagger_{g^{-1}h^{-1}}(k) + \beta \int F_{gh}(k) F^\dagger_{h^{-1}g^{-1}}(k) \tag{14}$$

So far we have not mentioned the relation between the gauge fields on discrete group and the Higgs doublets. In order to express the Higgs potential in terms of the Higgs doublets (in this model we assume that there are three Higgs doublets altogether) we make the assumption as follows:

(1) the quarks in same generation interact to ϕ_1; the quarks in two neighbourhood generations interact to ϕ_2; the quarks respectly in 1-st and 3-rd generation interact to each other through ϕ_3.

(2) the coefficients of the Yukawa coupling have the following factorized form following Hall and Weinberg[4].

The Hall-Weinberg model for the Yukawa coupling is as the following:

$$\mathcal{L}_Y = -\lambda^U_{ijn} \bar{Q}_{Li} U_{Rj} \tilde{\phi}_n - \lambda^D_{ijn} \bar{Q}_{Li} D_{Rj} \phi_n - \lambda^E_{ijn} \bar{L}_{Li} E_{Rj} \phi_n + h.c \tag{15}$$

with

$$Q_{Lj} \equiv \begin{bmatrix} U_{Lj} \\ D_{Lj} \end{bmatrix}, \quad \phi_n \equiv \begin{bmatrix} \phi^+_n \\ \varphi^0_n \end{bmatrix}, \quad \tilde{\phi}_n \equiv \begin{bmatrix} \phi^{0*}_n \\ -\phi^{+*}_n \end{bmatrix} \tag{16}$$

and

$$|\lambda^U_{ijn}| \approx Q_i U_j, \quad |\lambda^D_{ijn}| \approx Q_i D_j, \quad |\lambda^E_{ijn}| \approx L_i E_j \tag{17}$$

In terms of the knowledge of the gauge theory on discrete group we can write the Yukawa coupling as follows:

$$\bar{\Psi}(h)\Phi_g(h)\Psi(hg) = \bar{\Psi}_L \Phi_{ij} \Psi_R \tag{18}$$

where

$$\bar{\Psi}_L : (\bar{\Psi}(e), \bar{\Psi}(g_1), \bar{\Psi}(g_2))$$

and

$$\Psi_R : \begin{pmatrix} \psi_R(g_3) \\ \psi_R(g_4) \\ \psi_R(g_5) \end{pmatrix}$$

We can also write Yucawa coupling in matrix notion,

$$\begin{pmatrix} \bar{\Psi}_L & \bar{\Psi}_R \end{pmatrix} \cdot \begin{pmatrix} 0 & \Phi_{ij} \\ \Phi_{ij}^{\dagger} & 0 \end{pmatrix} \cdot \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \tag{19}$$

where

$$\Phi_{ij} = \begin{pmatrix} \Phi_{g_3}(e) & \Phi_{g_4}(e) & \Phi_{g_5}(e) \\ \Phi_{g_5}(g_1) & \Phi_{g_3}(g_1) & \Phi_{g_4}(g_1) \\ \Phi_{g_4}(g_2) & \Phi_{g_5}(g_2) & \Phi_{g_3}(g_2) \end{pmatrix} \tag{20}$$

Then the action of the quarks and leptons coupled to the Higgs can be expressed as

$$\mathcal{L}_Y = \mathcal{L}_Y^Q + \mathcal{L}_Y^L \tag{21}$$

$$\begin{aligned} \mathcal{L}_Y^Q &= \sum_{h,g} \bar{Q}_L(h)\Phi_g^Q(h)Q_R(hg) \\ &= Tr(\bar{Q}\Phi^Q Q) \end{aligned} \tag{22}$$

$$\begin{aligned} \mathcal{L}_Y^L &= \sum_{h,g} \bar{L}_L(h)\Phi_g^L(h)L_R(hg) \\ &= Tr(\bar{L}\Phi^L L) \end{aligned} \tag{23}$$

The numerical values of the factors are chosen, according to the values of the mass matrix of fermions, as follows:

$$Q_1 = 0.008, \quad Q_2 = 0.04, \quad Q_3 = 1,$$

$$U_1 = 0.004, \quad U_2 = 0.2, \quad U_3 = 1,$$

$$D_1 = 0.006, \quad D_2 = 0.025, \quad D_3 = 0.03,$$

$$L_1 = 0.0000028, \quad L_2 = 0.0006, \quad L_3 = 0.01.$$

From the above we can express the Φ_{ij}^Q and Φ_{ij}^L in terms of the three Higgs doublets ϕ_i. Then the Higgs potential, through the standard procedure of calculation[1] and [2], is deduced as follows:

$$\begin{aligned} V(\phi_1,\phi_2,\phi_3) = & \sum_{i,j} A_{ij}\phi_i\phi_i^{\dagger}\phi_j\phi_j^{\dagger} + \sum_{i\neq j} B_{ij}\phi_i\phi_j^{\dagger}\phi_i\phi_j^{\dagger} + \sum_{i\neq j} A'_{ij}\phi_i^{\dagger}\phi_i\phi_j^{\dagger}\phi_j \\ & + \sum_{i,j} C_{ij}\phi_i^{*}\phi_i^{*\dagger}\phi_j^{*}\phi_j^{*\dagger} + \sum_{i\neq j} C'_{ij}\phi_i^{*\dagger}\phi_i^{*}\phi_j^{*\dagger}\phi_j^{*} + \sum_{i,j} D_{ij}\phi_i^{*}\phi_j^{*\dagger}\phi_i^{*}\phi_j^{*\dagger} \\ & + \sum_{i,j} E_{ij}\phi_i\phi_i^{\dagger}\phi_j^{*}\phi_j^{*\dagger} + \sum_{i\neq j} F_{ij}\phi_i\phi_j^{\dagger}\phi_i^{*}\phi_j^{*\dagger} + \sum_{i\neq j} G_{ij}\phi_i\phi_j^{\dagger}\phi_j^{*}\phi_i^{*\dagger} \\ & - \sum_{i,j} H_{ij}\phi_i\phi_j^{\dagger} - \sum_{i,j} K_{ij}\phi_i^{*}\phi_j^{*\dagger} + 81 \end{aligned} \tag{24}$$

where we have chosen

$$\alpha = 0.0001, \quad \beta = 1.$$

in (14), and the coefficients matrices in (24) are

$$A_{ij} = \begin{pmatrix} 1.6\times10^{-6} & 5.6\times10^{-11} & 3.2\times10^{-12} \\ 5.6\times10^{-11} & 1.8\times10^{-10} & 4.5\times10^{-8} \\ 3.2\times10^{-12} & 4.5\times10^{-8} & 2.6\times10^{-13} \end{pmatrix},$$

$$B_{ij} = \begin{pmatrix} 0 & 3.6\times10^{-9} & 8.3\times10^{-12} \\ 3.6\times10^{-9} & 0 & 0 \\ 8.3\times10^{-12} & 0 & 0 \end{pmatrix},$$

$$A'_{ij} = \begin{pmatrix} 0 & 1.9\times10^{-13} & 5.2\times10^{-15} \\ 1.9\times10^{-13} & 0 & 4.3\times10^{-12} \\ 5.2\times10^{-15} & 4.3\times10^{-12} & 0 \end{pmatrix},$$

$$C_{ij} = \begin{pmatrix} 2. & 0 & 0 \\ 0 & 7.3\times10^{-7} & 7.4\times10^{-7} \\ 7.4\times10^{-7} & 0 & 8.0\times10^{-13} \end{pmatrix},$$

$$C'_{ij} = \begin{pmatrix} 0 & 4.2\times10^{-6} & 8.\times10^{-9} \\ 4.2\times10^{-6} & 0 & 7.4\times10^{-7} \\ 8.\times10^{-9} & 7.4\times10^{-7} & 0 \end{pmatrix},$$

$$D_{ij} = \begin{pmatrix} 0 & 1.3\times10^{-4} & 2.1\times10^{-9} \\ 1.3\times10^{-4} & 0 & 0 \\ 2.1\times10^{-9} & 0 & 0 \end{pmatrix},$$

$$E_{ij} = \begin{pmatrix} 2.7\times10^{-3} & 7.2\times10^{-9} & 2.9\times10^{-12} \\ 12.5\times10^{-8} & 23.2\times10^{-9} & 4.\times10^{-8} \\ 5.9\times10^{-9} & 5.8\times10^{-6} & 2.3\times10^{-13} \end{pmatrix},$$

$$F_{ij} = \begin{pmatrix} 0 & 4.2\times10^{-6} & 9.2\times10^{-9} \\ 4.2\times10^{-6} & 0 & 0 \\ 9.2\times10^{-9} & 0 & 0 \end{pmatrix},$$

$$G_{ij} = \begin{pmatrix} 0 & 3.0\times10^{-10} & 1.2\times10^{-11} \\ 3.0\times10^{-10} & 0 & 3.7\times10^{-10} \\ 1.2\times10^{-11} & 3.7\times10^{-10} & 0 \end{pmatrix},$$

$$H_{ij} = \begin{pmatrix} 3.6\times10^{-3} & 1.6\times10^{-3} & 3.8\times10^{-4} \\ 1.6\times10^{-3} & 2.5\times10^{-3} & 3.0\times10^{-4} \\ 3.8\times10^{-4} & 3.0\times10^{-4} & 1.4\times10^{-4} \end{pmatrix},$$

$$K_{ij} = \begin{pmatrix} 4. & 0.49 & 0.02 \\ 0.49 & 0.17 & 2.3\times10^{-3} \\ 0.02 & 2.3\times10^{-3} & 3.2\times10^{-4} \end{pmatrix}$$

The concrete form for the Higgs potential of three Higgs doublets model is main result of this paper. Obviously, it has the property for a breaking potential: the 2-nd order term is negative definite; and it has non-self-conjugate term (4-th order and/or 2-nd order). Although it is obtained at some special assumptions, it is the start-point to build a toy model for spontaneous CP violation.

3. A toy model for spontaneous CP violation

Now we have a Higgs potential at the hand, we can discuss the issue of spontaneous CP violation. The next step to build a spontaneous CP violating model is to decide the vacuum expecting values of the Higgs doublets. First, we assume the VEV's of the Higgs doublets take the form

$$<\phi_i>=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu_i exp(i\theta_i) \end{pmatrix} \tag{25}$$

where the phase angle of the 1-st Higgs doublet is assumed to be zero because the physical results connect to the relative phase angles only. So there are five parameters describing the VEV's which satisfy, something-like the Branco's[7], the following five constrain equations:

$$\begin{aligned}
F(1) &= \nu_3^2\nu_1 cos^2(\theta_3) + 4.750\times10^{-1}\nu_3^2\nu_1 - 1.485\times10^6\nu_3 cos(\theta_3) \\
&\quad + 6.237\times10^4\nu_2^2\nu_1 cos^2(\theta_2) - 3.068\times10^4\nu_2^2\nu_1 \\
&\quad - 2.958\times10^7\nu_2 cos(\theta_2) + 2.437\times10^8\nu_1^3 - 2.440\times10^8\nu_1 = 0. \\
F(2) &= \nu_3^2\nu_2 - 8.400\times10^2\nu_3 sin(\theta_3)sin(\theta_2) \\
&\quad - 8.400\times10^2\nu_3 cos(\theta_3)cos(\theta_2) + 4.774\times10^{-1}\nu_2^3 \\
&\quad + 3.346\times10^2\nu_2\nu_1^2 cos^2(\theta_2) - 1.646\times10^2\nu_2\nu_1^2 \\
&\quad - 5.563\times10^4\nu_2 - 1.587\times10^5\nu_1 cos(\theta_2) = 0. \\
F(3) &= \nu_3^3 + 1.355\times10^6\nu_3\nu_2^2 + 7.267\times10^3\nu_3\nu_1^2 cos^2(\theta_3) \\
&\quad + 3.452\times10^3\nu_3\nu_1^2 - 2.055\times10^8\nu_3 \\
&\quad - 1.138\times10^9\nu_2 sin(\theta_3)sin(\theta_2) - 1.138\times10^9\nu_2 cos(\theta_3)cos(\theta_2)) \\
&\quad - 1.079\times10^{10}\nu_1 cos(\theta_3) = 0. \\
F(4) &= \nu_3 sin(\theta_3)cos(\theta_2) - \nu_3 sin(\theta_2)cos(\theta_3) \\
&\quad + 3.983\times10^{-1}\nu_2\nu_1^2 sin(\theta_2)cos(\theta_2) \\
&\quad - 1.889\times10^2\nu_1 sin(\theta_2) = 0. \\
F(5) &= \nu_3\nu_1^2 sin(\theta_3)cos(\theta_3) - 1.566\times10^5\nu_2 sin(\theta_3)cos(\theta_2) \\
&\quad + 1.566\times10^5\nu_2 sin(\theta_2)cos(\theta_3) - 1.485\times10^6\nu_1 sin(\theta_3) = 0.
\end{aligned} \tag{26}$$

We solve these equations through the numerical calculation using the computer. The result of the numerical calculation shows that there are two types of solutions to them: one is the CP-conserving one, the other the CP-violating one.

The CP-conserving solution is

$$\theta_2 = 0, \quad \theta_3 = 0, \quad \nu_1 = 1.755, \quad \nu_2 = 15.57, \quad \nu_3 = 298.3 .$$

The CP-violating solution is

$$\theta_2 = -1.2289, \quad \theta_3 = -1.1295, \quad \nu_1 = 4.048, \quad \nu_2 = 349.5, \quad \nu_3 = 2.507 .$$

The energy level for the CP-violating solution is much lower than the one for the CP-conserving solution and it is ease to prove that the CP-violating solution is stable. Thus we have presented a toy model of spontaneous CP violation by use of the formalism of gauge theory on discrete group.

4. Discussion

In this paper we have constructed the Higgs potential of spontaneous CP vialation with use of the guage theory on discrete group. It is interesting to note that the spontaneous breaking of CP symmetry, as the symmetry breaking in the standard model of particle physics, may also be explained in the frame of the gauge theory on discrete group, although it is still far from a realistic model now. The conclusion which may be drewn from the results of this paper are:

(1) at the certain condition the gauge theory on a discrete group may leads to spontaneous CP violation, i.e, the Y-M action (Higgs potential) leads spontaneously to a complex VEV's;

(2) the factorization of the coefficients of the Yukawa coupling, the H-W assumption[4], and the values of the factors have connection to the concrete form of the Higgs potential and to the various CP violation properties.

It is interesting to investigate the phenomenological implication which is now going on.

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