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Fritzsch like quark mass matrices with decoupled u quark

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Abstract

Fritzsch like hermitian quark mass matrices with decoupled u quark as well as nonzero 22-elements, both in 'U' and 'D' sectors, have been considered to reconcile the data regarding m_t^{phys} and CKM matrix elements. Exploiting the idea of maximality of CP-violation, we find two interesting relations between CKM matrix elements, viz., $|V_{td}| \cong |V_{us}||V_{cb}|$, $|V_{ub}| \cong \frac{1}{2}(m_s/m_b)|V_{us}|$. Apart from obtaining $0.05 \le |V_{ub}/V_{cb}| \le 0.12$ and $90\text{GeV} \le m_t^{phys} \le 200\text{GeV}$, we find CP-violating rephasing invariant parameter $|J| \cong (3-5)\times 10^{-5}$. The predicted $|V_{td}|$ value implies almost maximal mixing in B_s^0-$\bar{B}_s^0$ system.

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The standard model of electroweak interactions does not shed any light on the quark mass matrices, as a consequence quark masses and weak mixing angles appear in the theory as free parameters. In the absence of any viable theory for quark mass matrices, several phenomenological models [1-5] have been considered with fair degree of success . The most successful as well as the most economical model is that of Fritzsch [1]. However, the recent data from colliders [6] have certainly ruled out some of these models, whereas it has made Fritzsch model untenable due to its inability to accommodate m_t^{phys}>90 GeV. Recently, several models [7—13] have been proposed, which can accommodate m_t^{phys}>90 GeV, giving several interesting clues to the likely nature of mass matrices as well as generating several useful relations amongst CKM matrix elements. In the absence of a viable dynamical theory for quark mass matrices, it is perhaps desirable to develop phenomenological models which are simple as well as predictive. In case such models, while remaining in tune with data, are able to generate simple relations amongst CKM matrix elements, then these models may provide vital clues for formulating a dynamical theory. We feel that in the formulation of a viable phenomenological scheme, ideas such as hierarchical structure of mass matrices, smallness of CP- violation, relatively smaller u quark mass compared to c and t quark masses, perhaps maximality of CP-violation etc., may play a vital role.

In the present letter, we consider hermitian mass matrices of the following form,

$$M^u = \begin{bmatrix} A^u & 0 & 0 \\ 0 & D^u & B^u \\ 0 & B^{*u} & C^u \end{bmatrix} \qquad (1)$$

and

$$M^d = \begin{bmatrix} 0 & A^d & 0 \\ A^{*d} & D^d & B^d \\ 0 & B^{*d} & C^d \end{bmatrix}, \qquad (2)$$

where the elements of M^i [i = u, d] are supposed to follow the hierarchical structure, e.g., $|A^i| << |B^i| \cong D^i < C^i$. This form of M^d has been considered earlier [12] but the present form of M^u has not been considered anywhere else.

At present we do not attempt to go into the origin of mass matrices, rather try to see the implications of these. In fact, we shall see that these structures not only fit the data but also give very simple relations amongst CKM matrix elements when the above mentioned simplifying assumptions are invoked.

The above matrices M^i can be expressed as

$$M^i = P^i \, \overline{M}^i \, P^{i\dagger}, \qquad (3)$$

where $\overline{M}^i$ are real matrices

$$\overline{M}^u = \begin{bmatrix} |A^u| & 0 & 0 \\ 0 & D^u & |B^u| \\ 0 & |B^u| & C^u \end{bmatrix}, \qquad (4)$$

and

$$\overline{M}^d = \begin{bmatrix} 0 & |A^d| & 0 \\ |A^d| & D^d & |B^d| \\ 0 & |B^d| & C^d \end{bmatrix}, \qquad (5)$$

where $A^i = |A^i| e^{i\alpha_i}$ with $\alpha_u = 0$; $B^i = |B^i| e^{i\beta_i}$ and

$$P^i = \text{diag}\left\{ 1,\ e^{-i\alpha_i},\ e^{-i(\alpha+\beta)_i} \right\}. \qquad (6)$$

The matrices $\bar{M}^i$ can be diagonalized exactly by orthogonal transformations, for example,

$$M^i = O^i M^i_{\text{diag}} (O^i)^\dagger , \qquad (7)$$

where

$$M^i_{\text{diag}} = \text{diag}\left\{ m_1\ ,\ -m_2,\ m_3 \right\} \qquad (8)$$

and subscripts 1,2 and 3 respectively refer to u, c and t quarks in the 'U' sector and d, s and b quarks in 'D' sector.

Using tr M^i, $\text{tr}(M^i)^2$ and det M^i, the matrix elements A^i, B^i, C^i can be expressed in terms of quark masses [14] and the diagonalizing matrices, in the leading order approximation, are

$$O^u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (1-\mathbb{R}_t)^{1/2} & \mathbb{R}_t^{1/2} \\ 0 & -\mathbb{R}_t^{1/2} & (1-\mathbb{R}_t)^{1/2} \end{bmatrix}, \qquad (9)$$

and

$$O^d = \begin{bmatrix} 1 & -b_0 & b_0 c_0^2 \mathbb{R}_d \\ b_0 (1-\mathbb{R}_b)^{1/2} & (1-\mathbb{R}_b)^{1/2} & \mathbb{R}_b^{1/2} \\ -b_0 \mathbb{R}_b^{1/2} & -\mathbb{R}_b^{1/2} & (1-\mathbb{R}_b)^{1/2} \end{bmatrix}, \qquad (10)$$

where $\mathbb{R}_b = D^d/m_b$, $\mathbb{R}_t = D^u/m_t$ $\mathbb{R}_d \cong \left\{ \mathbb{R}_b/(1-\mathbb{R}_b) \right\}^{\frac{1}{2}}$, $b_0^2 = m_d/m_s$ and $c_0^2 = m_s/m_b$.

The quark mixing matrix V_{CKM} in terms of O^i can be expressed as

$$V_{CKM} = O^{u\dagger} P^{ud} O^{d}, \qquad (11)$$

where $P^{ud} = P^{u\dagger}P^{d} = \mathrm{diag}\left\{1,\ e^{i\phi_1},\ e^{i\phi_2}\right\}$; $\phi_1 = -\alpha^d$, $\phi_2 - \phi_1 = \beta^u - \beta^d$.

By retaining terms of leading order, V_{CKM} can be expressed as

$$V_{CKM} \cong \begin{bmatrix} 1 & -b_0 & b_0 c_0^2 \mathbb{R}_L \\ b_0 g_1 & g_1 & g_2 \\ b_0 g_4 & g_4 & g_3 \end{bmatrix}, \qquad (12)$$

where

$$g_1 = \left\{(1-\mathbb{R}_t)(1-\mathbb{R}_b)\right\}^{1/2} e^{i\phi_1} + \left\{\mathbb{R}_t\,\mathbb{R}_b\right\}^{1/2} e^{i\phi_2}, \qquad (13)$$

$$g_2 = \left\{(1-\mathbb{R}_t)\,\mathbb{R}_b\right\}^{1/2} e^{i\phi_1} - \left\{\mathbb{R}_t(1-\mathbb{R}_b)\right\}^{1/2} e^{i\phi_2}, \qquad (14)$$

$$g_3 = g_1(\phi_1 \leftrightarrow \phi_2), \qquad (15)$$

$$g_4 = -g_2(\phi_1 \leftrightarrow \phi_2). \qquad (16)$$

Since the purpose here is not to delve into a detailed parameter fitting, therefore, we have considered acceptable central values for quark masses [14], m_u=.005 GeV, m_d=.009 GeV, m_s=.175 GeV, m_c=1.35 GeV, m_b=5.3 GeV. Before making predictions, it is perhaps essential to resort to some desirable simplifying assumptions. Without loss of generality one can assume phase elements β_1 to be zero [15], which effectively leads to only one phase being in the CKM matrix. To enhance the predictability of the model further, one can exploit the idea of maximality of CP-violation, as developed in a recent paper by Koide et al. [16]. This idea of maximality, developed in rephasing invariant manner, in the present context, leads to $\phi_1=\phi_2=90^0$ as well as

$$\mathbb{R}_b \cong 0.2. \tag{17}$$

After having fixed $\mathbb{R}_b$, we take $|V_{us}|$ and $|V_{cb}|$ as inputs to predict $|V_{td}|$, $|V_{ub}|$ and $|J|$ [17]. It is very easy to check that $|V_{ub}|$ and $|V_{td}|$ can be expressed in terms of $|V_{us}|$ and $|V_{cb}|$, for example,

$$|V_{td}| = b_0|g_2| \cong |V_{us}||V_{cb}| \quad , \tag{18}$$

and

$$|V_{ub}| = b_0 c_0^2 \mathbb{R}_d \cong \left({}^{m}s/m_b\right)\mathbb{R}_d|V_{us}|. \tag{19}$$

If we take the central values of $|V_{us}|$ and $|V_{cb}|$, for example, $|V_{us}|=0.22$ and $|V_{cb}|=0.045$ and $c_0^2=0.033$, we find

$$|V_{ub}/V_{cb}| \cong 0.085 \tag{20}$$

and

$$|V_{td}| \cong 0.010. \tag{21}$$

The above value of $|V_{ub}/V_{cb}|$ lies well within the expected experimental data [18,19]. To check the model further, we calculate Jarlskog's J-value [16,17] as a measure of CP-violation, for example,

$$|J_{max}| \cong 3.8X10^{-5} \quad , \tag{22}$$

which is in agreement with similar calculations [16]. However, if we use the full range of c_0^2 values, because of the error bars in the quark masses [14], we obtain $|V_{ub}/V_{cb}| \cong 0.07$—$0.11$ and

$$|J_{max}| \cong (3\text{--}5)\times 10^{-5}. \tag{23}$$

After having reproduced, by using approximate expressions (12)--(16), fairly good values of $|V_{ub}/V_{cb}|$, $|J_{max}|$ and $|V_{td}|$, we have undertaken detailed calculations of V_{CKM} matrix elements and

J values [20] as well as their dependence w.r.t. $\mathbb{R}_t$ and $\mathbb{R}_b$ which are constrained through the relation

$$\left\{\mathbb{R}_t(1-\mathbb{R}_b)\right\}^{1/2}-\left\{\mathbb{R}_b(1-\mathbb{R}_t)\right\}^{1/2}\cong|V_{cb}|. \qquad (25)$$

A general survey of the tables 1(a) and 1(b) indicates that we have been able to obtain fairly satisfactory values of $|V_{ub}/V_{cb}|$, $|J|$ and $|V_{td}|$. It is interesting to note that irrespective of the value of m_t^{phys} we can obtain a good fit of the CKM matrix elements by choosing a particular value of $\mathbb{R}_t$. Further, it is evident from the tables that all the $|V_{ub}/V_{cb}|$ values fall within the range 0.05--0.12 which is in agreement with the data [18,19]. Similarly for $|J|$, we obtain a range which fits well with other calculations [16,17]. The $|V_{td}|$ values also have a good overlap with the data [21,22].

After having calculated the CKM matrix elements, it is interesting to check the prediction regarding $B_s^0-\overline{B}_s^0$ mixing parameter x_s, for example,

$$x_s \cong 1.2x_d|{}^{V}ts/V_{td}|^2 \cong x_d/b_0^2 \cong 15 \qquad (25)$$

where x_d=0.69±0.13 [22]. This value of x_s, implies for the time-integrated quantitity $\chi_s \cong$ 0.5, indicating that mixing in this system is near maximal.

To conclude, we have considered Fritzsch like hermitian quark mass matrices with decoupled u quark as well as nonzero 22-elements, both in 'U' and 'D' sectors, to reconcile the data regarding m_t^{phys} and CKM matrix elements. The idea of maximality of CP-violation is used to find two interesting relations between CKM matrix elements, viz., $|V_{td}|\cong|V_{us}||V_{cb}|$, $|V_{ub}| \cong \frac{1}{2}({}^{m}s/m_b)|V_{us}|$.

Apart from obtaining $0.05 \leq |V_{ub}/V_{cb}| \leq 0.12$ and $90 GeV \leq m_t^{phys} \leq 200 GeV$, we find CP-violating parameter $|J| \cong (3-5) \times 10^{-5}$. Moreover, the predicted $|V_{td}|$ value implies almost maximal mixing in $B_s^0-\overline{B}_s^0$ system. It may be worthwhile to mention that all our basic conclusions remain valid for values of m_t^{phys} not only in the range 90—200 GeV but also beyond this range.

Acknowledgement:

The authors gratefully acknowledge a few useful discussions with Profs. M. G. Scadron and M. P. Khanna. PSG would like to thank the Chairman , Department of Physics for providing facilities to work in the department as well as Principal of his college for his kind cooperation.

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Table Captions

Table 1(a) Variations of $|V_{ub}/V_{cb}|$, $|V_{td}|$ and $|J|$ w.r.t. $\mathbb{R}_t$ when m_t^{phys}=90 GeV.

Table 1(b) Variations of $|V_{ub}/V_{cb}|$, $|V_{td}|$ and $|J|$ w.r.t. $\mathbb{R}_t$ when m_t^{phys}=200 GeV.

Table 1(a)

m_t^{phys} = 90 GeV					
$\mathbb{R}_t$	$\|V_{us}\|$	$\|V_{cb}\|$	$\|V_{ub}/V_{cb}\|$	$\|V_{td}\|$	$\|J\|$ $\times 10^5$
0.075	0.22	0.045	0.05	.012	2.3
0.10	0.22	0.045	0.06	.013	2.8
0.125	0.22	0.045	0.07	.013	3.1
0.150	0.22	0.045	0.08	.013	3.4
0.175	0.22	0.045	0.08	.014	3.8
0.200	0.22	0.045	0.09	.014	4.1
0.225	0.22	0.045	0.10	.014	4.5
0.250	0.22	0.045	0.11	.015	4.7
0.300	0.22	0.045	0.12	.015	5.4

Table 1(b)

			m_t^{phys}=200 GeV		
$\mathbb{R}_t$	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}/V_{cb}\rvert$	$\lvert V_{td}\rvert$	$\lvert J\rvert$ $\times 10^5$
0.050	0.22	0.045	0.05	.0118	2.0
0.075	0.22	0.045	0.05	.012	2.5
0.100	0.22	0.045	0.06	.013	2.9
0.125	0.22	0.045	0.07	.013	3.2
0.150	0.22	0.045	0.08	.014	3.6
0.175	0.22	0.045	0.09	.014	3.9
0.225	0.22	0.045	0.10	.014	4.4
0.250	0.22	0.045	0.11	.015	4.8
0.300	0.22	0.045	0.12	.015	5.5