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**Evidence for coexistence of two phases in relativistic  
heavy ion collisions and comparison with lattice  
results**

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# Evidence for Coexistence of Two Phases in Relativistic H.I. Collisions (RHIC) and Comparison with Lattice

## Results

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## Abstract

The combined CERN and Brookhaven heavy ion (H.I.) data supports a scenario of hadron gas which is in chemical and thermal equilibrium at a temperature  $T$  of about 140 MeV. Using the Brown-Stachel-Welke model (which gives 150 MeV) we show that in this scenario, the hot nucleons have mass  $3\pi T$  and the  $\pi$  and  $\rho$  mesons have masses close to  $\pi T$  and  $2\pi T$ . This provides a possible connection with Euclidean time lattice masses of quarks. A simple model with pions and quarks supports the co-existence of two phases in these heavy ion experiments, suggesting a second order phase transition.

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Stachel [1] has argued strongly in favour of a model of heavy ion collisions which suggests a hadron gas in chemical and thermal equilibrium with  $T = 140 \pm 20 \text{ MeV}$  confined in a volume of about  $2400 \text{ fm}^3$  in the mid-rapidity region. The support for this scenario comes from the absence of a peak at beam rapidity, the similarity of the data at CERN and Brookhaven at very different experimental conditions<sup>1</sup> and from a beautiful paper by Brown, Stachel and Welke (hence forth BSW in short)[3] where they use the experimental  $N\Delta$  (nucleon-isobar) spectrum to estimate the  $\Delta$ -s. From this percentage of the  $\Delta$ -s they predict  $T$ .

It is natural to ask the question whether this picture of a hadron gas has any connection with QCD, for example the recent first-principle lattice calculations at finite  $T$ , reviewed for example in Pettersson [4]. We calculate the energy of the hot nucleon in the model of BSW and find that in the relevant range of about  $T = 140 \text{ MeV}$  this goes like  $3\pi T$  just like the correlator mass in the lattice calculations. This supports the calculations of Huang [5] who found that in  $1+1$  dimensional Gross-Neveu model the imaginary time lattice and the real-time pole masses agree even at low  $T$ . Also since the lattice uses QCD and we (like BSW) have only used the hadron spectrum this also suggests that the hadron gas and QCD coexist in this realm of experience.

For completeness we give the relevant formulae that we use from BSW. The distributions are defined in terms of the chemical potential  $\mu$ , as

$$f_{\pm}(\mu) = 1 / \left( \exp \frac{\epsilon_i - \mu}{T} \pm 1 \right), \quad (1)$$

where the  $\pm$  signs stand for fermions/bosons and the states are defined in terms

<sup>1</sup>a very telling picture is told by figure 2 of [1] when compared to fig. 17 of [2]

of the hadron masses  $m_i$ :

$$\epsilon_i = (m_i^2 + k^2)^{1/2} \quad (2)$$

with  $k$  is the momentum in  $\text{GeV}$ .

The number and energy in any particular state is then (for a volume  $V(R)$ )

$$N_i = \frac{g_i V(R)}{2\pi^2} \int_0^\infty dk k^2 f_{\pm}(k, \mu), \quad (3)$$

We give our results in Table 1 for  $T = 140, 150$  and  $160 \text{ MeV}$ , where  $R_i$  is defined to be

$$R_i(T) = \frac{N_i}{\sum_i N_i}. \quad (4)$$

The results of [3] are reproduced for  $T = 150$ . The next obvious step is to take the  $\pi$  and  $\rho$  meson excitations from experiment [6], the  $R_i(T)$  are given in Table 2. For bosons the chemical potential  $\mu$  is set equal to zero. For  $T = 133.5 \text{ MeV}$  we reproduce the number of  $\pi^+$   $\pi^-$  reported in [1], namely 132. At this  $T$  the number of  $\rho$  mesons is 25, in agreement with data[7].

Now we calculate the hot masses for these particles :

$$E_i = \frac{g_i V(R)}{2\pi^2} \int_0^\infty dk k^2 \epsilon_i f_{\pm}(k, \mu). \quad (5)$$

which is given in Table 3. We find that the masses go like the lattice predictions:  $\pi T$  for the pion within 3  $\rightarrow$  5%,  $2\pi T$  for  $\rho$  within 10  $\rightarrow$  20%, and the nucleon within 1  $\rightarrow$  8% (Table 3). Note that the lattice results are valid at  $T$  equal and greater than  $T_c$ . So it appears that the hadron model is coexisting with the quark result at this  $T$ , in this volume. This agrees with the quark model calculations of [8]. To check this we calculate the entropy of the system. As in Stachel [1], the sum of baryon and pion entropy per baryon is listed in Table 3, and agrees

qualitatively with the number 12.2 given by her. We want next to compare it with the entropy of a simple quark model.

To construct a model of  $N\Delta$  in terms of the quarks we use the MIT bag. We will spend a little time justifying this. In their original paper the MIT group [9] had already hinted that the observed hadron density of states may be obtained from the bag correctly. Indeed Dey, Tomio and Dey [10] showed that the meson spectrum is essentially well-reproduced from the Laplace transform of the bag partition function and recently Dey, Dey and Tomio[11] extended the calculation to baryons. These calculations are reviewed in [12]. Colour projection was shown to improve the results marginally[13]. So with bag states we expect to get results similar to what we obtain from the experimental density of states.

We keep the pions as in the model BSW. The picture we have in mind is similar to the chiral/cloudy bag models. Only the nucleons are described as quarks, bound in a large bag. It was shown by Dey, Dey and Ghose [14] that such large bags are expected as the bag free energy shows extrema at two values of radius around 140 MeV. It was shown that colour projection does not change this character [15] and once again a lengthy recent letter reiterates the same result [16].

The bag states  $q_i$  and the degeneracies  $dq_i$  are given in [8], and the results for this model are in Table 4. The  $c_i$  of eq.(2) are now just  $q_i/R$  and we have

$$E(\mu) = \frac{12}{R} \sum_i (q_i/R) dq_i f_+(\mu), \quad (6)$$

and the corresponding entropy is:

$$S(\mu) = 12 \sum_i \left[ \frac{q_i - \mu/R}{RT} f_{\pm} - \ln(1 - f_{\pm}(\mu)) \right] dq_i \quad (7)$$

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Notice that the chemical potential here is defined in units of inverse radius  $R$  unlike in eq.(1)

The sum over states is kept over spin, colour and isospin and to produce the right density of nucleons given in by Stachel [1] we have to use a large  $\mu$ . We feel this is justified since the quarks in the nucleon move in an instanton field [17], and in the bag this field goes as  $1/Radius$  and is lumped with the chemical potential. There is a large literature on this [18] and a recent paper emphasizing the importance of instantons in the nucleon is Banerjee and Forkel[9]. The entropy in Table 4 is close to that of Table 3, showing the possible coexistence.

To summarize we find that if we excite the experimentally observed hadron states, in a volume of  $2400 fm^3$  at a  $T$  around 140 MeV, we get the nucleon, pion and  $\rho$  meson masses going like  $3\pi T$ ,  $\pi T$  and  $2\pi T$  respectively, as in the lattice, enforcing one to believe that at this  $T$  the hadron- and quark- pictures coexist. We check this by showing that the entropy in the hadron model agree with that of a simple quark model for the nucleon. In a first order phase transition a sharp increase in the entropy would be expected.

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Table 1: Occupation probabilities of the nucleon excitations

State	$J^P$	$g_i$	$R_i(140)$	$R_i(150)$	$R_i(160)$
$N(938)$	$\frac{1}{2}^+$	4	0.4772	0.4246	0.3776
$N(1440)$	$\frac{1}{2}^+$	4	0.0231	0.0260	0.0284
$N(1520)$	$\frac{3}{2}^-$	8	0.0281	0.0328	0.0370
$N(1535)$	$\frac{1}{2}^-$	4	0.0128	0.0150	0.0171
$N(1650)$	$\frac{1}{2}^-$	4	0.0062	0.0077	0.0092
$N(1675)$	$\frac{5}{2}^-$	12	0.0159	0.0120	0.0240
$N(1680)$	$\frac{5}{2}^+$	12	0.0151	0.0194	0.0233
$N(1700)$	$\frac{3}{2}^-$	8	0.0090	0.0115	0.0140
$N(1710)$	$\frac{1}{2}^+$	4	0.0042	0.0054	0.0066
$N(1720)$	$\frac{3}{2}^+$	8	0.0079	0.0102	0.0125
$\Delta(1230)$	$\frac{3}{2}^+$	16	0.3368	0.3434	0.3410
$\Delta(1620)$	$\frac{1}{2}^-$	8	0.0150	0.0183	0.0216
$\Delta(1700)$	$\frac{3}{2}^-$	16	0.0181	0.0230	0.0279
$\Delta(1900)$	$\frac{1}{2}^-$	8	0.0025	0.0035	0.0046
$\Delta(1905)$	$\frac{5}{2}^-$	24	0.0073	0.0102	0.0135
$\Delta(1910)$	$\frac{1}{2}^+$	8	0.0024	0.0033	0.0044
$\Delta(1920)$	$\frac{3}{2}^+$	16	0.0044	0.0062	0.0083
$\Delta(1930)$	$\frac{5}{2}^-$	24	0.0062	0.0088	0.0118
$\Delta(1950)$	$\frac{7}{2}^+$	32	0.0073	0.0105	0.0141

Table 2: Occupation probabilities of the excitations of the  $\pi$  and  $\rho$ 

State	$J^P$	$g_i$	$R_i(140)$	$R_i(150)$	$R_i(160)$
$\pi(138)$	0-	3	0.998	0.996	0.994
$\pi(1300)$	0-	3	0.002	0.002	0.004
$\pi(1670)$	2-	15	0.0007	0.001	0.002
$\pi(1770)$	0-	3	0.0001	0.0001	0.0002
$\pi(2100)$	0-	15	0.00004	0.0001	0.0002
$\rho(770)$	1-	9	0.9699	0.9569	0.9409
$\rho(1450)$	1-	9	0.0169	0.0229	0.02958
$\rho(1690)$	3-	21	0.0087	0.0132	0.01886
$\rho(1700)$	1-	9	0.0035	0.0053	0.00765
$\rho(2110)$	1-	9	0.0002	0.0005	0.00079
$\rho(2150)$	1-	9	0.0002	0.0004	0.00063
$\rho(2250)$	3-	21	0.0002	0.0005	0.00084
$\rho(2350)$	5-	33	0.0002	0.0004	0.00075

Table 3: Masses and entropy in the hadron model

$T(\text{MeV})$	133.5	140	150	160
$M_N/3\pi T$	1.091	1.068	1.036	1.007
$M_\pi/\pi T$	1.050	1.041	1.033	1.028
$M_p/2\pi T$	1.222	1.186	1.140	1.102
$(S_N + S_\pi)/n_N$	13.90	12.85	11.67	10.83

Table 4: Entropy in the  $\pi$ -quark model

$T(\text{MeV})$	133.5	140	150	160
$\mu$ (units $1/R$ )	6.4	6.5	6.7	6.8
$(S_q + S_\pi)/3n_q$	12.98	14.06	15.85	17.95